INDUCTIVE INTERFERENCE

Inductive Interference

between

Electric Power and Communication Circuits

Selected Technical Reports

with

Preliminary and Final Reports

of the

Joint Committee on Inductive Interference

and

Commission's General Order for Prevention or Mitigation of Such Interference

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Railroad Commission of the State of California
San Francisco, California,
April 1, 1919

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CONTENTS.

INTRODUCTION.

OWING to the great importance of inductive phenomena, from both scientific and practical standpoints, the California Railroad Commission deems it advisable to collect in a publication the most important of the Technical Reports of the Joint Committee on Inductive Interference, together with the Preliminary and Final reports by the committee to the commission, and the General Order resulting therefrom.

The technical reports selected for this publication comprise the most important ones, and will be, we believe, of permanent interest to all students of this subject. They constitute, collectively, a practically complete record of the principal activities and findings of the committee. The reports also include a large amount of fundamental data derived from actual tests on commercial lines.

As the subject of inductive interference is, inherently, a rather difficult one and the available information is very meager, the commission believes that those who are interested will welcome the opportunity to secure a record of these researches. Of course, the committee does not profess to have covered the subject completely. Certain important features it was impossible to take up with the means available. Still, the work has been carried considerably farther along several lines than in any other investigation of which the committee has knowledge and it is believed that much of the information herein contained is not obtainable elsewhere.

The extensive use of hydroelectric power in California has resulted in the development of large high voltage electric transmission and distribution systems comprising several thousand miles of lines. The telephone and telegraph services of the state likewise have developed into systems aggregating thousands of miles of communication lines. The present principal power transmission lines and their hydroelectric plants and the main telephone toll lines are shown on a map of the state on page 11. Another map, on page 12, shows the power and communication lines and the towns directly involved in the investigations of the Joint Committee on Inductive Interference.

References will occasionally be found in the text to technical reports not included in the publication. In such cases examination of the topical index will indicate where the subject matter is discussed in the published reports.

The credit for this work belongs to the members and technical staff of the Joint Committee. A chart of the committee organization is

shown on page 84. Particular credit is due the engineers who managed and conducted the various investigations and whose names appear as signatures to the various reports.

Much effort has been spent to eliminate errors. The commission will appreciate receiving notice of any that may be found.

Publication has involved the reproduction of a great many diagrams, curve sheets and record forms. Parties interested may secure blueprints or Van Dyke negatives from the California Railroad Commission, at cost.

Copies of the Preliminary and Final Reports and the commission's General Order No. 52 are available in pamphlet form, and may, upon request, be obtained from the commission.

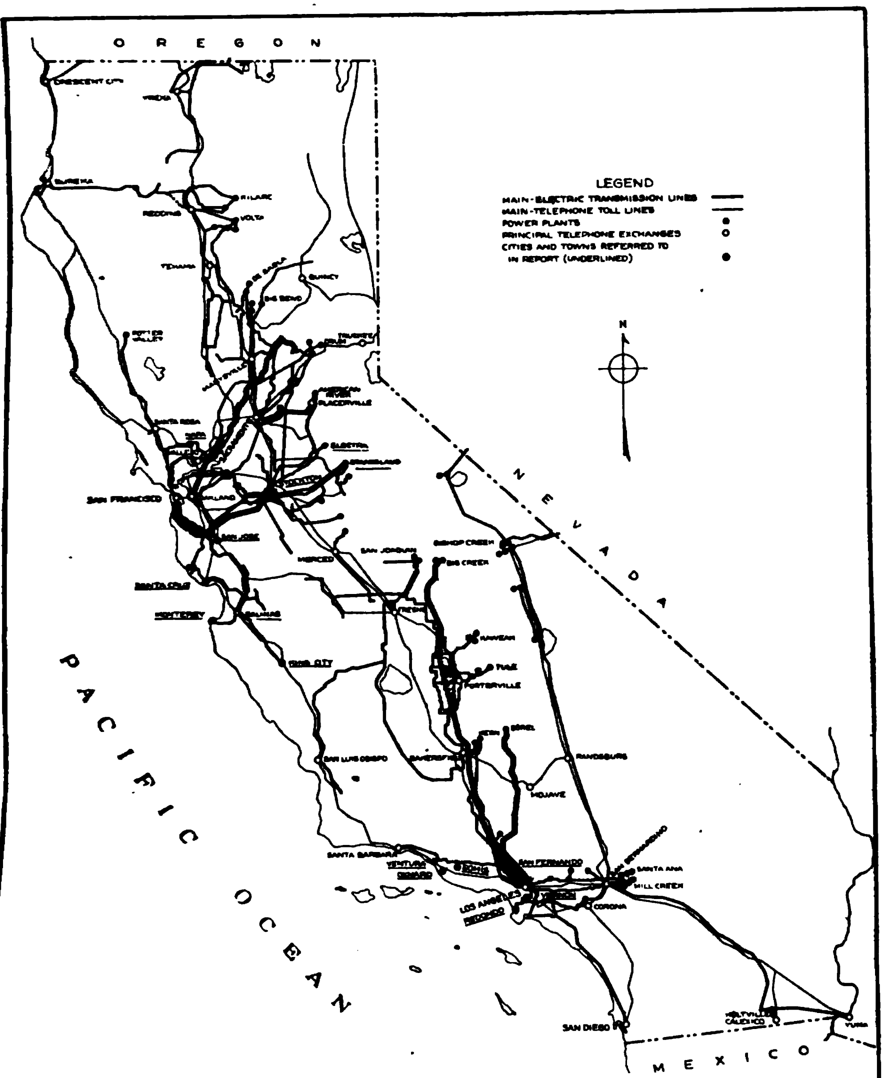
We desire to express our appreciation, and give credit, to our Engineering Department which has been in charge of and has made possible the publication of this work.

RAILROAD COMMISSION OF THE
STATE OF CALIFORNIA.

RICHARD SACHSE,
Chief Engineer.

EDWIN O. EDGERTON.
H. D. LOVELAND.
FRANK R. DEVLIN.
HARLEY W. BRUNDIGE.
IRVING MARTIN.

San Francisco, California,
June 1, 1919.



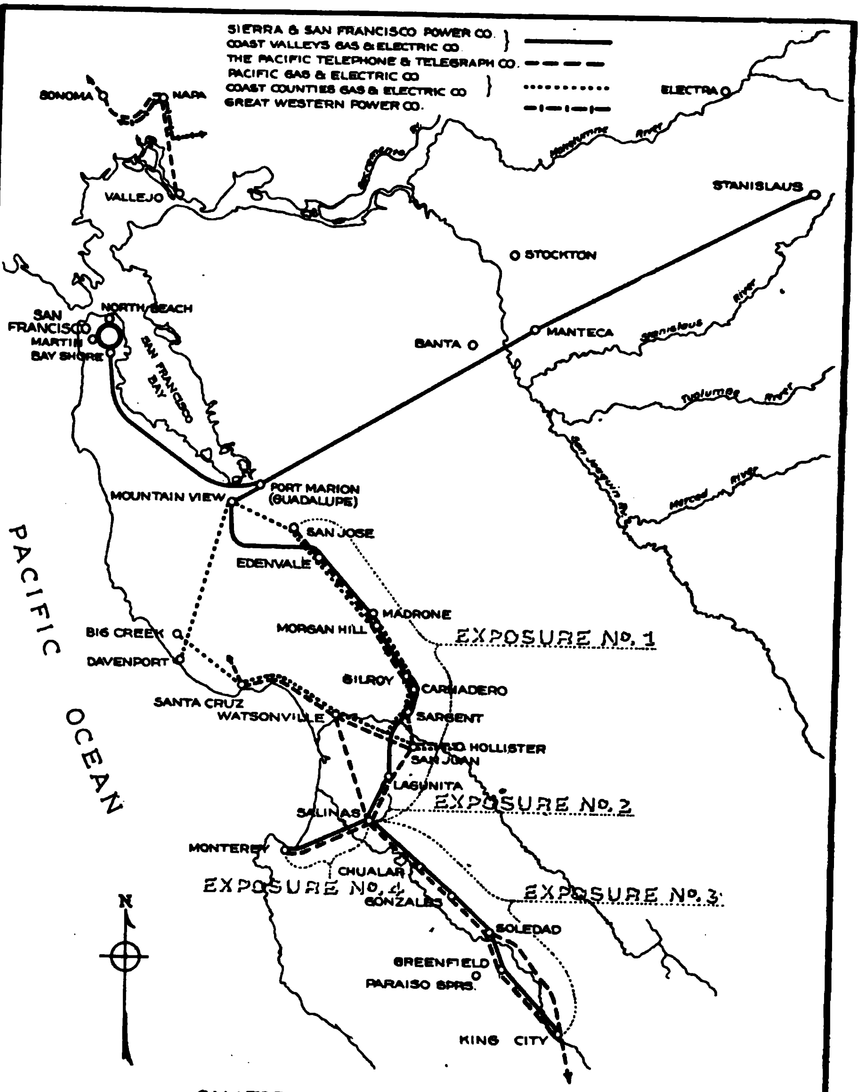
CALIFORNIA RAILROAD COMMISSION
ENGINEERING DEPARTMENT

MAIN ELECTRIC TRANSMISSION LINES
PRINCIPAL HYDRO-ELECTRIC PLANTS
AND
MAIN TELEPHONE TOLL LINES
IN
CALIFORNIA

ACCOMPANYING TECHNICAL REPORTS
OF
JOINT COMMITTEE ON INDUCTIVE INTERFERENCE

SAN FRANCISCO, CAL. RICHARD SACHSE CHIEF ENGINEER JULY 2ND, 1918

SCALE MILES



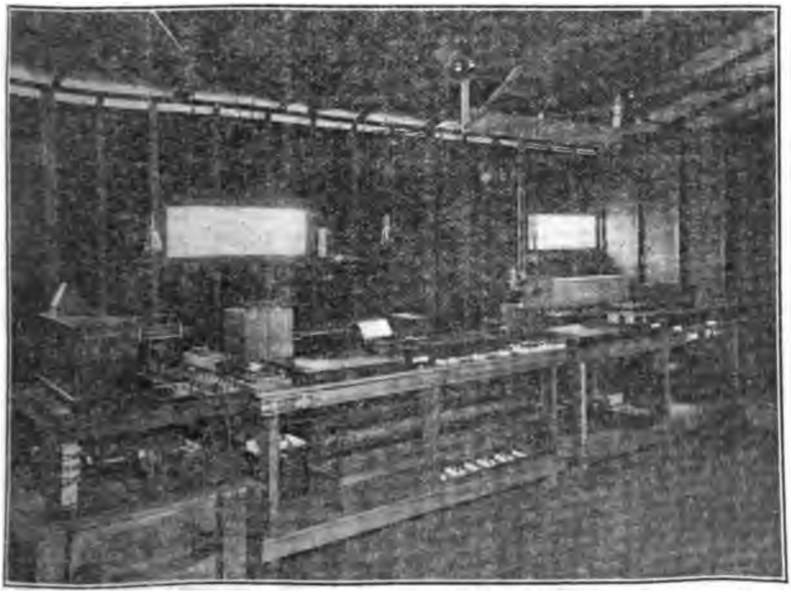
CALIFORNIA RAILROAD COMMISSION
ENGINEERING DEPARTMENT

POWER AND TELEPHONE LINES
OF
CENTRAL CALIFORNIA
INVOLVED IN TESTS OF
JOINT COMMITTEE ON INDUCTIVE INTERFERENCE

SAN FRANCISCO, CAL. RICHARD SACHSE, CHIEF ENGINEER JUNE 11TH, 1918

SCALE MILES

Field Laboratory.



Laboratory Test Room.

Final Report by the Joint Committee on Inductive Interference.

OUTLINE.

APPENDICES.

INTRODUCTION.

In July, 1914, the California Railroad Commission published the first report of the Joint Committee on Inductive Interference to the Commission and arranged for free distribution of the report to interested engineers and other parties. We then authorized the Committee to continue its work and the present final report is the result.

Complying with requests, not only of the members of the Joint Committee, but also from many other sources, we have decided to publish this final report in the same form as the first report was published. We have also concluded to publish in book form a number of the technical reports of the Joint Committee (see Appendix II of this report), and this publication is now in the course of printing and will be sold at cost by the Commission.

The rules proposed by the Committee have been adopted by the Commission and General Order No. 52, superseding General Order No. 39, "In the Matter of the Construction and Operation of Power and Communication Lines for the Prevention or Mitigation of Inductive Interference," effective August 1, 1918, has been issued.

The task of the Joint Committee is now completed. The Commission's appreciation of the work done was expressed in a letter of November 14, 1917, to the individual members of the Committee, reading in part as follows:

> "We acknowledge receipt of final report of the Joint Committee on Inductive Interference, dated September 28, 1917.
>
> In receiving this final report and in accepting the resignations of the members of the Joint Committee, we desire to express to each member of the Committee our very sincere appreciation for the splendid work which the Joint Committee has done during the last five years.
>
> The work in its complete form is a monument to this Committee of which each member must be justly proud.
>
> The Railroad Commission will give prompt consideration to the recommendations of the Joint Committee with reference to certain changes in General Order No. 39. The Commission is also taking up the question of the publication in book form of a number of the technical reports of the Joint Committee.
>
> As soon as the matter of the publication of the book has been finally determined upon, the Railroad Commission will also publish in pamphlet form the Joint Committee's final report.
>
> The Commission is asking the Chairman of the Joint Committee to make the necessary arrangements so that the records and correspondence files of the Joint Committee may be transmitted to the Railroad Commission for permanent custody."

Since the date of that letter the value of the labor accomplished by the Joint Committee has become even more apparent, and we take this occasion to express again our sincere appreciation.

CALIFORNIA RAILROAD COMMISSION.
E. O. EDGERTON,
H. D. LOVELAND,
ALEX GORDON,
FRANK R. DEVLIN,
Commissioners.

San Francisco, California,
September 13, 1918.

LETTER OF TRANSMITTAL.

September 28, 1917.

To the Railroad Commission of the State of California:

GENTLEMEN: Nearly five years ago the Joint Committee on Inductive Interference was organized under the auspices of the Railroad Commission of the State of California and began its investigations with the object of developing a better understanding of the subject of inductive interference and particularly of acquiring information necessary for establishing regulations which could be accepted by all interests concerned as being effective, comprehensive and reasonable.

In July, 1914, the Joint Committee rendered to the Railroad Commission a preliminary report containing an account of its investigations up to that time and including provisional rules which were recommended for immediate adoption. These rules were approved and are embodied in your General Order No. 39, which is now in effect in California.

In its preliminary report the Joint Committee outlined further investigations which it seemed important to make preparatory to putting the rules into more permanent shape and asked your authorization to carry on these investigations. This request being approved, the work was resumed and much additional information has since been obtained. The past few months have been devoted to the preparation of a report outlining briefly the main features of all work done by the Joint Committee. This report, which is submitted herewith, contains a draft of revised rules which are believed to represent a substantial improvement over the rules contained in the 1914 report. The Joint Committee recommends that these revised rules be approved by the Railroad Commission and issued as a new order to supersede General Order No. 39.

An attempt was made to meet the urgent demand for information defining the limiting relationships between power and communication lines which constitute a parallel. It was found, however, that sufficient information was not at hand to satisfy all parties as to a basis upon which these relationships could be set forth definitely.

While the subject of inductive interference offers an almost inexhaustible field for further investigation, the Joint Committee feels that the object for which it was formed has now been substantially accomplished and it seems unnecessary that its work should be further prolonged. The rules recommended will constitute, it is hoped, a satisfactory basis of procedure for several years at least, in dealing with cases of inductive interference, assuming a proper spirit of co-operation among the companies concerned. The rules themselves definitely call for and emphasize the necessity for this co-operation. In course of time, no doubt, the experience gained in the application of these rules, together with advances in the art, will make it desirable that the rules be changed in some respects, but such changes are thought to be a matter of the more or less distant future and not to require the continuance of the Joint Committee.

From time to time during the course of this investigation Technical Reports have been prepared setting forth in detail the matters which have been studied and the results and conclusions derived. There are in all 71 of these Technical Reports which are listed in Appendix II of the report herewith transmitted. As they contain a large amount of valuable information and are in fact the only records of most of the data derived by the Joint Committee, it is particularly important that they be rendered available to persons interested. To this end, the Joint Committee has selected 30 of the most valuable of these Technical Reports and earnestly recommends to the Railroad Commission that these be printed in a suitable volume by the State of California and offered for sale to libraries, colleges, companies, societies and individuals interested. The reports recommended for publication are so designated in the list given in Appendix II.

The official copies of the Technical Reports, other original records, and the correspondence files of the Committee are ready to be placed in the custody of the Commission.

In transmitting this report, which marks the conclusion of its work, the Joint Committee desires to express its satisfaction at the degree of success which has been reached in composing the differences formerly existing in California between the power and communication interests, differences due principally to lack of familiarity with the physical aspects of this subject. While it would be too much to say that there is now complete unanimity of opinion on every feature, still the Joint Committee has been able, after careful consideration of all information available, to agree unanimously upon the present report. In the opinion of the Committee the results accomplished emphasize strongly the superiority of a co-operative investigation of this kind, whereby the fundamental facts are ascertained and acted upon, as compared with litigation or other methods of arbitrary settlement without the benefits derived from such investigation.

In conclusion, the Joint Committee takes pleasure in acknowledging that the credit for what has been accomplished is due primarily to the Railroad Commission of the State of California which has consistently held to the policy of a co-operative investigation and has cordially supported this Committee in its work.

Respectfully submitted.

(Signed)

A. H. Griswold,
H. A. Barre,
J. E. Woodbridge,
J. L. Ord,
V. V. Stevenson,
R. W. Mastick,
Howard S. Warren,
C. H. Temple,
James T. Shaw,
Richard Sachse,
F. Emerson Hoar,
Arthur F. Bridge,
A. L. Wilson,
John A. Koontz,
J. P. Jollyman,
P. M. Downing,
A. H. Babcock.

FINAL REPORT

OF THE

Joint Committee on Inductive Interference to the Railroad Commission of the State of California

INTRODUCTION.

This report embodies the results of an investigation by the Joint Committee on Inductive Interference, extending over a period of approximately four years. The task undertaken by this Committee was a study of the problem of interference with communication (signal) circuits caused by the inductive effects of neighboring power (electrical supply) circuits, including field experiments and tests necessary to determine the underlying physical facts, and the preparation of recommendations to the Railroad Commission of the State of California for its guidance in making rulings designed to prevent or mitigate such interference.

A previous report by this Committee to the Railroad Commission was rendered on July 7, 1914, embodying the results of the first two years of the investigation and including provisional rules for the prevention or mitigation of inductive interference, based on the information available at that time. The rules therein recommended were adopted by the Railroad Commission in its General Order No. 39, effective August 20, 1914. The previous report also contains an outline of further investigations which this Committee considered essential in order that additional information might be acquired for amplifying and revising the rules to make them more definite and complete. These further investigations have now been carried out, as far as practicable, and certain additional work directed toward the same end has also been done.

Having completed the field investigations and having carefully analyzed and studied all information accumulated, the Committee now presents its final report, including revised rules for preventing or reducing inductive interference. The new rules are not radically different in substance from the rules formerly recommended and now in effect in California, but are considerably changed in form and arrangement. They are more specific as to certain of the requirements, more complete in several respects, less arbitrary in setting physical limits to be observed, more clearly expressed and, it is believed, better adapted to the conditions of practical use.

This report, which contains an account of the Committee's work from the beginning, including that covered by the preliminary report, is divided, for convenience, into three parts as follows:

Part One gives a historical account of the Committee's formation and activities.

Part Two presents, in nontechnical language, so far as possible, the nature of the subject, a brief resumé of the principal facts established or agreed upon by the Committee, and a concise statement of the physical principles underlying preventive or remedial measures.

Part Three contains the Committee's final recommendations for rules, together with explanations of the same in detail.

PART ONE.

HISTORY OF COMMITTEE'S ORGANIZATION AND WORK.

Formation of Committee.

The formation of the Joint Committee on Inductive Interference was the outgrowth of certain differences involving power, communication and railroad interests which were brought to the attention of the Railroad Commission of the State of California. As an alternative to contesting the issue at that time it was agreed by the power and communication companies, with the approval of the Commission, that a joint investigation should be made to obtain certain information essential to a proper solution of the difficulties due to inductive interference. The Commission desired that the matter be thoroughly investigated before passing upon the general principles involved in these difficulties. To this end a general conference was called to select representatives to form a "Joint Committee" empowered to conduct tests, experiments, and investigations, the results of which would serve as a basis of recommendations for rules and regulations to be issued by the Commission, tending to minimize inductive interference and physical hazard arising from parallelism of different classes of circuits. This conference was held December 16, 1912. As a result the Joint Committee on Inductive Interference, representing the Railroad Commission and railroad, power and communication interests of the state, was organized and authorized by the Commission to conduct the desired investigation.

Personnel.

The personnel of the Committee selected is given below.

Representing the Railroad Commission:

Mr. R. A. Thompson, Chief Engineer.
Mr. A. R. Kelley, Assistant Engineer.
Mr. James T. Shaw, Assistant Rate Expert.
Mr. F. Emerson Hoar, Assistant Rate Expert.

Representing Railroad Interests:

Mr. A. H. Babcock, Consulting Electrical Engineer, Southern Pacific Company.

Representing Telephone and Telegraph Interests:

Mr. A. H. Griswold, Plant Engineer, The Pacific Telephone and Telegraph Company.
Mr. R. W. Gray, Division Superintendent, Western Union Telegraph Company.
Mr. C. H. Temple, General Manager, United States Long Distance Telephone Company.
Mr. L. M. Ellis, General Manager, Union Home Telephone Company.

Representing Power Interests:

Mr. H. A. Barre, Electrical Engineer, Pacific Light and Power Corporation.
Mr. Louis Elliott, Engineer, Great Western Power Company.
Mr. P. M. Downing, Engineer, Pacific Gas and Electric Company.
Mr. J. E. Woodbridge, Chief Engineer, Sierra and San Francisco Power Company.

Since the formation of the Committee, through additions, resignation or death, the personnel of the Committee has changed as follows:

Mr. Louis Elliott resigned and Mr. J. A. Koontz, Engineer of the Great Western Power Company, was appointed in his place.

Mr. V. V. Stevenson, Electrical Engineer of the Postal Telegraph-Cable Company, and Mr. L. N. Peart, General Superintendent of the San Joaquin Light and Power Company, were added to the original membership by action of the Committee.

Mr. R. A. Thompson, Chairman of the Joint Committee, resigned. Mr. W. C. Earle, his successor as Chief Engineer of the Commission, was elected to membership and chairmanship. Subsequently Mr. Earle resigned and Mr. Richard Sachse, who succeeded Mr. Earle as Chief Engineer of the Railroad Commission, was elected a member and Chairman of the Committee.

Mr. L. M. Ellis resigned and Mr. R. W. Mastick, Transmission and Protection Engineer of the Pacific Telephone and Telegraph Company, was elected to membership.

Mr. H. S. Warren, Electrical Engineer of the American Telephone and Telegraph Company, was elected to honorary membership.

Mr. James T. Shaw, Secretary of the Joint Committee, resigned. Mr. A. R. Kelley was elected to the office of Secretary. The vacancy in membership created by the resignation of Mr. Shaw was later filled by the election of Mr. A. L. Wilson, Assistant Rate Expert of the Railroad Commission. Mr. James T. Shaw was elected to honorary membership.

The death of Mr. L. N. Peart created a vacancy which was filled by the election of Mr. J. P. Jollyman, Engineer of Electrical Construction of the Pacific Gas and Electric Company.

Mr. A. R. Kelley resigned, and Mr. A. F. Bridge, Assistant Electrical Engineer of the Railroad Commission, was elected to membership and to the office of Secretary.

Mr. R. W. Gray resigned and Mr. J. L. Ord, Division Plant Superintendent of the Western Union Telegraph Company, was elected to membership.

Organization.

The organization and personnel of the Joint Committee on Inductive Interference were approved by the Railroad Commission on January 6, 1913, and the Committee thereupon proceeded with its tests and investigations.

For the more efficient conduct of its work the Joint Committee was divided into several subcommittees, each assigned to and responsible for certain branches of the investigation. The present organization of the Committee is given on a chart presented as Appendix V.

Early in its work the Committee established a field engineering staff, reporting to the Subcommittee on Tests, to conduct the necessary tests and investigations. This field staff was at first composed of engineers in the employ of The Pacific Telephone and Telegraph Company and the American Telephone and Telegraph Company, and was later augmented by the addition of two engineers and a stenographer, engaged by the Committee. Since August, 1914, the stenographer has been provided by the Railroad Commission. In November, 1914, a third engineer was

engaged by the Committee and three were retained in its employ for nearly a year.

The Committee wishes to express its appreciation of the able manner in which Mr. L. P. Ferris has supervised the analytical and theoretical work.

Investigations.

Previous to the formation of the Joint Committee in December, 1912, The Pacific Telephone and Telegraph Company had started an investigation of inductive interference between the lines of the Coast Counties Gas and Electric Company and the lines of the telephone company in the neighborhood of Morgan Hill in Santa Clara County. This investigation was completed by the Committee and its results have been considered in connection with other work carried out by the Committee.

In January, 1913, the Committee established its field staff at Salinas, to investigate parallels on the line of the Sierra and San Francisco Power Company north of Salinas and on the line of the Coast Valleys Gas and Electric Company south of Salinas, both of these power lines being parallel with the lines of The Pacific Telephone and Telegraph Company, the Western Union Telegraph Company and the Southern Pacific Company's signalling system. The investigation at Salinas continued from January, 1913, until July, 1913.

The work undertaken at Salinas was for the purpose of determining (1) the magnitude and characteristics of the induction produced in the communication circuits, the factors in the power circuit causing this induction and the quantitative relationships involved; and (2) the effect of the condition of the neutral (grounded or nongrounded) of the autotransformers at Salinas, on the induction in the communication circuits.

Tests were made of the induction in the communication circuits, both north and south of Salinas, under operating conditions of the power circuits and with the neutral at Salinas alternatively grounded and nongrounded. Tests were also made with special methods of energizing the power circuit, in order to determine the relative importance of various factors in causing the induction. This determination was also made by theoretical methods, by computation of induction based upon the dimensions of the parallel involved, and the results compared with those of the tests. Instrument transformer equipment was investigated in order to determine the errors thereby introduced in measurements of the power-circuit voltages and currents.

In July, 1913, the field headquarters were moved to Santa Cruz. At this point the Committee desired to test the relative merits of various schemes of transpositions* for both power and telephone circuits, and to complete the investigation begun at Morgan Hill on the system of the Coast Counties Gas and Electric Company, which system is of a different character from that studied at Salinas. A mathematical study of transpositions in general, and particularly of those for the parallel between Santa Cruz and Watsonville was completed.

During the time the field headquarters of the Committee were at Santa Cruz, the report of the Committee to the Railroad Commission,

*For definition of "transposition," see page 44.

dated July 7, 1914, was presented. This report contained an account of the formation of the Committee, its activities and the results accomplished up to that date, and also included such recommendations for rulings by the Railroad Commission, as seemed justified to the Committee at that time. In addition, there was given a program of future work designed to put the Committee in possession of information which would permit making the recommended rulings more definite and complete.

In a letter of acknowledgment to the Committee the Railroad Commission approved the program of future work which was laid down in the report and authorized the continuance of the Committee's investigations. This program comprised experimental studies both of transpositions and of residual* voltages and currents of power circuits. The study of transpositions included: (1) the determination of the practical effectiveness, in reducing induction, of systems of power and communication circuit transpositions properly co-ordinated with each other, with consideration of different lengths of balanced sections; (2) the influence of imperfect electrical balance of communication circuits in impairing the effectiveness of transposition systems, and (3) practical effectiveness of transpositions in a power circuit isolated from ground in balancing the voltages between the several conductors and ground, with consideration of the relative efficiency of barrels** of different lengths.

The study of residual voltages and currents included an experimental investigation of different types of power system connections and apparatus with respect to the production of residual voltages and currents, of means to be employed to limit their magnitudes and the determination of the minimum values which will produce harmful inductive interference.

The work outlined in this program was continued at Santa Cruz until November 24, 1914. The experimental study of the effectiveness of transpositions in power and communication circuits undertaken at this point could not be carried out, due to lack of suitable equipment. An investigation of the effect of various transformer connections and of the magnetic density employed in transformer iron on the residual voltages and currents introduced in grounded-neutral networks by such transformers was begun. In addition, from Santa Cruz as headquarters, measurements were made with portable apparatus at various points on the systems of the Coast Counties Gas and Electric Company, the Sierra and San Francisco Power Company, and the Pacific Gas and Electric Company in order to study their characteristics with respect to residual voltages and currents.

On November 24, 1914, the field headquarters and laboratory of the Committee were moved to San Fernando. This location offered a number of advantages for experimental work, the chief one being the presence of an unused thirty-seven mile 15,000-volt line of the Pacific Light and Power Corporation which was available for testing at all times. A telephone circuit carried on the same poles with the 15,000-volt circuit was also available, constituting a parallel for experimental purposes. In addition, transformers were loaned by the Pacific Light and Power Corporation which, with other transformers already pro-

*For definition of "residual" see page 45.
**For definition of "barrel" see page 44.

vided by the Sierra and San Francisco Power Company, gave opportunity for carrying out transformer studies.

The principal experimental work undertaken at San Fernando comprised studies of the factors affecting the residual voltages and currents of power circuits, the effectiveness of transpositions in balancing power circuits and in neutralizing inductive effects, and the magnitude of inductive effects in short, uniform, nontransposed sections of parallel. Concerning the residual voltages and currents of power circuits isolated from ground, the investigations included the effects of transpositions, leakage, accidental grounds and frequency of alternations. Concerning the residuals of grounded-neutral circuits the investigation included the effects of magnetic density of the transformers and of various connections of the transformer banks. In preparing for the latter study a difficulty was encountered, due to large double-frequency residual voltages and currents, apparently peculiar phenomena previously unrecorded and probably of rare occurrence in practice. These were investigated to a very limited extent, for the purpose of devising means to overcome them so that the programmed tests might be carried out.

The availability of both an idle power circuit and an idle telephone circuit, the conditions of which could be varied for experimental purposes, gave an excellent opportunity for studying induction and the effect thereon of both power and telephone circuit transpositions and of telephone circuit unbalances. An extensive series of tests was made to determine the ratios of induced voltage in the telephone circuit to inducing voltage or current in the power circuit under different conditions of operation of the power circuit, for a short, uniform, nontransposed section of parallel. These ratios, termed coefficients of induction, were also obtained independently by calculations based upon physical dimensions of the line. The results of the two independent determinations were compared to ascertain the practicability of obtaining coefficients of induction for other cases by computations thereby eliminating the necessity for tests. Advantage was taken of the opportunity at San Fernando to measure the residual voltages and currents of the Pacific Light and Power Corporation's 15,000-volt system to supplement the similar measurements previously made on other systems.

On June 17, 1915, the headquarters of the field staff were moved to San Francisco, where the work of analyzing the San Fernando data was completed. At San Fernando the Committee's energies were largely centered on completing the experimental work, before the power line was required for service.

It was endeavored to make the analyses and reports as thorough and complete as possible. For each of the subjects experimentally investigated at San Fernando, theoretical studies were undertaken at San Francisco, which in some cases were much extended in scope over that of the corresponding experimental work. Where possible, the experimental and theoretical results were compared. The effects of circuit configuration, or arrangement and relative location of conductors, transpositions and frequency on the residual voltages and currents due

to the line unbalance of power circuits isolated from ground, and the effects of accidental grounds, were investigated from a theoretical standpoint. A study was made of the relation of magnetic density of the transformer iron and of transformer connections to the residual voltages and currents of triple frequencies thereby introduced into connected circuits. A report was prepared giving formulas for the computation of coefficients of induction in communication circuits paralleled by power circuits, including an explanation of the derivation of the formulas and convenient forms which had been developed for systematically carrying out such computations.

To determine the effect of configuration and relative position of power and communication circuits on the induction in the latter, an extensive series of computations based upon the dimensions of assumed cases of parallelism, was carried out. The results of this study comprise 214 curve sheets, containing over 3,000 curves, by the aid of which the values of the coefficients of induction for those cases of parallelism which occur most commonly may be determined.

Among the important reports prepared at San Francisco is one reviewing previous work and presenting new data on the subject of co-ordinating power-circuit and telephone-circuit transpositions as a means of reducing interference. A new telephone transposition system developed by the American Telephone and Telegraph Company, largely in response to the need for a system of telephone transpositions having increased flexibility in respect to co-ordination with power-circuit transpositions, is described and its use illustrated. As examples of co-ordinated transposition systems, plans are presented for all the parallels which have been experimentally investigated by the Committee.

Apparatus suitable for use in the experimental work of the Committee was not easily obtainable and in many instances it was necessary to design and develop special apparatus for certain of the tests. In cases where apparatus was not available for measuring desired quantities directly, it was necessary to develop methods of measurement whereby they might be obtained indirectly.

In deciding from time to time upon its program for future work, the Committee has found it necessary to formulate and consider in detail many plans of experimentation which have never been carried out. It has not always been easy to decide upon the best location for carrying on a particular investigation when each of the several different possible locations possessed certain advantages. To decide between them or to choose between different programs of work has meant that the several plans under consideration had to be worked up in considerable detail before the preponderance of advantage in favor of some one procedure could be established. In several cases plans for work regarded as particularly desirable had to be given up because they were found to be too laborious, or for other reasons were not feasible.

In the course of the investigation seventy-one Technical Reports have been prepared, which describe in detail the various features of the work, the method and apparatus employed and the results accomplished. These reports, some of which are recommended for publication, are listed in Appendix II, page 74.

At the request of the Committee, laboratory investigations were made in New York by the American Telephone and Telegraph Company, the Postal Telegraph-Cable Company and the Western Union Telegraph Company to determine the detrimental effects of extraneously induced currents on the operation of telephone and telegraph circuits. Reports of the results of these investigations were submitted to the Committee. These are also listed in Appendix II, page 74.

At various times during the course of its work, the Committee has contributed discussions before the American Institute of Electrical Engineers. The Committee's report of July 7, 1914, was presented at the Spokane Convention of the Institute in September, 1914, and later at meetings of the San Francisco and of the Los Angeles sections of the Institute. On each of these occasions considerable discussion was brought forth. In June, 1915, at the Deer Park Convention in connection with papers presented on the subject of irregular power-circuit wave-forms, the Committee submitted a discussion from the standpoint of inductive interference. In September, 1915, at the Panama-Pacific Convention the Committee submitted a discussion in which the progress of the work from July, 1914, to September, 1915, was described. In September, 1916, at the convention of the Institute held in Seattle, the Committee submitted a discussion of a paper presented on the subject of irregular wave-forms.

Finances.

The funds required for carrying on the work of the Committee were contributed by various telephone, power and telegraph companies. Such contributions were made at the start of the investigation and immediately after the report rendered to the Commission on July 7, 1914. Further support was given in the furnishing, by the railroad companies, of free transportation to the Committee members and employees while on Committee business, and of the Committee's equipment; by the telephone companies of the services of their engineers on the work of the field staff; and by the Railroad Commission of the stenographer and stationery supplies. The time which the Committee members devoted to the work was without cost to the Committee.

It is estimated that the total cost of the investigation is more than $100,000.

PART TWO.

EXPLANATION OF PROBLEM AND SUMMARY OF RESULTS.

Nature of subject.

The object sought herein is to describe what inductive interference is, using as far as practicable nontechnical terms, for the benefit of those not familiar with electrical theory.

The transmission of power electrically by wire circuits in either large or small quantities requires a *current* of electricity. Also, to make electricity flow, there must be in the circuit a *voltage* or, in other words, electric pressure, as all circuits offer more or less resistance or impedance to an electric current. If the voltage is produced directly by a battery it forces the electric current around the circuit in one direction only. Such current is called *direct* or *continuous*. Continuous voltage and current may also be produced by an electric generator, and this is the common practice for street railways, but on most other power lines the generators produce an *alternating* voltage,—that is a voltage which during each short interval of time known as a *period* (usually not longer than one twenty-fifth of a second) varies in value from zero up to a maximum, then diminishes to zero, increases to a maximum in the opposite direction, and then diminishes again to zero, repeating this cycle of variations through succeeding equal periods. Thus, the voltage and the corresponding current change in direction or alternate twice each period. The number of periods or cycles per second is called the frequency.

The voltage associated with any electric circuit is accompanied by an electric field of force, or condition of stress, in the surrounding space, whose intensity is proportional to the voltage. At the same time the corresponding electric current is accompanied by a magnetic field of force which occupies the same surrounding space and whose intensity is proportional to the current. Thus any changes in the magnitude or direction of the voltage and current, such as the alternations described above, are accompanied by corresponding changes in their fields. The intensity of these fields of force, in general, diminishes very rapidly with increasing distance from the circuit.

Conversely, any other circuit within these fields of force will have voltages and currents set up or "induced" in it, when changes occur in the fields, that is, when the voltage or current of the first circuit changes. Power circuits of the alternating-current type, most commonly employed in power transmission, having their voltages and currents continually varying, will continually induce voltages and currents in a neighboring communication circuit. These induced voltages and currents are evidence of the absorption of energy from the fields. Thus one circuit influences another by the transfer of energy from the one to the other, without any contact between the wires of the two circuits. This phenomenon, termed "induction," has long been known, and has many useful applications in electrical engineering.

To transmit signals over a communication circuit it is necessary that the power used, and thus the voltage and current, vary from instant to instant. In telephone circuits this variation is extremely complex, the

current which reproduces the human voice in a distant telephone consisting of a number of component simple currents varying in frequency from about 100 to 4,000 cycles per second. For telegraph circuits the variation is much less complex and the frequencies of the important components of the voltage and current are less than 300 cycles per second. In both cases the signalling impulses are sent and received by delicate mechanisms, and the amounts of power required are exceedingly small, particularly for telephone circuits. When communication circuits are in the field of influence of a power circuit the rate at which energy is transferred to them by induction may be comparable with, or even larger than, the power required for their operation, although entirely inappreciable compared to that of the power circuit. For example, the power required to operate a small incandescent lamp is sufficient, if directly applied, to cause a loud noise in several million telephone receivers.

For power circuits of the type most commonly used in California, the frequency is either 50 or 60 cycles per second. This is the fundamental frequency of the voltage and current, representing useful power, but there are also present in power circuits other voltages and currents, usually of relatively small magnitude, of various higher frequencies up to several hundred cycles per second. These higher frequencies or *harmonics* of the fundamental frequency, are the chief cause of interference to telephone circuits, since they are of the frequencies of the sound-waves of the human voice, at which the telephone is most sensitive. On the other hand, the chief interference with telegraph circuits is caused by the fundamental or useful frequency of the power circuits, which most nearly corresponds to the frequency of the telegraphic impulses.

The disturbances thus caused in telephone circuits manifest themselves as humming noises which impair the intelligibility of conversation and cause annoyance. In telegraph circuits, chattering of the relays is caused, the intelligibility of signals is impaired, and the speed and ease of transmission are reduced.

Under abnormal conditions the inductive disturbance due to a power circuit may be very greatly increased. When sudden changes take place in the conditions of the power circuit such as those caused by energizing or de-energizing the circuit, or when a wire breaks and falls to ground, relatively large amounts of energy may be suddenly introduced into the communication circuits. These momentary impulses may be sufficient to constitute a physical hazard, to operate protective devices or to cause severe acoustic shocks to telephone operators or users.

Briefly, then, inductive interference may be defined as the impairment of the serviceability of communication circuits resulting from the transference of energy into them, through intervening space, from near-by power circuits. The study of inductive interference deals with the factors affecting the magnitude and character of the induction and their relationships, the attendant detrimental effects on communication circuits and the means to be employed in overcoming or mitigating such interference.

Summary of facts established.

It seems desirable to summarize briefly the principal technical facts regarding inductive interference, which may now be considered as established. Only the most important points are mentioned here, a detailed technical discussion being given in the Technical Reports.*

1. *Primary Cause.*

As previously shown in discussing the "Nature of Subject," the primary cause of inductive interference is the presence, about the power circuits, of fields of influence which vary in intensity from instant to instant, usually in periodic or cyclic fashion. Communication circuits in regions where these fields are of appreciable strength absorb energy therefrom, by "induction." When the rate at which the energy is thus absorbed is of the same order of magnitude as the power required for the transmission of signals, the impulses received at the terminals of the communication circuit are distorted and the serviceability of the circuit is impaired.

2. *Interference to Telephone Circuits—Harmonics.*

Under normal operating conditions of the disturbing power circuits, interference to telephone circuits, manifested by a humming noise from the telephone receivers, is due almost entirely to the higher harmonics of the power-circuit voltages and currents; for the reason that such harmonics cover a considerable portion of the range of frequencies of human speech, at which telephone apparatus is most sensitive. Except when the interference is very slight or very severe, the detrimental effect of extraneous current in a telephone receiver increases approximately in direct proportion to the magnitude of the current. Increasing the frequency causes a very rapid increase in the detrimental effect (roughly as the square of the frequency) up to about 800 periods per second, beyond which there is a gradual decrease. When several frequencies are present in the extraneous current, the resultant detrimental effect is considered roughly proportional to the square root of the sum of the squares of the separate effects of the several single-frequency components, though this relation has not been definitely established.

The higher harmonics, which are irregularities of the voltage and current waves of power circuits, usually result from:

(*a*) design and construction of generators and motors, whereby pure sine wave shapes are only approximated;

(*b*) the use of iron in transformers under conditions approaching magnetic saturation, thereby causing distortion of the current and voltage waves;

(*c*) the presence of electric arcs in the circuit, as in some street-lighting systems.

The higher harmonics which commonly occur in alternating-current systems are odd integral multiples of the fundamental frequency. They are of sufficient magnitude to be of importance, often as high as nineteen times the fundamental frequency, and have been observed as

*Listed in Appendix II, page 74.

high as the 35th order. High frequency voltages and currents also occur in direct-current systems. Harmonics (other than the fundamental or first harmonic) are not essential to the functioning of power systems and may be sources of trouble therein.

Induction of the fundamental frequency of power circuits (below 100 cycles per second) is the cause of very little interference to telephone circuits, except when its magnitude is sufficient to constitute a physical hazard, or to operate grounded signalling devices, as both the human ear and telephone apparatus are much less sensitive to these relatively low frequencies.

3. *Interference to Telegraph Circuits.*

Under normal operating conditions of the disturbing power circuits, interference to telegraph circuits, manifested by the reduction in clearness and maximum speed of signalling, is due to induced currents of fundamental frequency and, to a limited extent, frequencies of the lower harmonics (chiefly the third).

Telegraph receiving instruments are readily responsive to these frequencies, because they approach the normal operating frequencies of telegraph transmission. Telegraph instruments are not sensitive to the higher harmonics. Other signal circuits (telephone circuits excluded), in general, resemble telegraph circuits in being most affected by induced currents of fundamental frequency.

4. *Balanced and Residual Components.*

In analyzing inductive effects, it is convenient to divide the power-circuit voltages and currents into two general classes: (1) "balanced," with respect to the earth as a neutral conductor or point of reference, and, (2) "residual," completely unbalanced with respect to the earth, *i.e.,* employing the metallic power-circuit conductors, as a group, for one "side" and the earth as the other side of their circuit.

"Balanced" current components in the several conductors of a power circuit are such that at every instant their algebraic sum is zero. The algebraic sum of the total currents in the several conductors of a power circuit at any instant is the "residual" current. Similarly, the "balanced" voltages of the several conductors are such that their algebraic sum is zero at every instant, while the algebraic sum of the total voltages to ground at any instant is the "residual" voltage.

As an example, a trolley circuit, consisting of an overhead trolley wire and "return" through rails and earth, is completely unbalanced with respect to the earth, its total voltage and current being residual. On the other hand, a two-wire circuit having no metallic connection to earth and its two sides symmetrical with respect to the earth's surface and not in close proximity to other circuits or objects, would have no residuals, the voltages to earth of the sides of the circuit being equal and opposite and the currents wholly confined to the metallic conductors and therefore equal and opposite, *i.e.,* in both cases balanced.

This classification of the voltages and currents is of basic importance, since there is no generally applicable relation between balanced and residual components or their inductive effects, and furthermore since the remedies for induction from balanced and residual voltages or currents are often fundamentally different.

The circuits commonly employed in power transmission and distribution ordinarily have both classes of voltages and currents in sufficient magnitude to require attention. With exceptions, such as trolley circuits above mentioned, the balanced components of fundamental frequency are the useful energy-transferring agents, while the residuals are the result of incidental differences between ideal design and construction of line and apparatus, giving perfect balance, and design and construction which approach this condition sufficiently for commercial operation, disregarding inductive effects.

Both balanced and residual voltages and currents contain harmonics, but the general tendency is that the residuals contain greater percentages of harmonics than do the balanced components. Besides, under some conditions (discussed in 5 below), a series of harmonics, odd multiples of three times the fundamental frequency, appear as residuals, but not in the balanced components.

Inductive effects from residuals are usually of greater intensity than those from balanced voltages or currents of equal magnitude. The ratio of effects from these two sources is exceedingly variable, ranging from about two to several thousand. The relatively greater induction-producing power of residuals is due to the fact that the residual components associated with the several conductors are all "in phase" and their inductive effects therefore cumulative, whereas the several balanced components are "out of phase" (by 120 degrees in a three-phase system) and hence their resultant induction is a differential effect, *i.e.*, the inductive effects due to the balanced components partially neutralize one another.

5. *Causes and Remedies for Residuals.*

Unbalances or inequalities among the admittances to ground of the several conductors of a power circuit cause residuals of the frequencies present in the voltages between conductors.* In a system without metallic connection to earth a residual voltage is produced. With a grounded-neutral system a residual current is produced and the residual voltage due to unbalanced line admittances is greatly reduced. Unbalanced admittances are caused by: (1) differences of position of the conductors with respect to ground and to one another, being a function of the configuration, height above ground, location of ground-wires and other neighboring objects, and to a small extent, of size of conductors; and (2) differences in insulation resistance, as may be due to defective insulators. Transposing the conductors, which tends to equalize their relations to ground and to one another, is an effective remedy for unbalanced capacitance. Such transpositions must be located with proper regard to changes in configuration, and at short enough distances from each other so that there is no material difference in the electrical conditions at two such points at any given instant. Of commonly occurring configurations the equilateral triangular is most nearly balanced, hence causes the least residuals due to unbalanced capacitances, while the plane configurations, especially the unsymmetrical horizontal, are the worst in this respect. The remedy for

*It is to be noted that the unbalances here referred to are not unbalances such as those due to single-phase loads between line conductors.

unbalanced insulation resistance lies in careful maintenance; for a well constructed and maintained system this is usually not an important source of residuals.

In a power system having loads connected between the several conductors and ground (as in a star-connected system with grounded neutrals), differences among the loads of the several phases may cause residual voltages and currents, due to part of the load being supplied through a circuit consisting of conductors with ground "return." Inequalities of ratios or impedances among the transformers of a bank also cause residuals in such a circuit. The evident remedy is careful equalization of loads, and the use of like transformers. Removal of the ground path for unbalanced load currents, allowing only one neutral ground, is the most effective and reliable remedy for this source of residual current.

When a transformer bank in a three-phase system is connected in star with neutral grounded, harmonics of three times the fundamental frequency, and odd multiples thereof, appear as residuals, on the grounded-neutral side. This is because of the variation of the permeability of the transformer iron with varying magnetic density, causing harmonics in transformer exciting currents or in their induced voltages. As the triple-harmonic components are "in phase" in the three transformers, triple-harmonic residual voltages and currents are produced if the neutral is grounded. Delta-connected windings on such a transformer bank provide a shunt path for these triple-harmonic components of the exciting current, and greatly lessen the residuals which might otherwise be caused on the grounded neutral side. Since the magnitude of these residuals decreases very rapidly as the maximum magnetic density is reduced, lowering the voltage impressed per turn of the transformer winding, or substituting transformers of lower magnetic density, is a very effective remedy. Isolating the neutral of a transformer bank eliminates it as a source of triple-harmonic residuals.

Generators with star-connected armature windings may cause residuals due to: (1) inequalities among the voltages induced in the several windings; (2) departure from ideal phase differences, 120 degrees for a three-phase generator; (3) triple-harmonic voltages of the three windings being in phase, between neutral and line terminals. When a generator is connected to the line either directly or through auto-transformers, residuals are thus caused only if the generator neutral is grounded. When connected to the line through transformers residuals will result from these causes if the transformer bank is star-star connected with line-side neutral grounded and station-side neutral connected to generator neutral. The remedies are: (1) careful design and construction, (2) avoidance of grounded neutral or transformer connections permitting transformation of generator residuals to line (as by the use of a delta connection on the generator side of the transformers).

The grounding of transformers, transformer banks or generators at unsymmetrical points of their windings unbalances the electrically connected circuit and thereby causes a residual voltage and current. The remedy is obvious.

6. *Factors Affecting Intensity and Magnitude of Induction.*

(*a*) *Dimensional Factors.* In general, as the horizontal separation of power and communication lines is increased, the induction decreases at a rate varying, roughly, from direct proportionality to about the third power of the separation. That is, doubling the separation reduces the induction from unity to some value between one-eighth or less and one-half. The rate of decrease is less for magnetic induction than for electric induction and less for induction in grounded circuits than for induction in metallic circuits. When the disturbed and disturbing circuits are very close together, the rate of decrease may be less than stated above and in some instances, notably with the vertical-configuration power circuit, there may be an increase of induction at first, as the separation increases.

Other things being equal, the magnitude of the induction increases nearly in direct proportion to the length of parallel.

The configuration of a power circuit has a large influence on the intensity of the induction from balanced voltages and currents, but a very small influence on induction from residuals. No one configuration commonly employed can be selected as universally superior, since the one giving the least inductive effect depends on the spacing of the conductors and the relative position of the two classes of circuits, also upon the type of induction, electric or magnetic, which preponderates.

Induction from balanced components increases nearly in direct proportion to the spacing of power conductors, but induction from residuals, particularly residual current, is only slightly affected by the conductor spacing.

The intensity of the direct induction in metallic communication circuits depends largely upon the arrangement of the conductors and increases in direct proportion to their spacing (for two-conductor circuits in a given plane); but the inductive effects on the conductors as a group, with reference to the earth as a neutral conductor, are only slightly affected by the spacing or arrangement.

(*b*) *Electrical Factors.* The induced current in a communication circuit increases in direct proportion to the magnitude of the voltage or current in the power circuit which causes it, and approximately in proportion to the frequency of the inducing voltage or current.

As stated above, residual voltages or currents produce much more intense inductive effects than balanced voltages or currents of the same magnitude.

The magnitude of the induced current is considerably affected by the amount and character of line and of terminal apparatus between the parallel or source of disturbance and the receiving instrument. The primary effect of such sections of unexposed line and apparatus is to diminish the received current.

Several communication conductors on one line tend to shield one another. It is generally assumed that ground-wires, as commonly employed for lightning protection on power lines, are shielding agents. This is true with respect to inductive effects from residual voltages and currents, but such ground-wires may increase the intensity of the induction from balanced voltages and currents, by distortion of the electric and magnetic fields about the power circuit.

Using the well-known laws of electricity and magnetism, it is possible to determine by computations the effect of these various factors, both dimensional and electrical, in simple practical cases. Even with the simplifying assumptions allowable, the work is usually tedious. In complex cases as when the simultaneous action of all factors is to be considered, quantitative results are best obtained by experimental means.

7. *Transpositions.*

One of the most valuable means of overcoming inductive interference under normal operating conditions of power circuits is to transpose the conductors of each circuit, so as to equalize their relations to all other circuits and to earth.

Transpositions in a power circuit tend: (1) to equalize the capacitances of its conductors to ground, thereby removing a source of residuals, and (2) to cause the inductive effects from the balanced voltages and currents to neutralize one another in neighboring lengths of a parallel communication line. Transposition of a power circuit does not reduce induction from residuals, except as it may do so indirectly by a reduction in the magnitude of the residuals as just noted.

Transpositions in a communication circuit tend: (1) to equalize the capacitances of its conductors to ground; (2) to lessen the induction among the several communication circuits of a line (known as "cross-talk" on telephone circuits); and (3) to equalize the inductive effects on the two sides of the circuits, due to near-by power circuits. Such transpositions do not protect the circuit against voltages induced between the circuit as a whole and ground or along the conductors as a group.

In order that transpositions shall be most effective they must be carefully located, within sections where the intensity of the inductive effects is uniform, with respect to points where the induction changes, called points of discontinuity. The transpositions in each class of line must also be located with regard to the transpositions of the other class of line, *i.e.*, the transpositions in the power and communication lines must be co-ordinated.

On account of the finite (though very short) time required for electric waves to travel along the conductors, the electrical conditions at a given instant will be different at different points along the lines, being practically opposite at points one-half wave length apart; hence transpositions laid out on a basis of uniform conditions do not produce perfectly neutralizing and equalizing effects in adjacent sections of a parallel. To be effective, therefore, the nominally balanced lengths of a transposition scheme should be very short as compared to a wave length at the frequencies of induction to be considered and guarded against. The impairment of balance due to this effect varies approximately as the square of the length of nominally balanced section and directly as the frequency of the induction. It is usually advantageous to omit transpositions at the junction points of successive balanced sections, or barrels of the power circuit, as this lessens the impairment of balance just mentioned.

The length of parallel within which a nominal balance should be obtained in a scheme of transpositions designed to adequately reduce

interference, is usually determined by the points of discontinuity, the lengths of sections thus required adequately meeting the requirement above mentioned of securing balance within a small fraction of a wave length. In long uniform parallels involving telephone circuits, balanced sections with barrels in the power circuits three miles in length are usually adequate. For such parallels involving telegraph circuits longer barrels are permissible as only the wave length of fundamental frequency need be considered.

Though transpositions afford a very practical and effective means of mitigation for some inductive disturbances, they cannot be considered as a complete remedy for interference even under normal operating conditions.

8. *Unbalance of Communication Circuits.*

Differences in the admittances to ground or series impedances of the two conductors of a metallic communication circuit cause currents in its terminal apparatus when a voltage is induced between its conductors and ground or along its conductors in multiple. These unbalances may be reduced to the smallest practicable values by transposing the conductors and by proper design, construction and maintenance of open-wire lines, cables and connected apparatus. A small amount of unbalance is, of course, unavoidable. Since the induced currents here considered are proportional to the product of the unbalance and the induced voltage, it is necessary to restrict the amounts of either or both these factors in order to sufficiently limit the induced currents in the terminal apparatus of metallic telephone circuits.

9. *Transients and Abnormal Conditions.*

When a section of power circuit is energized or de-energized a sudden change takes place in the electric and magnetic fields about the circuit. If the several conductors are not energized or de-energized at exactly the same instant, large residual voltages and currents exist momentarily. An extreme case of this sort occurs when single-pole switches are operated successively, or when one pole of a switch fails to operate.

When one conductor becomes grounded there is a sudden change from a condition of approximate balance to one of large unbalance which persists until the circuit is de-energized or the fault cleared.

At the time of such abnormal conditions of power circuits the induction in parallel communication circuits is greatly in excess of that experienced under conditions of normal operation, sometimes causing hazardous voltages, and acoustic shocks to telephone users. If the protective devices of communication circuits are operated, service interruption continues for a considerable period after the initial cause has subsided, until such devices are restored. Where telephone circuits are affected the operating personnel may be temporarily demoralized by severe acoustic shocks. An "arcing ground" on a power circuit not normally connected to ground may continue for some time with constant repetition of the accompanying transients, and corresponding severe disturbance.

The means sometimes employed in handling faults in power circuits of repeatedly re-energizing the faulty circuit, either to burn off a "ground" or locate it by sectionalization, often greatly aggravates the disturbance to communication circuits by repetition.

Abnormal conditions and severe switching transients are, aside from their detrimental effects on communication circuits, very undesirable from the standpoint of power-system operation. The frequency of their occurrence can only be lessened by high-grade design and construction and by careful operation and maintenance.

10. *Nonessential Features Cause Greatest Interference.*

It will be apparent from the foregoing that those features of power and communication circuits, which have the greatest tendency to result in interference, while not wholly avoidable, are nonessentials which serve no useful end in the normal functioning of the circuits, and that the necessary precautions for the prevention of inductive interference are not incompatible with a high grade of service but to some extent further that end. This circumstance is fortunate alike for the public, the power companies and the communication companies.

GUIDING PRINCIPLES FOR PREVENTING INTERFERENCE.

The following are the basic physical principles which underlie the rules recommended in Part Three and which should guide all efforts to prevent inductive interference.

1. Avoidance of close proximity.

By no other means can complete freedom from interference be secured.

2. Elimination or suppression of harmonics.

To the existence of harmonics is due practically all interference to telephone circuits under the normal operating conditions of parallel power circuits. Improvement in this respect may be effected by giving due regard to its importance in the purchase of new equipment.

3. Limitation of residuals.

The intensity of the induction due to residual voltages and currents is relatively more severe than that due to balanced voltages and currents and induction arising from residuals cannot be neutralized by power transpositions. They can be lessened by balancing the line and also the load, by the use of advantageous transformer connections and by the avoidance of excessive magnetic density in the iron cores of transformers.

4. Reduction of intensity of induction by favorable arrangements of conductors.

Within the latitude afforded by various practical configurations of power and communication circuits the induction with some arrangements is of much less intensity than with others. In cases of multicircuit power lines important advantage can be secured by care in fixing the phase relations of the conductors of the several circuits.

5. Neutralization of induction by co-ordinated transposition systems.

By means of transpositions in both classes of circuits within a parallel the phase or direction of the induction may be controlled so that mutually neutralizing effects are created in neighboring lengths of circuit. To be effective the transposition systems of the two classes of lines must be *co-ordinated.*

6. Balancing of metallic communication circuits.

Accurate balancing of metallic communication circuits, particularly telephone circuits, tends to reduce the disturbing effect of induction from parallel power circuits and other near-by communication circuits. Unbalances are reduced by transposing the conductors and by careful design, construction and maintenance of lines and apparatus.

7. High-grade construction and care in the operation and maintenance of power circuits.

No means are known, except increased separation of the two classes of lines, whereby the severe momentary disturbances to communication circuits, due to abnormal conditions on neighboring power circuits, can be prevented; hence the importance of minimizing such occurrences by high standards of construction, operation and maintenance.

PART THREE.

REVISED RULES RECOMMENDED BY COMMITTEE.

Reasons for Revising Rules.

Since submitting its preliminary report, dated July 7, 1914, recommending provisional rules for the prevention or mitigation of inductive interference which were later embodied in General Order No. 39, this Committee has greatly extended its investigations of some important branches of the subject. Considerable experience in the practical application of the rules has also been gained. In the light of the additional information thus made available and with due consideration of criticisms and suggestions which have been offered by others, the Committee has formulated, and herewith presents, revised rules which it recommends be embodied in a new order of the Railroad Commission to supersede General Order No. 39.

In formulating these revised rules, the Committee has endeavored to utilize the information obtained since its former report so that the rules may be, so far as practicable, definite and authoritative in respect to the specific limitations. The general arrangement of the rules has been modified in order to better meet the requirements of practical use. A detailed discussion of such of the provisions as seem to require it is given in a section immediately following the rules.

Text of revised rules.

RULES GOVERNING THE CONSTRUCTION AND OPERATION OF POWER AND COMMUNICATION LINES FOR THE PREVENTION OR MITIGATION OF INDUCTIVE INTERFERENCE.

I. GENERAL PROVISIONS.

(a) Applicability of rules.

These rules, except as otherwise provided in I (*e*) shall apply and be effective as follows:

1. Rules limited to lines involved in a parallel,* or to apparatus connected to such lines, shall apply only in case of parallels created hereafter; except that rules relating to operation or maintenance shall apply to all such lines and apparatus, both existing and new.

2. Rules not limited to lines involved in a parallel, or to apparatus connected to such lines, shall apply to new construction only, including, however, existing lines and apparatus when such are generally reconstructed or renewed.

(b) Co-operation.

Any party contemplating new construction which may create a parallel shall confer with the other party or parties concerned and they shall co-operate with a view of avoiding the parallel, or, if this be impracticable, of minimizing the resulting interference. Failure to comply with this requirement will receive consideration by this Commission in any subsequent issue involving such construction.

*For definition of "parallel" see page 44.

(c) Principle of least cost.

When there are two or more different practicable methods of avoiding or mitigating interference, the method which involves the least total cost shall in general be adopted irrespective of whether the necessary changes are made in the plant of the party creating the parallel or in the plant of the other party; provided, however, that preference shall be given to methods of avoiding a parallel over methods of mitigating interference; and provided, further, that as between different methods of mitigation having different degrees of effectiveness, the most effective method, the cost of which can be justified, shall be adopted. In estimating such costs, all factors of expense to both parties shall be taken into account.

(d) Existing parallels.

Parties operating power or communication lines shall exercise due diligence in applying measures, in general accordance with the principles of these rules, for mitigating inductive interference due to existing parallels. Any such parallels which now or hereafter cause excessive interference shall be attended to promptly.

When lines involved in existing parallels are added to, extended or generally reconstructed, or when additional apparatus is connected to such lines, or when apparatus now connected to such lines is renewed or rearranged, the new or changed plant shall thereafter conform to the provisions of these rules.

(e) Saving clause.

The Commission reserves the right to modify any of the provisions of these rules in specific cases, when in the Commission's opinion, public interest would be served by so doing.

II. DEFINITIONS.

Certain technical terms are employed herein in the senses set forth in the following definitions:

(a) Class H power circuit.

The term "Class H Power Circuit" means any overhead open-wire constant-potential alternating-current power transmission or distribution circuit or electrically connected network which has 5,000 volts or more between any two conductors or 2,900 volts or more between any conductor and ground; except railway trolley circuits and feeders electrically connected therewith.

(b) Electrically connected.

The term "Electrically Connected" means connected by a conducting path or through a condenser, as distinguished from connection merely through magnetic induction.

(c) Signal circuit.

The term "Signal Circuit" means any telephone, telegraph, messenger call, clock, fire, police alarm, or other circuit of similar nature used exclusively for the transmission of signals or intelligence, which

operates at less than 400 volts to ground, or 750 volts between any two points of the circuit, provided that if the voltage exceeds 150, the power transmitted shall not exceed 150 watts.

(d) Communication circuit.

The term "Communication Circuit" means any overhead open-wire signal circuit, except that, if such circuit be a telephone circuit, it is limited to inter-exchange metallic telephone circuits and to metallic telephone circuits operated by a railroad or other company for dispatching purposes, or for public use between separate communities.

(e) Line.

The term "Line" means any circuit or aggregation of circuits carried on poles or towers, and includes the supporting elements.

(f) Parallel.

The term "Parallel" means a condition where a Class H Power circuit and a communication circuit follow substantially the same course or are otherwise in proximity for a sufficient distance so that the power circuit is liable to create inductive interference in the communication circuit.

With some parallels interference occurs only at times of abnormal conditions on the power circuit in which case such of these rules as affect induction only under normal operating conditions do not apply. When the application of any rule is thus restricted, the condition under which the rule applies is referred to as a "normal" parallel.

(g) Configuration.

The term "Configuration" means the geometrical arrangement of a circuit or circuits, including the size of the wires, and their relative positions with respect to one another and earth.

(h) Transposition.

The term "Transposition" denotes an interchange of position of the conductors of a circuit between successive lengths thereof.

(i) Barrel.

The term "Barrel" means an arrangement of a section of power circuit of uniform configuration within which each conductor occupies each of the conductor positions for equal distances.

(j) Discontinuity.

The term "Discontinuity" means any abrupt change in the relative positions of a power and a communication circuit, or any abrupt change in configuration, line impedance or load along either such circuit (including such changes due to connected circuits, transformers, cables, loading coils or other apparatus) which materially affects the magnitude or phase of the induced voltages or currents per unit length or the capacitances of either circuit. Transpositions, however, are not considered to be discontinuities.

(k) Co-ordination.

The term "Co-ordination" as applied to transposition systems means that the transpositions in power and communication circuits involved in a parallel are efficiently located, with respect to each other and to the discontinuities, for reducing the inductive effects on the communication circuits.

(l) Balanced and residual voltages.

The voltages *to ground* of the several wires of a power circuit are divided for convenience into two classes of components, "balanced" and "residual."

The "balanced voltages" are those components which are equal in magnitude and have such phase relations that their algebraic sum is zero at every instant.

The remaining components of the voltages to ground, which exist under conditions other than perfect balance, are termed residual. They are equivalent to a single-phase voltage impressed between the power wires in multiple and ground. The sum of the residual components is termed the "residual voltage" of the circuit. In case of a three-phase circuit it is three times the equivalent single-phase voltage above mentioned.

Mathematically expressed, the residual voltage is the vector sum of the voltages to ground of the several wires of a power circuit, while the balanced voltages are those components whose vector sum is zero.

(m) Balanced and residual currents.

The currents in the several wires of a power circuit are divided for convenience into two class of components, "balanced" and "residual."

The "balanced currents" are those wholly confined to the wires of the circuit. Hence their algebraic sum is zero at every instant.

The remaining components of the currents in the several wires, which exist under conditions other than perfect balance, are termed residual. The sum of the residual components is the "residual current" of the circuit. It is equivalent to a single-phase current in a circuit having the power wires in multiple as one side, and ground as the other side.

Mathematically expressed, the residual current is the vector sum of the currents in the several power wires while the balanced currents are those components whose vector sum is zero.

III. LOCATION OF LINES.

(a) Avoidance of parallels.

Every reasonable effort shall be made to avoid creating parallels. If the parties concerned can agree upon a plan for providing an adequate separation of the two classes of lines so as to avoid interference, such plan shall be put into effect. In no case shall a parallel be created unless the cost of avoidance by separation is greater than the cost of the remedial measures required by these rules.

(b) Notice of intention.

The party proposing to build a new Class H power or a communication line which will create a parallel, or generally to reconstruct or

change the operating conditions of an existing line involved in a parallel, shall give due notice (at least sixty days where practicable but in any event not less than twenty days in advance of construction, except for minor extensions, for which notice shall be given immediately after the work is authorized) of such intention to the other party including full information as to the location within the parallel and such other features of the proposed line as would affect induction.

(c) Distance between lines.

Class H power lines and communication lines shall be kept as far apart as practicable. Their separation should be at least equal to the height above ground of the power wires, except when closer proximity is unavoidable.

If, in any case of inductive interference, it should be found impracticable to obtain a proper degree of relief by means of the remedial measures set forth in these rules or by other measures of a remedial nature, the parties concerned shall agree upon and put into effect a plan for increasing the separation of the lines within the parallel.

To promote the effective application of transpositions, both parties shall endeavor to maintain a uniform separation of the two lines throughout each normal parallel. However, in general, when it is feasible to secure more than a 20 per cent increase in separation, for a distance in excess of one mile, this shall be done.

(d) Length of parallels.

Parallels shall be made as short as practicable.

(e) Discontinuities.

In the location, construction and general reconstruction of lines within normal parallels every reasonable effort shall be made to avoid discontinuities (except those due to increases in separation as provided for in (*c*) above) which would interfere with the application of effective and economical co-ordinated transposition systems in the power and communication lines.

In the location and construction of the first line along a public highway, special effort shall be made to avoid crossing the highway and also to avoid other features which would result in unnecessary discontinuities in the event of the construction of another line along the same highway.

IV. DESIGN AND CONSTRUCTION OF LINES.

(a) General requirements.

The quality of material, workmanship, methods and grade of construction shall be in accordance with approved modern practice with special regard to the prevention of failures and the avoidance of features, such, for example, as inferior insulation, which would tend to cause or promote inductive interference.

(b) Arrangement and spacing of power conductors.

In the design for construction or general reconstruction of Class H power lines, consideration shall be given to the configuration of the lines

with a view to minimizing (1) throughout the entire length of the line inequalities among the capacitances to earth of the conductors; and (2) within normal parallels the intensity of the inductive effects. When two or more circuits are carried on one line the phase relations among the conductors of the different circuits should be chosen with the same purposes in view. The configurations to be preferred for three-phase lines under different conditions are discussed in the Exhibit attached hereto.

Excessive spacing of conductors should be avoided.

Two-wire branches electrically connected to a three-phase Class H power circuit should be avoided except those so short that they do not materially unbalance the three-phase circuit. Where such branches are employed they should be so distributed as to cause minimum unbalance.

No single-wire grounded Class H power circuits or branches of multi-wire Class H power circuits shall be employed.

(c) Transpositions—General.

All Class H power circuits and metallic communication circuits, or extensions of such circuits, hereafter constructed or generally reconstructed, shall be transposed throughout their entire lengths in such manner as to balance, as nearly as practicable, the capacitances to earth of their conductors. For single-circuit three-phase lines the maximum length of barrel for this purpose shall be twelve miles for circuits of triangular* configuration and six miles for other configurations.. For twin-circuit three-phase lines the maximum length of barrel shall be six miles; except that for circuits of the vertical type (including cases with the middle conductors displaced slightly outward) and the equilateral triangular type with vertices upward, nine-mile barrels may be used when the circuits are interconnected for minimum unbalances. The accompanying Exhibit contains information concerning the methods of interconnection giving minimum unbalances.

Exceptions. Power lines, located principally on private rights of way and not electrically connected to other lines, are exempt from this rule if separated from existing communication lines, and from highways required for the future construction of communication lines, by distances not less than those given below, except for crossings at angles over 30 degrees and other sections of unavoidable closer proximity not exceeding one mile in total length in each ten consecutive miles of line, provided, however, that such sections of closer proximity to any one such communication line or highway shall not exceed one mile in each thirty consecutive miles of line.

Voltage between power conductors	Minimum separation from highways and communication lines
Below 50,000	600 feet
50,000- 75,000	750 feet
75,000-100,000	850 feet
100,000-150,000	1,000 feet
150,000-200,000	1,200 feet

For power lines meeting all these conditions for exemption except that they are electrically connected to other lines through autotransformers, the maximum lengths of barrel may be twice those specified above.

*A triangular configuration as here used means one in which the altitude of the triangle exceeds one-half the length of the longest side as base.

The question of whether highways that may be involved will be required for future communication lines shall be settled by agreement between the power company contemplating construction and the communication companies operating within the territory to be traversed. In the event of disagreement or if there is no such communication company, the matter shall be referred to this Commission. In cases where the proposed use of a particular highway by a communication company would be the determining factor in deciding whether a given power line must be transposed, such communication company shall make an effort to locate its proposed line elsewhere and the decision shall be made in accordance with the principle of least cost laid down in I (*c*).

Existing Class H power circuits and those exempted under the preceding paragraph, which hereafter become involved in normal parallels, shall be transposed so as to balance their capacitances to earth, when necessary for limiting residual voltages and currents to amounts which can be tolerated. The location and number of transpositions for this purpose shall be determined by agreement of the parties concerned.

In the location and spacing of the transpositions due regard shall be paid to discontinuities which affect the capacitances of the circuit. Sections of circuit between such points of discontinuity should be treated independently.

In general, transpositions should be omitted at the junction points of successive barrels.

Metallic communication circuits, and single-phase and two-phase Class H power circuits, shall be transposed at intervals not exceeding four miles.

Power circuits less than three miles in length are not required to be transposed outside of parallels, except when the absence of transpositions would materially impair the balance of other circuits to which they are electrically connected.

Power circuits with grounded neutrals, having a voltage of less than 12,500 volts, between conductors, are not required to be transposed outside of parallels, except where the lack of such transpositions in any specific case is the cause of interference.

Within normal parallels the transpositions in the two classes of circuits shall be as provided in (*d*) below. When the transpositions required in a parallel impair the general transposition system of either line outside the limits of the parallel, the necessary readjustment of transpositions shall be made in the sections of line adjacent to the parallel, as a part of the remedial measures therefor.

(d) Transpositions—Inside limits of parallels.

Within each normal parallel an adequate scheme of transpositions, to neutralize, so far as practicable, the inductive effects, shall be installed in the power circuits, and also in the communication circuits, provided the latter are metallic. The transposition systems in the two classes of circuits shall be properly co-ordinated. The parties concerned shall co-operate to determine upon the transposition scheme to be employed. The transpositions required in the line last constructed shall be installed before it is placed in service.

In applying the foregoing, the following rules shall, in general, be observed:

1. For each normal parallel at least one barrel shall be installed in the power circuit. This applies also to a section of parallel where it is not practicable to obtain a balance by combining it with another section. In applying this rule it is not intended ordinarily to change the span lengths required for other purposes.

2. In long uniform parallels or sections of parallel, involving a telephone line at highway separation from the power line, the barrels shall be three miles in length, subject to such variation as may be necessary for co-ordination with the transpositions required in the telephone circuits. Transpositions should in general be omitted at the junction points of successive barrels.

3. Except as modified by (1) above, the number of transpositions required in power circuits paralleling telephone circuits shall be subject to the following limitations expressed in terms of the average distance between successive transpositions:

For power circuits of 50,000 volts or more between conductors, not less than one mile.

For power circuits of less than 50,000 volts between conductors, not less than one-sixth mile.*

4. In case of a parallel between a power line and a telegraph line or other grounded communication line, the transpositions in the power circuit shall be located with due regard to the limits of the parallels and to discontinuities, in order to form as nearly as practicable a balanced system, subject to the condition that the transpositions in the power circuit are not required to be less than one mile apart, except as modified by (1) above. In long uniform sections of parallel, barrels six miles in length should be sufficient. Transpositions should be omitted at the junction points of successive barrels.

5. The question of the most economical scheme to accomplish the purpose shall always be considered. Effort shall be made to utilize as many as practicable of the existing transpositions.

It is suggested that in case of a short section of new line, not sufficient of itself to require transpositions, but which is likely to be extended later so that transpositions would then be necessary, consideration be given to the advisability of installing one or more suitably located transpositions in the new section of line while it is being constructed in order to avoid interrupting the service by adding transpositions afterwards.

Exceptions. Cases of parallelism may occur where the interference is due almost wholly to residual voltages and currents in which event transpositions in the power circuit are not required, except as provided in IV (*c*).

V. DESIGN, CONSTRUCTION AND ARRANGEMENT OF APPARATUS.

(a) Quality and suitability.

In designing, specifying, or otherwise determining the quality or suitability of apparatus to be connected to Class H power or communication

*While barrels of approximately three miles, as provided in 2 above, are generally to be employed, the shorter barrels specified in 3 are sometimes necessary in short parallels and in short sections of parallels, in order to co-ordinate with the discontinuities and obtain a proper degree of balance.

circuits, and in arranging such apparatus for use, effort shall be made to avoid, so far as is reasonably practicable, all features which would tend to create or promote inductive interference under either normal or abnormal conditions. As instances in applying the foregoing, the following rules shall be observed.

(b) Rotating machinery.

In order to improve conditions generally, companies operating Class H power circuits shall make every effort to minimize the high frequency components of voltages and currents caused by rotating machinery. All new rotating machinery shall have as nearly as practicable a pure sine wave of voltage and shall not, in any case, deviate therefrom to exceed the limit set forth in the present standardization rules of the American Institute of Electrical Engineers.

No ground connection shall be used on the armature winding of an alternating-current generator or motor electrically connected to a power circuit involved in a normal parallel unless means are employed to avoid unbalancing the circuit and to reduce triple-harmonic residuals as far as may be necessary and practicable.

(c) Transformers and their connections.

In order that the wave-shape of voltage and current may be distorted as little as practicable by transformers, all new transformers on Class H power circuits should have an exciting current as low as is consistent with good practice, and which shall not, at rated voltage, exceed 10 per cent of the full load current; except that for transformers without neutral ground connections on the line side, the exciting current at rated voltage need not be less than 0.2 ampere.

Where three-phase transformers are employed with grounded neutrals the core type is preferable to the shell type.

Transformers or transformer banks shall not be grounded at such points of their windings as to unbalance a connected circuit involved in a normal parallel. As important cases under this rule, no grounded single-phase, grounded three-wire two-phase, or grounded open-star three-phase connection shall be so employed.

No star-connected transformers or autotransformers shall be employed with a grounded neutral on the side connected to a three-phase power circuit involved in a normal parallel, unless low-impedance delta-connected secondary or tertiary windings or other equivalent means are used for suppressing the triple harmonic components of the residual voltages and currents introduced by the transformers.

Care shall be taken that the individual units in each grounded-neutral bank of transformers, connected to a circuit involved in a normal parallel, are alike as to type and rating, including all electrical characteristics, and that they are similarly connected, so as not to unbalance the circuit.

Closed-delta connections shall be used wherever practicable in preference to open-delta connections on three-phase power circuits involved in normal parallels. When open-delta connections are employed, an effort shall be made to distribute such connections equally among the three phases.

Where triple harmonic residual voltages and currents due to star-connected transformer banks exist in amounts which can not be tolerated, and it is inexpedient to isolate the transformer neutrals, such residuals shall be limited by operating the transformers at reduced magnetic density or by other available means.

(d) Rectifiers.

Rectifiers and other apparatus tending to distort the alternating current wave when installed on power lines involved in normal parallels, shall, if necessary, be equipped with suitable auxiliary apparatus to prevent harmful distortion of the wave-form of power-circuit voltage or current.

(e) Switches.

Each oil-break switch in a power-circuit involved in a parallel, located between the source or sources of energy and the parallel, and used for energizing or de-energizing the circuit, shall have all poles mechanically interconnected for simultaneous action. There shall be at least one such switch so located as to control the supply of energy to each power circuit involved in a parallel, and, except at stations where an operator is constantly on duty, such switch shall be made automatic for short circuits, grounds, and in case of grounded neutral circuits, for abnormal neutral grounds.

Careful consideration shall be given to means of minimizing transient disturbances caused by switching operations on Class H power circuits, which would cause inductive interference. Wherever practicable provision shall be made for switching on the station-side rather than on the line-side of transformer banks.

Oil-break switches, having their poles mechanically interconnected for simultaneous action, shall be provided wherever the use of air switches or noninterconnected single-pole oil switches would cause harmful transient disturbances in parallel communication circuits.

(f) Fuses.

Switches shall be used instead of main line fuses wherever practicable in a power circuit involved in a parallel.

(g) Electrolytic lightning arresters.

When electrolytic lightning arresters are employed on a power circuit involved in a parallel they shall be equipped with auxiliary charging resistances and contacts so arranged that the horn gaps are short-circuited at the time of charging, to avoid, as far as possible, the production of arcs.

(h) Special instruments.

Reliable indicating devices shall be installed at the source of supply of power circuits involved in parallels, to inform the operators immediately of abnormal conditions, such as grounds, and wherever possible, open circuits, which have not operated automatic switches.

Whenever a neutral ground connection is employed on a circuit involved in a parallel, an ammeter, suitable for measuring the current in the neutral under normal operating conditions, shall be installed in

each neutral connection to ground at the main generating and main attended substations on the power system electrically connected to the circuit involved in the parallel.

(i) Communication apparatus.

All apparatus electrically connected to metallic communication circuits involved in parallels shall be designed and constructed so as to secure as nearly as practicable an accurate balance of the series impedances and the admittances to earth of the two sides of the circuits in order to minimize the detrimental effects of induction from parallel power circuits.

VI. OPERATION AND MAINTENANCE.

(a) General requirements.

Power and communication companies shall use all reasonable means to operate and maintain circuits involved in parallels in such a manner as to minimize interference under conditions of normal operation, and to avoid transient disturbances.

(b) Balance.

In the maintenance of both power and communication circuits involved in parallels special care shall be given to the prevention of mechanical and electrical failures which would cause or promote transient disturbances or unbalances such as those due to tree-grounds, defective or dirty insulators or other faults.

The voltages and currents of power circuits involved in parallels shall be kept balanced as closely as practicable and accidental unbalances shall be promptly corrected.

(c) Record of neutral current.

At all points on grounded neutral systems equipped as required in V (*h*), the power company shall observe and record daily the approximate maximum neutral current.

(d) Transformers.

No transformers connected to power circuits involved in normal parallels shall be operated at more than 10 per cent above their rated voltage. Wherever practicable in case of existing equipment and in all cases of new equipment, transformer banks with grounded neutrals on the side which is connected to a power circuit involved in a normal parallel shall not be operated at more than 5 per cent above their rated voltage.

(e) Switching.

In all switching operations care shall be taken to avoid, so far as possible, the production of harmful transient disturbances.

(f) Charging electrolytic lightning arresters.

When, notwithstanding compliance with V (*g*), interference is caused by charging electrolytic lightning arresters, such charging shall be done at night, so far as is possible, preferably between 2 a.m. and 4 a.m.

(g) Abnormal conditions.

Power companies shall adopt operating rules which shall specifically outline the procedure for their operators during times when a power circuit involved in a parallel is abnormally unbalanced, as will occur with an open, grounded or short-circuited line or transformer winding.

Such rules shall in general provide for the discontinuance of operation of the power line until the fault is remedied, excepting only those cases where it is clear that the service rendered the public by continuing operation of this section of power line is of greater importance than the communication service interrupted by such continued operation.

When it is necessary to energize a defective power line in order to locate a fault, care shall be taken to avoid, as far as possible, repeatedly energizing any section of such line which parallels communication circuits, until the fault has been cleared. Whenever possible the faulty section of line shall not be energized more than once until disconnected from the section of line involved in the parallel.

To facilitate the study and prevention of disturbances in communication circuits, occasioned by transient conditions of power circuits, accurate record shall be kept of the nature and time of occurrence of failures, changes in operating arrangements and all switching during times of abnormal conditions of Class H power circuits involved in parallels; and of all transient disturbances in communication circuits. These records shall be made available for use in tracing the causes of such transient disturbances.

EXHIBIT.

ARRANGEMENT AND SPACING OF POWER CONDUCTORS.

Supplementing IV (b) and IV (c).

The arrangement and spacing of the conductors of power circuits are of importance in determining (1) the unbalances or inequalities among the capacitances of the conductors to ground, which cause residual voltages and currents, and (2) the intensity of the inductive effects produced in communication circuits by the balanced voltages and currents of parallel power circuits. For sections of line within limits of parallels, consideration of the inductive effects should in general control rather than consideration of the capacitance unbalances. For sections of line outside the limits of parallels, consideration of capacitance unbalances should be given the greater weight, particularly for circuits operated without grounded neutrals.

The figures and comparisons given herein apply to nontransposed circuits, but the comparisons of different configurations hold also for transposed circuits, provided the circuits are transposed identically. If there were no irregularities or inexactnesses to impair the effectiveness of a transposition system, it would be possible theoretically, neglecting the effect of phase change and attenuation, to obtain a perfect balance by means of transpositions, irrespective of the arrangement of the conductors. Practically, however, circuits even when

carefully transposed have a material resultant unbalance, particularly at the frequencies of the higher harmonics, and this unbalance is proportional to the unbalance characteristic of the circuit configuration. In a similar manner the resultant induction due to a power circuit is proportional to the intensity of the induction characteristic of the configuration. Configurations differ widely in respect to their characteristic unbalances and intensities of induction, some arrangements, particularly of twin circuits, giving fully 90 per cent less unbalances or induction than others.

The effects of the arrangement and spacing of conductors on the unbalances of their capacitances to ground and on the induction produced in parallel communication circuits are discussed separately.

Effect on Capacitance Unbalance.

In general, the capacitances to ground of the conductors of a nontransposed multiconductor circuit are unequal, the magnitude of the percentage unbalances being determined by, and therefore characteristic of, the configuration of the circuit. This "characteristic unbalance" is an important factor in determining the residual voltage of a circuit isolated from ground, and in determining the residual current of a grounded neutral circuit, in so far as such current is caused by the line itself. Taking as a measure of the characteristic unbalance, the residual voltage of a short, uniform, nontransposed circuit without metallic connection to ground and energized with balanced three-phase voltages between conductors, termed the "characteristic residual voltage," the following table affords a comparison of various configurations of single-circuit power lines over the practical range of cross-sectional dimensions.

Characteristic Residual Voltage; Per Cent of Balanced Three-phase Voltage Between Conductors.

Configuration	
Equilateral triangle	0.5 to 4
Vertical	6 to 11
Horizontal—	
Symmetrical	5 to 9
Unsymmetrical	7 to 11
Isosceles triangle—	
Base horizontal	0 to 8
Base vertical	0.5 to 9
"L"	2 to 6
Inverted "L"	4 to 7

Triangular circuits have the smallest unbalances and characteristic residual voltages. Symmetrical horizontal and vertical circuits are about alike, the vertical having slightly the greater, and unsymmetrical horizontal circuits have the largest. The characteristic residual voltages of symmetrical horizontal and vertical configurations are from 2 to 8 times that of a corresponding equilateral triangular circuit, depending upon the spacing and height of the conductors. The characteristic residual voltages of unsymmetrical horizontal circuits are about 20 per cent greater than those of symmetrical horizontal circuits. They may, however, be reduced to those of the symmetrical cases

if the position of the intermediate conductor is alternated so that its average position is midway between the two outside conductors. (If the circuit is transposed this condition should be fulfilled in each section between transpositions.)

The characteristic residual voltages of equilateral triangular circuits are closely proportional to the conductor spacing, but the conductor spacing has but little effect in the cases of vertical and horizontal circuits.

With twin-circuit lines it is possible to interconnect the two circuits so that their unbalances tend to neutralize, giving smaller resultant unbalances among the capacitances of pairs of interconnected conductors than the unbalances among the conductors of individual circuits. For twin circuits of any type the maximum unbalances occur when conductors symmetrically located with respect to an intermediate vertical plane are at common potential. This arrangement should be avoided in all cases.

For circuits of the vertical type, or with top and lowest conductors in a vertical plane and middle conductors displaced outward a small distance, the minimum resultant unbalances are obtained when the top conductors of the two circuits are at common potential and the middle and lowest conductors of one circuit are at the potentials of the lowest and middle conductors respectively of the other. (See Figs. 1 and 2.) For triangular and horizontal circuits the minimum resultant unbalances are obtained when similarly placed conductors of each circuit are at common potential. (See Figs. 3, 4 and 5.) These figures are cross-sectional diagrams, the conductors at common potential being shown as interconnected.

The resultant unbalances with these arrangements are in some cases less than 10 per cent and in general less than 50 per cent of those with the worst condition described above. The arrangements indicated by Figs. 1, 2 and 3 give resultant unbalances of the order of magnitude of those of single-circuit equilaterial triangular lines of corresponding conductor spacing, while those of Figs. 4 and 5, in general, give greater unbalances. In all cases the characteristic residual voltage is taken as the measure of the unbalance.

Where ground wires are used or in cases where unsymmetrical circuits or more than two circuits are involved, special study is necessary to determine the best arrangement.

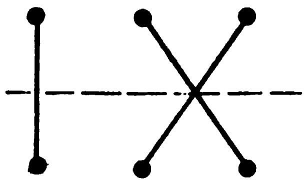


FIG. 1

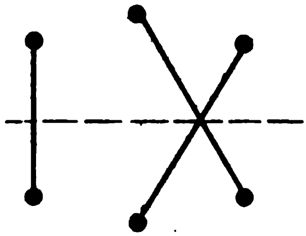


FIG. 2

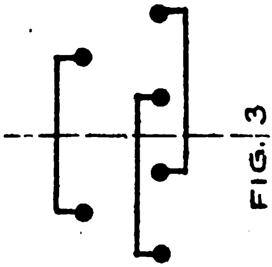


FIG. 3

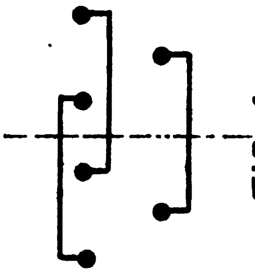


FIG. 4

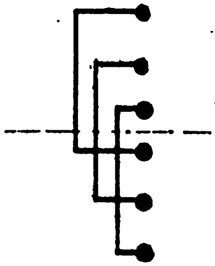


FIG. 5

METHODS OF INTERCONNECTION GIVING
MINIMUM RESULTANT CAPACITANCE UNBALANCES.

With twin circuits of any configuration if the interconnection giving maximum unbalance be altered by transposing the interconnecting wires the unbalance is halved. The two possible interconnections resulting from this procedure are shown in Fig. 6. This plan is useful when there is a doubt as to the best arrangement.

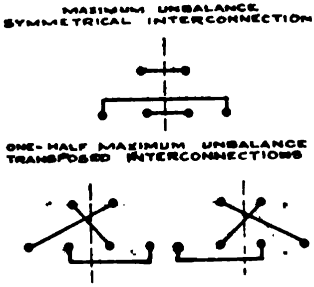


FIG. 6

To obtain the greatest advantage of arrangements giving small unbalances the twin circuits should be interconnected at both ends of the line and at intermediate substations where practicable. In cases where twin circuits are paralleled on the station side of transformer banks but can not be interconnected on the line side, it is still advantageous to fix the phase relation of the conductors as if they were to be interconnected for minimum unbalances.

When transposing twin-circuit lines to secure capacitance balance, the two circuits should be transposed at the same points and care should be taken to secure the condition for minimum unbalance in each section of line between transpositions. (See Fig. 9, below.)

The foregoing facts have an important bearing on the number of transpositions required to adequately balance different types of circuits, more frequent transpositions being necessary in circuits of large characteristic unbalances. This has been considered in IV (*c*).

Effect on Induction From Balanced Voltages and Currents.

The type of power circuit producing the least inductive effects in a parallel communication circuit depends upon the spacing of the conductors and the separation from the communication circuit. In general, for all types of circuit, an increase in the spacing of the power conductors causes a proportionate increase in the magnitude of the inductive effects. Excessive spacing should therefore be avoided. On the other hand, ample spacing to prevent short-circuits or grounds, due to snow, wind, birds, etc., is essential from the standpoint of inductive interference, as well as from that of power service.

For lines separated by the width of an ordinary highway, a vertical type of power circuit, in general, causes the smallest inductive effects, while the horizontal types cause the greatest effects, the triangular types being intermediate in this respect. The relative merits of different configurations vary somewhat with the separation of the two classes of lines and with the dimensions of the power circuit, depending also upon the relative importance of the balanced voltages and currents in producing induction.

For low-voltage horizontal lines, 15,000 volts or less, a symmetrical arrangement of the conductors is better than an unsymmetrical arrangement. For lines of any voltage, if an unsymmetrical arrangement is used, the intermediate conductor should be displaced toward the communication circuit. Hence, unsymmetrical horizontal power circuits along highways should have the intermediate conductor placed on the

side of the poles *toward the road*, where communication circuits are, or may be, located on the opposite side of the road.

When two or more synchronous circuits are carried on one line it is possible to interconnect the conductors of the two circuits or otherwise fix their phase relations so that a partial neutralization of the inductive effects takes place. For twin circuits of the vertical type, or with the top and lowest conductors in a vertical plane and the middle conductors displaced outward a small distance, the most favorable condition is in general, to have the diagonally opposite conductors at common potential. (See Figs. 7 and 8.)

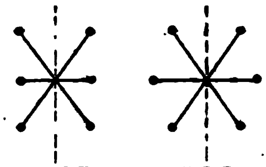


FIG. 7 FIG. 8

METHODS OF INTERCONNECTION CAUSING MINIMUM INDUCTIVE EFFECTS

For circuits of other types the most favorable method of connection varies with the spacing and height of the power conductors and with their position relative to the communication circuit. Thus it is not possible to give a general recommendation, since special study is required in each specific case to determine the most advantageous method of interconnection. Special study is also required for lines carrying more than two circuits of the same or different voltages, for unsymmetrical double-circuit lines, and in cases where ground wires are used.

In transposing twin-circuit lines to neutralize the inductive effects in parallel communication circuits, a similar precaution should be observed, as noted above, with respect to transpositions for capacitance balance. (See Fig. 9.)

Recommended Configurations.

Taking into account both effects above discussed and practical considerations of construction, the equilateral triangular configuration (either the "horizontal-base" or "wishbone" type) is in general recommended for single-circuit power lines; and the vertical configuration (including type of construction with middle conductors displaced slightly outward from vertical plane of the other two) for twin-circuit power lines.

The method of transposing twin vertical lines to preserve the best relation of interconnected conductors both outside and inside limits of parallels is illustrated in Fig. 9, one barrel being shown in each location.

Reference.

Further information concerning the subject discussed in this exhibit will be found in Technical Reports Nos. 51, 64 and 65 of the Joint Committee on Inductive Interference. These and other technical reports are to be published by the state of California.

COMMENTS ON RULES.*

Three principal features are to be noted in comparing the revised rules with General Order No. 39:

1. The arrangement has been entirely altered in order to group related provisions of the rules and facilitate finding any particular subject.

*Not intended to be included in proposed General Order.

TRANSPOSITION OF TWIN-CIRCUIT VERTICAL POWER LINES.

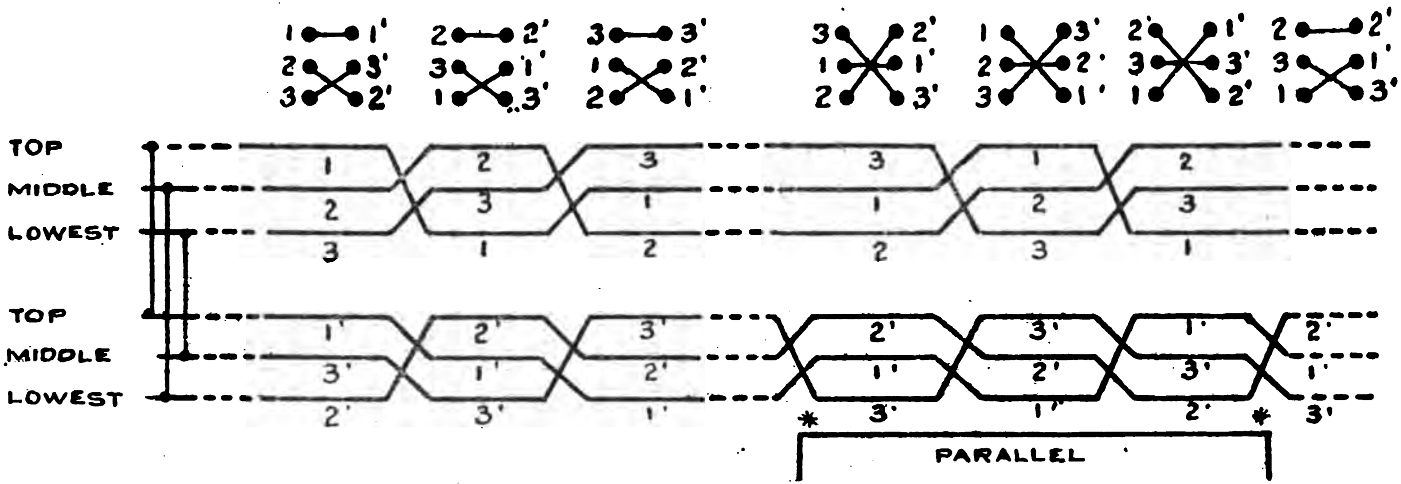


FIG. 9

* THESE TRANSPOSITIONS (IN CIRCUIT LAST INSTALLED) OCCUR ONLY AT THE ENDS OF THE PARALLEL AND ARE FOR THE PURPOSE OF CHANGING FROM THE BEST ARRANGEMENT OUTSIDE THE LIMITS OF THE PARALLEL TO THE BEST ARRANGEMENT INSIDE THE LIMITS OF THE PARALLEL AND VICE VERSA.

2. The principle of co-operation in the determination of means for avoidance and mitigation of interference has been emphasized throughout in conjunction with definite guiding rules where it is practicable to introduce them. An example of this is the specific procedure outlined in IV (*d*) for the transposition of lines within parallels.

3. Some of the rules of a precautionary nature are not limited to lines involved in parallels and apparatus connected thereto, but apply to all new construction.

Some of the rules are discussed in detail below:

I. General provisions.

(*a*) *Applicability of Rules.*

In adopting these rules in California, it would be inadvisable, in this Committee's opinion, to make them retroactive. To apply the rules to all existing construction would call for extensive changes in plant on the part of power and communication companies, particularly in transposing or retransposing power and telephone lines within parallels. Considerable time would be necessary for laying out the transposition schemes, as each parallel requires a special study in order to determine the most effective scheme. If the transposition work were to be carried out independently of other reconstruction, the extra cost and interruption to service would be important items and would involve an undue hardship, except in cases of serious inductive interference which the rules provide shall be attended to promptly. See I (*d*). It has accordingly been deemed advisable to limit the application of the rules generally to new construction, except those rules which deal with maintenance and operation where no reason for such limitation exists. The underlying principle is that all new and reconstructed plant, as it is built or installed, shall be made to conform with these rules.

(*b*) *Co-operation.*

Without a proper spirit of co-operation on the part of power and communication companies, no rules respecting inductive interference can be expected to have a full measure of success. To get the best results, it is essential that each party, operating either class of utility, recognize and have regard for the rights and interests of the other party, and also the convenience of the public as affected by the several kinds of service. An attitude of indifference, or of perfunctory adherence to the letter of the rules without giving consideration to all the various relevant circumstances, may to a large extent vitiate the results contemplated in these rules. The right procedure to employ in any case of interference, actual or impending, depends upon a variety of factors, such as the character and importance of the service involved, the effect on existing plant, the relation of present procedure to future plans of the parties concerned, besides, of course, the relative costs and effectiveness of different procedures, all of which demand co-operation.

(*c*) *Principle of Least Cost.*

Where there are two or more different methods available for avoiding or effectively mitigating an interference, public interest requires that the method of least total cost should be adopted. If alternative methods

afford different degrees of effectiveness, the method selected, if not the most effective, must at least be one which will give satisfactory results. In general, the most effective method should be chosen except where the saving in cost realized by another method is substantial and the sacrifice of effectiveness is not serious. In applying this principle to individual practical cases, the importance of the particular service affected should be taken into account and a grade of service maintained which is adequate and proper for the conditions of the case.

(d) *Existing Parallels.*

This rule has been drawn to conform to the principle explained in the discussion under (a) above. To care for cases of serious interference, provision is made that such cases shall be dealt with promptly.

II. Definitions.

(a) *Class H Power Circuits.*

This designates a restricted class of electrical supply circuits and is intended to accord with the terminology of other orders of this state and elsewhere relative to inductive interference and physical hazard. This class covers the great majority of circuits causing disturbances. Electric railway, series lighting and certain other circuits are treated separately in Appendix I.

(b) *Electrically Connected.*

"Electrical" connection is distinguished from "magnetic" connection. Electrically connected apparatus affects the balance of a circuit with respect to earth, while the corresponding effect of magnetically connected apparatus is negligible.

(c) *Signal Circuits.*

This is substantially the definition given in the National Electric Safety Code (Circular No. 54 of the U. S. Bureau of Standards).

(d) *Communication Circuits.*

A restricted class of signal circuits. Other signal circuits are considered in Appendix I.

(i) *Barrel.*

This term, while most commonly used in connection with three-phase three-wire circuits, applies in principle to any circuit having two or more metallic conductors, such, for example, as a four-wire circuit, in which case a barrel may be secured by three transpositions at quarter points in a uniform section of line.

(l) and (m) *Balanced and Residual Voltages and Currents.*

The definitions of residuals given are identical in meaning with those of General Order No. 39, but they are more fully explained and contrasted with balanced voltages and currents.

The division of voltages and currents of power circuits into balanced and residual components is fundamental to the consideration of effects of transpositions, power-circuit configuration, transformer connections, etc.

III. Location of lines.

(a) *Avoidance of Parallels.*

The first and most obvious means of preventing inductive interference is to avoid the close association of power and communication circuits. Further, it is recognized that in no other way can complete freedom from interference be secured. While, with the ever-increasing network of electrical circuits of all kinds, adequate separation to avoid interference is becoming increasingly difficult to maintain, the Committee feels that the importance of such separation justifies its being made the first premise in rules designed to prevent inductive interference.

(b) *Notice of Intention.*

In order to secure the time essential for determining the proper procedure in case of a parallel, advance notice is necessary. Through the opportunity for co-operative study thus afforded, means of avoidance or designs to reduce the interference can be worked out in advance and applied during the course of construction, at minimum cost and before interference develops.

(c) *Distance Between Lines.*

Since the inductive effects decrease rapidly with the separation of the parallel lines, the importance of this rule is evident. The effectiveness of transpositions depends upon the creation of mutually neutralizing inductive effects in neighboring sections of circuit and will in general be impaired if the intensity of induction is different in different sections of the parallel. Hence uniformity of separation is important.

(d) *Length of Parallel.*

Other things being equal, the inductive effects of a parallel increase in proportion to its length. Hence, parallels should be made as short as practicable.

(e) *Discontinuities.*

In general, the more discontinuities there are in a parallel the more transpositions are required in both power and communication circuits to provide effective balance, and since frequently it is difficult or impracticable, in the design and layout of transposition schemes, to take into account minor discontinuities which tend to nullify the effectiveness of the transpositions, all discontinuities should be avoided as far as practicable in the location and construction of lines.

Many instances have occurred where the first line along a road crossed at frequent intervals in order to cheapen construction. This procedure creates difficulties for any line subsequently constructed, besides greatly complicating the transposition scheme required to reduce inductive interference.

IV. Design and construction of lines.

(a) *General Requirements.*

As a primary consideration, power and communication lines which are liable to become involved in parallels should be designed and constructed in a proper manner, with due regard to features affecting interference, such as insulation. Severe transient disturbances to communication circuits due to abnormal conditions on power lines constitute a serious problem because there are no effective means of overcoming them. Hence, the importance of good construction in preventing the occurrence of failures on power circuits. Fortunately, construction which meets this important need will also best insure uninterrupted service of the power system itself.

(b) *Arrangement and Spacing of Power Conductors.*

The arrangement and spacing of the conductors of power circuits are of importance in determining the circuit unbalances which give rise to residuals, and in determining the intensity of the induction in parallel communication circuits due to balanced voltages and currents. By proper consideration of this matter in the design and construction of lines, the interference-producing characteristics of such lines may be materially lessened with little or no additional cost. This is particularly true of multi-circuit lines in which case great benefit may be secured simply by care in fixing the phase relations among the conductors of the different circuits. Due to lack of generally available information on this subject it is deemed advisable to give, in the form of an exhibit a brief resumé of the Committee's technical data bearing on the subject thus making possible intelligent compliance with the rule.

This rule is not mandatory with respect to any particular arrangement of conductors, for while some arrangements have important advantages over others it is inadvisable to thus restrict the type of line construction, when in many cases, by careful transposition of the line, the desired end can be attained more cheaply. Wherever practicable, however, the benefits derived by favorable arrangements of conductors should be sought as an additional safeguard against interference, particularly in case of major parallels. The great benefits obtainable by advantageous methods of interconnection of twin circuits should be secured in all such lines.

Two wire branches, metallically connected to a three-phase circuit, inherently unbalance the three-phase circuits. Even short branches of this character may cause an unbalance equal to that of the three-phase circuit without transpositions. The Committee realizes that the use of such branches offers an economical means of supplying small loads and that therefore they should not be absolutely prohibited. Every effort should, however, be made to avoid them.

Single-wire grounded power circuits are inherently completely unbalanced, that is, the entire voltage and current are residual, hence they are particularly troublesome to parallel communication circuits.

(c) *Transpositions—General.*

The unbalanced capacitances to ground of power circuits are an important cause of residual voltages and currents. This is true of grounded neutral as well as isolated circuits but is of greater importance with reference to the latter. While such unbalances may be lessened by observing certain precautions in the arrangement and spacing of the conductors as discussed above, this is not in itself sufficient and the circuit must, in general, be transposed so that each conductor occupies all of the conductor positions for equal distances, with due regard to discontinuities. In other words, an integral number of barrels, must be installed between terminals or switching points. The lengths of barrel specified are based on the unbalanced capacitances characteristic of the different configurations and upon phase-change and attenuation at the harmonic frequencies which cause the greater portion of the interference to telephone circuits.

In respect to metallic communication circuits, particularly telephone circuits, the more perfectly the capacitances of the conductors are balanced, the less the disturbing effects of induction from parallel power circuits. Unless the conductors of a circuit are symmetrically placed with respect to earth and all other conductors on the line, transpositions are necessary to equalize the capacitances to ground of the two sides of the circuit. Telephone circuits are usually transposed with respect to one another and these transpositions substantially meet the requirements of this rule.

Obviously, to undertake the transposition of the whole of an extensive power network at the time of the creation of a parallel, is very expensive, particularly in view of the interruption to service occasioned thereby. On the other hand, in the initial construction or during general reconstruction, an adequate transposition system can usually be installed at small cost as a definite part of the construction work. Such transpositions, besides serving to limit the residuals in all parallels with commercial communication circuits, are also beneficial in reducing the induction in private telephone circuits which are often on the same poles. It is recognized that some power circuits, particularly those of extra high voltage, are located on private rights of way, at such distances from highways that transpositions are not necessary for the protection of communication lines, and provision has accordingly been made for the exemption of such circuits from the general requirement of transpositions at the time of construction. In some cases, the proximity without transpositions permitted by the exception may result in serious interference, and special study should be given to cases which are on the border line.

In the matter of existing power circuits, and new power circuits coming under the above exception, which hereafter become involved in parallels, provision is made that they shall then be transposed sufficiently to limit their residual voltages and currents to amounts which can be tolerated. Since in these cases two or more parties are concerned, it is provided that the necessary transposition system shall be determined co-operatively.

(*d*) *Transpositions—Inside Limits of Parallels.*

Transposition systems in both power and communication circuits when properly co-ordinated offer the most reliable and effective means of preventing interference from the balanced voltages and currents of the power circuits.

In contrast with the corresponding rule of General Order No. 39, which allowed the communication company to specify the number and location of the transpositions in the power circuit, this revised rule provides that both parties shall co-operate to decide upon the transposition scheme to be employed. In the past most of the work of designing transposition schemes for parallels has been done by the communication companies, but so far as this Committee is aware the negotiations relating thereto have been generally conducted in a spirit of co-operation.

Three-mile barrels in three-phase power circuits co-ordinate satisfactorily with telephone transposition systems now available (designed particularly for circuits involved in parallels) and are short enough so that the effect of phase-change* along the line, in impairing the efficiency of the transposition scheme, is small. The omission of transpositions at the junction points of barrels does not usually impair the co-ordination of the transposition systems and is advantageous in reducing the effect of phase-change besides reducing the number of power transpositions. It is impossible in a rule to specify exactly the spacings of transpositions within parallels either in power or telephone lines since the spacings required vary with the circumstances being considerably influenced by the length of the parallel and by the discontinuities in both classes of line.

Barrels less than three miles in length are often very useful for securing economical and satisfactory schemes of transpositions, but in the higher voltage lines the difficulty and expense of installation are such that they are justified only in exceptional cases. For lines of lower voltages, parallels are more numerous and shorter barrels are practicable, also the greater liability of discontinuities (such as branch loads) in low-voltage lines renders necessary the greater flexibility of shorter barrels. The rules accordingly make distinction between lines above 50,000 volts and those below 50,000 volts in specifying minimum lengths of barrel. The limitations are identical with those of General Order No. 39.

Several schemes of co-ordinated transpositions can be designed for any given parallel depending upon the utilization of existing transpositions, length of barrel, type of telephone transposition system and discontinuities. Obviously, that scheme involving minimum total cost should usually be chosen. Generally in a new parallel it is economical to transpose the new line to co-ordinate in part, at least, with the existing transpositions of the prior line.

V. Design, construction and arrangement of apparatus.

(*a*) *Quality and Suitability.*

The same considerations that are mentioned under IV (*a*) which demand care in the design and construction of power and communica-

*The effect of phase-change is discussed in Technical Report No. 66.

tion lines to prevent inductive interference and to secure continuity of service, apply also to the apparatus connected to such lines.

(b) *Rotating Machinery.*

The elimination of higher harmonics from the wave form of rotating machinery strikes at the source of disturbance to telephone circuits by removing an underlying cause. It is obviously a matter which can best be cared for in specifications for new machinery.

In providing that the deviation from a pure sine wave shall not exceed the limit set forth in the present standardization rules of the American Institute of Electrical Engineers, it is recognized that this limit is unsatisfactory. This Committee has been in correspondence with a committee of the American Institute of Electrical Engineers which has under consideration the revision of the Institute's present rule on the subject. It is expected that the revised standardization rule will be more satisfactory, and it is recommended that this revised rule, when issued, be recognized by the Commission.

The improvement of wave-form in rotating machinery is essentially a problem for the manufacturer and all new machines should be designed with this in view. This Committee understands that substantial improvement in this direction can be effected at relatively small cost, hence this is one of the general precautionary rules made applicable to all new construction. For the improvement of conditions with respect to machines already manufactured and installed, the use of devices or "networks" external to the machine, designed to shunt the troublesome high-frequency components from the line, probably offers the most economical solution. The use of such devices may even prove economical with new machines.

The provision against grounding armature windings is designed to avoid a source of large harmonic residuals.

(c) *Transformers and Their Connections.*

If high magnetic densities be employed in transformers, the exciting current is large, and large higher harmonic components are introduced. The 10 per cent limit on exciting current here provided serves to guard against extreme designs but does not restrict present standard practice. It is sometimes economical to design small units for a greater exciting current. Banks of such small units, operated without neutral ground connections on the line side, do relatively little harm, hence the exception. Low exciting current is much more important for transformers of grounded neutral banks than for transformers operated without metallic connection to ground.

Core-type three-phase transformers tend to suppress their triple-harmonic exciting currents by mutual interaction in the cores. This is an important advantage over shell-type transformers, if there is a grounded neutral, as it helps to suppress triple harmonic residuals.

The grounding of transformer banks at such points of their windings as to unbalance power circuits involved in parallels is prohibited on account of the residuals thereby caused.

Star-connected transformers inherently tend to cause triple-harmonic residuals in connected transmission lines if the neutral be grounded. These residuals can be greatly reduced by providing a

shunt path for the triple-harmonic currents in the form of delta-connected windings, but such delta windings must be of relatively low impedance in order to be effective. Other means, viz, operation at low magnetic density, interconnected-star arrangements, or provision of star-delta connected transformers as shunts for the triple harmonics of star-star banks, may prove adequate in some cases.

Dissimilarities of electrical characteristics or connections among the different transformers of a grounded-neutral bank cause the power circuit to be unbalanced.

Open-delta connections cause triple-harmonic currents in transmission lines whereas in closed-delta transformer banks, such currents are locally confined.

The triple-harmonic residuals, which occur in grounded star-connected systems, practically disappear when the neutrals are isolated. However, it is not always desirable to isolate all neutrals. By reduction in operating voltage or by changes of transformer taps which reduce the magnetic density of connected transformers relatively large reduction in triple-harmonic residuals may be effected. This may in some cases prove the simplest remedy for excessive residuals.

(*e*) *Switches.*

The operation of switches is sometimes the cause of severe transient disturbances in parallel communication circuits. This is due in part to nonsimultaneous operation of the several poles of the switch which gives rise momentarily to large residuals. Oil-break switches usually complete the opening or closing of a circuit within a few cycles while air switches require a much longer time.

In order to prevent the continuance of abnormal conditions on power circuits with the consequent disturbances, lines involved in parallels should be disconnected immediately upon the occurrence of failures, and accordingly the rule provides that there shall be at least one automatic switch if prompt manual control be not available.

The transient disturbances are less severe when switching is performed on the station side of transformer banks, hence this procedure is recommended when practicable.

(*f*) *Fuses.*

The same consideration of avoiding the large residuals incident to nonsimultaneous opening of the several phases of a power circuit (referred to under "Switches"), dictates the preference for switches instead of fuses.

(*g*) *Electrolytic Lightning Arresters.*

Electrolytic lightning arresters not equipped as specified in this rule cause harmonic residuals, due to arcing, which may result in severe disturbances in parallel communication circuits. The use of charging contacts and auxiliary charging resistances largely prevents the arcing and lessens the rush of current so that such disturbances are greatly reduced.

(h) *Special Instruments.*

The object of this rule is to provide for immediate relief from the abnormal disturbances occasioned by grounds, open circuits, etc., which cause large residuals on power circuits, by giving notice of the existence of such unbalanced conditions.

Ammeters in the neutrals of transformer banks serve to indicate the degree of balance of the power circuit. By observing such meters, unbalances smaller than those which would operate circuit breakers can be detected. It is not necessary that such ammeters be constantly in circuit and suitable provision should be made for their protection under abnormal conditions.

(i) *Communication Apparatus.*

It is necessary to balance as accurately as practicable metallic communication circuits and electrically connected apparatus in order to minimize the disturbing effects of parallel power circuits. This requires that such apparatus be so designed and connected as not to introduce irregularities either in the series impedances of the two sides of the circuit or in their admittances to ground. This applies to all such apparatus, including that used for superposed grounded signalling systems (such as telegraph on telephone), although such balancing has no effect on interference with the superposed grounded system itself.

VI. Operation and maintenance.

(b) *Balance.*

It is to be expected that by careful inspection and maintenance incipient causes of failures can be detected and corrected before they develop serious consequences. In this way interruption to service can be avoided as well as severe disturbances to parallel communication circuits.

(d) *Transformers.*

This rule is designed to limit the introduction of harmonics due to excessive magnetization of transformers, which occur when they are operated at voltages above normal. Such overvoltage operation is particularly undesirable in the case of grounded-neutral banks of transformers.

(e) *Switching.*

Examples of compliance with this rule are, the use of oil-break switches with poles mechanically interconnected for simultaneous action instead of air-break switches, switching on station side of transformer banks, and exercise of special care in synchronizing.

(f) *Charging Electrolytic Lightning Arresters.*

In cases where noticeable interference is occasioned by charging electrolytic lightning arresters equipped as required in V (*g*), the disturbance can be made less troublesome by charging them during the early morning hours when telephone circuits are little used.

(*g*) *Abnormal Conditions.*

In General Order No. 39, a more specific procedure is outlined than is here given. While the general intent of the present rule is the same, more latitude is given, with the requirement that operating rules shall be developed with a view of minimizing the disturbing effects upon parallel communication circuits.

It is recognized that cases will arise where continuance of service over a faulty power circuit may be of greater importance than the interruption to communication service occasioned thereby and exception is made accordingly to the general rule that faulty power circuits shall not be re-energized for operation until the fault is remedied. It is, of course, contemplated in such cases, that the fault shall be remedied at the first opportunity.

The necessity is evident for having accurate records of abnormal occurrences and the switching incident thereto, for use in investigations to remedy such conditions.

Exhibit.

It is provided in rule IV (*b*) that in the design and construction of Class H power lines, consideration shall be given to the configuration of such lines with a view to minimizing their unbalances and their inductive effects in neighboring communication circuits. Configurations differ widely with respect to their characteristic unbalances and intensities of induction, some arrangements, particularly of twin circuits, giving only 10 per cent or less of the unbalances or induction given by others. In many cases, at little or no additional cost, interference may be materially lessened from what it otherwise would be by giving proper consideration to this matter.

As such information is not always easily available, this exhibit has been prepared to give a brief summary of the Joint Committee's technical data bearing on the subject. Doubtless many cases will arise which will be beyond the scope of the recommendations made in this Exhibit. In such cases a special study may be made. Methods for carrying on such studies are described in the Committee's Technical Reports which are referred to in the Exhibit.

APPENDIX I.

INTERFERENCE NOT COVERED BY RECOMMENDED RULES.

EXPLANATORY NOTE.—In the rules recommended, the term "communication circuit" is used in a restricted sense as defined in II (*d*) and the power circuits to which the rules apply are limited to those defined in II (*a*) as Class H. While these include the circuits most commonly involved in inductive interference, cases sometimes occur where either the power circuit causing the induction or the signal circuit affected by induction is of a type or character excluded under the above definitions. The Committee has not had an opportunity to make a special study of such cases. In the following the attempt is made to state the governing principles involved, for the assistance of the Commission in considering cases of this character.

(*a*) *Alternating Current Railways.*

Alternating current railway trolley circuits as now generally operated differ radically from other types of alternating current power circuits in that one side of the former is grounded throughout so that they are inherently unbalanced, and moreover, cannot be transposed. To such circuits the provisions of the foregoing rules in general do not apply, and are not so intended.

Where railway circuits of this character are operated, it is necessary to employ special measures in order to prevent inductive interference with neighboring communication circuits. Other than separating the two classes of lines where this is practicable, the most important of such measures can be embodied in the railway construction and should be included in the design of the electrification after a comprehensive study of the requirements of the particular case by the parties concerned. Also, the communication circuits, if metallic, should be properly transposed and otherwise balanced as closely as practicable. The parties should endeavor to agree as to the responsibilities involved and as to further measures to be adopted, if any such are necessary. In the event of failure so to agree the matter should be referred to this Commission.

NOTE.—This Committee has undertaken no investigation of cases of interference due to alternating-current railways, but as the seriousness of the inductive effects of such railways is recognized, provision is made for co-operation when such cases arise.

(*b*) *Constant-Current Lighting Circuits.*

Care should be taken in the location, design, construction, maintenance and operation of constant-current lighting circuits (both direct-current and alternating-current) to avoid, so far as practicable, inductive interference with communication circuits. In particular every reasonable effort should be made to avoid creating new conditions which would produce such interference, especially where interexchange telephone lines are affected. In cases where such conditions are unavoidable, remedial measures should be employed as may be necessary,

the details of which should be agreed upon by the parties concerned in general accordance with the following provisions:

1. Where necessary, the two sides of the lighting circuit should be run on one pole line within the section where the interference is set up and co-ordinated systems of transpositions applied to the lighting and telephone circuits.
2. Preference should be given to those types of lamps and other equipment which do not introduce high frequency components in the lighting current. The use of incandescent lamps instead of arc lamps is usually advantageous in this respect.
3. Due regard should be given to the insulation and balance of both the lighting and communication circuits. Balance of the lighting circuit requires equalization of the voltages to ground of the two sides of the circuit within the section where the two circuits are in proximity. This necessitates that the circuit be well insulated and in general that the lamps be similarly distributed in the two sides of the circuit with equal numbers of lamps in the two sides between the source of supply and the section of proximity.

NOTE.—It is common practice in city lighting to run single-wire circuits through many lamps in series, scattered widely, instead of carrying the return conductor on the same line, or where the two conductors are on the same line, without balancing their voltages to ground. Both of these features tend to create residuals and to cause severe inductive effects in neighboring signal circuits.

It should be practicable, by care in laying out such lighting circuits, and in locating important telephone lines, such as toll lines which occupy but a few streets, to avoid close proximity between these classes of circuits. In those cases where proximity is unavoidable, it is possible, by running both sides of the lighting circuit close together on the same line, by care in distributing the lamps and by transposing the circuit within the section of proximity, greatly to reduce the residuals.

A considerable difference exists among the various types of lamps used in that arc lamps introduce large harmonics into the lighting circuit, while incandescent lamps produce no appreciable distortion.

The balancing of a lighting circuit can be accomplished in many different ways depending upon the specific conditions. The simplest general procedure is outlined above.

(c) *Power Circuits of Lower Voltages.*

In case of interference with the operation of communication circuits by constant potential alternating-current circuits of voltage lower than the limits specified in the definition of a Class H power circuit, the parties concerned should agree upon remedial measures in general accordance with the rules recommended in this report and should co-operate in applying such measures to the extent that may be necessary, as follows:

1. Where practicable at reasonable expense, the lines should be separated sufficiently to avoid interference.
2. Co-ordinated systems of transpositions should be applied to both classes of circuits within the section where the interference is set up.

3. If practicable, the residual voltage and current of the power circuit should be reduced.
4. Due regard should be given to the insulation and balance of metallic communication circuits.
5. Consideration should be given to the reduction of the high-frequency components of the voltages and currents of the power circuit.

NOTE.—The physical principles upon which the rules as a whole are based apply in case of power circuits of all voltages, the differences being only quantitative in respect to the relative importance of different factors.

(*d*) *Cables.*

In case of inductive interference where either the power circuit or the communication circuit is carried in cable, consideration should be given to the employment of such remedial measures, included in the rules recommended or otherwise as may be reasonably applicable.

In such cases, particular features to which attention should be directed, are: (1) limiting the residual current of the power circuit, (2) balancing the communication circuits if they are metallic, and (3) transposing the communication circuits, if they are metallic and in open wire.

NOTE.—Where cables are used for either power or communication circuits within sections where these two clases of circuits are in proximity, there is, in general, far less liability of interference, and many provisions of the recommended rules are inapplicable. In some cases, however, residual currents in cabled power circuits may cause interference to either open-wire or cabled communication circuits, and open-wire power circuits sometimes cause severe disturbance to communication circuits which are in cable.

(*e*) *Telephone Subscribers' Circuits.*

In case of inductive interference by any electric power circuit with metallic telephone subscribers' circuits (which will usually occur only when the latter are long open-wire surburban loops) the parties concerned should agree upon a plan for avoiding the interference by removal of one of the lines or for mitigating the interference by remedial measures, as the circumstances may require, in general accordance with the recommended rules.

NOTE.—Telephone circuits falling within this class are far more numerous than those included under the definition of communication circuit—II (*d*). Fortunately, subscribers' telephone circuits are not as a rule seriously exposed to the influence of power circuits since they are generally short and are often run partly in cable. When, however, they are exposed, the disturbing effects are relatively more severe on account of the closer proximity of the parallel to the subscriber's station. A parallel involving a subscriber's circuit does not affect so large a portion of the public as a parallel involving an interexchange circuit and on such a basis the former is of less importance. However, this does not justify excluding such circuits from all protection against interference. Consequently, with due regard to the relative importance of the service affected, telephone subscribers' circuits should be afforded such protection as is necessary in general accordance with the principles governing the protection of other communication circuits.

(*f*) *Direct Current Circuits.*

In cases of inductive interference with communication circuits due to constant potential direct current circuits, usually occurring only where grounded railway trolley circuits and telephone circuits are in proximity, adequate remedial measures should be agreed upon and put into effect by the parties concerned. Where telephone circuits are involved, in addition to transposing and balancing such circuits, special consideration should be given: (1) to securing generators and motors having a voltage as free as practicable from high-frequency waves; and (2) to the use of special devices external to the generators, motors and rectifiers which tend to absorb the high-frequency currents and thereby prevent their appearance in the line.

NOTE.—High-frequency components may occur in constant-potential direct-current circuits and occasionally constitute a source of interference. This is particularly true of electrified railway circuits which use a large amount of power. It is therefore provided that effort be made to secure apparatus as free as possible from such high-frequency components and that if necessary, suitable shunt paths be provided to confine these high-frequency components to local circuits.

(*g*) *Other Cases of Interference.*

If any case of inductive interference, not otherwise covered by this report, should be experienced or become imminent, the parties concerned should endeavor to agree upon a procedure for avoiding preferably, or if avoidance be not feasible, for mitigating the interference by applying, to such extent as may be necessary, the measures set forth in this report, or by other means.

APPENDIX II.

LIST OF TECHNICAL REPORTS.

The following is a list of the technical reports which have been prepared in connection with the investigation by the Joint Committee on Inductive Interference. The reports recommended for publication are indicated by an asterisk (*) after the number.

Technical report number.	Subject or title.
1*	General outline of tests to be made at Salinas on parallels between lines of the Sierra and San Francisco Power Company, the Western Union Telegraph Company, the Southern Pacific Company, and The Pacific Telephone and Telegraph Company. (6 pages.) Dated January 6, 1913.
2*	Summary of results of tests at Morgan Hill on parallel between lines of the Coast Counties Gas and Electric Company and The Pacific Telephone and Telegraph Company between Morgan Hill and Gilroy. (8 pages.) Dated February 3, 1913.
3*	A description of the noise standard in use for measuring noise on telephone circuits in terms of a standard unit. (3 pages, 1 drawing.) Dated February 24, 1913.
4*	A description of the instruments and methods used for the measurement of effective values of induced voltages and currents. (2 pages.) Dated February 24, 1913.
5*	A description of apparatus and connections used in measuring line and residual currents and voltages of power circuits. (4 pages, 2 drawings.) Dated February 24, 1913.
6	Tests of the effects of opening the secondary delta of the autotransformer bank at Salinas. (7 pages.) Dated March 31, 1913.
7	Tests of the induction in the block signalling circuits of the Southern Pacific Company paralleled by the Salinas-King City circuit of the Coast Valleys Gas and Electric Company. (3 pages, 1 drawing.) Dated March 31, 1913.
8	Tests of the induction in the telephone circuits of exposure No. 2 at Salinas under normal operating conditions of the power system with particular reference to the effects of grounding and isolating the neutral of the Salinas autotransformers. (15 pages, 1 drawing.) Dated March 31, 1913.
9	Experimental determination of the coefficients of induction for residual currents and voltages in exposure No. 2 at Salinas. (4 pages.) Dated March 31, 1913.
10	Measurements of the harmonics of the neutral current at Salinas. (3 pages, 1 drawing.) Dated March 31, 1913.
11	Investigation of current transformers, ratios and errors due to the use of current transformers under the conditions of the tests. (17 pages, 4 drawings.) Dated April 7, 1913.
12	Formulæ for the computation of electrostatic and electromagnetic induction from power circuits in neighboring communication circuits. (14 pages, 4 drawings.) Dated March 31, 1913.
13.	An investigation of errors in measurements of residual voltage due to the potential transformers used and a discussion of the method of measurement at Salinas. (28 pages, 2 drawings.) Dated September 3, 1913.
14	Comparative tests of the noise in exposed telephone circuits with power on and off the 55,000-volt power circuit of the Sierra and San Francisco Power Company between Guadalupe and Salinas. (7 pages, 1 drawing.) Dated April 30, 1913.

Technical report number.	Subject or title.
15	Supplementary to Technical Report No. 8, differing from the earlier report in that the telephone circuits were shielded. Contains a discussion of transpositions. (22 pages.) Dated June 3, 1913.
16	Tests of induction in telephone circuits exposed to the Coast Counties Gas and Electric Company's 22,000-volt line between Morgan Hill and Gilroy with the power circuit untransposed and open at Gilroy. (4 pages.) Dated April 30, 1913.
17	Tests of the induction in telephone circuits exposed to the Coast Counties Gas and Electric Company's 22,000-volt line between Morgan Hill and Gilroy, before and after installing power-circuit transpositions. (24 pages, 1 drawing.) Dated April 30, 1913.
18.	Tests of the effect, on exposed telephone circuits, of grounding one phase of the Coast Counties Gas and Electric Company's 22,000-volt three-phase delta-connected line. (4 pages.) Dated April 30, 1913.
19	Tests of the combined effect of the Coast Counties Gas and Electric Company's and the Sierra and San Francisco Power Company's circuits on the telephone circuits in the exposure between Morgan Hill and Gilroy. (4 pages.) Dated April 30, 1913.
20	Tests of the effect on the residual voltage of transposing the Coast Counties Gas and Electric Company's 22,000-volt line within the exposure between Morgan Hill and Gilroy. (3 pages.) Dated April 30, 1913.
21	Tests to determine the comparative effect on the noise in the exposed telephone circuits of having the power on and off the Coast Counties Gas and Electric Company's 22,000-volt line between Morgan Hill and Gilroy, and the effect of shielding the telephone circuit under test by grounding other circuits on the lead. (4 pages.) Dated April 30, 1913.
22	Computation of the coefficients of induction from balanced and residual currents and voltages for the telephone circuits of exposure No. 2 at Salinas. (15 pages, 4 drawings.) Dated June 3, 1913.
23	Experimental determination of the coefficients of induction from residual currents and voltages, for the telephone circuits of exposure No. 2 at Salinas—more complete than Technical Report No. 9. (23 pages, 1 drawing.) Dated June 3, 1913.
24	Comparison of computations of Technical Report No. 22 with experimental data of Technical Report No. 23. (10 pages, 6 drawings.) Dated September 3, 1913.
25	Tests of induction in telephone circuits in exposure between Salinas and King City under normal operating conditions, with the neutral of the Salinas autotransformers grounded and isolated. (20 pages.) Dated June 3, 1913.
26	Tests of accuracy of measurement of residual current by certain current transformers. (3 pages, 1 drawing.) Dated June 3, 1913.
27	Tests of induction in telephone circuits in exposure No. 2 at Salinas with the North Beach steam station energizing the Sierra and San Francisco Power Company's line. Supplementary to Technical Reports Nos. 8 and 15, differing in the sources of supply of the power system. (21 pages.) Dated July 14, 1913.
28	Supplementary to Technical Reports Nos. 8 and 15. Voltage lowered 5 per cent at the Guadalupe autotransformers which supply the power circuit. (20 pages.) Dated September 3, 1913.
29*	Determination of impedances of lines, by computations and by measurements—numerous curve sheets and tables. (26 pages, 39 drawings.) Dated May 4, 1914.
30	Tests of induction in telephone circuits in exposure Nos. 1 and 2 at Salinas, with the neutral of the Salinas transformers grounded and isolated. (10 pages.) Dated September 3, 1913.

Technical report number.	Subject or title.
31	Supplementary to Technical Reports Nos. 8 and 15 and more complete. Includes tests with Salinas neutral grounded and isolated and with telephone circuits shielded and unshielded. (29 pages.) Dated September 2, 1913.
32	Supplementary to Technical Report No. 25. (22 pages.) Dated September 3, 1913.
33	Induction in test leads used at Salinas for connecting testing apparatus to the circuits of exposure No. 2, and the effect of such on the measurements of the induction from the exposure. (20 pages.) Dated September 3, 1913.
34	Effect of changes in the insulation resistance of the telephone line on the induction in telephone circuits of exposure No. 2 at Salinas. Also supplements Technical Reports Nos. 8, 15, and 31. (22 pages, 2 drawings.) Dated September 3, 1913.
35	General outline of tests to be made at Santa Cruz on the parallel between lines of the Coast Counties Gas and Electric Company and The Pacific Telephone and Telegraph Company. (4 pages.) Dated July 14, 1913.
36	Induction in telegraph circuits of the Western Union Telegraph Company and the Southern Pacific Company in exposure No. 1 between Salinas and San Jose. (8 pages.) Dated September 3, 1913.
37	Noise tests on telephone circuits radiating from Salinas, with the neutral of the Salinas autotransformers grounded and isolated. (4 pages.) Dated September 3, 1913.
38*	General review of tests at Salinas. Summarizing reports 1, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 36, 37. (52 pages, 1 drawing.) Dated October 13, 1913.
39	General consideration of transpositions and a study of the results to be expected from the application of various transposition schemes to the Santa Cruz-Watsonville exposure. (29 pages, 5 drawings.) Dated December 15, 1913.
40*	Method of measurement of capacitance and conductance unbalances. (1 page, 1 drawing.) Dated December 15, 1913.
41*	Harmonic analysis of alternating current waves by oscillograph and resonant shunt. Comparison of the methods. (27 pages, 3 drawings.) Dated March 16, 1914.
42*	Investigation of the current transformers in use at Santa Cruz, to determine their ratios of transformation and suitability for residual current measurements. (29 pages, 6 drawings.) Dated January 26, 1914.
43	Outline of tests to determine the effect of extraneous current on the intelligibility of telephone conversation. (7 pages, 1 drawing.) Dated March 16, 1914.
44	Induction in the telephone circuits of the Santa Cruz-Watsonville exposure and in the test leads, from sources other than the 22,000-volt line. (12 pages.) Dated March 16, 1914.
45	Induction in the telephone circuits of the Santa Cruz-Watsonville exposure under commercial operating conditions, with the original transpositions in both power and telephone lines. (15 pages.) Dated March 16, 1914.
46	Supplementary to Technical Report No. 39. A study of additional transposition schemes for the Santa Cruz-Watsonville exposure. (12 pages, 2 drawings.) Dated May 4, 1914.
47	Computation of the coefficients of induction for balanced and residual currents and voltages for the Santa Cruz-Watsonville exposures. (13 pages, 2 drawings.) Dated May 4, 1914.
48	Experimental determination of coefficients of induction in the Santa Cruz-Watsonville exposure, with the original transpositions. (42 pages.) Dated May 4, 1914.

Technical report number. Subject or title.

49 Further experimental determination of coefficients of induction for balanced voltages, in the Santa Cruz-Watsonville exposure, with the original transpositions. (13 pages.) Dated May 4, 1914.

50* Study of the influence of various transformer connections and flux densities on the third harmonic and its multiples in a three-phase circuit. (18 pages, 2 drawings.) Dated October 9, 1914.

51* Residual voltage due to the line unbalance of power circuits isolated from ground. Effect of circuit configuration transpositions and frequency. (69 pages, 38 drawings.) Dated September 30, 1916.

--* Memorandum supplementing Technical Report No. 51. (2 pages.) Dated February 28, 1917.

--* Memorandum supplementing Technical Report No. 51. (2 pages, 1 drawing.) Dated July 16, 1917.

52* Residuals produced by a ground on one phase of a normally isolated three-phase system. (29 pages, 2 drawings.) Dated July 28, 1915.

--* Memorandum supplementing Technical Report No. 52. (4 pages.) Dated July 3, 1917.

53 Investigation of current transformers—San Fernando. (7 pages, 1 drawing.) Dated July 29, 1915.

54* San Fernando-Somis parallel. Experimental determination of coefficients of longitudinal induction at 50 cycles. Effect of transpositions in power circuits. (30 pages.) Dated October 1, 1915.

55* Experimental determination of coefficients of induction from residuals at telephonic frequencies—Effect of admittance unbalance. San Fernando-Somis parallel. (26 pages, 6 drawings.) Dated January 7, 1916.

56* Determination of coefficients of induction in a short section of the San Fernando-Somis parallel. Measurements and computations. Effect of telephone transpositions. (41 pages, 6 drawings.) Dated February 10, 1916.

57 Determination of coefficients of transverse induction from residuals at telephonic frequencies in a short section of the San Fernando-Somis parallel. (8 pages, 3 drawings.) Dated February 14, 1916.

58* Investigation of potential transformers for residual voltage measurements. (42 pages, 4 drawings.) Dated April 14, 1916.

59* Relation of triple harmonic residuals in a transmission line to the magnetic density in connected transformer banks. Effect of the line characteristics. (61 pages, 14 drawings.) Dated September 6, 1916.

60* Triple-harmonic residuals as affected by certain types of three-phase star connection of transformers. (24 pages, 4 drawings.) Dated July 26, 1916.

61* Measurements of residual voltages and currents of several transmission systems. (29 pages, 1 drawing.) Dated July 29, 1916.

62* Double-frequency voltages and currents in a three-phase transmission line. (13 pages, 1 drawing.) Dated March 18, 1916.

63* Standard forms for recording data and computations. (22 pages.) Dated March 8, 1916.

64* Computation of induction between parallel power and communication circuits. (46 pages, 1 drawing.) Dated August 4, 1916.

65* Coefficients of induction for communication circuits paralleled by three-phase power circuits. Variation with relative position and configuration. (54 pages, 214 drawings, 92 tables.) Dated January 6, 1917.

66* Co-ordination of transposition systems for power and telephone circuits. (63 pages, 19 drawings.) Dated January 6, 1917.

67* Notes on the transposition of power circuits, and private telephone circuits. (9 pages, 3 drawings.) Dated August 14, 1917.

Technical report number.	Subject or title.
68*	Effect of protective ground wires of power lines on induction in parallel communication circuits. (7 pages, 1 drawing.) Dated June 28, 1917.
69*	Relation of currents in terminal apparatus of telegraph circuits to induced voltages and location of parallel. (8 pages, 2 drawings.) Dated July 19, 1917.
70*	The relative importance of the volt-mile (electric induction) and the volt (magnetic induction) in causing interference with telephone circuits. (10 pages, 1 drawing.) Dated August 17, 1917.
71*	The influence of wave-form on the detrimental effect of induction. (11 pages, 1 drawing.) Dated August 10, 1917.

The following reports, technical in character, were prepared at the request of the Joint Committee, but outside of its organization:

Tests to determine the effect of extraneous current of single frequency on the intelligibility of a telephone conversation. (13 pages, 14 drawings.) Dated December 22, 1914. Laboratory investigation by Engineering Department of American Telephone and Telegraph Company.

Tests to determine the effect of extraneous single frequency current on telegraph transmission. (40 pages, 29 drawings.) Dated September 14, 1916. Joint laboratory investigation by engineers of the Western Union Telegraph Company, the Postal Telegraph-Cable Company and the American Telephone and Telegraph Company.

The development of balance of telephone circuits. Practice of telephone companies in balancing their lines and apparatus as a means of reducing induction from foreign circuits and other telephone circuits. (61 pages, 20 drawings.) Dated June 15, 1915. Submitted by A. H. Griswold on behalf of the telephone interests.

APPENDIX III.

COMMENTS ON THE REPORT OF JULY 7, 1914, BY THE JOINT COMMITTEE ON INDUCTIVE INTERFERENCE TO THE RAILROAD COMMISSION OF THE STATE OF CALIFORNIA.

The investigations conducted since the issuance of the preliminary report in 1914 have greatly extended the detailed knowledge of the various factors affecting inductive interference, and have disclosed a few misstatements in the former report. The new matter is contained in Technical Reports Nos. 51 *ff.*, listed in Appendix II. The purpose of this appendix is to comment on certain statements of the preliminary report and to correct misstatements.

In Appendix I of the former report in the next to the last paragraph (on page 21)*, investigations are mentioned, to be conducted by the American Telephone and Telegraph Company, Western Union Telegraph Company and Postal Telegraph-Cable Company. The reports upon these investigations have been received and are listed in Appendix II of the present report.

In Appendix II of the former report under "3. Means for Preventing or Reducing Residual Voltages and Currents"—fifth paragraph, line 5, *ff* (beginning in the sixth line from the bottom of page 28)* it is stated that "With a horizontal arrangement of conductors, the capacities to ground are more nearly equal than with the triangular or vertical arrangement." As shown in Technical Report No. 51 (see also the exhibit of Part III of this report), the fact is that lines having their conductors in a plane, vertical or horizontal, present the worst unbalances of capacitances to ground experienced with ordinary configurations. Briefly, the triangular configuration is in general the best balanced; vertical and symmetrical horizontal configurations are very nearly alike with much greater unbalances than the triangular, and a little better than the unsymmetrical horizontal, which is the worst.

With respect to the remainder of the same paragraph, investigation has shown that the "electrostatic capacities" *are* "the controlling factors in determining the residual voltage and current of an isolated system under normal operation" with proper insulation; and that "properly spaced transpositions" *are* "substantially effective," thus confirming the statements previously made.

It should be noted that with a grounded neutral (see second paragraph of page 29) a residual current exists corresponding to the residual voltage of the isolated system, and is similarly controllable by transpositions. This matter is discussed in Technical Report No. 51.

In Appendix III of the former report, under "1. Effect of Transpositions in Reducing Induction," fourth paragraph, the words "series impedance" should be added before the word "capacity" in line 10 (the 11th line from bottom of page 32),* so the sentence will read "The voltage induced along the conductors of the telephone circuit and the induced voltage to ground would be present but would not be

*Page references are to edition of report published by the California State Printing Office, 1914.

effective in producing any voltage between the conductors of the telephone circuit, provided the series impedance, capacity and leakage to ground of each side of the telephone circuit were equal.''

In the same Appendix, in the next to the last paragraph of "1. Effect of Transpositions in Reducing Induction" (page 33)* the word "not" was omitted after "can." The first sentence should read "If the communication circuit has a ground return, it can not be transposed and the power circuit transpositions alone (referring to transpositions) will be effective in reducing interference arising from balanced currents and voltages."

With the foregoing changes, the discussion given in the preliminary report is considered correct, though much more complete discussions of various of its topics are to be found in the Technical Reports.

*Page references are to edition of report published by the California State Printing Office, 1914.

APPENDIX IV.

BIBLIOGRAPHY.

In the course of its investigation the Committee has collected numerous references to the published literature of subjects related to its work. A list of such references which deal directly with inductive interference is given below for the benefit of those interested in further study.

Many discussions on related subjects, such as transformer connections and design, switches and switching, protective devices, wave-form of electrical power apparatus, line construction, telephone apparatus, etc., may be found in textbooks, proceedings of societies and periodicals. No attempt has been made to include such references here.

It will, of course, be understood that the Committee in no way expresses opinion respecting statements made in the discussions listed, by referring to them here.

A list of references concerning the effect of transformers on the wave-form of currents and voltages, as influenced by magnetic saturation, is given in Technical Report No. 59; and a list of references on the computation of electric induction in Technical Report No. 64.

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Inductive Disturbances as Affecting Telephone and Telegraph Lines. P. J. Howe. Electrical World, May 31, 1913.
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Inductive Effects of Traveling Waves on Telephone Lines. K. W. Wagner. Elek. Zeit., June 4, 1914.
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Neutralization of Inductive Interference. Journal of Electricity, Power and Gas, November 21, 1914.
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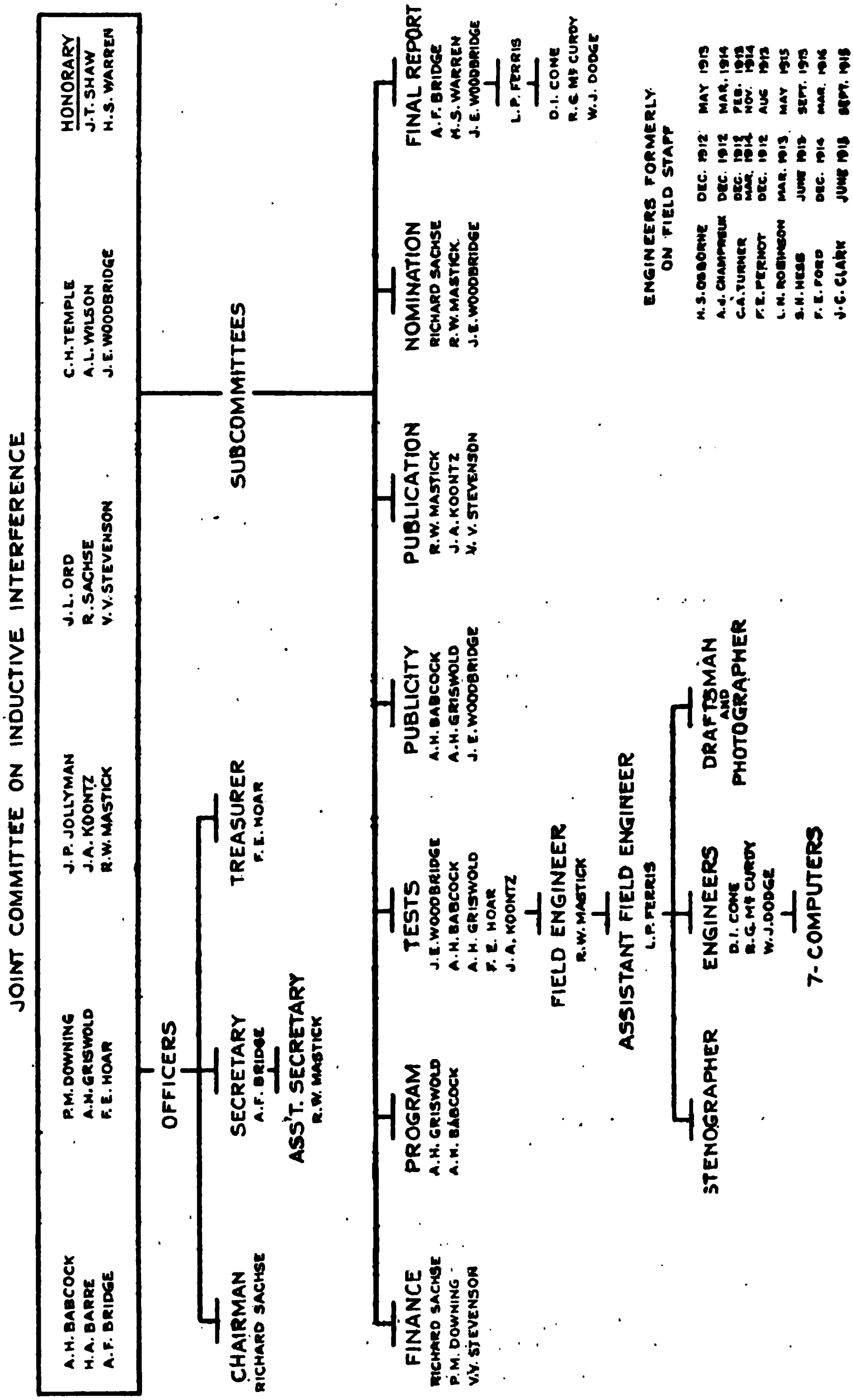
ABBREVIATIONS:

A. I. E. E.—American Institute of Electrical Engineers.
N. E. L. A.—National Electric Light Association.
Elek. Zeit.—Elektrotechnische Zeitschrift.

APPENDIX V
ORGANIZATION CHART



GENERAL ORDER No. 52.

(Supersedes General Order No. 39.)

Railroad Commission of the State of California.

IN THE MATTER OF THE CONSTRUCTION AND OPERATION OF POWER AND COMMUNICATION LINES FOR THE PREVENTION OR MITIGATION OF INDUCTIVE INTERFERENCE.

Approved July 3, 1918. Effective August 1, 1918.

It is hereby ordered that the following rules to govern the construction and operation of power and communication lines, subject to the jurisdiction of this Commission, in so far as that construction or operation applies to the prevention or mitigation of inductive interference, as hereinafter stated, be adopted and effective August 1, 1918.

OUTLINE.

IV. Design, Construction and Arrangement of Apparatus.
- a. Quality and Suitability.
- b. Rotating Machinery.
- c. Transformers and Their Connections.
- d. Rectifiers.
- e. Switches.
- f. Fuses.
- g. Electrolytic Lightning Arresters.
- h. Special Instruments.
- i. Communication Apparatus.

V. Operation and Maintenance.
- a. General Requirements.
- b. Balance.
- c. Record of Neutral Current.
- d. Transformers.
- e. Switching.
- f. Charging Electrolytic Lightning Arresters.
- g. Abnormal Conditions.

VI. Other Cases of Interference.

VII. Rules Subject to Laws and Orders of Commission.

Definitions.

Certain technical terms are employed herein in the senses set forth in the following definitions:

1. *Class H Power Circuit*: The term "Class H Power Circuit" means any overhead open-wire constant-potential alternating-current power transmission or distribution circuit or electrically connected network which has 5,000 volts or more between any two conductors or 2,900 volts or more between any conductor and ground; except railway trolley circuits and feeders electrically connected therewith.

2. *Electrically Connected*: The term "Electrically Connected" means connected by a conducting path or through a condenser, as distinguished from connection merely through magnetic induction.

3. *Signal Circuit*: The term "Signal Circuit" means any telephone, telegraph, messenger call, clock, fire, police alarm or other circuit of similar nature used exclusively for the transmission of signals or intelligence, which operates at less than 400 volts to ground, or 750 volts between any two points of the circuit, provided that if the voltage exceeds 150, the power transmitted shall not exceed 150 watts.

4. *Communication Circuit*: The term "Communication Circuit" means any overhead open-wire signal circuit, except that, if such circuit be a telephone circuit, it is limited to inter-exchange metallic telephone circuits and to metallic telephone circuits operated by a railroad or other company for dispatching purposes, or for public use between separate communities.

5. *Line*: The term "Line" means any circuit or aggregation of circuits carried on poles or towers, and includes the supporting elements.

6. *Parallel*: The term "Parallel" means a condition where a Class H power circuit and a communication circuit follow substantially the same course or are otherwise in proximity for a sufficient distance so that the power circuit is liable to create inductive interference in the communication circuit.

With some parallels interference occurs only at times of abnormal conditions on the power circuit in which case such of these rules as affect induction only under normal operating conditions do not apply. When the application of any rule is thus restricted, the condition under which the rule applies is referred to as a "normal" parallel.

7. *Configuration*: The term "Configuration" means the geometrical arrangement of a circuit or circuits, including the size of the wires, and their relative positions with respect to one another and earth.

8. *Transposition*: The term "Transposition" denotes an interchange of position of the conductors of a circuit between successive lengths thereof.

9. *Discontinuity*: The term "Discontinuity" means any abrupt change in the relative positions of a power and a communication circuit, or any abrupt change in configuration, line impedance or load along either such circuit (including such changes due to connected circuits, transformers, cables, loading coils or other apparatus) which materially affects the magnitude or phase of the induced voltages or currents per unit length or the capacitances of either circuit. Transpositions, however, are not considered to be discontinuities.

10. *Barrel*: The term "Barrel" means an arrangement of a section of power circuit within which each conductor occupies each of the conductor positions for such distances as will result in a maximum degree of balance.

11. *Co-ordination*: The term "Co-ordination" as applied to transposition systems means that the transpositions in power and communication circuits involved in a parallel are efficiently located, with respect to each other and to the discontinuities, for reducing the inductive effects on the communication circuits.

12. *Balanced and Residual Voltages*: The voltages *to ground* of the several wires of a power circuit are divided for convenience into two classes of components, "balanced" and "residual."

The "balanced voltages" are those components which are equal in magnitude and have such phase relations that their algebraic sum is zero at every instant.

The remaining components of the voltages to ground, which exist under conditions other than perfect balance, are termed "residual." They are equivalent to a single-phase voltage impressed between the power wires in multiple and ground. The sum of the residual components is termed the "residual voltage" of the circuit. In case of a three-phase circuit it is three times the equivalent single-phase voltage mentioned above.

Mathematically expressed, the residual voltage is the vector sum of the voltages to ground of the several wires of a power circuit, while the balanced voltages are those components whose vector sum is zero.

13. *Balanced and Residual Currents*: The currents in the several wires of a power circuit are divided for convenience into two classes of components, "balanced" and "residual."

The "balanced currents" are those wholly confined to the wires of the circuit. Hence, their algebraic sum is zero at every instant.

The remaining components of the currents in the several wires, which exist under conditions other than perfect balance, are termed "residual." The sum of the residual components is the "residual current" of the circuit. It is equivalent to a single-phase current in a circuit having the power wires in multiple as one side, and ground as the other.

Mathematically expressed, the residual current is the vector sum of the currents in the several power wires, while the balanced currents are those components whose vector sum is zero.

RULES.

I. General Provisions.

a. Applicability of Rules.

These rules, except as otherwise provided in I (*e*), shall apply and be effective as follows:

(1) Rules limited to lines involved in a parallel, or to apparatus connected to such lines, shall apply only in case of conditions of inductive interference created hereafter; except that rules relating to the operation or maintenance shall apply to all such lines and apparatus, both existing and new.

(2) Rules not limited to lines involved in a parallel, or to apparatus connected to such lines, shall apply to new construction only, including, however, existing lines and apparatus when such are generally reconstructed or renewed.

b. Co-operation.

Any party contemplating new construction which may create a parallel shall confer with the other party or parties concerned and they shall co-operate with a view of avoiding the parallel, or, if this be impracticable, of minimizing the resulting interference. Failure to comply with this requirement will receive consideration by this Commission in any subsequent issue involving such construction.

c. Principle of Least Cost.

When there are two or more different practicable methods of avoiding or mitigating interference, the method which involves the least total cost shall in general be adopted irrespective of whether the necessary changes are made in the plant of the party creating the parallel or in the plant of the other party; provided, however, that preference shall be given to methods of avoiding a parallel over methods of mitigating interference; and provided, further, that as between different methods of mitigation having different degrees of effectiveness, the most effective method, the cost of which can be justified, shall be adopted. In estimating such costs, all factors of expense to both parties shall be taken into account.

d. Existing Parallels.

Parties operating power or communication lines shall exercise due diligence in applying measures, in general accordance with the principles of these rules, for mitigating inductive interference due to existing parallels. Any such parallels which now or hereafter cause excessive interference shall be attended to promptly.

When lines involved in existing parallels are added to, extended or generally reconstructed, or when additional apparatus is connected to such lines, or when apparatus now connected to such lines is renewed or rearranged, the new or changed plant shall thereafter conform to the provisions of these rules.

e. Saving Clause.

Any party desiring to make a departure from these rules regarding the operation or reconstruction of lines now existing or believing that these rules work an injustice or an undue hardship, may file a written petition with the Railroad Commission, whereupon the Commission will take such action as may seem to it proper.

The Commission reserves the right to modify any of the provisions of these rules in any specific case or otherwise, when, in the Commission's opinion, public interest would be the better served by so doing.

f. Information for Commission.

Parties operating or constructing power or communication lines, subject to the jurisdiction of this Commission, involved in or which may become involved in a parallel, shall file with the Railroad Commission, as the Commission may require, information appertaining to measures for the prevention or mitigation of inductive interference agreed upon between said parties.

II. Location of Lines.

a. Avoidance of Parallels.

Every reasonable effort shall be made to avoid creating parallels. If the parties concerned can agree upon a plan for providing an adequate separation of the two classes of lines so as to avoid interference, such plan shall be put into effect. In no case shall a parallel be created unless the cost of avoidance by separation is greater than the cost of the remedial measures required by these rules.

b. Notice of Intention.

The party proposing to build a new Class H power or a communication line which may create a parallel, or generally to reconstruct or change the operating conditions of an existing line involved in a parallel, shall give due notice (at least sixty days where practicable but in any event not less than twenty days in advance of construction, except for minor extensions, for which notice shall be given immediately after the work is authorized) of such intention to the other party

and to the Railroad Commission, including full information as to the location within the parallel and such other features of the proposed line as would affect induction.

c. Distance Between Lines.

Class H power lines and communication lines shall be kept as far apart as practicable. Their separation should be at least equal to the height above ground of the power wires, except when closer proximity is unavoidable.

If, in any case of inductive interference, it should be found impracticable to obtain a proper degree of relief by means of the remedial measures set forth in these rules or by other measures of a remedial nature, the parties concerned shall agree upon and put into effect a plan for increasing the separation of the lines within the parallel.

To promote the effective application of transpositions, both parties shall endeavor to maintain a uniform separation of the two lines throughout each normal parallel. However, in general, when it is feasible to secure more than a 20 per cent increase in separation, for a distance in excess of one mile, this shall be done.

d. Length of Parallels.

Parallels shall be made as short as practicable.

e. Discontinuities.

In the location, construction and general reconstruction of lines within normal parallels every reasonable effort shall be made to avoid discontinuities (except those due to increases in separation as provided for in *c* above) which would interfere with the application of effective and economical co-ordinated transposition systems in the power and communication lines.

In the location and construction of the first line along a public highway, special effort shall be made to avoid crossing the highway and also to avoid other features which would result in unnecessary discontinuities in the event of the construction of another line along the same highway.

III. Design and Construction of Lines.

a. General Requirements.

The quality of material, workmanship, methods and grade of construction shall be in accordance with approved modern practice with special regard to the prevention of failures and the avoidance of

features, such, for example, as inferior insulation, which would tend to cause or promote inductive interference.

b. Arrangement and Spacing of Power Conductors.

In the design for construction or general reconstruction of Class H power lines, consideration shall be given to the configuration of the lines with a view to minimizing (1) throughout the entire length of the line inequalities among the capacitances to earth of the conductors; and (2) within normal parallels the intensity of the inductive effects. When two or more circuits are carried on one line the phase relations among the conductors of the different circuits should be chosen with the same purposes in view.

Excessive spacing of conductors should be avoided.

Two-wire branches electrically connected to a three-phase Class H power circuit should be avoided except those so short that they do not materially unbalance the three-phase circuit. Where such branches are employed they should be so distributed as to cause minimum unbalance.

No single-wire grounded Class H power circuits or branches of multi-wire Class H power circuits shall be employed.

c. Transpositions—General.

All Class H power circuits and metallic communication circuits, or extensions of such circuits, hereafter constructed or generally reconstructed, shall be transposed throughout their entire lengths in such manner as to balance, as nearly as practicable, the capacitances to earth of their conductors. For single-circuit three-phase lines the maximum length of barrel for this purpose shall be twelve miles for circuits of triangular* configuration and six miles for other configurations. For twin-circuit three-phase lines, the maximum length of barrel shall be six miles; except that for circuits of the vertical type (including cases with the middle conductors displaced slightly outward) and the equilateral triangular type with vertices upward, nine-mile barrels may be used when the circuits are interconnected for minimum unbalances.

Exceptions: Power lines, located principally on private rights of way and not electrically connected to the other lines, are exempt from this rule if separated from existing communication lines, and from highways required for the future construction of communication lines, by distances not less than those given below, except for crossings at angles over 30 degrees and other sections of unavoidable closer

*A triangular configuration as here used means one in which the altitude of the triangle exceeds one-half the length of the longest side as base.

proximity not exceeding one mile in total length in each ten consecutive miles of line; provided, however, that such sections of closer proximity to any one such communication line or highway shall not exceed one mile in each thirty consecutive miles of line.

Voltage between power conductors	Minimum separation from highways and communication lines
Below 50,000	600 feet
50,000–75,000	750 feet
75,000–100,000	850 feet
100,000–150,000	1,000 feet
150,000–200,000	1,200 feet

For power lines meeting all these conditions for exemption except that they are electrically connected to other lines through autotransformers, the maximum lengths of barrel may be twice those specified above.

The question of whether highways that may be involved will be required for future communication lines shall be settled by agreement between the power company contemplating construction, the communication companies operating within the territory to be traversed and the Railroad Commission. In the event of disagreement, or if there is no such communication company, the matter shall be referred to this Commission. In cases where the proposed use of a particular highway by a communication company would be the determining factor in deciding whether a given power line must be transposed, such communication company shall make an effort to locate its proposed line elsewhere and the decision shall be made in accordance with the principle of least cost laid down in I (*c*).

Existing Class H power circuits and those exempted under the preceding paragraph, which hereafter become involved in normal parallels, shall be transposed so as to balance their capacitances to earth, when necessary for limiting residual voltages and currents to amounts which can be tolerated. The location and number of transpositions for this purpose shall be determined by agreement of the parties concerned.

In the location and spacing of the transpositions, due regard shall be paid to discontinuities which affect the capacitances of the circuit. Sections of circuit between such points of discontinuity should be treated independently.

In general, transpositions should be omitted at the junction points of successive barrels.

Metallic communication circuits, and single-phase and two-phase Class H power circuits, shall be transposed at intervals not exceeding four miles.

Power circuits less than three miles in length are not required to be transposed outside of parallels, except when the absence of transpositions would materially impair the balance of other circuits to which they are electrically connected.

Power circuits with grounded neutrals having a voltage of less than 12,500 volts between conductors are not required to be transposed outside of parallels, except where the lack of such transpositions in any specific case is the cause of interference.

Within normal parallels the transpositions in the two classes of circuits shall be as provided in (*d*) below. When the transpositions required in a parallel impair the general transposition system of either line outside the limits of the parallel, the necessary readjustment of transpositions shall be made in the sections of line adjacent to the parallel, as a part of the remedial measures therefor.

d. Transpositions—Inside Limits of Parallels.

Within each normal parallel an adequate scheme of transpositions, to neutralize so far as practicable the inductive effects, shall be installed in the power circuits, and also in the communication circuits, provided the latter are metallic. The transposition systems in the two classes of circuits shall be properly co-ordinated. The parties concerned shall co-operate to determine upon the transposition scheme to be employed. The transpositions required in the line last constructed shall be installed before it is placed in service.

In applying the foregoing, the following rules shall, in general, be observed:

(1) For each normal parallel at least one barrel shall be installed in the power circuit. This applies also to a section of parallel where it is not practicable to obtain a balance by combining it with another section. In applying this rule it is not intended ordinarily to change the span lengths required for other purposes.

(2) In long uniform parallels or sections of parallel, involving a telephone line at highway separation from the power line, the barrels shall be three miles in length, subject to such variation as may be necessary for co-ordination with the transpositions required in the telephone circuits. Transpositions should, in general, be omitted at the junction points of successive barrels.

(3) Except as modified by (1) above, the number of transpositions required in power circuits paralleling telephone circuits shall be subject to the following limitations expressed in terms of the average distance between successive transpositions:

(*a*) For power circuits of 50,000 volts or more between conductors, not less than one mile.

(*b*) For power circuits of less than 50,000 volts between conductors, not less than one-sixth mile.*

(4) In case of a parallel between a power line and a telegraph line or other grounded communication line, the transpositions in the power circuit shall be located with due regard to the limits of the parallels and to discontinuities, in order to form as nearly as practicable a balanced system, subject to the condition that the transpositions in the power circuit are not required to be less than one mile apart, except as modified by (1) above. In long uniform sections of parallel, barrels six miles in length should be sufficient. Transpositions should be omitted at the junction points of successive barrels.

*While barrels of approximately three miles, as provided in (2) above, are generally to be employed, the shorter barrels specified in (3) are sometimes necessary in short parallels and in short sections of parallels, in order to co-ordinate with the discontinuities and obtain a proper degree of balance.

(5) The question of the most economical scheme to accomplish the purpose shall always be considered. Effort shall be made to utilize as many as practicable of the existing transpositions.

It is suggested that in case of a short section of a new line, not sufficient of itself to require transpositions, but which is likely to be extended later so that transpositions would then be necessary, consideration be given to the advisability of installing one or more suitably located transpositions in the new section of line while it is being constructed, in order to avoid interrupting the service by adding transpositions afterwards.

Exceptions: Cases of parallelism may occur where the interference is due almost wholly to residual voltages and currents in which event transpositions in the power circuit are not required, except as provided in III (c).

IV. Design, Construction and Arrangement of Apparatus.

a. Quality and Suitability.

In designing, specifying, or otherwise determining the quality or suitability of apparatus to be connected to Class H power or communication circuits, and in arranging such apparatus for use, effort shall be made to avoid, so far as is reasonably practicable, all features which would tend to create or promote inductive interference under either normal or abnormal conditions. As instances in applying the foregoing, the following rules shall be observed.

b. Rotating Machinery.

In order to improve conditions generally, companies operating Class H power circuits shall make every effort to minimize the high frequency components of voltages and currents caused by rotating machinery. All new rotating machinery shall have as nearly as practicable a pure sine wave of voltage and shall not, in any case, deviate therefrom to exceed the limit set forth in the present standardization rules of the American Institute of Electrical Engineers.

No ground connection shall be used on the armature winding of an alternating-current generator or motor electrically connected to a power circuit involved in a normal parallel unless means are employed to avoid unbalancing the circuit and to reduce triple-harmonic residuals as far as may be necessary and practicable.

c. Transformers and Their Connections.

In order that the wave-shape of voltage and current may be distorted as little as practicable by transformers, all new transformers on Class H power circuits should have an exciting current as low as is consistent

with good practice, and which shall not, at rated voltage, exceed 10 per cent of the full load current; except that for transformers without neutral ground connections on the line side, the exciting current at rated voltage need not be less than 0.2 ampere.

Where three-phase transformers are employed with grounded neutrals the core type is preferable to the shell type.

Transformers or transformer banks shall not be grounded at such points of their windings as to unbalance a connected circuit involved in a normal parallel. As important cases under this rule, no grounded single-phase, grounded three-wire two-phase, or grounded open-star three-phase connection shall be so employed.

No star-connected transformers or autotransformers shall be employed with a grounded neutral on the side connected to a three-phase power circuit involved in a normal parallel, unless low-impedance delta-connected secondary or tertiary windings or other equivalent means are used for suppressing the triple harmonic components of the residual voltages and currents introduced by the transformers.

Care shall be taken that the individual units in each grounded-neutral bank of transformers, connected to a circuit involved in a normal parallel, are alike as to type and rating, including all electrical characteristics, and that they are similarly connected, so as not to unbalance the circuit.

Closed-delta connections shall be used wherever practicable in preference to open-delta connections on three-phase power circuits involved in normal parallels. When open-delta connections are employed, an effort shall be made to distribute such connections equally among the three phases.

Where triple-harmonic residual voltages and currents due to star-connected transformer banks exist in amounts which can not be tolerated, and it is inexpedient to isolate the transformer neutrals, such residuals shall be limited by operating the transformers at reduced magnetic density or by other available means.

d. Rectifiers.

Rectifiers and other apparatus tending to distort the alternating-current wave when installed on power lines involved in normal parallels, shall, if necessary, be equipped with suitable auxiliary apparatus to prevent harmful distortion of the wave-form of power-circuit voltage or current.

e. Switches.

Each oil-break switch in a power-circuit involved in a parallel, located between the source or sources of energy and the parallel, and used for energizing or de-energizing the circuit, shall have all poles mechanically interconnected for simultaneous action. There shall be at least one such switch so located as to control the supply of energy to each power circuit involved in a parallel, and, except at stations where an operator is constantly on duty, such switch shall be made automatic for short circuits, grounds, and in case of grounded neutral circuits, for abnormal neutral currents.

Careful consideration shall be given to means of minimizing transient disturbances caused by switching operations on Class H power circuits, which would cause inductive interference. Whenever practicable, provision shall be made for switching on the station-side rather than on the line-side of transformer banks.

Oil-break switches, having their poles mechanically interconnected for simultaneous action, shall be provided wherever the use of air switches or non-interconnected single-pole oil switches would cause harmful transient disturbances in parallel communication circuits.

f. Fuses.

Switches shall be used instead of main-line fuses wherever practicable in a power circuit involved in a parallel.

g. Electrolytic Lightning Arresters.

When electrolytic lightning arresters are employed on a power circuit involved in a parallel they shall be equipped with auxiliary charging resistances and contacts so arranged that the horn gaps are short-circuited at the time of charging, to avoid as far as possible the production of arcs.

h. Special Instruments.

Reliable indicating devices shall be installed at the source of supply of power circuits, involved in parallels, to inform the operators immediately of abnormal conditions, such as grounds, and, wherever possible, open circuits, which have not operated automatic switches.

Whenever a neutral ground connection is employed on a circuit involved in a parallel, an ammeter, suitable for measuring the current in the neutral under normal operating conditions, shall be installed in each neutral connection to ground at the main generating and main attended substations on the power system electrically connected to the circuit involved in the parallel.

i. Communication Apparatus.

All apparatus electrically connected to metallic communication circuits involved in parallels shall be designed and constructed so as to secure as nearly as practicable an accurate balance of the series impedances and the admittances to earth of the two sides of the circuits in order to minimize the detrimental effects of induction from parallel power circuits.

V. Operation and Maintenance.

a. General Requirements.

Power and communication companies shall use all reasonable means to operate and maintain circuits involved in parallels in such a manner as to minimize interference under conditions of normal operation, and to avoid transient disturbances.

b. Balance.

In the maintenance of both power and communication circuits involved in parallels special care shall be given to the prevention of mechanical and electrical failures which would cause or promote transient disturbances or unbalances such as those due to tree-grounds, defective or dirty insulators or other faults.

The voltages and currents of power circuits involved in parallels shall be kept balanced as closely as practicable and accidental unbalances shall be promptly corrected.

c. Record of Neutral Current.

At all points on grounded neutral systems equipped as required in IV (*h*), the power company shall observe and record the approximate daily maximum neutral current.

d. Transformers.

No transformers connected to power circuits involved in normal parallels shall be operated at more than 10 per cent above their rated voltage. Wherever practicable in case of existing equipment and in all cases of new equipment, transformer banks with grounded neutrals on the side which is connected to a power circuit involved in a normal parallel shall not be operated at more than 5 per cent above their rated voltage.

e. Switching.

In all switching operations care shall be taken to avoid so far as possible the production of harmful transient disturbances.

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f. Charging Electrolytic Lightning Arresters.

When, notwithstanding compliance with IV (*g*), interference is caused by charging electrolytic lightning arresters, such charging shall be done at night, so far as is possible, preferably between 2 a.m. and 4 a.m.

g. Abnormal Conditions.

Power companies shall adopt operating rules which shall specifically outline the procedure for their operators during times when a power circuit involved in a parallel is abnormally unbalanced, as will occur with an open, grounded or short-circuited line or transformer winding.

Such rules shall, in general, provide for the discontinuance of operation of the power line until the fault is remedied, excepting only those cases where it is clear that the service rendered the public by continuing operation of this section of power line is of greater importance than the communication service interrupted by such continued operation.

When it is necessary to energize a defective power line in order to locate a fault, care shall be taken to avoid as far as possible repeatedly energizing any section of such line which parallels communication circuits, until the fault has been cleared. Whenever possible, the faulty section of line shall not be energized more than once until disconnected from the section of line involved in the parallel.

To facilitate the study and prevention of disturbances in communication circuits, occasioned by transient conditions of power circuits, accurate record shall be kept of the nature and time of occurrence of failures, changes in operating arrangements and all switching during times of abnormal conditions of Class H power circuits involved in parallels; and of all transient disturbances in communication circuits. These records shall be made available for use in tracing the causes of such transient disturbances.

VI. Other Cases of Inductive Interference.

If any case of inductive interference, not otherwise covered by these rules, shall be experienced or become imminent, such as interference from alternating-current railways operating with ground return, constant-current lighting circuits, power circuits carried in cables, power circuits of lower voltage than Class H power circuits, direct current circuits, or interference in communication circuits carried in cable or in subscribers' metallic telephone circuits, the parties concerned shall endeavor to agree upon a procedure for the prevention or mitigation

of the interference by applying remedial measures. In the event of disagreement between the parties concerned, the matter shall be referred to this Commission.

VII. Rules Subject to Laws and Orders of Commission.

These rules are to apply in all cases where there is no conflict with any law of this state or order of this Commission now or hereafter in effect. In case of conflict, where these rules add to the requirement of any law of this state or order of this Commission, these rules shall prevail; otherwise not.

RAILROAD COMMISSION OF THE STATE
OF CALIFORNIA,
By R. A. PABST, Asst. Secretary.

Preliminary Report by the Joint Committee on Inductive Interference.

OUTLINE.

LETTER OF APPROVAL.

San Francisco, July 30, 1914.

Joint Committee on Inductive Interference:

Gentlemen: We desire to acknowledge receipt of yours of the 7th instant, transmitting the report of the Joint Committee on Inductive Interference, and also of yours of the 23d instant, referring in greater detail to future work of your committee, and to thank you for the same.

The Commission realizes the arduous labor which your committee and the individual members thereof have performed in seeking to ascertain the causes of inductive interference between power and communication circuits, and to prescribe rules and regulations for preventing or minimizing such interference, and extends to the committee, and each member thereof, its congratulations on the results accomplished and its thanks for the scholarly, scientific, painstaking manner in which the work has been performed.

The Commission has adopted the rules as proposed by your committee and has added two new rules, one dealing with the applicability of the rules to existing and future construction and the other declaring the principle that these rules shall be subject to the laws of this State and the orders of this Commission, now or hereafter in effect. The Commission's order will be published as a general order.

Your report will be printed by this Commission for free distribution. While the general conclusions will be given to the press in the usual course, we shall be glad to have the report printed in full in the Proceedings of the American Institute of Electrical Engineers.

The Commission hereby requests your committee to continue its work along the lines indicated in your report and your letter of the 23d instant, and authorizes the raising of the necessary funds by assessment as heretofore, with the understanding that the Commission will assign one of its stenographers to the work of the committee, and that our engineering department will at all times co-operate with your committee.

With sincere appreciation for the work hitherto performed and assurances of continued interest in the work to which you will now devote yourselves,

We remain, respectfully,

RAILROAD COMMISSION,

John M. Eshleman,
H. D. Loveland,
Alex Gordon,
Max Thelen,
E. O. Edgerton,
Commissioners.

LETTER OF TRANSMITTAL.

SAN FRANCISCO, July 7, 1914.

To the Railroad Commission of the State of California,
San Francisco, California:

GENTLEMEN: The Joint Committee on Inductive Interference submits herewith a report based on its work to date, containing provisional rules which tend to improve conditions in respect to inductive interference. The investigation undertaken by the committee has not been completed, but the results already obtained serve to point out a number of requirements and precautionary measures which should be complied with in future work. These have been embodied in the rules presented herewith, and *it is the recommendation of the committee that these rules be made effective immediately without waiting for the completion of the investigation.*

The committee desires to explain, in respect to certain of the rules, that while the general character of their essential provisions is well understood, the information available at present is not sufficiently complete to make it possible to set definite quantitative limits and to make all the rules explicit, such as they should be in order to afford the maximum reduction of inductive interference consistent with the burdens imposed by the rules. In a few instances, rules have been drawn with definite limits which have been set somewhat arbitrarily, in accordance with the committee's best judgment. Therefore, the rules are not put forth as being complete or final, but must be regarded as provisional and subject to such change as the results of further investigation and experience may determine. They are, however, recommended unanimously by the committee as the best which can be formulated at this time, and thus having the support of all the interests represented on the committee, it is hoped that the rules will appeal to the Commission as being reasonable and proper.

The report also outlines other experimental work, some of which is now in progress, which the committee considers essential in order that additional information may be acquired for amplifying and revising these rules to make them more definite and complete.

Respectfully submitted.

(Signed) RICHARD SACHSE,
A. H. BABCOCK,
R. W. GRAY,
F. EMERSON HOAR,
J. E. WOODBRIDGE,
J. P. JOLLYMAN,
P. M. DOWNING,
H. A. BARRE,
C. H. TEMPLE,
A. H. GRISWOLD,
R. W. MASTICK,
V. V. STEVENSON,
J. A. KOONTZ,
A. L. WILSON,
Joint Committee on Inductive Interference.

Honorary Members:
HOWARD S. WARREN.
JAMES T. SHAW.

PRELIMINARY REPORT.*

BY THE

Joint Committee on Inductive Interference to the Railroad Commission of the State of California.

Scope.

This report presents briefly an account of the formation of this committee, its activities and results accomplished to date, and recommendations for such rulings by the Railroad Commission of the State of California as the committee believes are justified at this time; together with a technical discussion in explanation of the results and recommendations.

Historical.

Refers to section of Final Report entitled: "History of Committee's Organization and Work," pages 24 to 30 herein.

Results Accomplished.

The following paragraphs summarize very briefly the principal results accomplished to date. These statements of results are accompanied by brief explanatory comment upon the conclusions reached. The reasons for and explanations of these conclusions are given in more detail in the appendices, to which reference is made.

1. *Interference to telephone circuits under normal operating conditions of power circuits arises almost wholly from the harmonic voltages and currents of the power system.* (See Appendix I, page 117.)

This is due chiefly to the fact that the frequencies of the harmonics generally present in the voltages and currents of power systems cover a considerable portion of the range of the voice frequencies, particularly those frequencies at which telephone instruments and the human ear are of maximum sensibility. Extraneous currents of frequencies approaching the average voice frequency have a more injurious effect upon telephone conversation than currents of lower frequencies.

2. *The effect of induction of the fundamental frequency on telephone circuits is comparatively unimportant unless it is of magnitude sufficient to constitute a physical hazard.* (See Appendix I, page 117.)

This is due to the fact that the fundamental approaches the lower limit of audible frequencies, at which the telephone and the human ear are not efficiently responsive.

3. *Interference to telegraph and other signalling circuits is due principally to the fundamental and lower harmonics.* (See Appendix I, page 117.)

Telegraph receiving instruments are relatively insensitive, as compared with the telephone, to the higher harmonics, but are sensitive to

*See Appendix III of the Final Report, page 79.

disturbances of lower frequencies, such as the fundamental and lower harmonics which more nearly approach the normal operating frequency of such circuits.

4. *The power circuit currents and voltages may be divided into two factors: balanced and residual, of which, for equal magnitude, the latter in general produce the greater inductive interference.* (See Appendix II, page 119.)

Residual currents and voltages act inductively in a similar manner to single-phase currents and voltages acting in a circuit composed of the line conductors in parallel with earth return, which is a condition favorable to very large induction. Moreover, such a circuit which includes the earth as one side can not be transposed. Transpositions in the power circuit can not reduce the inductive effect of residuals except as they reduce the magnitudes of the residuals themselves, which they do in some cases. The inductive interference arising from such currents and voltages can be reduced only in the case of metallic circuits, such as telephone circuits, by transposing these circuits. It is, therefore, important that the telephone circuits be transposed at frequent intervals throughout parallels and carefully balanced throughout their entire length, and that the residual currents and voltages be kept sufficiently small to give negligible induction in telephone circuits so arranged.

5. *Inductive interference to communication circuits, arising from the balanced voltages and currents can in a large measure be prevented by means of an adequate system of transpositions applied to both power and communication circuits (assuming the latter are metallic) and located with due regard to each other.*

This is accomplished partly by creating mutually neutralizing inductive effects in neighboring lengths of each side of the communication circuit or circuits by transposing the power circuit, and partly by equalizing the inductive effects on the two sides of the communication circuit or circuits by exposing each side equally to the influence of the power circuit by transposing the communication circuit.

6. *Abnormal conditions and at times switching operations produce transient disturbances of a very severe character.*

This is due to the fact that abnormal conditions almost invariably give rise to residuals of large magnitude, often including high harmonics. Abnormal occurrences incident to electrical power transmission do not give warning of their occurrence, and since they can not be produced artificially on transmission systems without subjecting the apparatus to great risk or danger, it has been deemed unwise to attempt any experimental tests of these effects. This conclusion is therefore drawn from general experience and data of actual occurrences collected by the committee.

Rules Recommended by the Committee.

The following are the rules which the committee, as the result of its study to date, recommends be issued at this time to govern the future construction and operation of power and communication circuits which

are or are proposed to be so located as to create a parallel, as hereinafter defined:

OUTLINE OF RULES.

DEFINITIONS.

The following definitions are given of certain technical terms employed herein:

a. *Power Circuit.* The term "power circuit" includes any overhead constant potential alternating current power transmission or distribution circuit or electrically connected network which has a voltage of five thousand volts or more between any two conductors, or of three thousand volts or more between any conductor and ground.

b. *Communication Circuit.* The term "communication circuit" includes any overhead, open wire telephone, telegraph, or signalling circuit which is used in the service of the public.

c. *Telephone Circuit.* The term "telephone circuit" includes any inter-exchange metallic telephone circuit, and therefore excludes subscribers' circuits. This term also includes any metallic telephone circuit operated by any railroad or other company for dispatching purposes or for public use between separate communities.

d. *Line.* The term "line" means any circuit or aggregation of circuits carried on poles or towers.

e. *Parallel or Parallelism.* The terms "parallel" or "parallelism" refer to cases where a power line and a communication line follow substantially the same course, or are otherwise in proximity for a sufficient distance, so that the power circuit is liable to create inductive interference in the communication circuits.

f. *Residual Current.* The term "residual current" denotes the vector sum of the currents in the several conductors of a power circuit.

g. *Residual Voltage.* The term "residual voltage" denotes the vector sum of the voltages to ground of the several conductors of a power circuit.

h. *Transposition.* The term "transposition" denotes the interchange of position of the several conductors of a circuit.

I. Avoidance of Parallelism.

Every reasonable effort shall be made to avoid new parallelism. The party proposing to build a new communication or power line, which will create a parallel, or generally to reconstruct an existing line involved in a parallel shall give due notice (at least thirty days wherever possible) of its intention to the other party, including detailed information as to the location and character of the proposed line. If a plan can be devised and agreed upon by the two parties for maintaining an adequate separation between the two classes of lines so as to avoid interference, this shall be done. In case it is impracticable to secure adequate separation between a power line and a communication line, parallelism will be permitted, subject to the conditions set forth in II.

II. Conditions Under Which Parallelism Will be Permitted.

a. *Minimum Horizontal Separation.* The minimum horizontal separation between the power line and communication line shall be equal to the height of the taller line. The only exceptions to this provision are angle crossings and other unavoidable cases of close proximity, and in all such cases the power line shall be kept above the communication line and constructed in conformity with the National Electric Light Association's specifications for overhead crossings or other approved equivalent which may be agreed to by both companies.

b. *Balance of Power System.* The power company shall exercise due diligence to keep the currents in, and the voltages to ground of, the conductors of any power circuit involved in a parallel as closely balanced as practicable. In all cases where telephone circuits are involved, special consideration shall be given to the prevention or elimination of harmonics in the residual current and in the residual voltage.

c. *Limitation of Residual Currents and Voltages.* Pending additional rules on specific means other than those given herein, the parties concerned shall endeavor to agree upon the means to be employed for the prevention or limitation of residual currents and voltages, and in the event of disagreement the matter shall be referred to the Railroad Commission of the State of California.

d. *Transpositions Inside Limits of Parallel.* An adequate system of transpositions shall be installed in the power circuit (or circuits), and

in the communication circuit (or circuits) provided the latter is metallic. When both circuits are transposed the transpositions in both the communication and power circuits shall be located with due regard to each other.

Every reasonable effort shall be made by both parties concerned to fix the limits of the parallel and the location of crossings, branch lines, and connected apparatus so as to facilitate the application of an effective transposition scheme.

In the case of a parallel between a power line and a telephone line the company owning or operating the telephone line involved shall have the right to specify the number, type (in respect to electrical characteristics) and location of the transpositions in the power circuit, subject to the following limitations:

1. For power circuits of 50,000 volts or over the average distance between successive transpositions shall not be required to be less than one mile and the minimum distance between any two successive transpositions shall not be required to be less than two-thirds of a mile.
2. For power circuits of less than 50,000 volts the distance between successive transpositions shall not be required to be less than one-sixth of a mile.

The transposition system of the telephone circuits shall be modified where necessary in order that the power and telephone circuits shall be, as nearly as practicable, mutually non-inductive.

For short parallels less than six miles in length (or short sections of longer parallels which have to be treated independently because of abrupt change in conditions) with power circuits of 50,000 volts or over, where it is impracticable to obtain an adequate balance by the location of transpositions in accordance with the limit specified above, the company owning or operating the telephone line involved shall have the right to specify the number, type and location of transpositions, provided the distance specified between successive transpositions is not less than one-half mile.

When necessary (due to variations in lengths of telephone transposition sections) in order to secure an adequate balance, a reduction of 10 per cent in the limiting distances between successive power circuit transpositions as given above shall be allowed.

In the case of a parallel between a power line and a telegraph line or other grounded communication circuit, the location of the transpositions in the power line shall be with due regard to the limits of the parallel in order to form as nearly as practicable a balanced system. The location and type of such transpositions shall be as specified by the communication company, subject to the condition that the transpositions in the power circuit may not be required to be less than one mile apart.

In no case shall the power company be required to relocate poles or towers for the transpositions.

The parties concerned in any proposed parallel shall endeavor to agree upon a transposition scheme for such parallel in accordance with the above. In the event of a disagreement, the matter shall be referred to the Railroad Commission of the State of California.

e. *Transpositions Outside Limits of Parallel.* In addition to transpositions within the limits of a parallel, as provided in "d" hereof, each new power circuit isolated from ground (or extension of such existing circuit) which is constructed subsequent to the date when these rules become effective, shall be transposed throughout its entire length in such manner as to balance the electrostatic capacities to earth of its several conductors, so as to avoid inequalities among the voltages to earth of the several conductors, which would create inductive interference. Such transpositions shall not be more than eight miles apart, provided, however, that circuits less than three miles in length are not required to be transposed until they are extended to a greater length; except that extensions or spurs from existing lines, the electrostatic capacities to earth of whose conductors are balanced, shall be so constructed as not to change materially the balance of the existing lines to which they are connected.

f. *Uniformity of Parallel.* To facilitate the application of effective transpositions, both parties shall endeavor to maintain uniform separation, uniform arrangement of conductors and uniform relative location of the two classes of circuits within the limits of a parallel. However, when it is feasible to secure a substantial increase of separation between the two lines for a considerable portion of a parallel this shall be done, as such an increase of separation is of more benefit than uniformity.

g. *Transformer Connections.* (1) On any power circuit involved in a parallel, no grounded single-phase, or grounded open-star transformer connections shall be employed.

NOTE.—This does not apply to railroads operating alternating current trolleys with ground return which are covered by V.

(2) On a power circuit involved in a parallel no star-connected transformers or auto-transformers with grounded neutral shall be employed, unless delta-connected secondary or tertiary windings or other equivalent means are used of suppressing the third harmonic components of the residual voltages and currents introduced by the transformers.

(3) Where single-phase loads are connected to a polyphase power circuit involved in a parallel the power company shall endeavor to arrange successive connections of this type so as to equalize the loads upon the several phases.

(4) On a three-phase circuit involved in a parallel, the power company shall use, wherever practicable, a closed-delta connection in preference to an open-delta connection, and where the latter is employed an effort shall be made to distribute such connections equally upon the several phases.

h. *Switch Equipment.* A power circuit involved in a parallel shall be equipped, between the source of supply and the parallel, with oil switches, all poles of which shall be mechanically interconnected for simultaneous action. With the exception of stations where an operator is constantly on duty, these switches shall be rendered automatic for short-circuits, grounds, and abnormal neutral currents.

i. *Switching.* All switching on all parts of a system connected to a circuit involved in a parallel, which causes harmful transient disturbances in communication circuits, shall be done by means of oil switches, all poles of which are mechanically interconnected for simultaneous operation.

j. *Use of Air Switches.* The use of air switches, on a power circuit involved in a parallel, is prohibited except for purposes of isolating sections of dead line, or for disconnecting transformers under no load. This applies to the entire power system, any circuit of which is involved in a parallel, unless such switching is so remote as not to cause harmful transient disturbances in the communication circuits.

k. *Abnormal Conditions.* A power circuit involved in a parallel shall not be operated at any time with an open, grounded or short-circuited line wire or wires or transformer winding.

l. *Devices for Indicating Abnormal Conditions on Systems Isolated from Ground.* If a power circuit involved in a parallel is electrically isolated from ground, reliable indicating devices shall be installed at its source of supply to inform the operator immediately of abnormal conditions, such as grounds and wherever possible, open-circuits, which have not operated automatic switches. Upon indication of trouble by such devices, the operator shall immediately open the oil switches and proceed in the manner outlined in "m."

m. *Procedure Under Abnormal Conditions.* In case of the opening of an oil switch due to an abnormal condition in a power circuit involved in a parallel, or any circuit supplying or supplied by the same, such switch may be closed once; if opened a second time due to the continuance of the fault or abnormal condition, said switch shall not be closed again until the line has been sectionalized. The fault may then be located by energizing sections of line, provided that further sectionalization of the line be done in such sequence as to cause the minimum disturbance to parallel communication circuits, and provided further that where practicable the faulty section of line shall be energized but once in this process of sectionalization, where the fault exists within or beyond the parallel, until such fault is remedied.

n. *Ammeters in Neutral Ground Connections.* Wherever a neutral ground connection is employed on a circuit involved in a parallel an ammeter, suitable for measuring as accurately as practicable the current in the neutral under normal operating conditions, shall be installed in all neutral connections at the main generating and substations on the power system electrically connected to the circuit involved in the parallel. The power company shall maintain a record of hourly measurements of the neutral current at all such points.

o. *Charging Electrolytic Lightning Arresters.* Where a power system is equipped with electrolytic lightning arresters so charged as to cause inductive interference in communication circuits, the method of charging the arresters shall be modified to eliminate the disturbances as far as possible. The charging of such lightning arresters shall be done at such time as to give the minimum liability of interference with communication circuit operation, preferably between the hours of 2 a.m. and 4 a.m.

p. *Wave Form of Rotating Machines.* The power company shall make every effort to obtain generators and synchronous motors for use on all parts of the system, giving, as nearly as reasonably possible, pure sine waves of voltage at fundamental frequency. In no case shall the deviation from a pure sine wave exceed the limit set forth in the Standardization Rules of the American Institute of Electrical Engineers.

q. *Exciting Current of Transformers.* In order that the wave shapes of voltage and current may be distorted as little as practicable

by transformers, the main line transformers employed on circuits involved in a parallel and on future extensions of such circuits shall have an exciting current as low as is consistent with good practice, and in no case shall the exciting current at rated voltage exceed ten per cent of the full load current. Such transformers shall not be operated at more than ten per cent above their rated voltage.

III. Provisions Applying to Existing Parallels.

The following sections of II shall apply also to power circuits involved in existing parallels: b, i, j, k, l, m, o, p, and q. Also, g-3 and g-4 shall apply to existing parallels to the extent that transformers added hereafter shall be connected as provided in said rules.

IV. Waiver of Conditions by Communication Company.

At the option of the company operating the communication circuit or circuits any of the provisions of II and III may be waived, provided that such waiver does not increase the hazard.

V. Parallelism With Alternating Current Railways.

It is recognized that railroads operating alternating current trolleys with ground return create serious inductive interference with parallel communication circuits. In the present state of the art, no means for completely overcoming inductive interference from such parallels is known, hence, they are to be avoided if possible and where unavoidable, the responsibilities arising therefrom must be settled by mutual agreement or in case of inability to agree the matter shall be referred to the Railroad Commission of the State of California.

Discussion of Rules. (Page 106.)

It will be noted from the definitions that the terms "power circuit" and "telephone circuit" are used in these rules in a special, restricted sense.

(I) The first and most obvious means of preventing inductive interference is to avoid the close association of power and communication circuits. Further, it is recognized that in no other way can complete freedom from interference be secured. While, with the ever increasing network of electrical circuits of all kinds, adequate separation to avoid interference is becoming increasingly difficult to maintain, the committee feels that the importance of such separation justifies its being made the first premise in rules designed to prevent inductive interference.

Notice, sufficiently in advance, should be given the other party or parties concerned in any proposed parallel in order that thorough consideration may be given by both parties to possible means of avoiding the parallel or, in case the parallel can not be avoided, to the necessary remedial measures to be employed.

(II-a) The best insurance against physical hazard in case of close proximity is to maintain a separation equal to the height of the taller line, thus avoiding the possibility of physical contact in case of failure. In the case of crossings and unavoidable cases of close proximity for short distances extra strength construction is necessary as a precaution against failure.

(II-b-c) As has been pointed out under the heading "Results Accomplished," and more fully explained in Appendix II, page 119, residual voltages and currents are particularly troublesome factors in causing interference. Means to eliminate or reduce such residuals in power systems are highly important, and while information at this time does not enable the committee to formulate as explicit a rule as is desirable, yet the importance of the subject justifies its inclusion in the rules. The acquisition of further information on which to base a more explicit rule upon this subject is a most important problem, the experimental study of which is discussed in the following section of this report.

(II-d) Transpositions properly located in both power and communication circuits offer the most reliable and effective means for preventing interference from balanced voltages and currents of power circuits. While the inductive effects increase in severity for the higher voltage circuits, due in part to the increased separation of the line conductors, which renders more frequent transpositions desirable, the mechanical difficulties involved are so great as to overbalance the other reasons and the rules, therefore, provide for less frequent transpositions in the higher voltage circuits than in the lower voltage circuits. A further reason for frequent transpositions in the lower voltage circuits is the necessity of a flexible system of transpositions applicable to short parallels which generally occur with such circuits.

(II-e) The provision requiring transpositions outside the limits of a parallel on systems electrically isolated from ground is an explicit measure for carrying out the purpose of the more general provision given under II-b-c, "Balance of Power System" and "Limitation of Residual Voltages and Currents."

(II-f) Non-uniformity of separation and type of construction within the limits of a parallel are inequalities which can not in many cases be taken into account in the design and layout of transposition schemes. Such inequalities tend to nullify the effectiveness of the transpositions, hence it is desirable that they be avoided. A precautionary statement is included in the rule in order that the possibility of securing a wide separation for a considerable portion of a parallel may not be sacrificed for the sake of absolute uniformity throughout the entire length.

(II-g) Some types of transformer connections and methods of operation give rise to large residual voltages and currents and certain provisions of the rules are designed to prohibit or restrict the use of such connections and methods of operation. These rules may be considered as explicit provisions complying with the general provision in II-b-c, "Balance of Power System" and "Limitation of Residual Voltages and Currents." The sufficiency of these specific provisions as an insurance against harmful residual voltages and currents is subject to future determination.

The present information of the committee does not warrant the definite recommendation of any one type of connection or method of operation as best from the standpoint of inductive interference. This is true as to the relative merits of the two general types of systems, the grounded neutral and the isolated system. The advantages and disadvantages of these general types and any modifications of these types are dependent upon their inherent characteristics in respect to residuals and the limitations and control of residuals under both normal

and abnormal conditions. Both types are on an equality with respect to the interference caused by balanced voltages and currents.

(II-k) Continued operation under certain abnormal conditions is possible in some power systems. In particular, it is possible to operate a grounded star-connected system with one phase open, and it is possible to continue the operation of an isolated system when one phase becomes grounded accidentally. The former gives rise to a large residual current and the latter to a large residual voltage, both of which are liable to render parallel communication circuits inoperative. For these reasons the rule prohibits such operation which, aside from the consideration of inductive interference, does not constitute good practice in power system operation.

(II-h-l-n) To provide that operation under the abnormal conditions mentioned above may not continue without the knowledge of the power company, the rules specify that devices for indicating grounds shall be installed on isolated systems. With respect to grounded star-connected systems, the rules specify with certain exceptions the automatic opening of switches by abnormal neutral currents. In such systems ammeters are required in all main neutral ground connections. Such ammeters, read regularly, afford means of detecting abnormal neutral currents and are of value in showing the degree of balance of the system, as the neutral current is easily affected by unbalanced conditions.

(II-m) Accidental causes give rise to occasional abnormal conditions. These can only be guarded against by good construction and maintenance, and careful operation which, however, can not prevent entirely such occurrences. When trouble develops on a power circuit involved in a parallel, it is always liable to cause serious interference to the communication circuits, if the exposure is severe. In the present state of the art, the method of fault location on power circuits is a process of repeated sectionalization and energization of the faulty line until the fault is located within certain limits. This process causes repeated interruptions with loss of time in the operation of the communication circuits, and in the case of telephone circuits is accompanied sometimes by injury to the operators. It should be explained that the loss of time is much greater than the duration of the disturbance, owing to the time required to restore the protective devices on the communication circuits to their normal condition. No method of locating faults on power circuits is known which meets the requirements of practice and yet avoids the disadvantages of the present method. The inductive disturbances due to fault location can be to a considerable degree ameliorated by disconnecting the faulty line from the rest of the system and energizing this line by a single generator at such excitation as may be necessary to overcome the insulation of the fault. Whenever practicable this method is employed by power companies; hence, it has not been thought necessary to cover it by a specific rule.

In view of these facts, the committee is recommending the limitation of the present practice in this regard so as to avoid, as far as seems practicable, the repeated interruptions to communication circuit operation. It is highly desirable that some better method of fault location be developed, not only because of the attendant consequences of the present method on communication circuits, but also because of the abnormal strains to which the power apparatus is necessarily subjected.

(II-h-i-j) Normal switching operations on power circuits produce at times severe transient disturbances in parallel communication circuits. The commonly recognized fact that oil switches produce less severe transient disturbances in power circuits, affords the basis for the provisions in the rules dealing with switches and switching. The automatic features required are designed to prevent continued operation under abnormal conditions.

(II-o) Transient disturbances of severe nature to telephone circuits are sometimes caused by the charging of electrolytic lightning arresters. There are available methods of diminishing the transients due to this cause, and a general provision to the effect that such methods shall be employed when necessary is included in this rule. It is further provided that the charging of arresters should be done at times when the telephone circuits are least used.

(II-p-q) Fundamentally, interference to telephone circuits by power circuits in normal operation is largely due to the existence of harmonics in the currents and voltages. While the complete elimination of these harmonics seems impracticable, still beneficial results may be obtained by practical efforts in this direction, and the committee feels that the two general provisions as to the wave form of rotating machines and the exciting current of transformers are of great importance both from a practical standpoint and also as enunciating a general principle. The matter of generator wave form particularly is of importance for all types of systems. The provision with reference to the exciting current of transformers, while desirable in all cases, is particularly so on grounded star-connected systems.

(III) Certain of the measures in II, particularly those referring to power system operation, which are helpful in mitigating inductive interference have been recommended to apply to existing parallels.

(IV) Since these rules are designed for the protection of communication circuits, it is proper that the companies operating such circuits be given the right to waive any measures of protection which they may in any particular case consider unnecessary.

(V) The committee has undertaken no investigation of cases of parallelism with alternating current railways, but as the seriousness of this class of exposure is recognized, it was thought desirable that it be referred to specifically.

Future Work.

The further work necessary in order to secure the information essential as a basis of determining more explicit and effective rules than those herein recommended, is particularly concerned with the subjects of transpositions and residual voltages and currents. In order to cover these subjects in as effective and economical a manner as possible it is thought that the procedure should be along the following lines:

1. Experimental study of transpositions which includes the determination of:

(*a*) The practical effectiveness of transpositions in both power and communication circuits as a means of reducing induction arising from balanced voltages and currents; involving considerations

of different co-ordinated transposition schemes, particularly with different lengths of power circuit barrels.

(*b*) The practical effectiveness of transpositions in communcation circuits as a means of reducing inductive interference arising from residual voltages and currents; involving considerations of different systems, particularly different lengths of balanced communication circuit transposition sections.

(*c*) The influence of imperfect electrical balance of communication circuits in impairing the effectiveness of transpositions.

(*d*) The practical effectiveness of transpositions in a power circuit isolated from ground as a means of balancing the electrostatic capacities to earth of the several conductors, and thereby reducing residual voltages and currents; involving considerations of the relative efficiency of different lengths of power circuit barrels.

2. Experimental study of the causes and effects of residuals, including:

(*a*) A comparison of the different types of power system connection and apparatus in common use and their characteristics in respect to the production of residuals, particularly harmonic residuals.

(*b*) Means to be employed in limiting residual voltages and currents.

(*c*) A determination of the minimum values of residual voltages and currents which will produce harmful inductive interference.

It is thought that these two studies could progress simultaneously. The work indicated under (1) could best be done on an actual parallel selected to be as uniform and as free from secondary disturbances as possible. Some preliminary work has been done along these lines which indicates the best methods of procedure, and this should facilitate the carrying out of the investigation.

The study mentioned under (2) consists in part of an investigation of the characteristics and magnitudes of residual voltages and currents in typical power systems, both those with grounded neutrals and systems entirely isolated from ground. A part of the study of residuals is logically related to the study of transpositions and could be carried out in connection with the study outlined under (1) and at the same time and place.

In addition to the above the committee has already arranged for the investigation of the two following subjects:

1. A determination of the detrimental effect of extraneous currents on a telephone circuit as a function of the frequency including a determination of the maximum amount of extraneous current, of different frequencies and combinations of frequencies, which is allowable in a commercial telephone circuit.

2. A determination of the effects of extraneous current of different amounts and characteristics, in limiting the speed of telegraph operation.

This work is now in progress.

APPENDIX I.

HARMONICS.

Any complex electrical wave of periodic structure may be resolved into component sine waves of suitable amplitudes and phase differences, having frequencies which are in integral relation to the fundamental frequency. The simple sine wave of lowest frequency is termed the fundamental, and those of higher frequency are termed harmonics of the fundamental wave. The fundamental may be considered the first harmonic. The analysis of a periodic wave into its constituent sine waves or harmonics is not merely a mathematical conception or process but is in accordance with the facts of electricity and acoustics.

In general, alternating current systems, by virtue of their inherent characteristics, do not permit the existence of harmonics other than odd integral multiples of the fundamental frequency, *i. e.*, 3d, 5th, 7th, 9th, 11th, etc., harmonics. Such harmonics may exist in either or both the current and voltage waves of a power system.

Commercial frequencies of power transmission in California are 25, 50 and 60 cycles per second. The power systems, so far investigated, operate at a fundamental frequency of 60 cycles per second. The investigation has shown harmonic currents and voltages of appreciable magnitude up to the 35th harmonic. On one system the 23d (corresponding to a frequency of 1,380 cycles per second) has been found to be prominent. Induced currents and voltages in parallel communication circuits have been observed corresponding to these harmonics.

The detrimental effect of the induced voltages and currents in parallel communication circuits depends, in general, upon their magnitude and upon the frequency of the induction as compared with the operating frequency of the communication circuit. The presence of extraneous current of a frequency approaching that of normal operating frequency of the communication circuit has a more injurious effect than the same amount of current of a frequency far removed from the operating frequency of the circuit.

The frequency of the voice currents flowing in a telephone circuit ranges from about 200 cycles per second up to possibly 2000 cycles per second. The average voice frequency is considered to be approximately 800 cycles per second, and at about this frequency the telephone receiver is most sensitive. It is on account of these considerations that extraneous currents of the higher frequencies, arising from the harmonics of a power system, are relatively more detrimental to telephone service. The harmonics of the power systems have been found to be responsible for the greater portion of the inductive interference to telephone service, under normal operating conditions of parallel power circuits. Any extraneous current of a frequency within the audible range produces a disturbance which impairs the efficiency of a telephone circuit. The combined effects of all extraneous currents present, of frequencies within the range of audition, constitute the humming "noise" heard in the receiver of a telephone circuit which is subject to induction.

The effect of currents of the fundamental frequency (60 cycles or less) on telephone circuits is relatively unimportant as compared to that of higher harmonics, owing to the fact that the fundamental approaches

the lower limit of audible frequencies. However, if the induction due to the fundamental becomes sufficiently great, constituting a physical hazard, or of such magnitude as to operate the protective devices on the telephone circuits or interfere with superimposed telegraph service or other grounded signalling devices, it is then of great importance from the standpoint of interference.

In regard to the effect of extraneous currents on the operation of telegraph circuits, for reasons analogous to those given above, such circuits are relatively more affected by extraneous currents of fundamental frequency or of the frequencies corresponding to the lower harmonics such as the 3d and 5th.

At the present time the American Telephone and Telegraph Company is undertaking, on behalf of the Joint Committee on Inductive Interference, an extensive series of tests in regard to the detrimental effect of extraneous currents of various frequencies on the intelligibility of telephone conversation. In addition, this company, in conjunction with the Western Union Telegraph Company and the Postal Telegraph Cable Company, is undertaking an investigation of the effect of extraneous currents on the operation of telegraph circuits and apparatus of different types.

Harmonic currents and voltages in power circuits arise from many causes. Generators or other rotating machines do not, in general, produce pure sine waves of fundamental frequency. This is due to several features in the design of the apparatus. A certain amount of distortion of wave form, with the consequent introduction of disturbing harmonics, is inherent with the use of transformers. This distortion of wave form is due to hysteretic action in the iron core of the transformer. The distortion varies in character and magnitude with the saturation and characteristics of the iron employed. Certain connections of transformers are possible which will suppress the third harmonic and its multiples in a three-phase power system. The fact that practically all inductive interference to telephone circuits is due to the harmonic currents and voltages, renders it important that an effort be made to obtain rotating machinery for use in power systems which produces as nearly as is reasonably possible pure sine waves of fundamental frequency, and also that an effort be made to obtain transformers and to arrange connections of the same in such a manner as to reduce as far as practicable the distortion of wave form.

APPENDIX II.

BALANCED AND RESIDUAL VOLTAGES AND CURRENTS.

This appendix comprises the four following sections:

1. Analysis of Voltages and Currents and Discussion of the Effects of Their Components.
2. Causes of Residual Voltages and Currents.
3. Means for Preventing or Reducing Residual Voltages and Currents.
4. Discussion of Tests.

1. Analysis of Voltages and Currents and Discussion of the Effects of Their Components.

To facilitate the analysis of inductive effects in parallel communication circuits, arising from a power circuit, the voltages and currents of the power circuit can be conveniently regarded as consisting of components which exhibit distinct characteristics and which may be treated separately.

Considering a three-phase circuit having equal voltages between any two conductors, the voltages to ground from the conductors can be resolved into two sets of components, balanced components and residual components. Since the voltages between any two conductors are equal, the voltages between the conductors may be graphically represented by three vectors forming an equilateral triangle. The potential of the ground may be represented by a point which may be inside or outside of the triangle depending on the magnitude and character of the residual voltage, and the actual voltages to ground from the conductors may be represented by three vectors drawn between the point representing the ground potential and the corners of the triangle. The balanced components of the voltages to ground from the conductors consist of three equal voltages whose vector sum is zero and which are therefore displaced one-third cycle in time phase with respect to one another. These balanced components may be represented by three vectors drawn from the center of the equilateral triangle to the corners. The residual components of the voltages to ground from the conductors consist of three equal voltages which are in phase with one another and which may be represented by three identical vectors drawn from the point representing the ground potential to the center of the equilateral triangle. If the residual voltage is zero the point representing the ground potential will be at the center of the triangle. The residual voltage of the system is defined as the vector sum of the voltages of the three conductors to ground. It is, therefore, by definition, three times the residual voltage of the individual conductors, or three times the equivalent single-phase voltage of the three conductors in parallel with respect to the earth. It should be noted that the inductive effect of the residual voltage is equal to that of a single-phase voltage between ground and the three conductors in parallel, equal to the residual voltage of the individual conductors, or to one-third the residual voltage of the system.

If one conductor is grounded the residual components (assuming the voltages between wires remain unchanged) will each equal the voltage between conductors divided by the square root of three, and the residual voltage of the system will be equal to the voltage between conductors multiplied by the square root of three.

If a power circuit consists of a single conductor with ground return, the residual voltage will be equal to the voltage from the conductor to ground.

The currents flowing in the three wires of a three-phase, three-wire circuit can be considered to be composed of three sets of currents; namely, (1) balanced components consisting of equal currents in each of the three line wires whose vector sum is zero, and which are, therefore, displaced one-third cycle in time-phase with respect to one another; (2) a single-phase current flowing in a loop composed of two of the line wires; (3) a residual current divided equally between the three line wires and returning through the earth. The residual current of the three-phase circuit is defined as the vector sum of the three line currents. It is, therefore, the equivalent of a single-phase current flowing through the three line conductors in parallel, with the earth completing the circuit.

In the case of a power circuit consisting of a single conductor with a ground return the entire current flowing in the conductor is residual.

In the above discussion, reference is made to three-phase, three-wire power circuits, but the analysis there given may be generalized so as to apply to a power system of any number of phases. Most electrical power transmission systems are of the three-phase, three-wire type and subsequent statements will apply particularly to such systems, unless otherwise stated.

At a point in the vicinity of a power circuit, such as might represent the location of an element of a communication circuit conductor, the resultant electromagnetic field due to the balanced currents would be zero if the power circuit conductors were equidistant from the point (disregarding the effect of the earth). In general, the power circuit conductors are not exactly equidistant from such point, and therefore the resultant electromagnetic field due to balanced currents is not zero. For this reason, the balanced currents in the power circuit have unequal effects on the communication circuit, hence there is a resultant induction. For residuals, there is, in general, a much greater inequality in the distances between the affected conductors (or circuits) and the sides of the residual circuit (power conductors in parallel one side, earth other side) than in the distances to the several power conductors, which constitute the circuit for the balanced components. Thus the resultant electromagnetic field due to residual currents is large in comparison with the field set up by balanced currents of the same magnitude. It may be noted that the electromagnetic forces at any point due to residual currents in the different power conductors are in the same time-phase, hence the inductive effects of all the residual components are cumulative and not differential as in the case of the balanced components.

In a similar way it may be shown that residual voltages produce proportionately far greater inductive effects than balanced voltages.

Computations based on the physical characteristics of two of the parallels investigated show that, for an exposure near Salinas for eight miles with a 55,000-volt line on the opposite side of the county road from a communication line, one ampere of residual current produces as much induction in a ground return communication circuit as would forty amperes of balanced current; and one volt residual produces as

much induction as one hundred and ten volts balanced. Similar computations based on the physical characteristics of an exposure between Santa Cruz and Watsonville, where the communication circuits are paralleled for seventeen miles by a 22,000-volt line on the opposite side of the county road, show that one ampere residual produces as much induction in a ground return communication circuit as would two hundred and forty amperes of balanced current; and one volt residual produces as much induction as ten volts balanced. All of the above comparative values are for currents and voltages of sixty cycles frequency.

The above values illustrate the relative induction-producing powers of balanced and residual currents and voltages in two specific cases. Such values will vary considerably for different parallels, but these cited may be taken in a general way, as indicative of the relative severity of the effects on a single conductor produced by these two factors. Such values for a unit length of non-transposed circuit in any given parallel, are dependent upon the separation, height, and configuration of the conductors of the two classes of circuits, and upon the character and condition of the ground and neighboring objects. For the entire parallel, or total length of exposure, these values are further dependent upon transpositions. The actual amount of induction arising from each of the two components depends also upon the actual magnitudes and the frequencies of the components in the power circuit.

It will be shown in Appendix III, page 130, that inductive interference arising from balanced currents and voltages can be reduced by proper transpositions in the power circuit, but that power circuit transpositions do not reduce the inductive interference produced in a parallel communication circuit by residuals. Residual currents and voltages act inductively to produce the same effects as a single-phase grounded circuit operating with the three line conductors in parallel. This generally represents the worst possible condition from the standpoint of inductive interference. Transposing the conductors of the power circuit can not reduce the inductive interference arising from residuals, except in so far as the magnitude of the residual voltages and currents is reduced by such transpositions. The effect of power circuit transpositions on the magnitude of these components is discussed below.

In the detailed discussion of transpositions in Appendix III, page 130, it is shown that transpositions in a communication circuit can reduce the induced voltages from residuals only as between the two sides of a metallic circuit.

In view of the above it is evident that attention must be given to the problem of restricting residuals to amounts which do not cause material interference either to grounded communication circuits or to properly transposed and balanced metallic circuits.

2. Causes of Residual Voltages and Currents.

While a degree of balance of the voltages and currents of the power system may be obtained which satisfies all the practical demands of power operation, this may not be sufficient to prevent the production of residuals sufficient to cause serious inductive interference to parallel communication circuits.

Residual currents and voltages may arise from one or more causes which act singly or together. The principal sources of residual currents and voltages are,

1. Unbalanced loads between the three phases and the neutral of a grounded star-connected system.
2. The introduction of the third harmonic and its odd multiples as residual current and voltage due to certain apparatus and connections employed on a grounded star-connected system.
3. Unbalanced capacity and leakage between the several phases of the system and ground. This applies more particularly to systems isolated from ground.

There are two principal types of commercial three-phase power circuits used in California.

1. The grounded neutral circuit or network, in which all important generating points have a grounded neutral and in which all or part of the receiving points may be connected with a grounded neutral. No resistances are inserted between the neutrals and ground.
2. The isolated circuit or network, which normally has no metallic connection to ground at any point.

The characteristics of the grounded neutral system with particular reference to residuals are as follows:

Under Normal Conditions.

(*a*) The impedances between line conductors and ground are determined very largely by the load impedances of the transformers. With balanced loads the residual voltage other than the third harmonic and its odd multiples may be eliminated.

(*b*) The effect of unbalanced loads on the residual voltage is small, as the tendency of generators and transformers is to maintain equal voltages between the several conductors and ground.

(*c*) With balanced loads the residual current, other than the third harmonic and its odd multiples, may be eliminated.

(*d*) Unbalanced loads between line and neutral cause corresponding residual currents, which will be large if the unbalance is large, as such unbalanced load currents flow through the neutral to earth.

(*e*) The varying permeability of the iron in star-connected transformers with grounded neutrals introduces the third harmonic and its odd multiples as residual voltages and currents. The use of delta-connected secondary windings reduces this effect greatly below that of star to star-connections.

(*f*) Grounded star-connected generators connected directly to the line or through grounded star to star-connected banks of transformers, may introduce the third harmonic and its odd multiples as residual voltages and currents.

Under Abnormal Conditions.

(*g*) A ground on one phase short circuits that phase through the neutral connection and causes a residual current throughout the whole length of the circuit, this current being practically equal to the short-circuit current to ground on that portion of the circuit between the

sources of power supplying the fault and the point where the circuit is grounded. A large residual voltage (approaching as maximum 58 per cent of the voltage between phases) will be created in proximity to the fault and, if the low tension side of the receiving transformers is star-connected, throughout that portion of the circuit between the fault and such receiving transformers. If the neutral of the receiving transformers is isolated, the short-circuit current will exist only between the source of supply and the fault and there will be no residual current between the fault and such receiving transformers. The above mentioned residual voltage will in this case exist not only in proximity to the fault on the supply side but also throughout the length of circuit from the fault to the receiving transformers. The power circuit is rendered inoperative.

(*h*) An open condition of one phase causes a large residual current, as the unbalanced load currents of the other two phases must flow through the neutral to earth. A large residual voltage will exist beyond the fault if the low tension side of the receiving transformers is star-connected. The power circuit may not be rendered inoperative for three-phase supply beyond the fault, in case the receiving transformers are grounded star-delta connected.

The characteristics of the isolated system with particular reference to residuals are as follows:

Under Normal Conditions.

(*a*) The impedances between line conductors and ground are determined by the electrostatic capacities and the leakage between the several conductors and ground. With balanced loads a residual voltage may exist, due to unbalanced capacity and leakage. Such residual voltage as is due to unbalanced capacity may be eliminated by transposing the circuit so as to equalize the electrostatic capacities to ground of the several phases. If there are single-phase branches making the total lengths of the three conductors unequal, this will introduce inequalities among the capacities to ground which it may not be possible to balance by transpositions. Inequalities in capacity or leakage result in unequal voltages between the different line conductors and ground.

(*b*) The effect of unbalanced loads on the residual voltage is very slight.

(*c*) With balanced loads a small residual current consisting of unbalanced charging current may flow due to non-uniform distribution of unbalanced capacity and leakage.

(*d*) Unbalanced loads have but a slight effect upon the residual current.

(*e*) The transformers can not introduce the third harmonic and its odd multiples as residual voltages or currents.

NOTE.—Due to unsymmetrical three-phase connections sometimes employed (such as open-delta and Scott connections) the third harmonic and its odd multiples may appear in the voltages between lines and in the line currents, creating dissimilarities in the wave forms for the several phases. These harmonic components of the line voltages and currents are affected by unbalanced capacity and leakage in the same way as any other components as may appear in the residuals. It should be noted, however, that such harmonics are not impressed directly upon the line as residuals, as in the case with grounded neutral systems.

(*f*) The generators can not introduce the third harmonic and its odd multiples as residual voltages and currents.

NOTE.—If a two-phase generator containing a third harmonic in its voltage wave supplies the line through Scott or other two to three-phase transformer connections the third harmonic will appear in the voltage between lines. Subject to the conditions of the circuit as regards capacity and leakage balance, this harmonic along with all others may or may not appear in the residuals.

UNDER ABNORMAL CONDITIONS.

(*g*) A ground on one phase causes a large residual voltage (173 per cent of the voltage between phases) throughout the entire length of the circuit. A residual current will be created in proximity to the fault, its magnitude increasing with the extent, voltage and frequency of the system. The power circuit may not be rendered inoperative and the power company operators may be unaware of the existence of the abnormal condition. In some cases the residual voltage and currents are greatly augmented by the resonant effects accompanying arcing grounds.

(*h*) An open condition of one phase may cause a large residual voltage, a certain amount of residual current will flow, due to the interchange of unbalanced charging current, between sections of line on either side of the fault. The power circuit is rendered inoperative for three-phase supply beyond the fault.

A consideration of the characteristics of the two types of systems indicates that under normal operating conditions with balanced loads upon all phases, the residuals of the grounded neutral system may be limited to the third harmonic and its odd multiples. The magnitude of these harmonics is dependent largely on the type of connection on the low tension side of the transformer banks, the delta being preferable to star-connection. Under the same condition the residuals of the isolated system may be limited to those resulting from unbalanced leakages to ground, which should be small on a well maintained system. The effect of an unbalance in the loads connected between conductors upon the residuals of either type of system is small, while the effect of an unbalance in the loads connected between conductors and ground upon a grounded neutral system is to cause a residual current which is proportional to the amount of such unbalance which will be large if the unbalance is severe. The residual current, due to this cause, consists of the fundamental and all harmonics present in the line currents, in addition to which the third and its odd multiples are introduced as before by the varying permeability of the transformer iron, and in some cases by the generators.

Under abnormal conditions both types of systems give rise to residuals which are liable to cause interruption and damage to parallel communication circuits. The most frequent abnormal condition which produces severe interference is an accidental ground. A ground on one phase of a grounded star-connected system creates a severe and widespread electromagnetic unbalance, giving rise to corresponding inductive effects. This is accompanied by an electrostatic unbalance in the vicinity of such ground. On the lower voltage systems this latter effect is relatively of little importance. On the other hand, a ground on one phase of an isolated system creates a severe and widespread electrostatic unbalance, giving rise to corresponding inductive effects. This is

accompanied by an electromagnetic unbalance in the vicinity of the ground. On small low-voltage isolated systems, such electromagnetic unbalance is relatively of little consequence, but it should be noted that with increased voltage and extent of the system such effects do become of great importance, giving rise to electromagnetic disturbances in exposed communication circuits in addition to the electrostatic disturbances.

The magnitude of the inductive effects from either type of system is dependent upon the character of the exposure, extent of the power circuit and other factors which render it impossible with the information at hand to draw a definite conclusion as to the relative total amounts of interference inherent with the two types of system. Furthermore, it is not necessarily true that either type of connection has an advantage from the inductive interference standpoint for power systems of all sizes and voltages.

3. Means for Preventing or Reducing Residual Voltages and Currents.

To miniimize or prevent residual voltages and currents due to cause 1, it is necessary to equalize as closely as practicable at all points the load between the several phases of the circuit and the neutral, or to remove the ground path for unbalanced load currents, thus allowing a grounded neutral at one end of the circuit only. As it is difficult, if not impossible, to maintain all loads in a state of equilibrium at all times, the latter method has the advantage of greater reliability.

Single-phase connections to ground should not be employed. Where single-phase loads or unbalanced three-phase loads must be supplied, the transformers supplying such loads may be connected across the line wires, or may be connected star to delta, with the neutral not grounded. It should be noted that single phase or unbalanced three-phase loads on the low tension or delta side of grounded star to delta-connected transformers produce effects on the high tension side similar qualitatively to single-phase loads between line and ground, but these effects are greatly reduced in magnitude by the inherent balancing influence of transformers so connected, due to the fact that all three transformers participate in supplying such a single-phase load.

Residuals which arise from cause 2 may be greatly reduced by means of certain types of connections for generators and transformers. Thus, for example, connecting the secondary windings of the transformer banks in delta largely suppresses these components of the residual voltage and current but does not entirely prevent them. Where the transformers are connected grounded star to star, these components can be, to a certain extent, kept out of the line by the use of a second bank of transformers having a delta connection on one side and a star connection on the side in common with the first bank with the neutrals interconnected.

The possibility of the introduction of third harmonic residuals on the line due to the use of grounded star-connected generators may be avoided by the employment of transformers between generators and line, the windings on the generator side of the transformers being isolated from ground.

To eliminate or reduce residual currents and voltages which may be due to cause 3, it is necessary to transpose the conductors of the power circuit so as to equalize the electrostatic capacities of the several phases to ground, and this equalization must be attained within distances sufficiently short to prevent the accumulation of large unbalances. With a horizontal arrangement of conductors, the capacities to ground are more nearly equal than with the triangular or vertical arrangement.* It is probable that the electrostatic capacities are the controlling factors in determining the residual voltage and current of an isolated system under normal operation, and while an investigation of the extent to which such residuals may be reduced by properly spaced transpositions has not as yet been made, it is reasonable to suppose that transpositions will be substantially effective. The effect of unbalanced leakage can not be controlled, except through proper construction and maintenance of the power system. It is to be noted that the maintenance of the system free from accidental grounds and partial grounds becomes increasingly difficult the larger the extent of the power network.

On a grounded star connected system, the electrostatic capacity and the leakage of the several phases to ground are relatively less effective in producing residual voltage, as on such systems the voltages to ground are determined almost entirely by the generators and transformers.

4. Discussion of Tests.

Having given a general analysis of the causes and effects of and means to reduce residual currents and voltages, it is desirable to call attention to the results of tests which have been conducted, which have a bearing on this subject.

At Salinas the effect of grounding or isolating the neutral of the auto-transformers, which have also a secondary delta winding, was investigated. These auto-transformers are supplied at 55,000 volts over a transmission line which parallels the circuits of The Pacific Telephone and Telegraph Company in what have been termed exposures No. 1 and No. 2. These auto-transformers in turn supply a 33,000-volt line of the Coast Valleys Gas and Electric Company, extending from Salinas to King City, a distance of approximately 45 miles, and paralleling throughout practically this entire length, the coast route toll lead of The Pacific Telephone and Telegraph Company. These same telephone circuits are involved in the parallels with the 55,000-volt line north of Salinas. In addition to supplying the King City line, this bank of auto-transformers at Salinas supplies a 22,000-volt line extending to Monterey, a distance of approximately 18 miles. Aside from the ground on the transformer neutral at Salinas, there are no grounds on either the 33,000-volt line or the 22,000-volt line. The 55,000-volt line supplying the Salinas transformers is energized at the Guadalupe substation of the Sierra and San Francisco Power Company, approximately 73 miles distant from Salinas through grounded star-connected auto-transformers, which have delta-connected secondary windings, and which are supplied by the 104,000-volt line of this same system which operates with grounded neutral connections at its main generating station and substations. It will be understood from this statement of conditions that the neutral current at Salinas is not identical with the residual current

*See T. R. No. 51, page 266.

of any one of the three high-tension lines which are connected together by these auto-transformers. The condition of the Salinas neutral affects the induction arising from the several exposures through its effect on the residual currents and voltages of the high tension lines connected to the auto-transformers at that point. A representative value of the neutral current at Salinas during these tests is 0.3 ampere. It is composed almost entirely of the ninth harmonic, the fundamental and the third harmonic, their magnitudes decreasing in the order named. With the power system in normal operation, isolating the neutral of the auto-transformers at Salinas did not greatly affect the resultant induction in the particular exposures under observation. The values in the following table, taken from the data of the tests, indicate the effect of the condition of this neutral on the residual currents of the 55,000-volt and the 33,000-volt lines.

Residual Current at Salinas—Amperes.

Order of harmonic	55,000-volt line		33,000-volt line	
	Neutral at Salinas		Neutral at Salinas	
	Grounded	Non-grounded	Grounded	Non-grounded
1	0.120	0.057	0.061	0.073
3	0.054	0.160	0.075	0.120
9	0.078	0.100	0.120	0.075

Two reasons may be given for the fact that the condition of the Salinas neutral does not greatly affect the resultant residual current of these lines: (1) The load balance on these lines is such that a relatively small amount of load current flows through this neutral; (2) As three high tension lines are connected together by these auto-transformers, opening their neutral connection to ground does not completely eliminate the path for the residual current of any one of the three lines, since it may then flow to earth through the admittance to ground of the other two lines.

These particular conditions are not commonly found but a similar condition, in that there is a path to ground for residual current aside from the neutral connection, prevails in any case where the power circuit extends for a considerable distance beyond such neutral connection. The investigation showed, for the conditions which applied to the 55,000-volt line, that removing the neutral ground connection beyond the parallel decreased the fundamental and increased the third and ninth harmonics in the residual current, as shown in the above table. It is not to be concluded, however, from this one case that the third harmonic and its odd multiples in the residual current would in all cases be increased by removing the neutral ground connection of a bank of receiving transformers where the circuit extends beyond the point of measurement of such residual current. If the circuit is terminated at the transformer bank, the removal of the neutral ground connection must eliminate the residual current at that point.

In the case of the 33,000-volt line, the grounding of the neutral at Salinas merely gave another and nearer grounded neutral point on the line supplying power, but did not give a grounded neutral point in

each direction from the point of measurement of the residuals, as it did in the case of the 55,000-volt line. As the 33,000-volt line has no ground connection beyond Salinas, the residual current must flow to ground entirely through the admittance of this line to ground. The residual current, therefore, diminishes to zero at the King City end of the line. Isolating the neutral of the Salinas transformers affects the constituents of the residual currents in this line arising from the Salinas transformers and those impressed by the 55,000-volt line, in such a way that they combine vectorially to give a different resultant from that with the Salinas neutral grounded. The result is to increase the fundamental and third harmonic and to decrease the ninth harmonic when the neutral is isolated. The residual current in the 22,000-volt line was not determined, but residual voltage measurements were made with the Salinas neutral isolated and grounded and the results are included in the following table, from which it may be noted that the fundamental, third and ninth harmonics were all greater with the Salinas neutral isolated.

The banks of star-connected auto-transformers at the Guadalupe and Salinas substations are provided with closed-delta secondary windings, which in the case of Salinas supply power for local consumption. An experimental opening of the delta at Salinas demonstrated, as would be anticipated, that the use of such delta-connected secondary windings reduces, in a large measure, the third harmonic introduced by these transformers in comparison with its value without the use of such delta-connected windings. If grounded star-connected transformers are used, it is important, therefore, from the standpoint of induction, to provide such transformers with closed-delta connected secondary windings or with other means of reducing the third harmonic and its odd multiples. Such means may, however, in some cases be insufficient to reduce the residuals to such low values that they will not produce harmful inductive interference to parallel communication circuits.

The investigation on the system of the Coast Counties Gas and Electric Company shows results which are summarized in the following table with reference to the residual current and residual voltage. Santa Cruz, where the measurements were made, is 20 miles from one source of supply and 75 miles from the other end of the line where power was also supplied. For the sake of comparison the averages of the residual voltage of the 22,000-volt line between Salinas and Monterey, a distance of 18 miles, are also given:

Order of harmonic	Residual voltage—volts			Residual current amperes
	Santa Cruz	Salinas		
		Neutral		
		Grounded	Non-grounded	Santa Cruz
1	360	50	90	0.094
3	—	150	320	—
9	19	40	50	0.021
11	14	—	—	0.017
13	10	—	—	—
23	14	—	—	—

The system of the Coast Counties Gas and Electric Company is isolated from ground and employs a number of Scott-connected and open delta-connected transformers. The residuals at Santa Cruz on this system are composed principally of fundamental, ninth and eleventh harmonics. The fundamental is predominant. The third harmonic is absent or too small to measure accurately. It should be noted here that the use of Scott and open delta-connected transformers permits the third harmonic and its odd multiples to exist in the line voltages and currents of a three-phase isolated system. In all probability the residuals on this system are caused by unbalanced admittances to ground of the power line conductors. As has already been pointed out, that part of the unbalance due to electrostatic capacity could be greatly reduced by properly spaced transpositions in the power circuit. In contrast to the results at Salinas, the residuals of this system exhibit a prominent fundamental and the absence of, or relatively small amounts of, the third harmonic and its odd multiples.

APPENDIX III.

TRANSPOSITIONS.

The sources of the disturbances in communication circuits, which arise from parallel power circuits, have been treated in the first section of the preceding appendix. The effect of transpositions on the induction in communication circuits produced by parallel power circuits will now be considered.

This appendix comprises the four following sections:

1. Effect of Transpositions in Reducing Induction.
2. Characteristics of Present Transposition Systems.
3. Characteristics of Proposed Transposition Schemes.
4. Results of Tests.

1. Effect of Transpositions in Reducing Induction.

Transposing a circuit is the interchanging of the positions occupied by the conductors.

By transposing a power line the phase of the resultant electromagnetic field, due to balanced currents and the phase of the resultant electrostatic field due to balanced voltages is changed, and the induction is reduced by the production of neutralizing effects in the neighboring lengths of a parallel conductor. Thus, by locating the power circuit transpositions so that each conductor occupies all of the several possible conductor positions for equal distances, a section or "barrel" is obtained within which the resultant induction on a parallel conductor due to balanced currents and voltages is completely neutralized, neglecting attenuation and remanent electrostatic effect and assuming the parallel is uniform throughout the barrel.

Inasmuch as residual currents and voltages are in phase in the several conductors, the transposition of the power circuit does not reduce the inductive effects therefrom in a parallel conductor, except as the magnitudes of the residual currents and voltages are reduced by the power circuit transposition. (See Appendix II, page 119.)

As usually constructed, the conductors of a telephone circuit are close together as compared with their distances to a power line, and the circuit is usually isolated from ground. Could the conductors of a metallic communication circuit be located at the same point in space, as is approximately true of a pair of wires twisted together, the resultant electromagnetic and electrostatic induction between the sides of the communication circuit would be zero. The voltage induced along the conductors of the telephone circuit and the induced voltage to ground would be present but would not be effective in producing any voltage between the conductors of the telephone circuit, provided the capacity and leakage to ground and series impedance of each side of the telephone circuit were equal. On overhead lines the conductors of a metallic communication circuit must be at least several inches apart, hence in general when paralleled by a power line, the resultant electromagnetic and electrostatic induction in the two conductors will be unequal in magnitude. The result is that a voltage exists between the sides of the circuit which causes a current to flow in apparatus connected between the conductors, such as a telephone receiver.

Transpositions in communication circuits tend to equalize the induction in the two sides of the circuits by exposing each side equally to the influence of the power circuit, that is, by reversing in successive lengths the phase of the induction between the two sides of the circuit.

In an exposure where the induction from balanced currents and voltages would be completely neutralized by the power circuit transposition system if there were no communication circuit transpositions, or where such induction would be completely equalized by the communication circuit transpositions, if there were no power circuit transpositions this induction will practically always be partially cumulative if both power and communication circuit transpositions are installed without due reference to each other. It should be noted, however, that the maximum disturbances which may be set up in a parallel communication circuit by balanced currents and voltages in the power circuit will be present when neither the power circuit nor the communication circuit is transposed. Hence it is very important that the power and communication circuit transpositions be properly located with respect to each other, and in this way only can the maximum benefits from the transpositions be derived.

Induction from residual currents and voltages is reduced by communication circuit transpositions.

If the communication circuit has a ground return, it can not be transposed and the power circuit transpositions alone will be effective in reducing interference arising from the balanced currents and voltages. Also, the induction into a ground return communication circuit from residual currents and voltages is not affected by transpositions, except indirectly as previously stated. It is possible, though not of general practical application, to obtain the effect of a transposition in a grounded alternating current power or communication circuit by means of a transformer or repeating coil.

Induction between wires and ground is harmful to metallic as well as to ground return circuits, for in case the metallic circuit is not perfectly balanced electrically, such induced voltage forces a current to circulate in the metallic circuit through the terminal apparatus. It is not practical to maintain communication circuits in a state of perfect balance at all times.

2. Characteristics of Present Transposition Systems.

The transposition systems used on long distance metallic telephone circuits are designed primarily to reduce the "cross-talk" or induction from one telephone circuit into another, and provide for a high degree of balance between any circuit and all others on the line.

The length of standard balanced telephone transposition sections used by The Pacific Telephone and Telegraph Company is approximately eight miles (more exactly, 41,600 feet) and this is representative of the length of sections of the transposition systems used by other companies operating similar lines. To improve the transmitting qualities of telephone circuits used for long distance work, loading coils are introduced in certain circuits at the ends of the standard transposition sections. Uniform spacing of the telephone "S" poles (end poles of transposition sections) is an important consideration in the application of loading. It is important that the induction be neutralized in each sec-

tion between loading points, as these are points of discontinuity in the circuits.

The system now used also provides for the transposition of every circuit at actual intervals ranging from one-quarter mile to two miles, the average intervals for different circuits varying from approximately one-quarter mile to three-quarters of a mile, hence every circuit is to a certain extent balanced to induction from parallel power circuits.

In addition to the metallic circuits composed of two conductors, the telephone companies employ phantom circuits which are made up from two physical (two wire) circuits. Each "conductor," or side of the phantom circuit, consists of the two conductors which form one physical circuit. As usually made up, the physical circuits occupying adjacent horizontal positions are used for the phantom circuit. Hence, the average distance between the sides of the phantom circuit is equal to twice the distance between the conductors of the physical circuits. Due to the greater distance between the sides of the phantom circuit as compared with the physical circuits, the phantom circuits are more subject to inductive interference than the physical circuits. The phantom circuit possesses marked advantages in economy and transmission efficiency over the physical circuits composing it, hence is extensively used for the longer distances. The transpositions in the phantom circuits are spaced at average intervals for different circuits, varying approximately from three-quarters of a mile to two miles.

The purpose of transposition systems applied to power circuits has been to reduce the disturbance in parallel communication circuits and in some cases to equalize the separation of the pairs of conductors forming the several phases. Usually when transpositions have been applied to power circuits to reduce the disturbance in existing parallel communication circuits, one or more complete barrels have been provided within the total length of the exposure. The best obtainable results from power circuit transpositions will be had only when they are located with due regard to the transposition points of the communication circuit. No such practice as this has been followed in the past. The transposition systems heretofore applied to parallel power and communication circuits have therefore failed to meet the requirements for maximum effectiveness. Hence, balanced currents and voltages in the power circuits have, in general, caused more disturbance than necessary in parallel communication circuits.

3. Characteristics of Proposed Transposition Schemes.

It would be possible to fulfill the conditions for balance with regard to induction arising from balanced currents and voltages, by cutting a "barrel" into the power circuit between successive communication circuit transpositions. Inasmuch as telephone transposition points are ordinarily spaced at one-fourth mile intervals, this solution in the case of a three-phase power circuit would necessitate transpositions at an average spacing of one-eighth mile and a minimum spacing of one-twelfth mile, which is impracticable in most cases.

It would be possible to satisfy the conditions for balancing the induction in metallic circuits, from both balanced and residual currents and voltages, by installing any completely balanced system of communication circuit transpositions between each two successive power circuit trans-

positions. Assuming twelve mile "barrels" in the power circuit, the conditions for balance could be fulfilled with the present standard telephone transposition system. However, with power circuit barrels of a length such as is essential in most parallels, this solution would require the redesign and relocation of all telephone transpositions in the exposure, involving several times as many transpositions as are normally required, with the liability of interference with the location of loading coils.

Both the above solutions satisfy the conditions for balancing the induction in metallic circuits, arising from residuals, in lengths of circuit equal to or twice the distance between successive communication circuit transpositions, assuming these are uniformly spaced. In the standard transposition section as now used, balance is thus obtained in distances varying from an average of approximately one-fourth to four miles.

Between these two comparatively simple but extreme solutions the practical but more complicated solution for general cases is to be obtained. This involves the combination of power circuit "barrels" of moderate length with a modified communication circuit transposition system designed to procure balance as far as practicable for all circuits. In this way co-ordinated transposition systems may be designed which are sufficiently flexible to meet the requirements of short parallels and portions of longer parallels separated by points of discontinuity.

In the discussion above with reference to schemes of transpositions the balances or unbalances mentioned are those which would occur, due solely to the relative locations of transpositions in an exposure whose physical characteristics are uniform throughout. Even with a scheme of transpositions, balanced in the sense described, applied to both power and communication circuits involved in an actual parallel, there are a number of factors as noted below, which in general are not capable of being taken into account quantitatively and because of which effective neutralization may not be obtained. These factors are:

1. Non-uniformity of separation, configuration and other physical characteristics.
2. Variation in magnitude and phase of the inductive effects along the exposure (applying particularly to the higher frequencies).
3. Inherent inability of transpositions to completely neutralize electrostatic induction (this remanent effect can be reduced as far as desired by inserting a sufficient number of transpositions).
4. Imperfect electrical balance of the communication circuit.

While these factors which prevent complete neutralization of the induction can not be entirely eliminated, their effects can be abated by reducing the length of balanced transposition sections. Thus it is not sufficient merely to install transpositions in both lines so that they are balanced to each other; but, also, it is necessary to take into consideration the length of section within which balance is obtained and to make this length as short as the conditions of the particular case require.

Points of discontinuity, such as abrupt changes in power line current where a material amount of load is taken off, cross-overs, or substantial changes in separation, should, if practicable, be made neutral points

(junction points of balanced sections) in the transposition scheme. Where cross-overs occur balance should in general be obtained independently for the portions of the communication line on each side of the power circuit.

The transposition system and the location and spacing of transposition poles are factors of prime importance in the successful operation of telephone lines, on account of the mutual effects among the many circuits carried on such lines. On the other hand, transpositions in power circuits are, relatively, of minor importance in the operation of a power system and from this standpoint the effect of small changes in the location of such transpositions is negligible. Hence, in general, the requirements of the communication circuits are the chief factors which should govern the location of all transpositions in both power and communication circuits.

An individual study is necessary to determine the best procedure for any given parallel owing to the wide variation in conditions. Thus only is it possible in each case to determine the best location and method of transpositions with regard to the requirements of both power and communication systems.

4. Results of Tests.

The investigation at Salinas demonstrated that the induction in a ground return circuit in the exposures concerned arises principally from the residual voltages and currents, while the induction in a metallic circuit shows principally the characteristics of the balanced voltages and currents, together with some effect from the residuals. This result was to be expected as there are power circuit transpositions which reduce the induction in the conductors used as ground return circuits, due to the balanced components, but these transpositions and the transpositions in the telephone circuits are improperly located with respect to each other and therefore are inefficient as regards the induction in the metallic circuits. On the other hand, the telephone transposition system tends inherently to reduce the induction in the metallic circuits, arising from residuals. A study of the relative location of power and telephone circuit transpositions for exposure No. 2 at Salinas, indicated that by modifying the present transpositions of both circuits, it is possible to reduce materially the induction from balanced currents and voltages. Had it been feasible to take the power circuit out of service for the purpose of experimental retransposition, the above scheme as well as one for the King City exposure, would probably have been installed and the effects thereof experimentally determined. Under the conditions existing, however, it was deemed advisable to postpone the matter of transpositions for both these exposures, pending the acquisition of further information as to the extent to which retransposition would be warranted as a permanent improvement.

The experimental study of transpositions was, therefore, transferred to another point where a power line is not the sole source of supply and can, therefore, be shut down for alterations and tests under special conditions.

The experimental determination of the practical effectiveness of transpositions has not been completed. However, an extended theoretical study of transpositions has been made, including the design of a

modified telephone transposition system. This system, which requires many additional transpositions, is more flexible in its properties of coördination with different lengths of power circuit "barrels."

A study made to determine the relative efficiency of various schemes of transpositions designed for the Santa Cruz-Watsonville exposure of The Pacific Telephone and Telegraph Company's toll lead to the 22,000-volt line of the Coast Counties Gas and Electric Company, emphasizes the following general principles:

1. The necessity of proper relative location of power and telephone circuit transpositions.
2. The importance of the effect of cross-overs and the desirability of making them neutral points in the transposition scheme.
3. The necessity of some modifications of the telephone transposition system.

APPENDIX IV.

APPARATUS.

For the proper conduct of its tests and experiments the Joint Committee on Inductive Interference has secured, either through purchase or on loan account from various power and communication interests, apparatus of an aggregate value of over twelve thousand dollars.

The following is a brief schedule of the property in use by this committee, together with its estimated replacement value:

Buildings (portable laboratory)		$480 00
Furniture and fixtures		128 00
Apparatus—		
Oscillograph	$1,115 00	
Oscillator	600 00	
Motor generator set	260 00	
Meters	1,202 50	
Batteries	100 00	
Condensers	990 00	
Bridges	675 00	
Galvanometers	265 00	
Rheostats	734 80	
Switchboards	135 40	
Miscellaneous apparatus	1,505 00	
Coils and relays	645 00	
Transformers	2,412 50	
Miscellaneous	787 00	
Photographic	293 60	11,820 80
Grand total		$12,428 80

The above property is owned by the Joint Committee on Inductive Interference and various power and communication companies as follows:

Joint Committee on Inductive Interference	$1,251 15
The Pacific Telephone and Telegraph Company and American Telephone and Telegraph Company	8,293 65
Sierra and San Francisco Power Company	2,002 50
San Joaquin Light and Power Company	300 00
Pacific Gas and Electric Company	110 00
Western Union Telegraph Company	235 00
Testing force	256 50
Total	$12,428 80

APPENDIX V.

LIST OF TECHNICAL REPORTS.

Refer to complete list given in Appendix II of Final Report, pages 74 to 78.

APPENDIX VI.

ORGANIZATION—JOINT COMMITTEE ON INDUCTIVE INTERFERENCE.

Refer to Appendix V of Final Report, page 84.

*Technical Report No. 1.

January 6, 1918.

GENERAL OUTLINE OF TEST NO. 2 TO BE MADE AT SALINAS ON PARALLELS BETWEEN THE SIERRA AND SAN FRANCISCO POWER COMPANY, THE WESTERN UNION TELEGRAPH COMPANY, THE SOUTHERN PACIFIC COMPANY, AND THE PACIFIC TELEPHONE AND TELEGRAPH COMPANY.

Your subcommittee on tests have considered in some detail the tests which can profitably be made at Salinas on the above-mentioned parallels, and have adopted the tests which are outlined in this memorandum. We feel that the details of the tests should be open to modification as the work progresses, and that the work which we are here outlining may suggest further tests which it will be of importance to make on these parallels. The report which we are submitting now for your approval should, therefore, be considered as a general outline rather than as a detailed description of the tests.

Method of Making Tests.

In the following paragraphs we are outlining the tests which should be made on the signalling circuits for each condition of the power line for which tests are made.

A—Tests of Telephone Circuits.

The tests on telephone circuits are designed to give quantitative information regarding the total effect of the power lines on the telephone circuits and also regarding the effect of each harmonic in the power system on the telephone circuits.

The tests which we are outlining here are those which the past experience of the Pacific Telephone company indicates will most efficiently and completely give the information required.

Preliminary Work.

During the tests all of the circuits on the telephone pole lead will be disconnected from the rest of the telephone system at each end of the exposure in order to obtain definite testing conditions and to avoid the effects of secondary induction from other parallels in the system. As this procedure completely interrupts service over the telephone lead, it will ordinarily be necessary to make the tests at night. Good grounds will be made at the test poles at each end of the exposed section of the

*Technical Reports Nos. 1 to 50, inclusive, were signed by Mr. J. E. Woodbridge, as the Chairman of the Subcommittee on Tests, and subsequently approved by the Joint Committee. Technical Reports Nos. 51 to 71, inclusive, were signed by Assistant Field Engineer, Mr. Livingston P. Ferris, subsequently approved and signed by Field Engineer, Mr. R. W. Mastick, then by Mr. Woodbridge and finally by the Secretary of the Joint Committee, Mr. Arthur E. Bridge, upon approval by the Joint Committee.

telephone line. Two pairs of wires well insulated and well transposed will be run from the station where the tests are to be made to the telephone line.

Condition Tests.

Two telephone lines will ordinarily be connected in for test at the same time, and where possible these two circuits should be side circuits of a phantom so that these tests may be made at once on two telephone circuits and one phantom circuit. The tests will be made on a large enough number of telephone circuits to insure that an idea has been obtained of the extreme and average conditions of the lead. The tests which we suggest are merely outlined here and will be described in detail in memoranda, which will be later submitted to the Committee for its records.

1. Tests of the capacity and insulation balance of each telephone circuit.
2. Tests of the total noise induced between the wires of the telephone circuits measured by the noise standard.
3. Tests of the current flowing between the telephone wires and ground on short circuit and of the voltage induced between telephone lines and ground on open circuit.
4. Observations of the voltage from each power line to ground and also of the residual voltage; that is, the vector sum of these three voltages. Observations also of the current in each line wire and of the residual current; that is, the vector sum of these three currents.
5. Simultaneous resonance analysis of the induction between telephone wires and from telephone wires to ground on the one hand, and of the above named voltages and currents in the power system on the other hand. It may be found unnecessary to make resonance analysis of all of the the voltages and currents in the power system, but analysis should be made of the residual current and voltage, and also of enough of the single currents and voltages to make sure that the characteristics of these currents and voltages have been determined.
6. Oscillograms made simultaneously with tests No. 5 and of the same voltages and currents in the power and telephone systems. Enough oscillograms should be taken in each test to get records of the distinctive wave shapes and to check the results of the resonance analysis. The oscillograms should be purposely distorted by one of the methods outlined in the detail memorandum so as to make prominent the higher harmonics. Oscillograms of the true wave shapes should also be made and in each set of tests a direct current calibration should be taken.

B—Tests on Telegraph Circuits.

Arrangements will be made by which all of the telegraph circuits on the telegraph line may be opened at the same time for short periods

during which tests are being made in order to obtain definite testing conditions, and to eliminate the effect of secondary induction. Arrangements will also be made so that the telegraph wires may be looped in the substation one at a time while the tests are being made.

Tests should then be made on each telegraph wire of the voltage to ground with both ends open circuited and with the distant end connected to ground, and of the current flowing through the circuit with both ends connected to ground. A sufficient number of oscillograms should be taken to determine the wave shape of the induced disturbance under different conditions.

These tests would be carried out on a large enough number of telegraph wires to determine to what extent the disturbance is different for circuits on different pin positions of the telegraph lead.

Simultaneously with the above tests, observations should be made of the voltages and currents in the power line as outlined in paragraphs 4, 5 and 6 under "Tests on Telephone Circuits." These observations need, however, include only the magnitudes of the fundamental and of the one or two most prominent harmonics.

In order to determine to what extent the telegraph service is disturbed by the measured extraneous currents, it will be necessary to make tests of the effect of given extraneous currents on telegraph equipment of various kinds. Your subcommittee has now under advisement the best method of procedure for making these tests and will report on this question at a later date.

C—Tests on Railroad Signalling Circuits.

Observations and oscillograms should be made of the maximum alternating current induced in the railroad signalling circuits under different conditions. These observations will be supplemented by tests of the effect of the alternating current on the relays of the signalling circuits. These tests will be similar to the tests of the effect of alternating current on telegraph apparatus, and the best method of making them will be later reported by this subcommittee.

D—Miscellaneous Tests.

It may be found desirable to make further tests on some of the types of equipment used by the Southern Pacific Company, such as the telegraphone, or of other types of circuit which are not considered above. Your subcommittee has this matter under advisement and will report any recommendations for such tests at a later date.

Conditions of the Power Circuits.

The following is a general outline of the different conditions of the power circuits under which tests should be made. In this outline no attempt has been made to go into detail, as it is felt that the detailed tests will be modified according to the exigencies of operation and that additional tests will be suggested by the results of the tests here outlined.

1. Tests with the power line cut dead, to determine that under these conditions the signalling circuits are free from disturbances.
2. Tests of the induction under normal operating conditions, both light load and heavy load.
3. The effect of delta transformer windings on the induction in signalling circuits. These tests will include the effect of opening the delta winding of the transformers at Guadalupe substation, and if it can be arranged, opening the delta winding at Salinas substation.
4. The effect of grounding the neutrals. These tests will include the effect of varying the number of points at which the neutrals of star connected transformers are grounded.
5. Tests of the effect of the operation of the Monterey steam station. These tests should be made with the Monterey station paralleled with the Stanislaus station and also with the Monterey station alone supplying power past certain of the parallels involved.
6. Tests of the effect of charging aluminum lightning arresters at Stanislaus power house and at the Bay Shore substation.
7. Tests of the effect of switching high tension lines.
8. Tests of the disturbance caused by known residual voltages and currents. In these tests the conductors of the power line involved in a given parallel will be cut dead and a known voltage will be impressed between the three power wires in parallel and the ground. These tests will be made with 60 cycles impressed voltage and, if it is practicable to obtain it, with 180 cycles impressed voltage. Tests of the effect of unbalanced current will be made by passing a known current through each of the line wires with a ground return.
9. Tests with one line wire grounded. In these tests the method of procedure will probably be to create a ground on one wire when the line is dead and then to switch potential on the circuit.

We are making preparations for the test according to this outline, and respectfully submit it for your approval.

(Signed) J. E. Woodbridge,
Chairman Subcommittee on Tests Nos. 1 and 2.

Technical Report No. 2.

February 3, 1913.

PROGRESS REPORT OF THE TESTS OF THE PARALLEL BETWEEN THE SYSTEMS OF THE COAST COUNTIES GAS AND ELECTRIC COMPANY AND THE PACIFIC TELEPHONE AND TELEGRAPH COMPANY BETWEEN MORGAN HILL AND GILROY.

It is the purpose of this memorandum to briefly summarize the results and conclusions which can at present be drawn from the tests of inductive interference which have been made at Morgan Hill on the above-named parallel.

Method of Testing.

The tests on this exposure were made in the month of December according to the outline of the method of making tests on telephone circuits given in the report of the subcommittee on tests No. 2 concerning the tests to be made at Salinas, dated January 6, 1913,* with the exception that no quantitative measurements were made according to the test described in paragraph 5 of the outline, and no oscillograph records were obtained of currents induced in the telephone lines during normal operation of the power system.

Effect of the Power Line on Telephone Circuits.

The tests have shown that under normal operating conditions of the power system, the potentials induced between the telephone wires and ground are not dangerous or such as to interfere with telegraph operation. The noise induced in the telephone circuits varies in magnitude between five micro-amperes or less and one hundred micro-amperes.† The amount of noise is different for different telephone circuits and for different conditions of the power line, and is generally greater when the power line is fed from the south than when it is fed from the north. Under the abnormal condition of operation, when one wire is grounded, a potential of two or three hundred volts is induced between the telephone wires and the ground, and the noise in the telephone circuits varies between twenty-five micro-amperes and a value considerably beyond the range of the testing instruments, *i. e.*, 150 micro-amperes.

The continued fair weather during this test did not permit of any measurements being taken under rainy conditions.

*T. R. No. 1.
†Equivalent, at 240 cycles per second.

Harmonics in the Power System.

The tests have shown as a fundamental fact that the noise occasioned in the telephone circuits by this parallel is due to a considerable number of harmonics in the power system. In addition, it is found that the harmonics which are prominent in the power current and voltage, and which are important in producing noises in the telephone circuits, are quite different under different conditions of the power system. This fact is illustrated, for example, by oscillogram of power waves No. 36, which shows a heavy thirteenth harmonic and oscillogram of power waves No. 42, which shows heavy fifth and eleventh harmonics. The work has shown further that the harmonics present in the power wave vary considerably from minute to minute.

In order to make possible the measurements of these important harmonics, the oscillograms of potential were taken with the vibrator of the oscillograph in series with a condenser which is placed across the secondary terminals of a potential transformer. Similarly oscillograms of current were taken with the oscillograph placed in series with a condenser which is shunted across the terminals of a resistance inserted in series in the secondary of a current transformer. By this means the high harmonics in both the current and potential waves were exaggerated on the oscillograms so that it is possible to determine their magnitude to a fair degree of approximation. The amount by which a given harmonic is exaggerated can easily be computed if it is assumed that the exaggeration is not affected by the instrument transformers.

Some work was done during the tests at Morgan Hill and more is now being done to determine to what extent the exaggeration of the higher harmonics is affected by the instrument transformers. In connection with the potentials this work involves placing a high voltage condenser between line and ground, passing the current flowing through the condenser through one vibrator of the oscillograph, and at the same time connecting another vibrator of the oscillograph in series with a condenser and across the secondary terminals of a potential transformer which is in parallel with the high-tension condenser. Two high-tension condensers were most kindly loaned for this work by the Federal Telegraph Company. During the period in which the condensers were available, however, we were not successful in obtaining a conclusive test of the sort outlined above. Further work on this matter is postponed until the completion of some more adequate condensers which are now being built by the Federal Telegraph Company, and which they have kindly offered to loan for this work.

The accurate analysis of an oscillogram such as those which we have obtained in this work, and which contain a large number of harmonics, is nearly a day's work for one man. In view of the variations in the

harmonics existing in the power system under different conditions, the analysis of the tests on this parallel involves a considerable amount of work. We are hoping that the amount of this work will be considerably reduced in subsequent tests by the use of the method of harmonic analysis which is mentioned in the report of your subcommittee of January 6 outlining the tests at Salinas.

The Effect of Transposition of Power Wires.

Tests were made on this parallel both when the power lines were entirely nontransposed opposite the parallel and after the two barrels had been cut into the power line in the length of the exposure between Morgan Hill and Gilroy. This exposure is composed of two sections in which the telephone line is respectively east and west of the power line, and one barrel was cut into each of these sections. In order to make clear the effect of these transpositions, it is necessary to briefly summarize here some of the characteristics of electrostatic induction between power and signalling wires, ignoring, for the purpose of this discussion, the effects of electromagnetic induction.

In any three-phase system in which equal voltages are impressed between the three wires, the potentials between the three wires and ground may be considered to be composed of two sets of potentials. One of these sets of potentials, which we will call the balanced voltages, consists of three equal voltages between the three wires and ground whose vector sum is zero. The other set of voltages, which we will call the residual voltages, consists of three equal voltages between the three wires and ground, which are in phase with each other, and whose vector sum is therefore equal to three times any single residual voltage.

The inductive effect of the balanced voltages on parallel circuits can be made as small as is desired by the insertion of a sufficient number of properly located transpositions. The effect of the residual voltages on outside circuits, however, is not reduced by transpositions in the power circuits except in so far as the magnitude of the residual voltages is affected by the transpositions. The residual voltages give the same inductive effect as a single-phase grounded circuit operating with the three power conductors in parallel. For this reason a residual voltage which is small in comparison with the balanced voltages has a proportionally very large effect in producing a disturbance in near-by circuits.

The balanced voltages and the residual voltages each act by induction on parallel circuits in two ways, by longitudinal induction and by transverse induction. By longitudinal induction is meant the creation of a voltage between the wires of the parallel circuits and ground. By transverse induction is meant the creation of a voltage between the two conductors of a parallel metallic circuit.

In a power transmission system operating without a grounded neutral, the voltages between the wires and ground are affected by the capacities between the wires and ground. In order, therefore, to have the voltage between the wires and ground equal—that is, in order that there may be no residual voltage—it is ordinarily necessary for the capacity to ground of all phases to be equal throughout the entire transmission system. That is, it is necessary to have a certain number of transpositions cut into sections of the power lines not directly involved in parallels with signaling circuits, in order to balance the capacity between the power wires and ground. It is to be expected that this consideration will ordinarily be much less important in a system operating with a grounded neutral.

In the system of the Coast Counties Gas and Electric Company, as at present operated, there are nontransposed sections of line connected to the section between Morgan Hill and Gilroy, both when the section is fed from the north and when it is fed from the south. Tests have shown that in each case there is a residual voltage on the wires of approximately 400 volts between each wire and ground. We understand from the Coast Counties Gas and Electric Company, that when this section of line is fed from the south the only nontransposed section of line connected to it is between Gilroy and Watsonville, and that the company will cut in such transpositions as we require for further tests on request. When the section of line between Morgan Hill and Gilroy is fed from the north, it is also connected to a line of the Pacific Gas and Electric Company between San Jose and Morgan Hill which, we are informed, is nontransposed.

After the completion of a set of tests on the exposure between Morgan Hill and Gilroy, during which the section of power line directly concerned in the parallel was nontransposed, transpositions were cut into the power line which were designed to remove simply the longitudinal induction due to the balanced components of the voltages. To eliminate the transverse induction requires a larger number of transpositions than have been inserted. There remains also the above noted residual voltage on the line, and the induction from this voltage, as noted above, can not be eliminated by transposition.

The results of the tests, before and after these transpositions were cut into the power line, are somewhat difficult to interpret without an analysis of the oscillograms of the power waves which were taken at the time of the tests because of the variations in the disturbing harmonics. Without such analysis, however, the result seems to indicate that by removing the longitudinal induction due to the balanced voltages an appreciable decrease was effected in the noise induced in the telephone circuits, but that apparently the induction due to the residual voltage

and the transverse induction from the balanced voltages are both important. The residual voltage during normal operation can probably be removed by a proper transposition of the sections of the system which are at present nontransposed, and we shall possibly wish to ask the Pacific Gas and Electric Company to transpose the section of line between San Jose and Morgan Hill in order that we may obtain more conclusive information of this point. Before doing so, however, we are proceeding with a theoretical analysis of the relative importance of the various factors in producing noise. This analysis will include the effect of electromagnetic induction from the current in the power conductors.

Investigation of Power System.

Since your last meeting an investigation has been made of the principal generating and substations of the Coast Counties' system, and analyses of the wave forms of voltage and current at various places have been made by means of a portable resonance analysis apparatus. The diagrams of connections of the stations visited and the results of the analyses have been entered in the book containing the results on the tests of the parallel at Morgan Hill in accordance with the memorandum of December 16 from the assistant secretary transmitting that book to your files. We are also attaching to that book maps giving complete information regarding all of the exposures between the Coast Counties Gas and Electric and The Pacific Telephone and Telegraph Company. We ask your acceptance of the new information thus added to your files.

Below is a summary of interesting points shown by the analyses which were made in this investigation:

Current to a 50-light mercury arc rectifier at Santa Cruz—Fairly large 3d, 5th and 7th harmonics but none particularly noteworthy.

Voltage wave of 2-ph. turbine driven generator at Santa Cruz—18 slots per pole, 3d harmonic fairly large, none particularly prominent.

Voltage wave of 400-kilowatt 2-ph. synchronous motor at Santa Cruz—6 slots per pole, open slot construction, 3d, 5th and 15th harmonics fairly large, 11th and 13th harmonics very large.

Generators at Big Creek* rotating armature, 2-ph., 76 and 104 slots respectively, with 14 poles—No harmonics particularly prominent.

Outgoing current at Big Creek* under full load—No harmonics particularly prominent.

Voltage wave of 750-kilowatt generator at Watsonville, 3 slots per pole, closed slot construction reconnected from 11,000 volts to 2,200 volts—Very large 5th harmonic, most others rather large.

Bus voltage at Watsonville generator disconnected—large 3d, 5th, 11th and 13th harmonics.

*Big Creek, Santa Cruz County, not the Big Creek station of the Pacific Light and Power Company, in Fresno County, Cal.

It should be noted that during the tests at Morgan Hill very large 5th, 11th and 13th harmonics were observed at various times both in the power system and in the induced current in the telephone lines. It is our intention to make some further tests at Morgan Hill to determine to what extent the large harmonics observed there are due to the operation of the synchronous motor at Santa Cruz and the generator at Watsonville. It is also to be noted that the small effect attributed to 3d harmonics in the power and telephone circuits at Morgan Hill is due to the delta type of connection which is used on this system.

As is indicated in detail in the report above, the work which has been done to date in the analyses of the results obtained at Morgan Hill already indicates further simple tests by which we can obtain more information regarding the effect of the parallel tested. We are proceeding with the work on the analyses along the lines indicated above with the idea that it will give us further information regarding the effect of this parallel and may suggest further tests which we can profitably make on this Coast Counties system. Our suggestions for these further tests will be reported to you at a later meeting.

Respectfully submitted.

(Signed) J. E. Woodbridge,

Chairman Subcommittee on Test No. 1 at Morgan Hill.

Technical Report No. 3.

February 24, 1913.

MEASUREMENTS OF NOISE IN TELEPHONE CIRCUITS.

In the report of January 6* outlining the tests to be made at Salinas, Item 2 of the tests outlined under the heading "Condition Tests" is as follows:

> "2. Tests of the total noise induced between the wires of the telephone circuits measured by the noise standard."

It is the purpose of this memorandum to briefly outline the methods and apparatus used in making these tests.

The quantity which is measured in these tests is the effect on the ear caused by the complex current wave which is induced in the telephone receiver by the power circuits. As the effect of a current of a given magnitude in producing noise depends largely upon its frequency, and as the induced current is compounded in general of a large number of frequencies, the effective value of the induced current is not in general an exact measure of the noise produced by that current. The method which has been employed for many years by the telephone companies in measuring the noise consists in comparing noise caused by the induced current in a telephone receiver with a standard noise, the magnitude of the standard noise being changed until the two noises, standard and induced, are judged to have the same detrimental effect on a telephone conversation.

The circuit which is used for these tests is shown diagrammatically in the attached sketch P. I. C. 32.

The generator which produces the standard noise is connected to a shunt box, by means of which various amounts of the standard current are shunted through a telephone receiver. The telephone receiver is connected alternately to the standard noise shunt and to the line under test by means of a mercury switch, and the magnitude of the standard noise is adjusted by means of the shunt until the point of equality is found.

The standard noise generator is an electrical generator of the inductor type composed of a disk of nonmagnetic material in which are inserted a number of soft iron pole pieces and which revolves between the poles of a permanent magnet. Coils are wound on the poles of the magnet and as the disk revolves an alternating current is induced

*T. R. No. 1.

in this winding by the pulsations of magnetism caused by the motion of the soft iron pole pieces under the poles of the permanent magnet. The voltage of the generator and the resistances in the shunt are so adjusted that when the generator is operating at 240 cycles, the calibrations on the shunt give directly the effective value of the current through the telephone receiver in microamperes. In the standard apparatus this shunt is provided with steps up to 150 microamperes. The noise induced in the circuits under test in this investigation is so great that this range has been found to be insufficient and an addition has been built for the shunt by which its range is increased to 500 microamperes.

The pitch and quality of the noise induced in telephone circuits differs a good deal in different cases of parallelism. The noise standard has, therefore, in general, a quality and pitch different from that of the noise being measured. For this reason the amount of standard noise which is considered to be equal to the noise induced in the circuit is somewhat a matter of individual judgment and the method cannot hope to be precise. The steps on the noise standard are accordingly adjusted so that there is a large difference in the amount of standard noise in the telephone receiver with the shunt on adjacent steps. Men who are accustomed to measuring noise in this way ordinarily agree as to which point on the shunt most nearly represents the magnitude of the noise being measured.

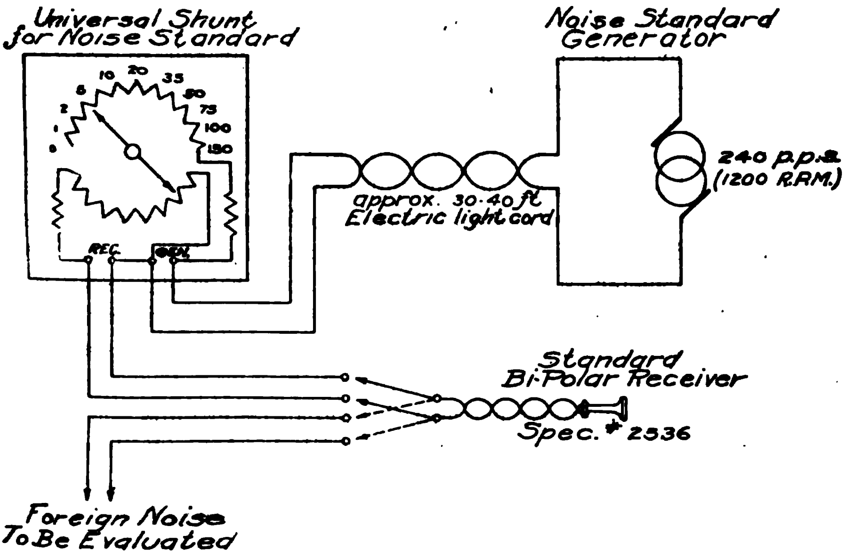
Respectfully submitted.

(Signed) J. E. Woodbridge,

Chairman Subcommittee on Tests No. 1 and No. 2.

P.I.C. 32
FEB. 24, 1913.

Noise Standard
Circuit For Evaluating Noise in
Terms of Noise Units



Technical Report No. 4.

February 24, 1913.

MEASUREMENTS OF THE EFFECTIVE VALUES OF VOLTAGES AND CURRENTS INDUCED IN SIGNALLING CIRCUITS.

Item 3 under the heading "Condition Tests" in the memorandum of January 6,* outlining the tests to be made at Salinas, reads as follows:

> "3. Tests of the current flowing between the telephone wires and ground on short circuit and of the voltage induced between telephone lines and ground on open circuit."

It is the purpose of this memorandum to briefly outline the methods and apparatus which are used in making these tests.

These tests are carried out on all types of signalling circuits and are of interest as indicating in a rough way the severity of the disturbance induced on telephone circuits and also as giving a fairly good measure of the degree to which the disturbance will affect the operation of telegraph apparatus or other signalling apparatus on the circuits. They also indicate the degree of approach to conditions under which the protective devices of the signalling circuits would be operated and the conditions which would involve risk of fire or injury to employees.

For the measurement of the current flowing in the circuit between the signalling wires and ground a series of ammeters and milliammeters have been provided. The most sensitive of these is the Paul Electrodynamometer Milliammeter with a full scale of deflection of 20 mils. Tests of the current flowing in the circuit are made both with the distant end open circuited and with the distant end grounded.

For tests of the voltages between the signalling circuits and ground on open circuit a Kelvin Multicellular Electrostatic Voltmeter has been provided having a range from 20 to 80 volts. The range may be increased by the use of series and parallel condensers with the voltmeter.

Respectfully submitted.

(Signed) J. E. Woodbridge,

Chairman Subcommittee on Tests No. 1 and No. 2.

*T. R. No. 1.

Technical Report No. 5.

February 24, 1913.

MEASUREMENT OF CURRENTS AND VOLTAGES IN THE POWER CIRCUIT.

In item 4 under the heading "Condition Tests" in our memorandum of January 6*, outlining the method of making tests at Salinas, mention is made of the measurements of effective voltages and current to be made on the power system as follows:

> "4. Observations of the voltage from each power line to ground and also of the residual voltages, that is, the vector sum of these three voltages. Observations also of the current in each line wire and of the residual current, that is, the vector sum of these three currents."

These measurements are in general made by the aid of instrument transformers. The type of connection used in making these voltage and current measurements is indicated in the attached sketches P. I. C. 4 and P. I. C. 5.

For the measurement of potentials in a three-phase circuit three potential transformers are connected with their primaries between the three wires of the circuit under test and ground, as is indicated in the sketch. The secondaries are connected so that measurements may be made of the voltage across each secondary which represents the voltage between the corresponding power wire and ground, or the secondaries may be connected in delta with one corner open and a measurement made of the voltage across that open corner. This voltage represents the vector sum of the voltages between the three wires and ground.

Similarly for the current measurements the primary of a current transformer is connected in each wire of the power circuit under test and the secondaries of these three transformers are so connected that an ammeter may be inserted in the circuit of any one secondary or an ammeter may be inserted in the common wire which carries the vector sum of the currents in the three secondary windings.

We are dependent upon the exact equality of the ratios of the three potential and the three current transformers which are used for these measurements to obtain true residual currents and voltages. The equality of the ratios of the potential transformers may readily be checked by putting the primaries of the transformers in parallel across the same voltage and measuring the resultant voltage across pairs of the second-

*T. R. No. 1.

ary windings connected in opposition. Another good way of checking the potential transformers is to place the three primaries in delta on the high tension circuit and measure the potential across the open corner of the secondary delta. Under these circumstances the vector sum of the three primary voltages is necessarily zero.

Check tests of this sort are carried out on each bank of potential transformers used. As an indication of the accuracy to be obtained by these methods, tests at Salinas made on pairs of the 200 watt G. E. potential transformers which are being used there for the tests show a maximum difference in secondary potential of 0.15 volts out of the total secondary voltage of 55 volts, or about 0.3 per cent. Tests of similar potential transformers were made at Morgan Hill by connecting primaries in delta and the voltage across the open corner of the secondary delta was found to be 0.4 volts, with 110 volts across each secondary. A part of this resultant voltage was due to the third harmonic in magnetizing current of the potential transformers, which can be analyzed and eliminated.

Comparative tests of the current transformers can be made by putting the primaries in series and putting the pairs of the secondary windings in parallel opposing across an ammeter. We were unable to make any residual current measurements at Morgan Hill because of the lack of suitable current transformers and have as yet no test ready to report of the current transformers at Salinas.

It may be noted that with the type of connection used for the potential transformers they form a star connected system of transformers to ground with a secondary delta which is open. Under these circumstances the potential transformers themselves produce a small third harmonic residual voltage on the power wires because of the third harmonic in the magnetizing current. This third harmonic component of the magnetizing current of the potential transformers is so small, however, that a little consideration indicates that its effect is entirely negligible.

The importance of the residual voltage in producing disturbances in signalling wires is indicated in the technical report of your subcommittee on the progress of tests on the parallel at Morgan Hill dated February 3.* The importance of the residual current in producing disturbances in parallel signalling circuits is due to the fact that the residual current represents current which flows through the power wires as one side of the circuit and through the earth as the other side. The earth current would flow along the surface of the earth if it were of infinite conductivity. Actually, because of the resistance of the

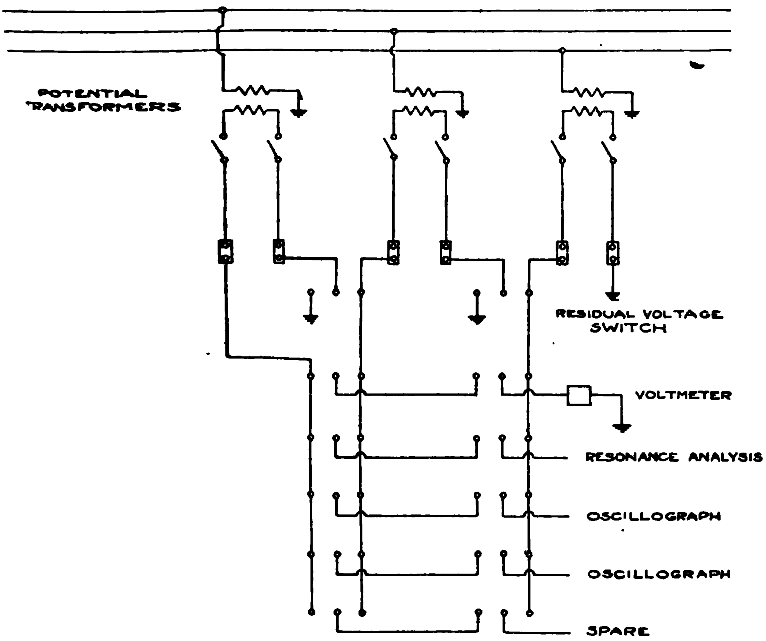
*T. R. No. 2.

earth near its surface, the return current ordinarily flows at a considerable distance below the surface. In some cases it has been found that the current produces the same effect as though it were concentrated one or two miles below the surface of the earth. In other words, the residual current flows in a very large inductive loop and has a correspondingly great inductive effect on parallel circuits.

Respectfully submitted.
(Signed) J. E. Woodbridge,
Chairman Subcommittee on Tests No. 1 and No. 2.

JAN 3, 1913. P.I.C. 4.

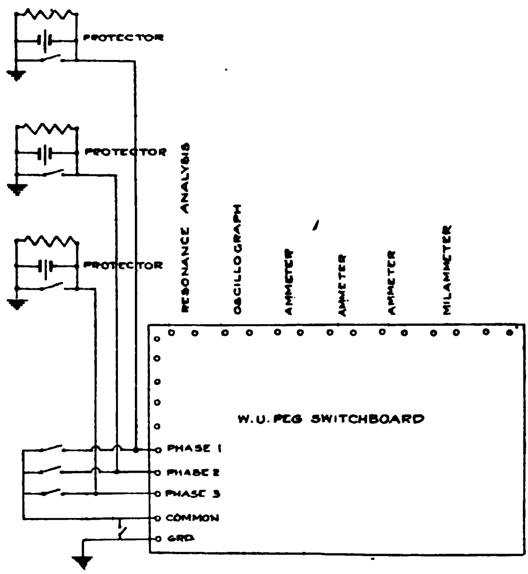
METHOD OF TESTING
CONNECTIONS FOR POTENTIAL TRANSFORMERS



JAN 3 1913

FIG 5

METHOD OF TESTING
CONNECTIONS FOR CURRENT TRANSFORMERS



Technical Report No. 29.

May 4, 1914.

DETERMINATION OF LINE IMPEDANCES.

1. Purpose.

In order to determine from observations the induced voltages on communication circuits, and to predict the current which will flow under given conditions, a knowledge of the circuit impedances at the various frequencies is required. It is the purpose of this report to discuss the methods used in this investigation for the determination of such impedances, and to present the data obtained at Salinas* and Santa Cruz.

2. Methods.

(a) Measurement.

The equivalent impedance of a communication circuit with any given load at the distant or "receiving" end may be measured by an impedance bridge. Two forms of such a bridge, with the corresponding formulæ, are shown on P. I. C. No. 186, attached, the form of bridge being dependent on whether the equivalent reactance of the circuit is inductive or condensive (capacitive).† As a source of single-frequency alternating current the Vreeland sine wave oscillator is employed. The bridge in use is made from apparatus at hand, no portion of it being shielded except the two ratio arms A and B. A telephone receiver is used to indicate balance by the null method. If the supply electromotive force contains more than one frequency, or if the bridge, or the circuit whose impedance is to be measured, is subject to induction, there will be noise heard in the receiver, even though the bridge is balanced. To eliminate such noise from extraneous currents, which is sometimes very troublesome, the receiver circuit may be made selective for the measuring frequency, by introducing into it capacity and inductance in such amounts as to cause the circuit to resonate at the measuring frequency. This procedure may increase the impedance of the receiver circuit and thus decrease the sensitiveness of the bridge. Such added inductance and capacity must be shielded or placed at some distance from other portions of the bridge. To lessen errors due to the capacity between different portions of the bridge and ground, the bridge proper is isolated from the generator and receiver by transformers as shown. For the best results the different portions of the bridge should be shielded. It has been found that this unshielded bridge is especially liable to error in

*Omitted in publication.
†See also P. I. C. 246 attached to T. R. No. 63.

making measurements of circuits which have ground as one side. This error is a minimum when the ground side of the circuit to be measured is connected to the bridge at the same terminal as the energy supply.

It has been found possible to measure transverse, but not longitudinal, impedances on telephone circuits in exposure No. 2 at Salinas and in the Santa Cruz-Watsonville exposure, with the power lines in operation. Trouble from power circuit induction in such measurements was obviated by using even harmonics of the fundamental power line frequency. Satisfactory longitudinal measurements on exposed circuits can, in general, be made only when the power lines are not in operation.

(*b*) *Computation.*

The formulæ for the impedance of a circuit with uniformly distributed constants are well known.* On P. I. C. No. 187, attached, are given the formulæ used for the computation of line circuit impedances. It will be seen that the impedance for a given frequency depends on four independent primary constants, the resistance, the inductance, the conductance or leakage, and the capacity per unit length; hence the value of the results hinges directly on the accuracy with which these quantities are known or can be determined. For open-wire lines, particularly grounded circuits, data from which to determine all of these quantities accurately, for the particular conditions at hand, are not in general available. The number, position and condition of nearby circuits and other objects, humidity, condition of insulators, frequency, and depth of equivalent ground plane affect these primary constants.

3. Data.

On P. I. C.'s. No. 77 to No. 94, inclusive (on file with the Joint Committee on Inductive Interference), are given the computed impedances for the Salinas-King City telephone circuits as listed below. These cir-circuits are of No. 12 N. B. S. G. hard-drawn copper wire, 41 miles in length. In the tables, values are given for odd harmonics of 60 cycles. The frequency range covered is from 0 to 2,100 cycles.

P. I. C. No.	Subject.
79	Table of resistance, reactance and impedance of two wires in parallel to ground, with far end open; grounded; and grounded through 100 ohms resistance.
77, 78, 80	Curves of resistance, reactance and impedance of two wires in parallel to ground, with far end open.
81, 82, 83	Curves of resistance, reactance and impedance of two wires in parallel to ground, with far end grounded through 100 ohms.

*For the mathematical development, see Fleming, Propagation of Electric Currents in Telephone and Telegraph Conductors; Steinmetz, Transient Electric Phenomena and Oscillations, Section 111; Kennelly, The Application of Hyperbolic Functions to Electrical Engineering Problems.

P. I. C. No.	Subject.
84, 85, 86	Curves of resistance, reactance and impedance of two wires in parallel to ground, with far end grounded.
87	Tables of resistance, reactance and impedance of one wire to ground, with far end open; and grounded.
88, 89, 90	Curves of resistance, reactance and impedance of one wire to ground, with far end open.
91, 92, 93	Curves of resistance, reactance and impedance of one wire to ground, with far end grounded.
94	Table of resistance, reactance and impedance of a phantom, with far end open; and short circuited.

The following are some miscellaneous impedance computations:

P. I. C. No.	Subject.
95	Table of resistance, reactance and impedance of a phantom No. 12 N. B. S. G. copper, 50 miles long, with far end open; and closed through a No. 291A subscriber's telephone set (with receiver on hook).
97	Table of resistance, reactance and impedance of one No. 9 B. and S. G. copper wire to ground, 71 miles long, with far end open; and grounded.

On P. I. C.'s No. 109 to No. 111, inclusive, are given the results of impedance measurements on exposure No. 2 at Salinas. The conductors are No. 12 N. B. S. G. copper. The equivalent resistance, reactance, impedance, and inductance or capacity are plotted as functions of the frequency over the range from 0 to 1,500 cycles.

P. I. C. No.	Subject.
109	Impedance of a phantom, with far end open.
110	Impedance of a phantom, with far end short circuited.
111	Impedance of two wires in parallel to ground, with far end open. (Various conditions of shielding by neighboring conductors.)

On P. I. C.'s No. 163 to No. 176, inclusive, are given the results of impedance measurements on the Santa Cruz-Watsonville exposure. The circuits are No. 12 N. B. S. G. copper, 17.07 miles in length. The resistance, reactance, impedance, and equivalent capacity or inductance, are given for the frequency range, 0 to 1,500 cycles. The transverse measurements were made for two conditions as regards insulation resistance of the circuits. These measurements include the impedance of the test leads.

P. I. C. No.	Subject.		
163	Transverse	Physical	Far end open
164	Transverse	Physical	Far end short circuited
165	Transverse	Phantom	Far end open
166	Transverse	Phantom	Far end short circuited
167	Longitudinal	Single wire	Far end grounded
168	Longitudinal	Physical	Far end clear
169	Longitudinal	Physical	Far end grounded
170	Longitudinal	Phantom	Far end clear
171	Longitudinal	Phantom	Far end grounded
172	Longitudinal	Eight wires	Far end clear
173	Longitudinal	Single wire	Far end clear
175	Longitudinal—Transverse	Test leads	Far end clear
177	Longitudinal—Transverse	Test leads	Far end short circuited

P. I. C.'s No. 174 and No. 174A give the inductance per mile of one, two and four wires in parallel to ground, as a function of the distance from the wires to the equivalent ground plane.

P. I. C. No. 176 shows the variation of the open circuit impedance of the test leads with the insulation resistance.

Using the values obtained by measurements of the open-circuit and short-circuit impedance of the Santa Cruz-Watsonville circuits, the primary constants were computed for a few cases, principally to determine the variation in the depth of the equivalent ground plane with frequency. To obtain the values of open and short circuit impedance for the circuits within the exposure alone, formula (8), P. I. C. No. 187, is used. By this means error is avoided in cases where the test leads have different electrical characteristics than the line. The results of these computations are given in the following table:

Line Constants Computed from Impedance Measurements.

Frequency	Primary constants (per mile) R ohms	L milli-henrys	S micro-mhos	C micro-farads	Propagation constant $P = \alpha + j\beta$	Z ohms	h*—feet
	One No. 12 N. B. S. G. copper wire to ground.						
0	4.8	3.9	2.0	—	— —	—	300
240	5.3	3.9	2.5	0.0122	$0.0121 \angle 65^\circ.2 = 0.0051 + j0.0110$	$654 \angle -17^\circ.1$	300
720	6.0	3.8	3.6	0.0120	$0.0313 \angle 78^\circ.4 = 0.0063 + j0.0307$	$576 \angle -7^\circ.8$	230
1200	6.8	3.7	4.8	0.0122	$0.0513 \angle 81^\circ.7 = 0.0074 + j0.0507$	$558 \angle -5^\circ.3$	170
	Four No. 12 N. B. S. G. copper wires (1 phantom) to ground.						
0	1.2	2.4	5.6	—	— —	—	280
240	1.6	2.4	7.8	0.0270	$0.0127 \angle 72^\circ.7 = 0.0038 + j0.0122$	$307 \angle -6^\circ.4$	280
1200	3.4	2.2	13.9	0.0251	$0.0565 \angle 82^\circ.1 = 0.0077 + j0.0559$	$297 \angle -3^\circ.7$	150

*h is the distance from the wires to the equivalent ground plane—a plane conductor of infinite extent and conductivity so located that the inductance of the wires with it as a return path is the same as that of the actual circuit. The "image" conductors used in computations of electromagnetic induction are the images of the actual conductors with respect to this plane.

The depth of the equivalent ground plane is determined from the values of inductance given in the table above and the curves given on P. I. C. No. 174. The measurements, it will be seen, show a consistent decrease in the depth of the equivalent ground plane with an increase in frequency. This is in accord with theoretical considerations.

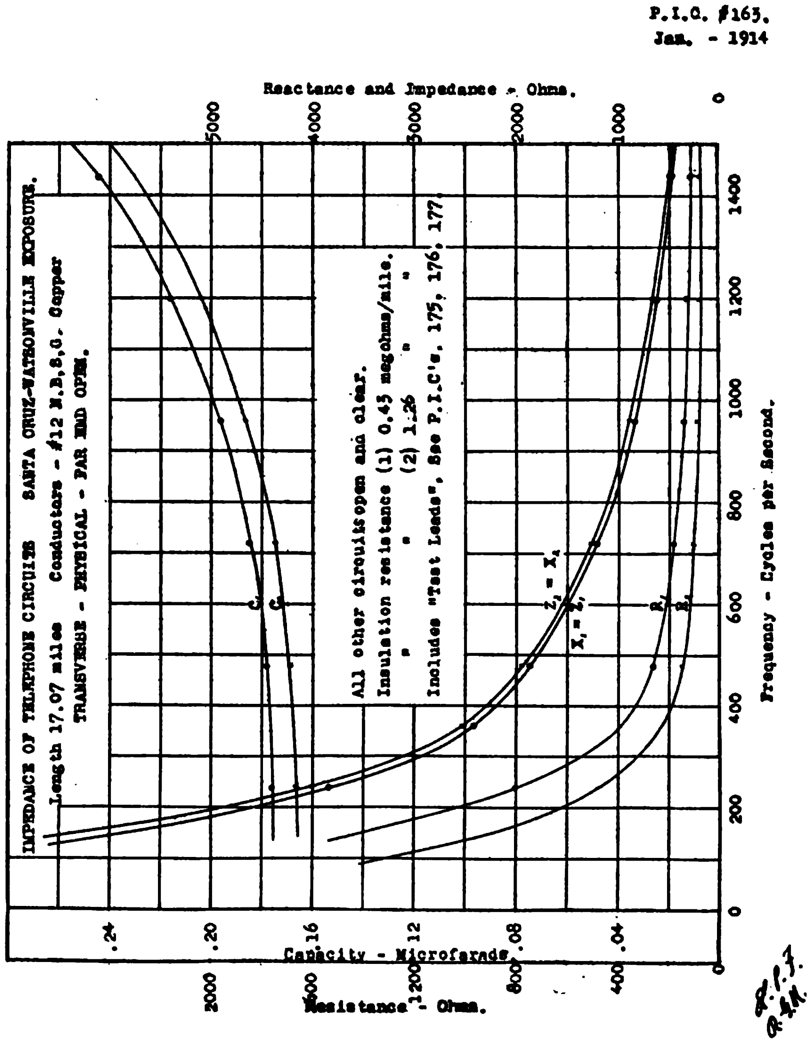
4. Conclusion.

It is evident that wherever possible the open-circuit and short-circuit impedances should be measured, since some measurements would, in general, be necessary in any case to determine the primary constants with a fair degree of accuracy. Having measured the open and short-circuit impedances of a circuit, the effects of apparatus of known impedance added to its ends may be determined by the formulæ given. Measurements of the circuit impedance will, as a rule be more accurate and require less time than computations, which are somewhat lengthy.

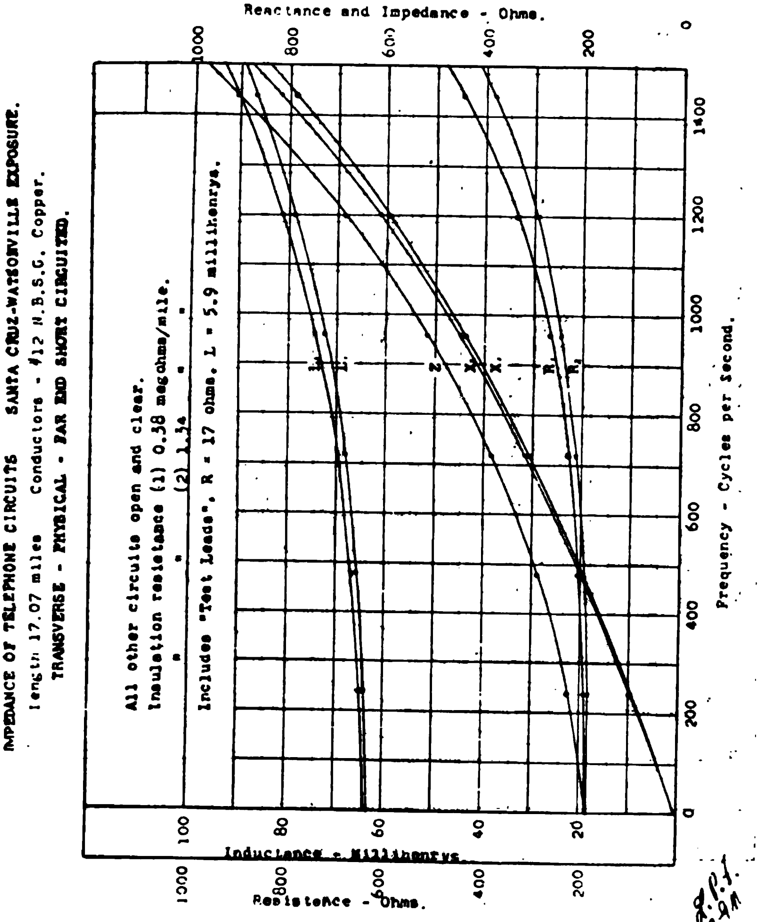
Respectfully submitted.
(Signed) J. E. Woodbridge,
Chairman Subcommittee on Tests No. 1 and No. 2.

Attached: P. I. C. Nos. 163 to 173 inc., 174A, 186, 187.

In Files of the Joint Committee: P. I. C. Nos. 77 to 94 inc., 95, 97, 109, 110, 111, 174, 175, 176, 177 and tables to accompany 163 to 177 inc.

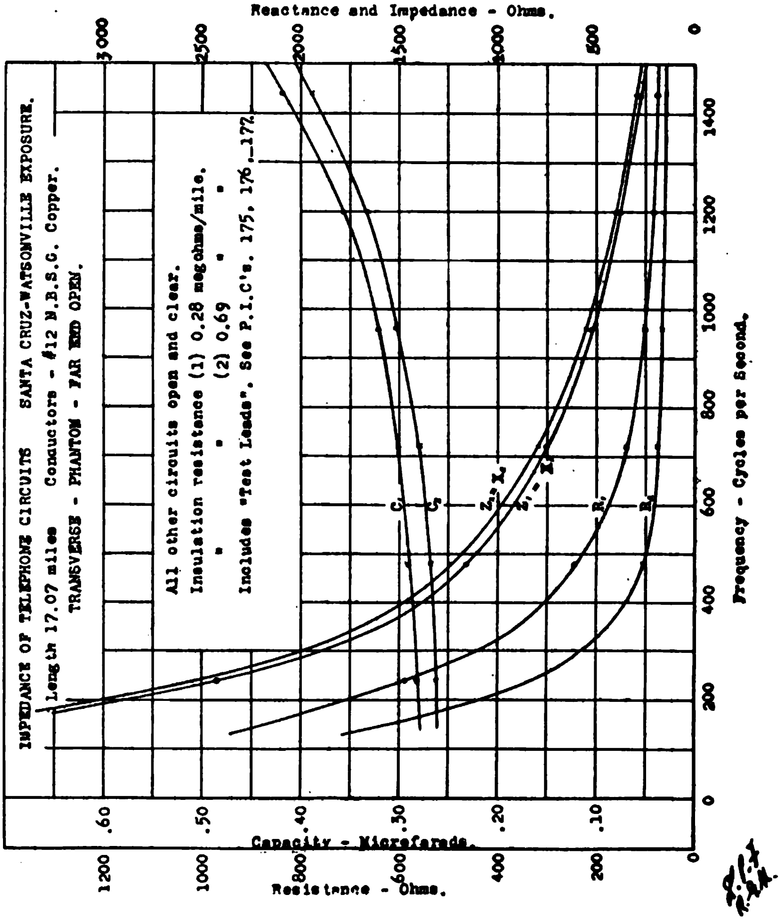

P.I.C. #163.
Jan. - 1914
IMPEDANCE OF TELEPHONE CIRCUITS SANTA CRUZ-WATSONVILLE EXPOSURE.
Length 17.07 miles Conductors - #12 N.B.S.G. Copper
TRANSVERSE - PHYSICAL - FAR END OPEN.
All other circuits open and clear.
Insulation resistance (1) 0.45 megohms/mile.
" " (2) 1.26 " "
Includes "Test Leads", See P.I.C's. 175, 176, 177.
Reactance and Impedance - Ohms.
Capacity - Microfarads.
Resistance - Ohms.
Frequency - Cycles per Second.

P.I.C. #164.
Jan. - 1914

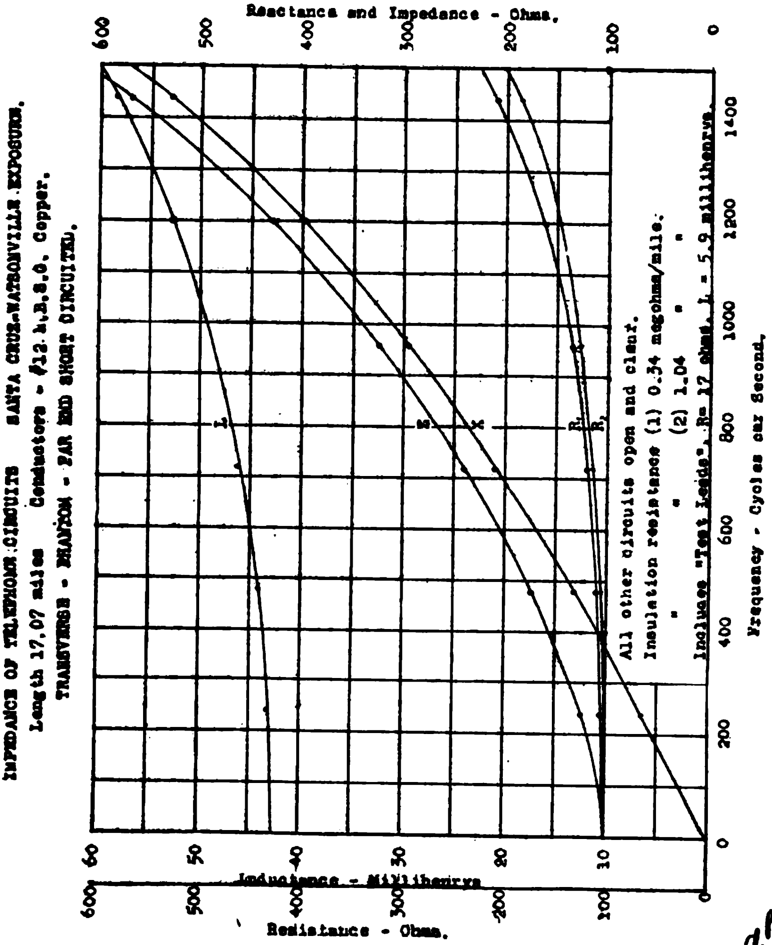


11—30069

P.I.C. #165.
Jan. 1914.

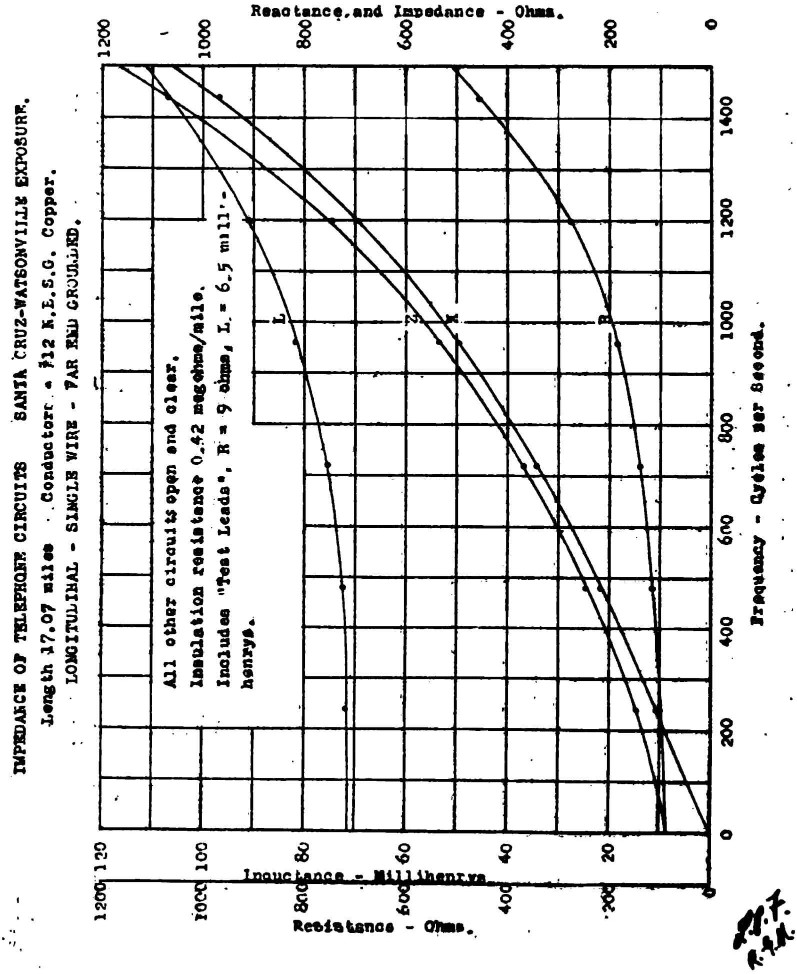


P.I.C, #166.
Jan, - 1914.

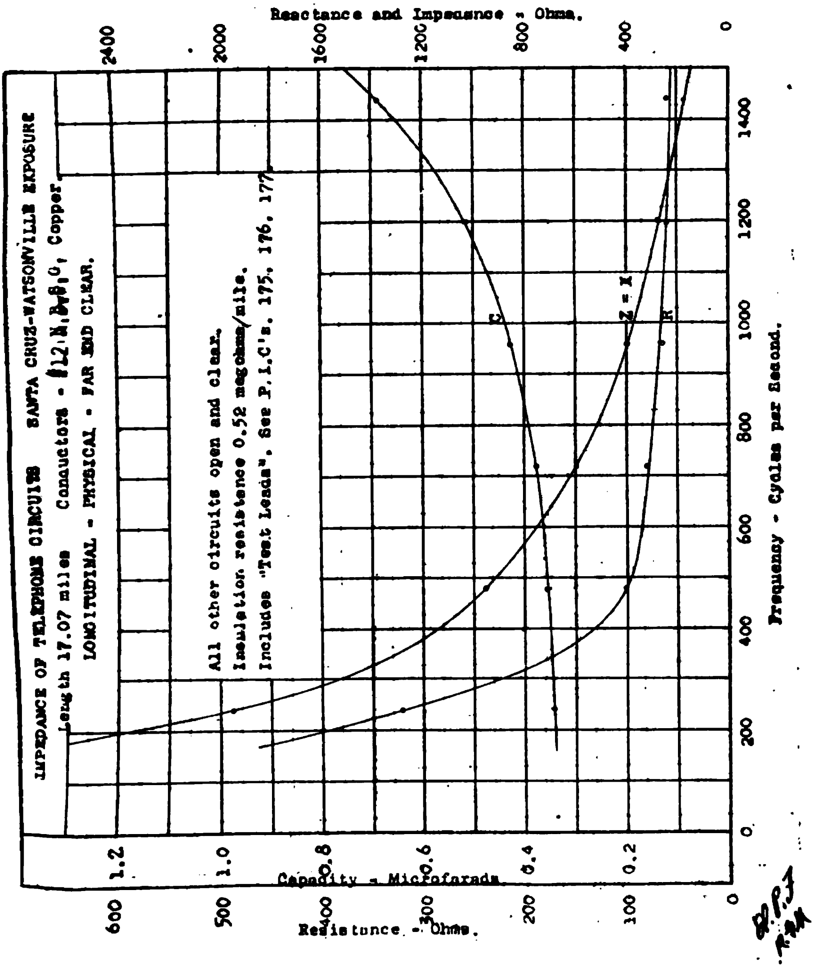


J.P.J.
R.A.M.

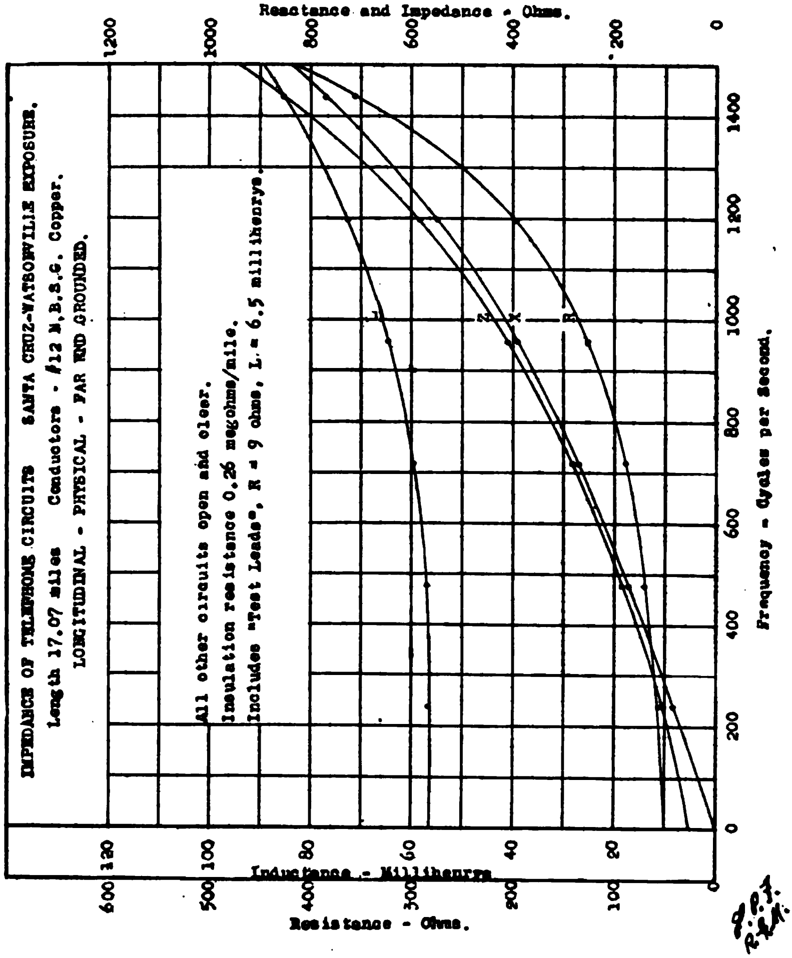
P.I.C. #167.
Jan. - 1914



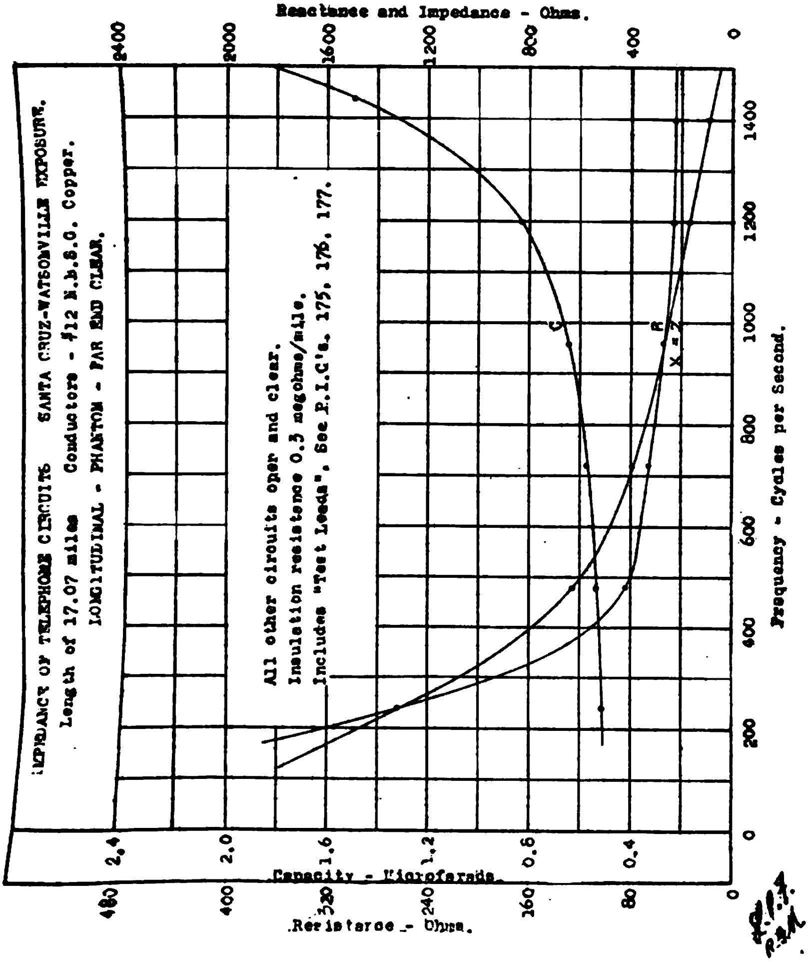
P.I.C. #168.
Jan. - 1914



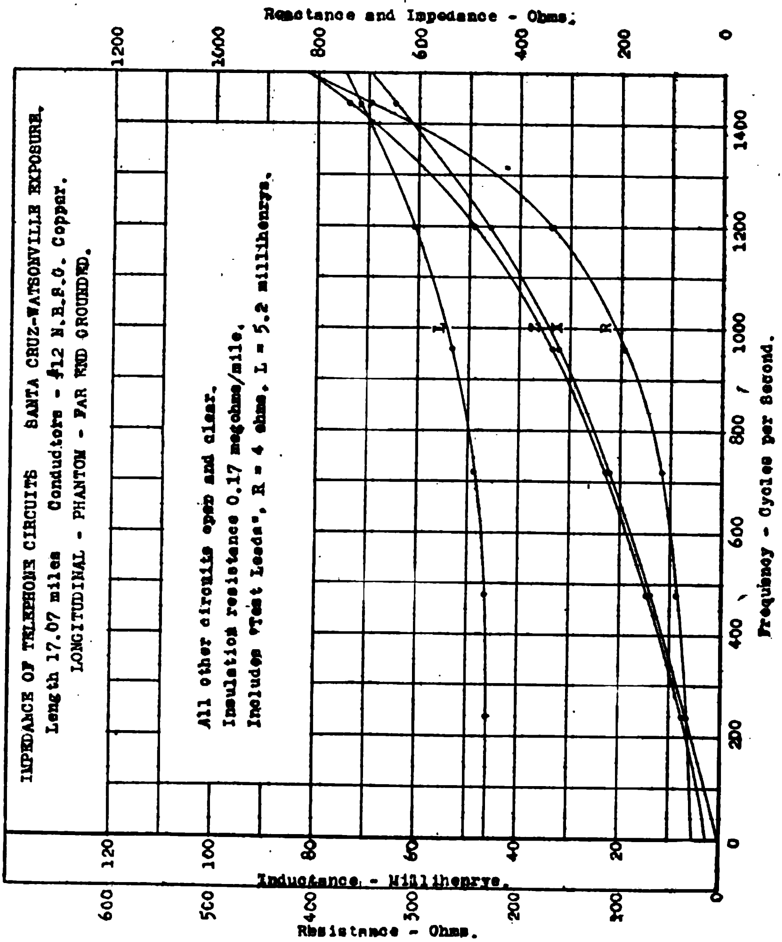
P.I.C. #169.
Jan. - 1914.

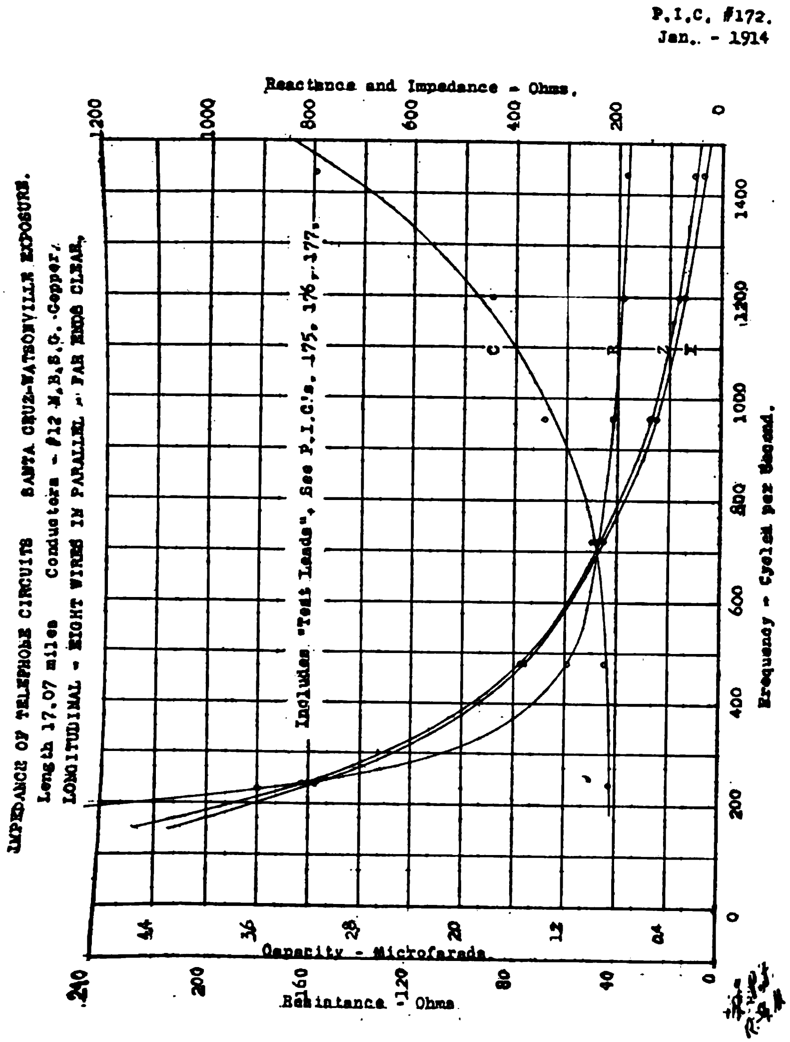


P.I.C. #170.
Jan. - 1914.



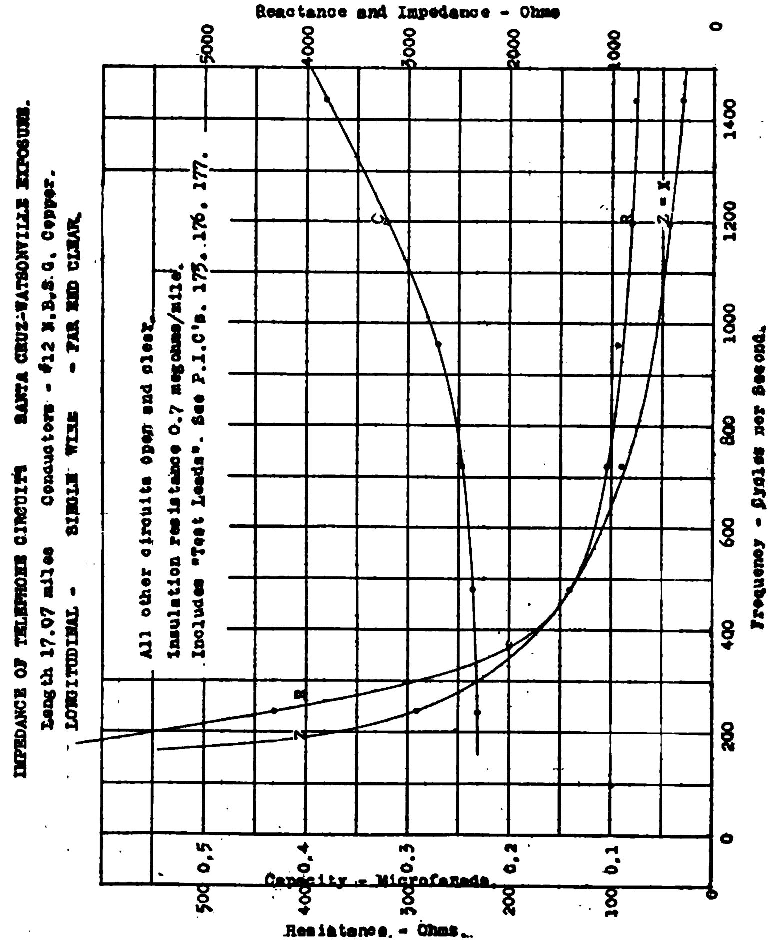
F.I.C. #171.
Jan. - 1914

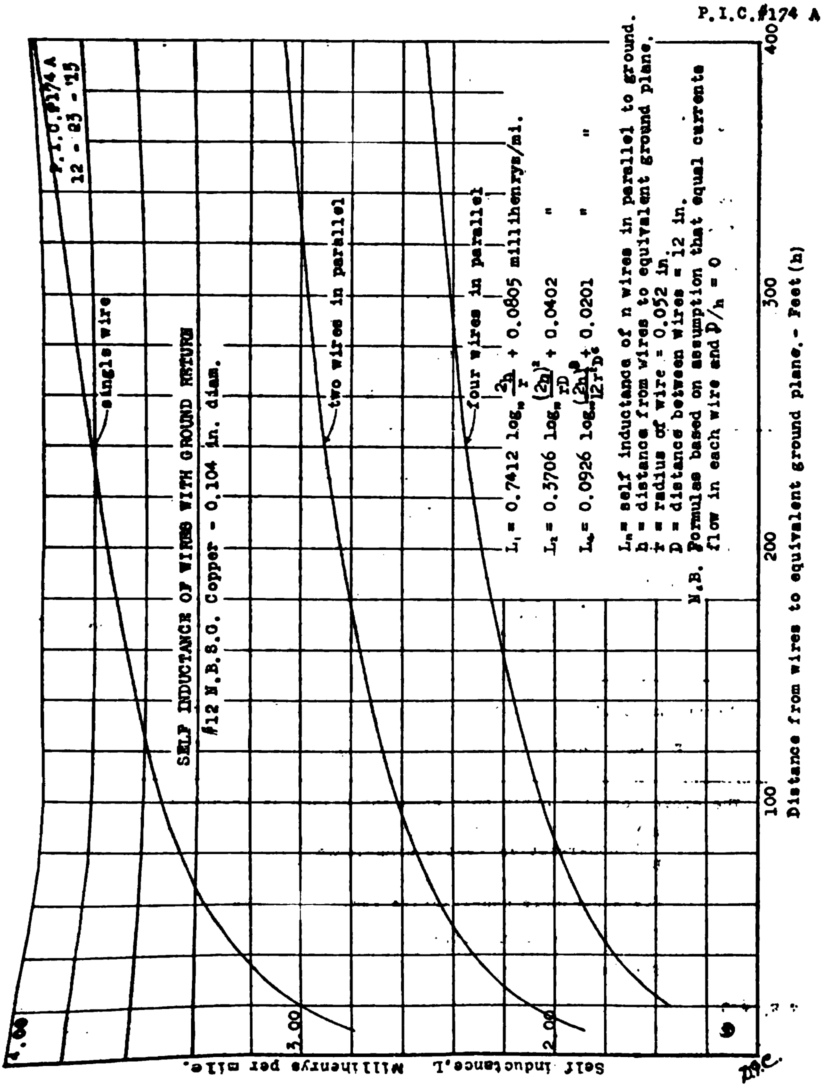



P.I.C. #172.
Jan. - 1914
IMPEDANCE OF TELEPHONE CIRCUITS SANTA CRUZ-WATSONVILLE EXPOSURE.
Length 17.07 miles Conductors - #12 N.B.S.G. Copper.
LONGITUDINAL - EIGHT WIRES IN PARALLEL - FAR ENDS CLEAR.
Includes "Test Leads", See P.I.C.'s. 175, 176, 177.
Reactance and Impedance - Ohms.
Frequency - Cycles per Second.
Capacity - Microfarads.
Resistance - Ohms
C
R
Z
X

13—20069

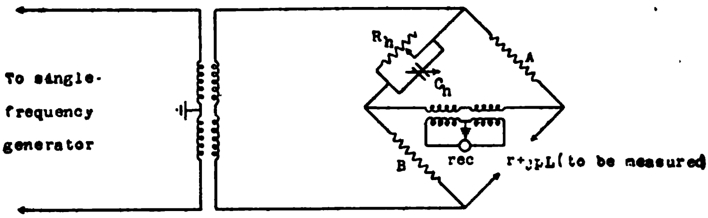
P.I.C. #173.
Jan. - 1914.




P.I.C. #174 A
P.I.C. #174 A
12 - 23 - '15
SELF INDUCTANCE OF WIRES WITH GROUND RETURN
#12 N.B.S.G. Copper - 0.104 in. diam.
single wire
two wires in parallel
four wires in parallel
$L_1 = 0.7412 \log_e \frac{2h}{r} + 0.0805$ millihenrys/mi.
$L_2 = 0.3706 \log_e \frac{(2h)^2}{rD} + 0.0402$ "
$L_4 = 0.0926 \log_e \frac{(2h)^8}{12 r^4 D^6} + 0.0201$ "
L_n = self inductance of n wires in parallel to ground.
h = distance from wires to equivalent ground plane.
r = radius of wire = 0.052 in.
D = distance between wires = 12 in.
N.B. Formulae based on assumption that equal currents flow in each wire and $D/h = 0$
Distance from wires to equivalent ground plane. - Feet (h)
100
200
300
400
Self inductance, L Millihenrys per mile.
2.00
3.00
4.00
D.C.

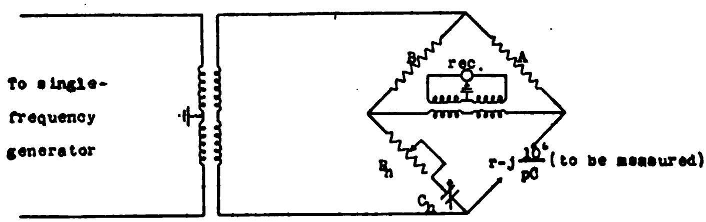
SCHEMATIC DIAGRAM OF IMPEDANCE BRIDGE

P.I.C. #186
5-12-14



Measurement of Impedance containing Inductive Reactance.

$r = \frac{AB}{R_h}$, ohms $\qquad L = C_h \cdot \frac{AB}{10^6}$, henrys



Measurement of Impedance containing Capacity Reactance.

$r = R_h \frac{A}{B}$, ohms $\qquad C = C_h \frac{B}{A}$, microfarads

The above formulae apply only when the bridge is balanced

$\left.\begin{array}{l} A \\ B \end{array}\right\}$ non-reactive "ratio arms", ohms

R_h = adjustable non-reactive resistance, ohms

C_h = adjustable capacity, microfarads

p = $2\pi \cdot$ frequency

T.R. #29

P. I. C. No. 187
Mar. 13, 1914.

Impedance of Circuits Having Uniformly Distributed Constants.

Nomenclature:

R = resistance, ohms per unit length of circuit.
L = inductance, henrys per unit length of circuit.
S = conductance, ohms per unit length of circuit.
C = capacity, farads per unit length of circuit.
p = 2π frequency.
l = length of the circuit.
P = propagation constant.
α = attenuation constant.
β = wave length constant.
Z_o = initial sending end impedance (also called surge impedance and iterative impedance).
Z_r = impedance of load at receiving end.
Z_1 = equivalent impedance of line and receiving end load.
Z_{oc} = Z_1 for the case when $Z_r = \infty$ (receiving end open).
Z_{sc} = Z_1 for the case when $Z_r = 0$ (receiving end short-circuited).
Z_2 = equivalent impedance of line, receiving end load and test leads.
Z'_{oc} = open circuit impedance of test leads.
Z'_{sc} = short circuit impedance of test leads.

Formulæ:

$$Z_o = \sqrt{\frac{R + jpL}{S + jpC}} \tag{1}$$

$$P = \sqrt{(R + jpL)(S + jpC)} = \alpha + j\beta \tag{2}$$

$$Z_1 = Z_o \frac{Z_r \cosh Pl + Z_o \sinh Pl}{Z_o \cosh Pl + Z_r \sinh Pl} \tag{3}$$

$$Z_{oc} = Z_o \coth Pl \tag{4}$$

$$Z_{sc} = Z_o \tanh Pl \tag{5}$$

For computation of the fundamental constants from measured values of Z_{sc} and Z_{oc}.

$$R + jpL = \frac{\sqrt{Z_{oc} \quad Z_{sc}}}{l} \tanh^{-1} \sqrt{\frac{Z_{sc}}{Z_{oc}}} \tag{6}$$

$$S + jpC = \frac{1}{l\sqrt{Z_{oc} \quad Z_{sc}}} \tanh^{-1} \sqrt{\frac{Z_{sc}}{Z_{oc}}} \tag{7}$$

To eliminate the effect of the impedance of the test-leads.

$$Z_1 = Z'_{oc} \frac{Z_2 - Z'_{sc}}{Z'_{oc} - Z_2} \tag{8}$$

Technical Report No. 38.

October 18, 1917.

GENERAL REVIEW OF TESTS AT SALINAS.

OUTLINE.

1. PURPOSE.
2. DESCRIPTION OF EXPOSURES.
3. TECHNICAL REPORTS (List).
4. EXPOSURE No. 1.
   (a) The Effect of Opening the Secondary Delta at Salinas (T. R. No. 6).
   (b) Effect of Grounded and Nongrounded Neutral on Telephone Circuits in Exposures No. 1 and No. 2 (T. R. No. 30).
   (c) Induction in telegraph circuits between Salinas and San Jose (T. R. No. 36).
5. EXPOSURE No. 2.
   (A) Tests Under Operating Conditions of the Power System.
      (a) Discussion of Reports.
         Technical Report No. 8.
         Technical Report No. 15.
         Technical Report No. 27.
         Technical Report No. 28.
         Technical Report No. 31.
      (b) Summary of Results.
   (B) Special Tests and Computations.
      (a) Tests of Known Residual Currents and Voltages (T. R. No. 9).
      (b) The Effect of Known Residual Current and Residual Voltage in Exposure No. 2 at Salinas (T. R. No. 23).
      (c) Computations of the Effect of Different Factors in the Production of Inductive Disturbances in Telephone Circuits for Exposure No. 2 at Salinas (T. R. No. 22).
      (d) Comparison of Technical Reports No. 22 and 23 (T. R. No. 24).
6. SALINAS-KING CITY EXPOSURE.
   (a) Induction in Telephone Circuits Between Salinas and King City (T. R. No. 25 and T. R. No. 32).
   (b) Induction in Block Signalling Circuits of Southern Pacific Co. (T. R. No. 7).
7. MISCELLANEOUS.
   (a) Neutral Current at Salinas (T. R. No. 10).
   (b) Comparative Noise Tests in Telephone Circuits North of Salinas. Sierra and San Francisco Power Company's 55-kv. Line Dead and Energized (T. R. No. 14).
   (c) Noise Tests on All Toll Circuits Radiating from Salinas (T. R. No. 37).
   (d) Arc Circuit Induction (T. R. No. 33).
   (e) Investigation of Current Transformers (T. R. No. 11 and T. R. No. 26).
   (f) Investigation of Potential Transformers. Measurement of Residual Voltage (T. R. No. 13).
   (g) Effect of Low Insulation (T. R. No. 34).
   (h) Formulæ for the Computation of Induction Between Power and Signalling Wires (T. R. No. 12).
8. CONDENSED SUMMARY OF CONCLUSIONS.
9. DEFINITIONS OF TERMS USED IN TECHNICAL REPORTS.
10. P. I. C. INDEX.

1. Purpose.

The object of this report is to review and discuss all the results obtained, which are recorded in various technical reports, and to present those conclusions which appear at this time to be justified by the investigation at Salinas.

2. Description of Exposures.

The several exposures are listed below, in tabular form, together with descriptive data of the particular parállels under test.* The following abbreviations are used in this tabulation:

S. and S. F. ---------------Sierra and San Francisco Power Company
C. C. G. and E. -------------Coast Counties Gas and Electric Company
C. V. G. and E. -------------Coast Valleys Gas and Electric Company
P. T. and T. ---------------The Pacific Telephone and Telegraph Company
W. U. ---------------------Western Union Telegraph Company
S. P. ---------------------Southern Pacific Railroad Company

Power company			Signal company	Miscellaneous data
Name	Type of system	Voltage		
Exposure No. 1—Between Edenvale and Salinas—Distance 50 Miles.				
S. & S. F.	3 phase grounded neutral	55 kv.	P. T. & T. W. U. S. P.	Exposures in this distance are of various lengths, as shown by P. I. C. No. 45. P. I. C. No. 119 (attached to this report) gives typical pole diagrams with separations, heights and circuit numbers, for exposures Nos. 1, 2 and 3.
C. C. G. & E.	3 phase isolated from ground	22 kv.	P. T. & T. W. U. S. P.	
Exposure No. 2—Lagunita to Salinas—Length of Exposure 8 Miles.				
S. & S. F.	3 phase star grounded neutral	55 kv.	P. T. & T.	P. I. C. No. 44 gives a diagrammatic plot of this exposure. Exposure No. 2 is a portion of Exposure No. 1, but P. T. & T. circuits are the only signal circuits involved. P. I. C. No. 61 shows the type and relative location of telephone and power transpositions.
Exposure No. 3—Salinas to King City—Length of Exposure 41 Miles.				
C. V. G. & E.	3 phase grounded neutral	22 kv. to Feb. 6, '13, 33 kv. from Feb. 6, '13.	P. T. & T. W. U. S. P.	
Exposure No. 4—Salinas to Monterey—Distance 20 Miles.				
C. V. G. & E.	3 phase grounded neutral	22 kv.	P. T. & T.	Noise tests only were made on this exposure.

*See Technical Report No. 66 for further description with diagrams.

3. Technical Reports.

The following is a list of the various technical reports covering the work at Salinas:

No.	Date presented	Title.
1	Jan. 6	General Outline of Test No. 2 to be Made at Salinas on Parallels Between the Sierra and San Francisco Power Company, the Western Union Telegraph Company, the Southern Pacific Company, and The Pacific Telephone and Telegraph Company.
6	April 7	Tests of the Effect of Opening Secondary Delta at Salinas, Feb. 3, 1913.
7	April 7	Tests on Southern Pacific Block Signalling Circuits, Jan. 20, 1913.
8	April 7	Test of the Effect of Opening and Grounding the Neutral at Salinas, March 14, 1913.
9	April 7	Tests of Known Residual Currents and Voltages, Jan. 29, 1913.
10	April 7	Neutral Current at Salinas.
11	April 7	Investigation of Current Transformers.
12	April 7	Formulæ for the Computation of Induction Between Power and Signalling Wires.
13	Sept. 3	Tests of Potential Transformers, April 20 and 23, and July 4, 1913.
14	May 5	Test of Noise on Telephone Circuits at Salinas, March 30 and April 13, 1913.
15	June 9	Effect of Grounding the Neutral of the Power Transformers at Salinas—Telephone Circuits Shielded, April 4, 1913.
22	June 9	Computation of the Effect of Different Factors in the Production of Inductive Disturbances in Telephone Circuits for Exposure No. 2 at Salinas.
23	June 9	The Effect of Known Residual Current and Residual Voltage in Exposure No. 2 at Salinas, May 10–11, 1913.
24	Sept. 3	Comparison of Technical Reports No. 22 and No. 23.
25	June 9	Induction in Telephone Circuits Between Salinas and King City, April 24, 1913.
26	June 9	Comparison of Current Transformers for Residual Current.
27	July 14	Induction in Telephone Circuits in Exposure No. 2 at Salinas with the North Beach Steam Station Energizing the Power System, May 16 and 17, 1913.
28	Sept. 3	Induction in Telephone Circuits in Exposure No. 2. Five Per Cent Lower Voltage Taps at Guadalupe, May 19, 1913.
30	Sept. 3	Effect of Grounded and Nongrounded Neutral on Telephone Circuits in Exposures No. 1 and No. 2, June 13, 1913.
31	Sept. 3	Effect of Grounding the Neutral of the Power Transformers at Salinas—Telephone Circuits Shielded and Nonshielded, June 13 and 14, 1913.
32	Sept. 3	Induction in Telephone Circuits Between Salinas and King City, June 18, 1913.
33	Sept. 3	Induction in Test Leads Used at Salinas for Connecting the Testing Apparatus to the Toll Circuits of Exposure No. 2 and the Effect of Such on the Measurements of Induction from this Exposure, June 26 and 27, August 25 and 28, 1913.
34	Sept. 3	Effect of Low Insulation Resistance on the Induction in Exposure No. 2, at Salinas, June 30, 1913.
36	Sept. 3	Induction in Telegraph Circuits Between Salinas and San Jose, January 16 to February 3, 1913.
37	Sept. 3	Noise Tests on All Toll Circuits Radiating from Salinas, July 2 and 3, 1913.

4. Exposure No. 1.

(*a*) *The Effect of Opening the Secondary Delta at Salinas* (T. R. No. 6).

This test was made on telegraph and telephone circuits of exposures No. 1 and No. 2 to determine the effect of opening the secondary delta winding of the load transformers at Salinas. During this test the 55-kv. power line between Guadalupe and Salinas was energized from Guadalupe in the normal way. All load was removed from the power transformers at Salinas, both from the taps on the high potential star connected winding and from the 2,300 volt delta connected winding. The neutral of the transformers was grounded and one corner of the delta winding was opened. This condition does not exist at Salinas on the system of the Sierra and San Francisco Power Company with the present method of operation at that point. It was introduced for experimental purposes. The object of this test was to obtain some data as to the effects of operating a transmission line with the secondary delta of the transformers open. This type of connections offers a high impedance secondary path to the third harmonic of the magnetizing current.

By comparison with the data of other tests at Salinas this test shows that opening the secondary delta caused—

A large increase in the residual voltage and residual current of the 55-kv. line, particularly the third harmonic.

An increase in the longitudinal, induced voltage in the telegraph circuits in exposure No. 1.

An increase in the longitudinal, induced voltage in the telephone circuits in exposure No. 1 and No. 2 combined.

An increase in the longitudinal, induced voltage in the telephone circuits in exposure No. 2, measured with the far end of the circuit open.

(No data are recorded of the effect on the circuits in exposure No. 2 alone measured with the far end of the circuits grounded.)

Predominant third harmonic in the induced voltage in the signalling circuits, both longitudinal and transverse.

(*b*) *Effect of Grounded and Nongrounded Neutral on Telephone Circuits in Exposure No. 1 and No. 2* (T. R. No. 30).

This test was made to determine the effect of grounding the neutral of the autotransformers at Salinas on the induction in the telephone circuits which parallel the 55-kv. line of the Sierra and San Francisco Power Company in exposures Nos. 1 and 2. It should be noted that these telephone circuits are also paralleled by the 22-kv. line of the Coast Counties Gas and Electric Company for a part of their length, and this 22-kv. line was presumably in normal operation during the tests. Telephone circuits other than the test circuits were in normal operation.

The effects of grounding the neutral of the autotransformers at Salinas may be summarized as—

> No noticeable change in the magnitude of the noise in the telephone circuits.
>
> A slight change in the quality of the noise in the telephone circuits.
>
> Increased longitudinal induction in the telephone circuits measured with the far end of the circuits open.
>
> Large increase in ninth harmonic in the transverse induction both with the far end of the circuit open and with it shorted through a subscriber's set.

For either condition of the neutral the following observations were noted:

> The noise in the telephone circuits was greater with the far end of the circuit open than with it shorted through a subscriber's set.
>
> The presence of prominent third and ninth harmonics in the transverse induced voltage indicates that either the residual voltage or the residual current or both are important in the production of noise in these exposures.
>
> No definite statement can be made as to the magnitude of the induction due solely to the 55-kv. line of the Sierra and San Francisco Power Company in exposure No. 1 because the telephone circuits under test are paralleled also by the 22-kv. circuit of the Coast Counties Gas and Electric Company from Morgan Hill to Sargent. Exposure No. 2 is not subject to this difficulty.

(c) *Induction in Telegraph Circuits Between Salinas and San Jose* (T. R. No. 36).

These tests were made at Salinas on the telegraph circuits of the Western Union Telegraph Company and the Southern Pacific Company in exposure No. 1, to determine the magnitude of the induction in these circuits. The telegraph circuits under test are paralleled in this exposure by the 22-kv. power line of the Coast Counties Gas and Electric Company from Morgan Hill to Sargent, and the 55-kv. power line of the Sierra and San Francisco Power Company from Edenvale to Gilroy and from Carnadero to Sargent.

On account of the incompleteness of the measurements of the induced voltages and currents in the telegraph circuits, it is impossible to draw any conclusions as to the absolute value of the induction, and the effect of either of the power lines on the telegraph circuits. The fundamental and third harmonic frequencies are predominant in the voltage induced in the telegraph circuits.

A test was also made to determine the effect of opening one corner of the secondary delta of the transformers at Salinas. The results of this test are given in the discussion of technical report No. 6.

5. Exposure No. 2.

A—Tests Under Operating Conditions of the Power System.

(Technical Reports 8, 15, 27, 28 and 31.)

(a) Discussion of Reports.

(T. R. No. 8) This report records the results of the first of a number of tests made upon this exposure under operating conditions of the power system. The object of this test was primarily to determine the effect of the condition of the neutral, grounded or nongrounded, of the autotransformers on the induction in the telephone circuits. The power system was energized in the normal manner. The neutral was grounded during the first series of tests. The same tests were repeated later with the neutral ground connection removed. The telephone circuits not under test were open and clear at both ends of the exposure, thus exercising the minimum shielding effect upon the test circuits. Mention of this fact was unintentionally omitted from the report.

A close study of this report in connection with other and later reports made on this exposure shows several inconsistencies in the data and conclusions. These are pointed out below.

> In the noise tests the amount of noise measured on circuit No. 1673 appears to be much greater than in other tests and the marked effect of grounding the neutral has not been observed since. In explanation, it is thought that the circuit or apparatus was grounded at some point.
>
> In table 4 (*c*) the short circuit current recorded for circuit No. 151 at 12.45 a.m. is 5 milliamperes. The circuit was supposed to be clear at the far end. Such a large current has not been observed for the same conditions in other tests and is very large compared with that observed for similar circuits and conditions in this test. A ground on the circuit is supposed to have been the reason for the large current obtained. In the same table the value of short circuit current for circuit No. 159 at 4.45 a.m. is given as 3.4 milliamperes. The far end of this circuit was supposedly grounded. It is evident by comparison of the data for this and later tests that the far end of this circuit must have been clear and not grounded as reported, probably a mistake on the part of the lineman at the distant end.
>
> It is quite evident now that the residual current as given by oscillogram No. 176 is incorrect. The oscillogram shows the wave forms of the residual current and line current to be practically the same. An error in connections probably accounts for this result. The induced currents and voltages on circuit No. 151, as given by this same oscillogram, appear rather small for the conditions as stated and are therefore doubtful.
>
> On oscillogram No. 174, the amplitude and wave form of the current in circuit No. 269 indicate that the circuit was in reality grounded at the far end, and not clear as reported.

Many of the conclusions given in technical report No. 8 rest upon these results and are therefore regarded as erroneous or doubtful. This applies, in particular, to the conclusions in regard to the effect of grounding the neutral of the power transformers on the induction and upon the power system itself. As a number of other and more reliable tests have been made on this exposure, reference should be made to the results of those tests in regard to this question. It should be noted that means have since been provided for quickly checking the condition of the test circuits, thus avoiding some of the mistakes to which attention has been called.

(T. R. No. 15) The tests described in this report were made to determine the relative induction in the telephone circuits under the conditions of grounded and nongrounded neutral of the autotransformers. They were intended to be complementary to those described in technical report No. 8, and differed from the earlier tests in that the circuits not used for testing purposes were grounded at the far end of the exposure, thus exercising an electrostatic shielding effect upon the test circuits. Conditions on the power system were the same as obtained in the former tests.

As shown by the tests of this date, the condition of the neutral made no appreciable difference in the magnitude or character of the noise in the telephone circuits. In the tests reported in technical report No. 8, a considerable difference in the magnitude and character of the noise was noted for the two conditions of the neutral, grounded and nongrounded. At the time technical report No. 15 was prepared the difference in the effects of grounding the neutral as observed in the two tests could only be ascribed to the effect of electrostatic shielding. As this was not considered a satisfactory explanation of this difference, further tests were deemed advisable. These further tests have confirmed the assumption that shielding was not the cause of the difference in the effects noted, which can now be explained on the basis of errors discovered or assumed in the earlier tests.

On account of the marked difference in the results of the two tests, and having no satisfactory explanation of this difference, no attempt was made in technical report No. 15 to discuss in detail the effect of grounding the neutral. This omission was, also, partly due to the difficulty and uncertainty of comparisons owing to the length of time between comparable observations. The effects of grounding the neutral as shown by an inspection of the tables in this report are—

By comparison of resonant analyses:

A marked decrease in the prominent harmonics, third and ninth, of the longitudinal induction both electromagnetic and electrostatic;

A decrease in the third harmonic of the transverse electrostatic induction:

A decrease in the third and ninth harmonics of the residual current;

By comparison of oscillograms:

An increase in the fundamental and a decrease in the third and ninth harmonics of the longitudinal induction, both electromagnetic and electrostatic;

An increase in the fundamental, a decrease in the third harmonic and a slight increase in the ninth harmonic of the residual current. It should be noted that this effect on the ninth harmonic does not agree with the effect noted by the resonant shunt. The two methods are in agreement as to the other points mentioned. It should be remembered that the fundamental is not measured by the resonant shunt.

In connection with technical report No. 15 there is given a discussion of the effects of transpositions in the power and telephone circuits. The results of a study of the existing transpositions and of a modified system designed to reduce the induction are given. As some of the principles outlined in this discussion are fundamental to this and other exposures, no attempt is made to abstract them here. Reference should be made to the full discussion in technical report No. 15.*

(T. R. No. 27) During these tests the power system was energized from the steam station at North Beach, San Francisco, and measurements made of the induction with the neutral of the autotransformers at Salinas grounded and nongrounded. The tests are complementary to those recorded in technical reports No. 8 and No. 15, except in regard to the power system energy supply.

No characteristic change attributable to the North Beach station was observed in power line voltages and currents, except in the neutral current, in which the third harmonic is reduced to one-half its normal value.

No effects of shielding were observed.

The line voltage to ground at Salinas was reduced 3%, for a short time, by lowering the generator voltage at North Beach, and its effect on the neutral and line currents observed. For the discussion of these data, see review of technical report No. 28, below.

(T. R. No. 28) To determine the effect on the induction of lower flux density in the Salinas autotransformers, and all the transformers supplied from them, these tests were made with the power line voltage reduced 5% by change of taps at Guadalupe.

The results indicate that no marked change in the power line voltages and currents, or in the induction, occurred as a result of this alteration in the operating condition.

The method of lowering the voltage here used does not alter the flux density of the Guadalupe transformers, and no effect was observed on

*See T. R. No. 66.

the neutral current at Salinas. Lowering the voltage at North Beach, as described in technical report No. 27, caused a reduction in the primary voltage and the flux density of the Guadalupe transformers. The neutral current at Salinas in this case was lowered 25% in effective value, and the third and ninth harmonics, about 40%. This reduction, therefore, appears to be due to a lowering of the flux density in the generators and main transformers of the system. Unfortunately, no data on residual current were obtained, under the lowered voltage condition.

(T. R. No. 31) The object of the test was to determine the features and relative magnitude of the induction under the conditions of grounded and nongrounded neutral, the effect of the condition of the neutral on the power system itself, and the effect of electrostatic shielding on the telephone circuits.

During the tests the power system was energized from its normal source of supply. The neutral was grounded or ungrounded at will, thus obtaining comparable observations with a minimum of elapsed time. The results of this test can, therefore, be relied upon to show the effect of the condition of the neutral with greater accuracy than those of previous tests. In other respects, too, this report gives more complete information in regard to the different features of the induction.

Reference to the rather complete discussions contained in the report can profitably be made in considering this general review of the results obtained at Salinas.

(*b*) *Summary of Results* (T. R. No. 8, No. 15, No. 27, No. 28 and No. 31).

The most reliable information, gathered from the results of the various tests, as to the effect of grounding the neutral of the Salinas autotransformers, on the power system, and on the induction in the telephone circuits of this particular exposure (No. 2 at Salinas) is summarized in the following table. It can not be said that these results will obtain for the exposure under any other condition of the power system, and these facts can not be generalized to apply to any other exposure.

Inspection of the table shows, that under existing conditions, for this exposure, the induction in the telephone circuits is somewhat less with the neutral at Salinas grounded than with it nongrounded.

Effect of Grounding Neutral of Salinas Autotransformers.

Power system	Current	Line	Eff. value	No effect.
			Harmonics	No appreciable effect.
		Residual	Eff. value	—10%.
			Harmonics	1+, 3—, 9—.
	Voltage	Line	Eff. value	No effect.
			Harmonics	No appreciable effect.
		Residual	Eff. value	Probably decreases.
			Harmonics	1—, 3—, 9—.
Telephone system	Electro-magnetic induction	Transverse	Noise	—20%.
			Harmonics	3—, 9—.
		Longitudinal	Eff. value	—50%.
			Harmonics	3—, 9—, 15+.
	Electro-static induction	Transverse	Noise	No effect.
			Harmonics	No effect.
		Longitudinal	Eff. value	Slight decrease.
			Harmonics	3—, 9 inconsistent.

(Negative sign indicates a decrease.)

The following table of the prominent harmonics, in the currents and voltages of the power system and in the induction in the telephone circuits, is valuable in showing the relative importance of different factors in producing the induction observed.

Prominent Harmonics.

Power system	Currents	Line	1, 3, 5, 7, 11, 13.
		Residual	1, 3, 9.
	Voltages	Line	1, 5, 7, 11.
		Residual	1, 3, 9.
Induction in telephone system	Electro-magnetic	Transverse	3, 9 (1 not measured).
		Longitudinal	1, 3, 9.
	Electro-static	Transverse	5, 7 (1 not measured).
		Longitudinal	3, 9 (1 not measured).

The fundamental and the third and ninth harmonics are prominent in the neutral current. The effect of the neutral in so far as this exposure is concerned is through the residual current of the 55-kv. line. The above tabulation of prominent harmonics holds for either condition of the neutral, grounded or nongrounded, though the relative magnitudes of these harmonics do not remain the same under the two conditions.

It is to be noted that the line currents and voltages exhibit chiefly the characteristics of the balanced currents and voltages owing to the predominance of the balanced over the residual components.

From a comparison of the prominent harmonics of the power and telephone systems (currents and electromagnetic induction, voltages and electrostatic induction) the following conclusions are drawn: (*a*) the transverse and longitudinal electromagnetic induction is due largely to the residual current; (*b*) the transverse electrostatic induction is due largely to the balanced voltages; (*c*) the longitudinal electrostatic induction is due largely to the residual voltage.

Values of the mutual inductance, computed on the assumption that the total third and ninth harmonic voltages of the longitudinal electromagnetic induction arise entirely from the residual current, are of the same order of magnitude as those obtained by the residual current test described in technical report No. 23 and therefore confirm the above statement as to the origin of longitudinal electromagnetic induction.

The results of various analyses of the residual current are collected in a table below. Similar collections made for residual voltage and for neutral current are given under 7 (*f*) and 7 (*a*) respectively.

Oscillograms of Residual Current—55-kv. Line. Milliamperes. Neutral Nongrounded.

Date, 1913	T. R. No.	Osc. No.	Order of harmonic								
			1	3	5	7	9	11	13	15	17
March 15	8	176	600	6	13	9	4	10	11	4	3
June 3	15	189	58	136	16	*	82	6	*	6	*
May 17	27	252	61	130	15	17	86	*	12	12	13
May 17	27	256	59	146	13	*	99	20	*	13	*
May 19	28	269	61	166	17	*	122	*	14	*	*
May 19	28	272	57	188	17	13	124	12	3	9	*
June 14	31	282	61	200	18	21	140	15	16	*	*
June 14	31	284	46	174	*	14	92	*	*	15	*
June 14	31	287	56	170	*	12	94	*	*	*	*
Average			57	160			100				

* Oscillogram No. 176 in error, not included in the average.

Oscillograms of Residual Current—55-kv. Line. Milliamperes. Neutral Grounded.

Date, 1913	T. R. No.	Osc. No.	Order of harmonic 1	3	5	7	9	11	13	15	17
March 15	8	172	23	28	2	3	16	3	2	*	1
June 3	15	191	70	92	8	6	96	5	*	5	*
May 16	27	241	150	60	*	19	78	12	*	*	*
May 17	27	260	200	68	*	21	66	12	*	10	*
May 19	28	265	180	40	11	18	87	12	11	14	*
May 19	28	276	176	30	*	9	89	*	*	*	*
June 14	31	281	129	78	*	23	84	*	*	14	*
June 14	31	283	150	40	*	20	66	*	*	*	*
June 14	31	286	150	23	*	*	69	*	*	*	*
June 30	34	310	78	61	*	7	73	*	11	8	*
June 30	34	313	69	73	11	19	71	*	8	8	*
June 30	34	314	81	74	10	20	74	9	14	9	*
June 30	34	318	81	54	*	*	77	*	11	13	*
June 30	34	321	100	48	12	*	70	*	*	11	*
Average			120	54			73				

The lengths of the unbalanced exposures, resulting from the existing transpositions in the power and telephone circuits, have been computed and a study has been made to see how the present transposition system can be modified so as to reduce the transverse induction. The following table gives the lengths of unbalanced exposures due to the present system.

Type of induction	Residual current and voltage North, feet	Residual current and voltage South, feet	Balanced current and voltage North, feet	Balanced current and voltage South, feet
Longitudinal	23,400	18,000	4,500	2,600
Transverse-Phantom No. 1680	+2,300	—2,300	10,200	11,500
Transverse-Phantom No. 1673	+2,600	—2,700	12,000	11,500

The exposure is divided into two parts, "North" and "South," with reference to the crossover between the two lines. The phase angles of induction are different for these different sections, and they are, therefore, separately considered.

The unbalanced exposures, resulting from the proposed system of transpositions designed with particular reference to the transverse unbalance, are tabulated below. These unbalances are worked out for the balanced currents and voltages only. Included in the table in order to facilitate comparison are the corresponding unbalances of the existing system.

Type of induction	Present power transpositions		Proposed power transpositions	
	North	South	North	South
Longitudinal	4,500	2,600	2,300	0
Transverse-Phantom No. 1680	10,200	11,500	2,400	0
Transverse-Phantom No. 1673	12,000	11,500	2,200	3,200

The proposed system of transpositions requires that the existing transpositions of the power line be removed and that the line be re-transposed, locating power transpositions opposite the following telephone poles: 49/33, 50/16, 51/4, 51/26, 52/11, 53/19, 54/4, 54/24, 55/8, 55/23.

It is very probable that an application of this proposed system of power transpositions would greatly reduce the transverse induction in the telephone circuits resulting from this exposure and would give much valuable information regarding the effect of a proper relative location of power and telephone transpositions. A more nearly complete discussion of the points to be considered in the re-design of the present transposition system is given in the resumé of technical report No. 15. It should be noted that it was impracticable to try out this proposed system of transpositions owing to the fact that the power line could not be shut down for a sufficient length of time.*

No marked change in the power line voltages and currents or in the induction were observed with 5% reduction in the line voltage from Guadalupe south, obtained by changing the autotransformer taps.

With the North Beach steam station energizing the power system, and its voltage reduced so as to cause a 3% reduction at Salinas there was a marked reduction (40% in third and ninth harmonics) in the neutral current.

With the North Beach steam station energizing the system at normal voltage the only change observed, which could be attributed to the difference in generators, was a reduction of about one-half in the value of the third harmonic in the neutral current.

With the Monterey steam station energizing the autotransformers there was a noticeable lack of high harmonics in the line voltages and currents.

In regard to shielding the test circuits electrostatically by grounding the other circuits on the telephone lead at one end, there is some indication that both longitudinal and transverse electrostatic induction are somewhat less with shielding. No accurate estimate can be made as to

*See T. R. No. 66.

the magnitude of this reduction owing to the long elapsed time between comparable observations, with consequent change in other conditions.

No data were obtained in this exposure regarding electromagnetic shielding.

On this length of exposure, namely 8 miles, it is possible to measure separately, under different circuit conditions of the signalling circuits, the electrostatic and electromagnetic induction. This statement should be interpreted to mean that certain circuit conditions and apparatus favor the predominance of one factor, while making the other negligibly small. It will not in general be possible to differentiate in this manner between electromagnetic and electrostatic induction, in longer exposures.

From noise measurements, the ratio of the electromagnetic to the electrostatic transverse induced current is from one-third to one-half. The effective value of the longitudinal voltage, arising from electrostatic induction, is much greater than that due to electromagnetic induction. Resonant analyses show that this relation holds for the third harmonic voltage; however, in general, the reverse appears to be true, particularly for the ninth and higher harmonics. For transverse induction the resonant analyses show that the harmonic voltages due to the electrostatic effect of the power system, are, in general, greater than those due to the electromagnetic effect.

It must be noted, in considering the above paragraphs, that the relative importance of electrostatic and electromagnetic induction in causing a disturbance in a circuit under commercial operation, is not given by the ratio of the induced voltages arising from the two types of induction. This depends, not only upon the induced voltages, but also upon the impedance of the nonexposed sections of the circuit (including terminal apparatus), and upon the open and short circuit impedances of the exposed section. The induced voltages are, of course, of fundamental importance. These considerations are in line with the report of the Program Committee, presented on April 7, as to the necessity of investigating the effect of nonexposed sections of circuit and terminal apparatus in reducing disturbances arising from a given exposure.

The introduction of the terminal apparatus described in technical report No. 8 (see P. I. C. No. 42), between the circuit under test and the receiver of the noise standard, caused a reduction in the noise amounting to approximately 50% of that measured without this terminal apparatus. It is planned to obtain much more complete information on this subject in carrying out the suggestions of the Program Committee.

B—Special Tests and Computations.

(Technical Reports 9, 22, 23, 24.)

(a) *Test of Known Residual Currents and Voltages.* (T. R. No. 9).

The results of this test indicated discrepancies between the measured and computed values of the voltages in the telephone circuits induced by residual currents and residual voltages, in exposure No. 2 at Salinas. The data acquired were not sufficient to warrant final conclusions, so that it was decided to supplement it by further tests. This was done as reported in technical report No. 23.

(b) *The Effect of Known Residual Current and Residual Voltage in Exposure No. 2 at Salinas* (T. R. No. 23).

RESIDUAL CURRENT TEST.

The data derived from this test show that the longitudinal induced voltage on the telephone circuits in this exposure corresponding to any harmonic, is 2.8n volts per ampere of the same harmonic in the residual current of the power line. n is the order of the harmonic, being unity for the fundamental frequency.

RESIDUAL VOLTAGE TEST.

The data derived from this test show that the longitudinal induced voltage on the telephone circuits, in this exposure, corresponding to any harmonic is 1.5 per cent of the same harmonic in the residual voltage of the power line.

The above coefficients, for electromagnetic and electrostatic induction, were derived with the other circuits on the telephone lead open and clear.

(c) *Computation of the Effect of Different Factors in the Production of Inductive Disturbances in Telephone Circuits for Exposure No. 2 at Salinas* (T. R. No. 22).

From the formulæ, quoted in technical report No. 12*, the coefficients for longitudinal and transverse electromagnetically and electrostatically induced voltages due to balanced and residual currents and voltages have been computed for exposure No. 2 at Salinas.

The computations are based on an average, uniform separation between the power and telephone lines, uniform configuration of the two lines, uniform height above ground for the conductors, and do not take into account the shielding effects of neighboring objects. The

*A more complete collection of formulas is given in T. R. No. 64.

surface of the earth† was taken as the neutral plane for the determination of the positions of the images.

(*d*) *Comparison of Technical Reports No. 22 and No. 23* (T. R. No. 24).

The principal points to be noted in comparing technical reports No. 22 and No. 23 are the discrepancies between the computed and measured coefficients of induction. The computed electromagnetically induced voltage is less than that which was measured. The computed electrostatically induced voltage is greater than that which was measured.

The discrepancies in the coefficients for electrostatic induction are probably due to the shielding effects of neighboring objects.

The discrepancies in the coefficients for electromagnetic induction are probably due to the return path of the current through the earth being at a considerable depth below the earth's surface.

The computed coefficients, for electromagnetic induction, have been revised to accord with actual values determined by test. The revision was accomplished by assuming images in accord with the equivalent depth of earth return currents indicated by the tests recorded in technical report No. 23. The most probable coefficients, thus derived, are tabulated in technical report No. 24. With these revised coefficients it is possible to estimate the longitudinal voltages arising from electrostatic and electromagnetic induction in a telephone wire in exposure No. 2 for given values of harmonics in the residual voltages and currents of the power system.

6. Exposure No. 3, Between Salinas and King City.

(*a*) *Induction in Telephone Circuits Between Salinas and King City* (T. R. No. 25 and T. R. No. 32).

The purpose of these tests was to determine the disturbances in the telephone lines of this exposure under normal operating conditions of the power line, with the neutral of the Salinas autotransformers either grounded or nongrounded, and the effect of partially shielding the telephone circuits under test by grounding other wires on the lead.

Almost all the evidence regarding the effect of the neutral connection is from technical report No. 32, as in the tests there reported it was possible to ground or unground the neutral at will throughout the tests, while in the tests of technical report No. 25, it was nongrounded for one interval of a few minutes only. A few observations recorded in technical report No. 25, with one test circuit grounded at both ends, show apparently considerable electromagnetic shielding of the others. Enough has not been done to warrant any conclusions regarding this type of shielding.

†An erroneous assumption for magnetic induction. See T. R. No. 64.

It was necessary to compute the telephone line impedances, as noise on the lines prevented measurement. The line is a quarter wave-length for a frequency near the fifteenth harmonic, so due to the large variations in impedance for small changes in frequency, and the approximations involved in assuming line constants there may be considerable error in the results for the fifteenth and neighboring harmonics.

In computing the results of the tests, the impedances of the telephone circuits have been assumed the same, whether the circuits were shielded or not. This assumption will introduce some error, particularly in considering the effect of shielding on the induction.

No study has been made of the transpositions in the power and telephone circuits, and no computations made regarding the relative importance of various factors in the induction.

The exposure is relatively long (41 miles) and the total capacity and leakage of the telephone circuits are large. Hence electromagnetic and electrostatic effects are observed with the far end of the circuit either open or closed, and it is impossible to differentiate between them in the measurements.

No characteristic effect of the condition of the neutral on the line currents in the power system is evident, as may be seen from oscillograms of line current.

The following table gives the characteristics of residual current in the 33-kv. line.

Residual Current. Amperes. Neutral Nongrounded.

Date, 1913	T. R. No.	Osc. No.	Order of harmonic							
			1	3	5	7	9	11	13	15
April 25	25	205	0.077	0.093	0.004	0.009	0.056	0.004	0.004	0.003
June 18	32	291	0.074	0.150	0.024	0.019	0.100	*	0.015	0.014
June 18	32	301	0.069	0.120	0.016	*	0.070	*	0.019	*
Average			0.073	0.120	0.015		0.075		0.018	
Residual Current. Amperes. Neutral Grounded.										
April 25	25	204	0.069	0.073	*	0.014	0.123	0.012	*	*
April 25	25	207	0.065	0.083	0.010	0.017	0.140	0.008	*	*
June 18	32	290	0.050	0.068	*	0.020	0.130	*	0.014	*
June 18	32	300	0.059	0.075	0.011	*	0.100	*	0.010	*
Average			0.061	0.075			0.123			

*Too small to measure.

The effects applying to this exposure, of grounding the neutral of the Salinas autotransformers, may be summarized as follows:

> None of the harmonics of the line voltages to ground of the power system are measurably affected.

No consistent effect on the harmonics of the line currents of the power system was observed.

The residual voltage of the power system is very probably reduced in magnitude, though the amount is uncertain. (See discussion of technical report No. 13.)

In the residual current of the power line, all harmonics are reduced in value, except the ninth, which shows a large increase. (As elsewhere noted, the neutral current at Salinas has the ninth harmonic predominant, with prominent fundamental and third harmonics.)

The magnitude of the noise in the telephone circuits is not measurably affected, but a rise in the pitch of the noise was observed. Measurements of transverse induced voltage with the resonant shunt do not show any effect, especially on the prominent harmonics.

The longitudinal induced voltage is slightly increased in effective value. With the far end of the circuit grounded, all harmonics are reduced, except the ninth, which increases. Thus the longitudinal induced voltage follows the changes noted in the residual current. With the far end of the circuit open, the fundamental and the harmonics above the ninth sometimes increase. The ninth and lower harmonics behave, as with the far end grounded, like the residual current.

In conclusion, the effect of the Salinas neutral connection on induction in this exposure is slight. Grounding it reduces the lower harmonics and increases the ninth in magnitude.

Shielding, by grounding at one end, circuits not under test on the telephone lead, reduces the transverse induction, as shown by noise measurements. It reduces the longitudinal induced voltage about 25% with the far end of the test circuit grounded, and about 40% with the far end open.

The magnitude of the longitudinal induced voltage is approximately the same with the far end of the test circuit either open or grounded.

The noise measured through terminal apparatus (see P. I. C. No. 42, T. R. No. 8) is about one-half that observed when a receiver only is connected across the circuit.

The residual voltage, with neutral grounded, is about 2% of the balanced line voltage to ground. The residual current is roughly 2% of the line current. In both residual voltage and current, the fundamental and the third and ninth harmonics are prominent.

In the line voltages to ground and in the currents, beside the fundamental, the fifth, seventh and eleventh are the prominent harmonics.

The longitudinal induction arises principally from the residual voltage and current of the power system, while the transverse induction shows principally the characteristics of the balanced voltages and currents, together with some effect from the residual voltage and current.

(*b*) *Induction in Block Signalling Circuits of Southern Pacific Company* (T. R. No. 7).

The purpose of these tests was to determine the amount and character of the current induced in the block signalling circuits between Salinas and Soledad under normal operating conditions of the power system.

Tests were made with the signalling circuits connected as in normal operation, except that the battery leads were disconnected from the batteries and connected together. Tests were made with the far end of the circuit open, with it closed, with one side grounded, and of the induction from wires to ground.

In each case, except the track circuit test, there were several circuits in parallel with the meter. It is, therefore, not certain that the measurements are true indications of the total inductive effects. However, the connections were those of actual operation.

The track circuit test is of little value on account of direct current leakage from adjacent circuits, except to indicate that the induced current is but a few milliamperes.

The electrostatic induction is much greater than the electromagnetic. As electromagnetic induction causes, in general, a much greater disturbance than electrostatic, in this type of circuit, future tests can more profitably be made where the electromagnetic induction is large.

The transverse induced currents in the signalling circuits are very small (1.5 milliamperes or less) with only the fundamental prominent, and are larger with the far end of the circuit open than with it closed. Grounding one side of the circuit about doubled the induced current. The longitudinal induced current may be several times greater than the transverse.

Under normal operating conditions of the power line, the inductive disturbance in the signalling circuits should cause no interference.

7. Miscellaneous.

(*a*) *Neutral Current at Salinas* (T. R. No. 10).

On March 7 and 15 tests were made to determine the relative magnitudes of harmonics present in the neutral current wave. The analyses of the neutral current wave forms indicate the presence of a large ninth harmonic associated with the fundamental, third, and seventh harmonics, whose magnitudes are less than that of the ninth,

decreasing in the order named. The results as obtained by analyses of the two oscillograms are:

Oscillograms of Neŭtral Current.

Harmonic	Magnitude milli-amperes	Ratio to ninth	Magnitude milli-amperes	Ratio to ninth
1	158	0.92	200	0.56
3	48	0.26	98	0.27
5	4	0.023	8	0.022
7	22	0.13	50	0.14
9	172	1.00	360	1.00
11	18	0.10	33	0.091
Oscillogram No.	166		172	
Date, 1913	Mar. 7		Mar. 15	

The total neutral current at the time oscillogram No. 166 was taken was 0.24 amperes and for oscillogram No. 172 0.43 amperes.

The ratios of the magnitudes of other harmonics to that of the ninth agree very closely for corresponding harmonics in the two cases with the exception of the fundamental. The magnitude of the ninth harmonic compared to the third is somewhat surprising and the explanation is not at all obvious. The length of the power circuit between Gaudalupe and Salinas is approximately a quarter wave length for the ninth harmonic. This fact may explain its large magnitude due to the condition of natural resonance for the ninth harmonic.*

In connection with other tests made at Salinas, a large number of oscillograms of the neutral current have been taken and several resonant analyses made. The results from the various oscillograms and resonant analyses are given in tables below. All values are expressed in milliamperes.

*See T. R. No. 61.

Oscillograms of Neutral Current—Milliamperes.

Date, 1913	T. R. No.	Osc. No.	Order of harmonic									
			1	3	5	7	9	11	13	15	17	19
March 7	10	166	158	48	4	22	172	18	*	*	*	*
March 15	8	172	200	98	8	50	360	33	9	8	2	*
April 15	15	193	335	52	9	22	194	12	5	*	*	*
May 16	27	240	160	*	*	56	270	18	*	*	*	*
May 16	27	242	160	39	*	52	270	*	*	*	*	*
May 17	27	246	170	62	*	29	230	*	*	*	*	*
May 17	27	247	150	37	*	23	150	*	*	*	*	*
May 17	27	257	174	64	*	39	289	*	*	*	4	*
May 19	28	265	130	120	*	*	280	*	*	*	*	*
May 19	28	276	168	110	16	*	270	*	*	*	*	*
June 14	31	283	160	120	*	28	160	*	*	*	*	*
June 18	32	293	160	130	38	63	240	*	*	*	*	*
June 18	32	295	160	88	80	21	190	*	*	*	*	*
June 18	32	296	150	96	30	30	220	*	*	*	*	*
June 18	32	300	180	120	25	18	210	*	*	*	*	*
June 30	34	314	47	140	18	21	210	11	11	*	*	*
June 30	34	315	24	110	18	*	200	*	*	*	*	*
June 30	34	319	48	120	19	25	260	15	*	*	*	*
June 30	34	320	57	130	26	*	220	18	*	*	*	*
June 30	34	321	81	110	23	17	190	*	*	*	*	*
Average			130	92			220					

Resonant Analyses of Neutral Current—Milliamperes.

Date, 1913	T. R. No.	Order of harmonic									
		1	3	5	7	9	11	13	15	17	19
March 15	8	*	41	12	20	370	9	2	4	1	1
April 4	15	*	85	9	32	340	4	4	5	2	3
May 17	27	*	33	*	27	280	11	7	4	1	1
Average			53		26	330	8	4	4	1	2

*Not measured.

(b) *Comparative Noise Tests on Telephone Circuits North of Salinas. S. & S. F. P. Co.'s 55-kv. Line Dead and Energized.* (T. R. No. 14.)

The purpose was to determine the effect of the operation of the 55-kv. line on the noise in the telephone circuits north of Salinas when the circuit under test was connected through to its regular northern terminal and all other circuits were in normal operation. The measurements were made on T. P. T. & T. Co.'s San Francisco-Los Angeles toll lead. Under these conditions the telephone lines are involved in a number of parallels at different points in addition to those with the Sierra and San Francisco Power Company. The telephone circuits were opened one at a time and connected to the testing apparatus. During one series of measurements the 55-kv. line was dead. A second

series of measurements was taken with this line energized and in normal operation. The results of both sets of measurements are summarized graphically on P. I. C. No. 63, attached to technical report No. 14. The tests show in general a large increase in the noise of the circuit with the power line energized.

In connection with these tests, note was made of the transient disturbance in the telephone circuits produced by switching operations on the 55-kv. line. These are described in detail in the report. It is sufficient to repeat here that the indications were that telephone subscribers using these circuits at such times would be subjected to a deafening noise from the receiver.

(c) *Noise Tests on All Toll Circuits Radiating from Salinas* (T. R. No. 37).

The results of these tests indicate that phantom circuits are more noisy than physical circuits when subject to induction. The condition of the neutral, grounded or nongrounded, of the autotransformers at Salinas had practically no effect on the magnitude of the noise induced in the telephone circuits, and only a slight effect on the quality of the noise in a few circuits. Reference should be made to the table of results in Technical Report No. 37.

(d) *Arc Circuits Induction* (T. R. No. 33).

Tests were made on the leads used for connecting the testing apparatus to the toll circuits of exposure No. 2. These test leads are paralleled for approximately one thousand feet by two series arc circuits. The results indicate appreciable longitudinal induction arising from this short exposure. The electromagnetic induction is due principally to that circuit which runs as a single wire along most of the exposure, the other side of the circuit being remote. The transverse induction from this exposure is entirely negligible.

Direct and immediate comparison of measurements of the induction from exposure No. 2 with arc circuits on and off showed no effect whatever due to the arc circuits. In consequence it may be stated that measurements made on exposure No. 2 are not in error due to the influence of the arc circuits.

(e) *Investigation of Current Transformers.*

(T. R. No. 11 and T. R. No. 26).

Tests were made to determine the ratio of transformation of the fundamental and harmonics with various current strength and amounts of resistance in the secondary. These tests were made upon the bank

of current transformers for the 33-kv. circuit between Salinas and King City.

The results of these tests show:

> The ratio of transformation of all harmonics up to and including the 19th harmonic, and for current up to full load rating does not depart greatly from the rated ratio providing the impedance in the secondary does not exceed 10 ohms.
>
> With resistances and currents such that the secondary voltage is between 50 and 150 volts, the ratio of transformation of the harmonics may deviate as much as 30 per cent from the rating.

Tests were made on this same bank of current transformers and on those of the 55-kv. line to determine their suitability in connection with measurements of residual current. The results of the tests show that these banks of current transformers are suitable for such measurements, provided certain precautions are observed. These precautions are:

> Equal external impedances in all three secondary circuits, not to exceed 10 ohms.
>
> Low impedance meter for measurement of residual current.

While these tests are not assumed to constitute a complete investigation, they are considered sufficient to indicate that the current transformers tested are satisfactory for the purpose of this study of inductive interference.

(*f*) *Investigation of Potential Transformers.* Measurement of Residual Voltage.* (T. R. No. 13.)

The name plate data of the three potential transformers tested are as follows:

G. E. Type P. Form C. 200 Watts. 13,200—110/55 volts.

In service, at Salinas, these were connected to the 22-kv. line because of the lack of potential transformers suitable for use on the 55-kv. and 33-kv. lines. The primaries were connected in star with their neutral grounded. For measuring residual voltage the secondaries were delta connected, one corner of the delta being closed through a measuring instrument. Under these conditions, with transformers of exactly equal ratios of transformation and with no distortion due to magnetization, a true measure of the residual voltage (vector sum of the line voltages to ground) will be obtained. Inequalities in the ratios of transformation and distortion of wave shape due to magnetic hysteresis

*Refer to T. R. No. 58 for more complete discussion.

will introduce errors. It was the purpose of the investigation to determine the magnitude of such errors.

The effect of inequalities in the impedances of the transformers, in introducing a residual voltage on the line, was not considered in the investigation. This effect is probably small.

The results show that the ratios of transformation differ slightly, and, in consequence of this slight inequality, an apparent residual voltage, equal to 0.25 per cent of the value of the corresponding balanced leg voltage (fundamental or any harmonic), will exist. The phase position of this apparent residual voltage will affect the error introduced in the measurement, which error may vary from zero to the full value of the apparent residual voltage. On this basis the maximum error arising from this cause is:

80 volts fundamental for the 55-kv. line.
50 volts fundamental for the 33-kv. line.

These values are closer approximations than those given in technical report No. 13.

Due to the third harmonic circulating current in the secondary delta, an error of approximately 60 volts may be introduced into the third harmonic residual voltage of the 55-kv. line. For the 33-kv. line this would be equivalent to 36 volts.

The investigation throws no doubt on the accuracy of the measurements of the ninth harmonic in the residual voltage.

As stated above, the potential transformers were connected to the 22-kv. line for measuring the residual voltage of the 33-kv. and 55-kv. lines. The 22-kv. and 33-kv. lines are supplied from the 55-kv. line by star connected autotransformers. It was desired to measure the residual voltage, both when the autotransformer neutral was grounded and when it was nongrounded, so as to determine the effect, on the residual voltage, of grounding the neutral. When the neutral of the autotransformers is grounded, the residual voltage of the 55-kv. line is transformed to the 33-kv. and 22-kv. lines, and vice versa. When the autotransformer neutral is not grounded, the same transformation ratios for residual voltage do not hold. This conclusion was reached during the compilation of technical report No. 13. The practice, in reports of tests at Salinas, had been to use the same ratio in reducing readings to terms of the high-tension lines for both conditions of the neutral, grounded and nongrounded. Hence, the records of residual voltage, with the neutral of the autotransformers nongrounded, are in error. The magnitude of this error has not been determined because the impedance of the autotransformer windings between taps and the admittances to ground of the high-tension transmission lines are not

known. As a consequence, the effect of grounding the neutral of the autotransformers on the magnitude of the residual voltage of the 55-kv. and 33-kv. lines can not be definitely stated.

Below are given the results of the analyses of all oscillograms of residual voltage. These results have been reduced so as to apply to the 22-kv. line, thus eliminating considerations of autotransformer ratios. These tables indicate, on the whole, markedly less third harmonic and somewhat less ninth harmonic and fundamental, with grounded neutral. It is reasonable to suppose that grounding the neutral of the autotransformers will have a similar effect on the residual voltage of the other lines.

Oscillograms of Residual Voltage (Volts)—Neutral Grounded.

Date, 1913	T. R. No.	Osc. No.	Order of harmonic								
			1	3	5	7	9	11	13	15	17
March 15	8	173	61	196	3	8	45	2	*	2	2
April 15	15	192	33	148	5	10	45	4	8	*	*
April 25	25	201	65	201	*	*	40	*	*	*	*
April 25	25	209	29	155	*	9	43	8	6	8	*
May 16	27	242	45	155	*	18	29	*	*	*	*
May 19	28	266	89	180	*	*	53	*	*	*	*
May 19	28	275	45	123	7	10	45	9	6	6	6
June 14	31	281	49	123	*	*	34	*	*	*	*
June 14	31	283	40	114	*	*	22	*	*	*	*
June 14	31	286	37	114	*	*	22	*	*	*	*
June 18	32	290	75	213	*	24	64	*	*	*	*
June 18	32	298	68	218	*	*	48	*	*	*	*
June 30	34	313	40	110	*	*	27	*	*	*	*
June 30	34	318	39	123	*	*	41	*	*	*	*
Average			49	150			41				
Possible error per cent			67	16			†				
Per cent effective value balanced voltage to ground			0.3	1.2			0.3				

*Too small to measure.
†NOTE.—The known errors in the ninth harmonic are negligible.

Oscillograms of Residual Voltage (Volts)—Neutral Nongrounded.

Date, 1918	T. R. No.	Osc. No.	Order of harmonic								
			1	3	5	7	9	11	13	15	17
March 15	8	174	65	147	9	12	22	3	2	1	*
April 15	15	186	72	274	27	*	65	9	7	2	*
May 16	27	244	81	202	*	11	44	*	*	*	*
May 19	28	270	90	319	21	*	53	*	*	*	*
May 19	28	273	90	290	23	*	45	*	*	*	*
June 14	31	282	94	344	*	*	57	*	*	*	*
June 14	31	284	102	286	*	*	29	*	*	*	*
June 14	31	287	86	305	*	*	31	*	*	*	*
June 30	32	291	123	554	47	*	89	*	*	*	*
June 30	32	299	143	450	75	*	82	*	*	*	*
Average			94	320			53				
Possible error—per cent			35	7.7			†				
Per cent effective balanced voltage to ground			0.7	2.5			0.4				

*Too small to measure.
†Note.—The known errors in the ninth harmonic are entirely negligible.

(*g*) *Effect of Low Insulation* (T. R. No. 34).

The object of the tests described in this report was to determine the effects of a reduction in the telephone line insulation on the induction observed in the circuit, with particular regard to a possible increase in the transverse induction accompanying low insulation.

From an induction standpoint, the insulation of a telephone circuit becomes of particular importance in so far as it contributes to the admittance unbalance between the sides of a circuit and ground. Such an unbalance, in connection with longitudinal induction, will cause a current to circulate around the telephone circuit. This current will combine vectorially with any due purely to transverse induction. The considerations which are pertinent to this test are:

The effect of a reduction in insulation on the conductance unbalance of the circuit.

The relative magnitude of the prominent harmonics of the induction under the two conditions of insulation.

The relative magnitude of the transverse induction observed under the same two conditions, particularly of those harmonics prominent in the longitudinal induction.

The condition of the power system.

The results of this test applying to the four points above mentioned indicate that:

> There is apparently no decided change in the conductance unbalance produced by the marked reduction (25 to 1) in the insulation of both sides of the circuit to ground.
>
> The effective value of the longitudinal electrostatic induced voltage is less with low insulation. The third harmonic of the longitudinal electromagnetically induced voltage is slightly less in the observations made under the conditions of low insulation. The ninth harmonic is unchanged. The third and ninth harmonics are the prominent harmonics in the longitudinal induction.
>
> Noise measurements indicate less transverse induction with low insulation when the far end of the circuit is open, and, for other conditions of the circuit at the far end, noise measurements show no change in the transverse induction accompanying a change in the insulation. With the far end of the circuit open, measurements show a somewhat greater third harmonic with low insulation. For other conditions of the circuit at the far end, the third harmonic was too small to measure under both conditions of insulation. For all conditions of the circuit at the far end, the change in insulation is unaccompanied by any change in the ninth harmonic.
>
> There was no material difference in the condition of the power system for the two series of tests, with high and low insulation.

It is evident from the results of this test that a 25 to 1 reduction in the insulation resistance between both sides of the circuit and ground had practically no effect upon the transverse disturbance.

Further work of this nature can profitably be done in other exposures and particularly in regard to the effect of various known admittance unbalances introduced for experimental purposes.

Attached to technical report No. 34 are curves showing a very large diurnal variation of insulation resistance for telephone circuits in the neighborhood of Salinas. These curves are P. I. C. No. 103A and P. I. C. No. 103B.

(*h*) *Formulæ for the Computation of Induction Between Power and Signalling Wires* (T. R. No. 12).*

This report is a discussion of the factors and a tentative compilation of formulæ necessary in computing the induction produced by power lines in adjacent telephone circuits. The formulæ given are general, and to facilitate computation it is necessary in some cases to modify them.

*See T. R. No. 64.

The factors necessary for such computation are listed below:

Power System.

Type of system
Number of conductors
Size of conductors
Material of conductors
Height of conductors
Configuration of conductors
Location of transpositions
Type of transpositions
Length of exposure
Balanced voltages
Residual voltages
Balanced current
Residual current
Wave shape of voltages and currents
Effective value of each harmonic
Phase relation of voltages and currents
Phase relation of harmonics

Signalling System.

Number of circuits
Type of circuits
Metallic or grounded
Number of conductors
Size of conductors
Material of conductors
Height of conductors
Configuration of conductors
Location of transpositions
Type of transpositions
Length of exposure
Loading data
Insulation conductance
Circuit impedances

General.

Relative location of signalling and power transpositions
Separation of signalling and power pole lines
Depth of image
Shielding effects of neighboring objects

The last named factor can be applied as a correction to the final computed values of electrostatically induced voltage. This factor depends on the number and proximity of shielding objects, and the factor must be experimentally determined for any given exposure.

To determine the current which will flow in a signalling circuit due to an induced voltage, it is necessary to know the impedance of the circuit and the apparatus connected to it. Except in cases where it is possible to measure the impedances, it becomes necessary to compute them for the various frequencies under consideration. In most exposures induction and difference of earth potential prevent measurements of the impedances.

8. Condensed Summary of Conclusions.

A representative value of the neutral current at Salinas is 0.3 ampere. It is composed almost entirely of the ninth harmonic, together with the fundamental and third harmonics, decreasing in magnitude in the order named.

Under existing conditions, with the power system in normal operation, the effects of grounding the neutral of the Salinas autotransformers appear to be as follows:

1. Signalling circuits.

(*a*) For exposure No. 1 (including No. 2): Grounding the neutral produces no noticeable change in the magnitude of the

noise in the telephone circuits, a slight change in the quality, an increase in the ninth harmonic of the transverse induction, and an increase in the effective value of the longitudinal induction. On the whole, grounding the neutral has a slight detrimental effect on the signalling circuits.

(*b*) For exposure No. 2: Grounding the neutral diminishes the induction in the telephone circuits and is, therefore, beneficial from an induction standpoint.

(*c*) For exposure No. 3: The effect of the neutral is slight. Grounding it reduces the magnitude of the lower harmonics in the induction, and increases the magnitude of the ninth harmonic.

(*d*) For exposure No. 4: Only noise measurements were made on this exposure, and grounding the neutral decreases the noise slightly.

2. Power system.

(*a*) For the 55-kv. line: The condition of the neutral has no appreciable effect on the line voltages and currents. Grounding the neutral diminishes the effective value of the residual current, increases the fundamental and decreases the third and ninth harmonics.

Grounding the neutral probably decreases the effective value and harmonics of the residual voltage a slight degree.

(*b*) For the King City line: The condition of the neutral has no appreciable effect on the line voltages and currents. Grounding the neutral reduces the magnitude of all the harmonics in the residual current except the ninth which shows a large increase. Grounding the neutral probably decreases the effective value and harmonics of the residual voltage.

(*c*) For the Monterey line: Grounding the neutral decreases the effective value and harmonics of the residual voltage. No other measurements were made on this line.

The fundamental, fifth, seventh, and eleventh harmonics are prominent in the balanced currents and voltages while the fundamental, third, and ninth harmonics are prominent in the residual currents and voltages of the power system.

By a comparison of the prominent harmonics in the currents and voltages of the power system with those in the induction in the telephone circuits the following conclusions are drawn:

The longitudinal induction arises principally from the residual voltages and currents, while the transverse induction shows principally the characteristics of the balanced voltages and currents, together with some effect from the residual voltages and currents.

A study of the proper relative locations of power and telephone transpositions for exposure No. 2 indicates that it is possible to modify the present system so as to materially reduce the transverse induction.

It was impracticable, however, to install this proposed system for testing purposes owing to the fact that the power line could not be shut down for a sufficient length of time. It is understood that the effectiveness of the proper relative location of power and telephone transpositions will be determined at other points on more flexible power systems. With this in view no particular study has as yet been made of a transposition system for the other exposures investigated at Salinas.

The importance of maintaining a well balanced condition on the power system is emphasized by the effects resulting from the relatively small residual currents and voltages of the power lines involved in the exposures investigated. This fact is further emphasized by the impossibility of reducing the longitudinal induction arising from residual currents or voltages by any system of transpositions of the power and telephone circuits, except in so far as the power line transpositions reduce the magnitude of the residual currents and voltages.

Comparative noise tests on the telephone circuits north of Salinas with the 55-kv. line dead and energized showed in general a large increase in the noise with the power line energized.

Electrostatic shielding by grounding other circuits on the telephone lead reduces the electrostatic induction in some degree, particularly the longitudinal induction. No data were obtained as to the effect of electromagnetic shielding.

Opening the secondary delta of the autotransformers at Salinas caused a large increase in the residual voltage and current with a consequent increase in the induction. In all cases the increase applied particularly to the third harmonic.

Tests upon the block signal system of the Southern Pacific Company between Salinas and King City indicate that under normal operating conditions of the power system the induction is insufficient to cause interference with the operation of the signals.

Investigation of the effect of known residual currents in exposure No. 2 leads to the conclusion that the equivalent depth of earth's surface is one hundred and fifty meters below the actual earth's surface.

9. Definitions of Terms Used in Technical Reports.

Balanced—Whose vector sum in the phases is zero. (In contradistinction to residual.)

Capacity Unbalance—Difference between capacities to ground of two sides of a circuit. Sometimes called capacity balance.

Conductance Unbalance—Difference between conductances to ground of two sides of a circuit. Sometimes called conductance balance.

Distortion Circuit—Inductance coil shunted by oscillograph vibrator and condenser in series: Used to magnify higher harmonics in oscillograms.

Exposure—A parallel of power and signalling circuits. (Length of unbalanced-) That length of exposure between a signalling circuit and a power line, neither of which is transposed, which will produce inductive effects of the same magnitude

as those produced in the actual exposure with the given transpositions in the power and telephone circuits.

Longitudinal—Applied to induction in signalling circuits, from the side of a circuit to ground or along the side of a circuit. (See Transverse.)

Noise Standard—An instrument for measuring noise. (See P. I. C. No. 32 for diagram.)

Noise Tests—Measurements of noise in signalling circuits.

Noise Units, Standard—Microamperes at 240 cycles.

Open Circuit Voltage (abbreviated Eoc.)—Voltage measured by meter of infinite impedance.

Oscillator—Vreeland mercury arc sine wave generator.

Residual (voltage)—Vector sum of the line voltages to ground.
(current)—Vector sum of the line currents.

Resonant Analysis—Analysis of a wave shape by means of a resonant shunt.

Resonant Shunt—An instrument consisting of a circuit, in series with a telephone receiver, containing inductance and capacity so chosen that it can be made to resonate for any particular frequency or harmonic, the magnitude of which it is desired to measure. The amount of noise in the receiver produced by the unknown harmonic is equated with the noise produced by a known current of the same frequency.

Shielded (abbreviated Shld.)—Protected to some extent against induction. (Electrostatically-) (abbreviated Es.-Shld.)—Protected to some extent against electrostatic induction. (Electromagnetically-) (abbreviated Em.-Shld.)—Protected to some extent against electromagnetic induction.

Short Circuit Current (abbreviated Isc.)—Current measured by meter having zero impedance.

Terminal Apparatus—Apparatus connected at end of signalling circuit, particularly used in some of the noise tests on telephone circuits. (See P. I. C. No. 42 for terminal apparatus used at Salinas.)

Transverse—Between sides of a signalling circuit.*

Abbreviations Used in Technical Reports.

Ckt.—Signalling circuit.
Em.-Shld.—Electromagnetically-shielded.
Eoc.—Open circuit voltage.
Es.-Shld.—Electrostatically-shielded.
Grd.—Grounded, connected to ground.
Isc.—Short circuit current.
N. N. G.—Neutral nongrounded.
N. G.—Neutral grounded.
Non-Grd.—Nongrounded.
Non-Shld.—Nonshielded.
Osc.—Oscillogram.
Ph.—Phase.
P. I. C.—Designating blue prints.
Shld.—Shielded.
T. R.—Technical report.
Vib.—Oscillograph vibrator.

10. P. I. C. Index.

P.I.C. No.	Attached to T. R. No.	Title
2	12	Formulæ for Electrostatic Induction.
3	12	Formulæ for Electrostatic Induction—Figure Illustrating Notation Used.
4	5	Method of Testing—Connections of Potential Transformers.
5	5	Method of Testing—Connections of Current Transformers.
7	12	Formulæ for Electrostatic Induction—Transverse Induction.
9	12	Formulæ for Electromagnetic Induction.
32	3	Noise Standard—Circuit for Evaluating Noise in Terms of Noise Units.
38	11	Circuit Diagram—Tests on Current Transformers.
39	11	Diagram of Connections—Resonant Shunt Method of Testing Current Transformers.

*See T. R. No. 70.

P.I.C. No.	Attached to T. R. No.	Title
40	11	Calibration of Current Transformer No. 904475 (G. E.).
42	8	Diagram of Connections for Noise Measurements.
44	--	Detail of Exposure No. 2, Test No. 2, at Salinas. (NOTE.—P. I. C. No. 44 is superseded by P. I. C. No. 61.)
45	14	Details of Exposure No. 1, Test No. 2, at Salinas.
50	11	Transformation Ratio of Current Transformers for High Harmonics.
51	10	Neutral Current—Circuit Diagram for Oscillograms.
53	26	Diagram of Connections—Comparison of Current Transformers for Residual Current.
55	7	Circuits Used in Tests of Induction into S. P. Signal Circuits at Salinas.
58	22	Diagram of Pole Lines in Exposure No. 2.
61	22	Transpositions in Power and Telephone Circuits Exposure No. 2, at Salinas.
62	24	Total Mutual Inductance Between Three Power Conductors in Parallel and Telephone Wires Distant 17 Meters and 7.44 Meters Above Ground Exposure No. 2.
63	14	Results of Comparative Noise Tests on T. P. T. & T. Co.'s Circuits North from Salinas—55 kv. Line Dead and Energized.
66	22	Electrostatic Induced Voltage to Ground.
67	13	Connections for Potential Transformer Tests.
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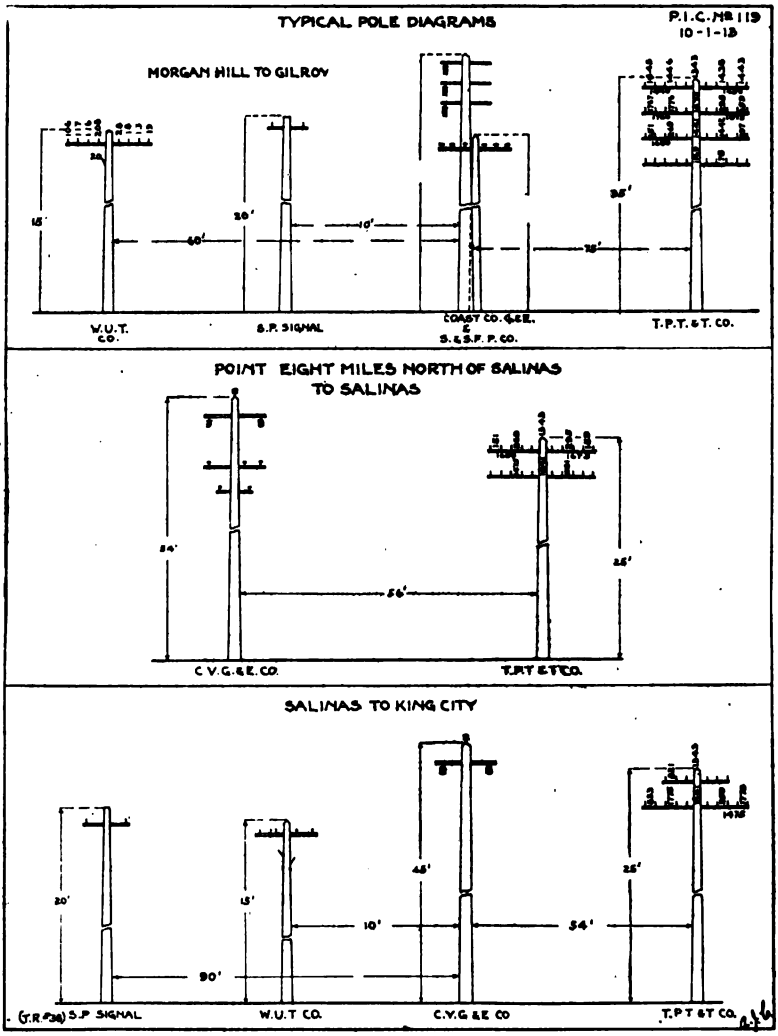
70	22	Electromagnetically Induced Voltage to Ground for Configuration by P. I. C. No. 58.
71	22	Constants for Induction Due to Residual Voltages and Currents.
72	22	Phase of Induction to Ground.
74	23	Connections of Power Apparatus—Special Tests on Exposure No. 2 at Salinas.
100	24	Longitudinal Induced E. M. F. for Side Circuit "A" Due to Line Currents as Given in Table V.
101	24	Transverse Induced E. M. F. Between Side Circuits "A" and "B" Due to Line Currents as Given in Table V.
102	13	Transformer Connections at Salinas and Guadalupe.
103-A-B	34	Insulation Resistance of Circuit No. 295.
112	24	Values of "A" in $K = A + jB = E/1000'$.
113	24	Values of "B" in $K = A + jB = E/1000'$.
119	38	Typical Pole Diagrams.

Respectfully submitted.

(Signed) J. E. WOODBRIDGE,
Chairman Subcommittee on Tests No. 1 and No. 2.

ATTACHED: P. I. C. 119.


TYPICAL POLE DIAGRAMS
P.I.C. No. 119
10-1-13
MORGAN HILL TO GILROY
15'
40'
20'
10'
35'
75'
W.U.T. CO.
S.P. SIGNAL
COAST CO. G.&E. & S.&S.F. P. CO.
T.P.T. & T. CO.
POINT EIGHT MILES NORTH OF SALINAS TO SALINAS
34'
56'
25'
C.V.G.&E.CO.
T.P.T.&T.CO.
SALINAS TO KING CITY
20'
15'
45'
25'
10'
54'
90'
(G.R.#38) S.P. SIGNAL
W.U.T. CO.
C.V.G.&E. CO.
T.P.T.&T. CO.

Technical Report No. 40.

December 15, 1913.

CAPACITY AND CONDUCTANCE UNBALANCE MEASUREMENTS.

1. Purpose.

This report is to describe the method in use for the measurement of the capacity and conductance unbalances between the sides of a circuit and ground.

2. Description of Method.

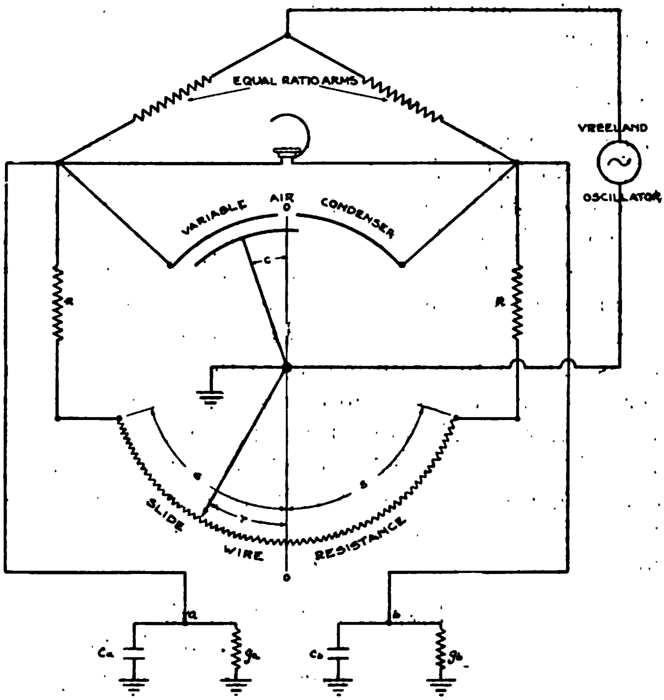
The method in use is indicated clearly by the schematic diagram given on P. I. C. No. 134, attached to this report. In principle, the method is a Wheatstone bridge arrangement. A telephone receiver is used as a detector and a Vreeland Oscillator as a source of electromotive force. A balance is obtained by adjusting the variable air condenser and the setting on the slide wire resistance until no sound is heard in the telephone receiver. When connected to a circuit subject to induction a balance is difficult to obtain, owing to the extraneous currents in the receiver. When not subject to errors from such a cause, the method is quite accurate. It gives directly and simultaneously the difference in the capacities and conductances between the two sides of a circuit and ground. The method gives no indication of the absolute magnitude of either of the capacities or conductances whose differences are measured.

Respectfully submitted.
(Signed). J. E. Woodbridge,
Chairman Subcommittee on Test No. 3.

Attached: P. I. C. 134.

P. I. C. No. 134
11-22-18.

MEASUREMENT OF CAPACITY AND CONDUCTANCE UNBALANCES
BETWEEN SIDES OF A CIRCUIT AND GROUND
SCHEMATIC DIAGRAM



C = CAPACITY ADDED TO ONE SIDE AND SUBTRACTED FROM THE OTHER IN ORDER TO OBTAIN BALANCE.
r = RESISTANCE " " " " " " " " " " " " " "

CAPACITY UNBALANCE = $C_b - C_a = 2C$

CONDUCTANCE " $= g_b - g_a = \frac{2r}{(R+S)^2 - r^2}$

T. R. No. 40

J.B.J.

Technical Report No. 41.

March 16, 1914.

HARMONIC ANALYSIS OF ALTERNATING CURRENT WAVES.

OUTLINE.

1. Introduction.

The purpose of this report is to outline and illustrate the methods used in determining the values of the various harmonics of the voltage and current waves in power and communication circuits and to discuss the relative accuracy of the methods.

The first method involves the use of an oscillograph by means of which a photographic record of the wave is obtained. This wave form is analyzed by a mathematical process into its component harmonics. The use of the oscillograph and the method of analyzing the wave forms are given in detail in section II.

The second method involves the use of a resonant shunt, consisting of a telephone receiver, an inductance coil, and a variable condenser in

series. By means of the variable condenser the shunt is made to resonate for an harmonic whose magnitude is to be measured. By this method the magnitudes of the harmonics of a complex wave are experimentally determined. The use of the resonant shunt is described in detail in section III.

The results of a comparative test of the two methods are given in section IV, which includes, also, a discussion of the accuracy of the two methods and their adaptability for different purposes.

II. Oscillograph.

(a) *Description of the Instrument and Its Operation.*

The oscillograph used is an electromagnet type instrument manufactured by the General Electric Company. The free period of the vibrators is approximately 6,000 cycles per second. In use, the vibrator is immersed in a damping fluid whose density is such as to make its motion aperiodic. The sensitivity of the vibrator may be judged from the fact that a current of approximately 1.7 milliamperes produces a deflection of 1 millimeter.

The operation of the oscillograph is as follows: the reaction between the alternating field, set up by the current in the vibrator, and the unidirectional field of the electromagnet, causes the vibrator to oscillate in response to the fluctuations of current in the vibrator, the angular deflection being proportional to the instantaneous value of the current. A mirror, of negligible mass, is mounted on the vibrator and its oscillations cause a reflected beam of light to vibrate in a plane. This beam of light is focused on a rotating photographic film, thus recording the alternating current wave.

(b) *Circuit Arrangements for Oscillograms.*

The circuit arrangements which are used in determining the wave forms of the voltages and currents of power circuits and of the induced currents in neighboring circuits are described below. Diagrams of these circuits are given on P. I. C. No. 124, attached.

1—Voltage—Power Circuits.

Figures No. 1 and No. 2 of P. I. C. No. 124 show the circuit arrangements for obtaining the wave forms of voltages. The vibrator is supplied through a potential transformer and the amount of current is controlled either by an adjustable noninductive resistance (Fig. 1) or by an adjustable condenser (Fig. 2) in series with the vibrator.

In most instances it is necessary to magnify the harmonics of the line voltages in order to obtain satisfactory measurements. The series capacity produces magnification approximately in proportion to the

frequency, provided that the impedance of the remainder of the circuit is small in comparison to the impedance of the condenser. When capacity is used, care must be taken to avoid resonance at the frequency of any harmonic present in the voltage wave.

For oscillograms of residual voltage, where the fundamental is generally small and the harmonics prominent, the other arrangement, series resistance, is usually employed. This arrangement produces no distortion of the wave form. The series resistance should be as large as possible consistent with obtaining a satisfactory amplitude.

The computations necessary in order to obtain the primary voltages corresponding to the different harmonics present in the wave, are indicated below each diagram (P. I. C. No. 124). For this purpose the value of the current through the vibrator for each harmonic is required. This latter is obtained from the analysis of the oscillogram as described in another portion of this report.

2—Current—Power Circuits.

Circuit arrangements for obtaining oscillograms of currents are shown by figures No. 3 and No. 4 of P. I. C. No. 124. The supply is through a current transformer.

As the maximum allowable current through the vibrators of the oscillograph is small and as it is important that the total impedance in the secondary of the current transformers be as small as possible, the vibrator circuits are in most instances shunted by a low noninductive resistance (Fig. 3) or low impedance air core inductance coil (Fig. 4). The use of the inductance coil in combination with the condenser, greatly distorts the wave form, magnifying the harmonics in the vibrator circuit; due to the fact that the impedance of the inductance coil increases and that of the condenser decreases with an increase in the frequency. A magnification greater than the square of the frequency can be obtained by this arrangement which is used particularly to obtain oscillograms of line currents.

For oscillograms of residual current the harmonics are usually sufficiently prominent to enable an accurate determination to be made of them without distortion. For this purpose the circuit shown by Fig. 3 is used. If the residual current is sufficiently small it may be passed directly through the vibrator, dispensing with the shunt resistance, in which case the series resistance should, also, be eliminated.

Expressions for the value of the primary current corresponding to any harmonic through the vibrator accompany each diagram.

3—Induction—Communication Circuits.

Oscillograms of the induced current in telephone circuits (or circuits of any other type) are obtained by passing the current directly through

the vibrator or through the vibrator shunted by a noninductive resistance if the current is large, Fig. 5. Distortion is unnecessary as the harmonics are usually prominent. Expressions for the open-circuit voltage and short circuit current corresponding to any particular harmonic are given with Fig. 5. A knowledge of the impedance of the telephone circuit is required in addition to the constants of the vibrator circuit.

Oscillograms have been obtained only of longitudinal induction as the oscillograph is not sensitive enough to record the magnitudes of transverse induction encountered thus far.

4—Calibration.

In order to use the oscillograph as an ammeter, *i. e.,* express the results of the analysis of the wave form in electrical units, it is necessary to calibrate the vibrators at frequent intervals. This is accomplished by passing a known amount of direct current through the vibrators and taking an oscillogram showing both the direct and reversed as well as the zero deflections. From such an oscillogram the amount of current necessary to produce unit deflection is determined.

The damping of the vibrators varies somewhat with the frequency, even with the best proportioned damping liquid, and as a result the current per unit deflection also varies and differs from the value obtained with direct current. To correct for this a series of oscillograms of known amounts of single frequency alternating currents should be taken over the working range of frequencies. For this purpose use is made of the Vreeland Oscillator as a source of supply. The results of such a series of oscillograms are shown in a table given in section IV (a). It is not necessary to repeat such a series of oscillograms unless the condition of the vibrators or damping fluid is changed.

(c) *Analysis of Oscillograms.*

1—Method.

There are many known methods* of analyzing periodic waves into their component parts when the graph of the wave has been obtained. Some of the methods are graphical while others are analytical in their nature. We have adopted, with some modification of the mechanical details, the analytical method published by Prof. Alan E. Flowers.† This and all other methods are based on the fact that any complex

*Thompson, Dynamo Electric Machinery; Steinmetz, Engineering Mathematics; Fleming, Propagation of Electric Currents in Telephone and Telegraph Conductors. These may be consulted for the fundamental principles upon which the various methods of analysis are based.

†Engineering Quarterly, University of Missouri; May, 1907.

wave of periodic structure may be represented by a Fourier series of the form

$$Y = + B_0 + B_1 \cos x + B_2 \cos 2x + B_3 \cos 3x + \dots + B_n \cos nx + A_1 \sin x + A_2 \sin 2x + A_3 \sin 3x + \dots + A_n \sin nx$$

The coefficients $A_1 \dots A_n$ and $B_0 \dots B_n$ in the above equation are sufficient to determine the effective value and the phase of any component of the complex wave. For alternating current waves which contain only odd harmonics, all coefficients with even subscripts become zero and the constant term B_0 disappears if there is no unidirectional component present in the wave. The remaining coefficients may then be computed if one knows the values of n equi-spaced ordinates for a half wave length of the complex wave.

2—Measurement of Ordinates.

The oscillogram, or photographic record of the complex wave, provides a graph of the wave from which the values of n equi-spaced ordinates can be obtained. For this purpose, the original film, contact print, or an enlargement is used, the choice being determined by the judgment of the computer. He also determines by inspection the character of the harmonics in the wave and accordingly uses the form of analysis best suited. If there are even harmonics in the wave, they are subtracted out and, if desired, analyzed for separately. It has been found convenient to measure either 18 or 36 ordinates* and thus resolve the complex wave into 17 or 35 component waves. If there are no harmonics of order higher than the 17th, the eighteen ordinate method should be used. If there are harmonics higher than the 17th and none higher than the 35th, the thirty-six ordinate method should be used. Whenever possible, the eighteen ordinate method is used because of its advantages of greater simplicity and less labor required. The labor estimated for the two methods is 1.5 to 2.0 man-hours for each wave by the eighteen ordinate, and 4 to 6 man-hours for each wave by the thirty-six ordinate method.

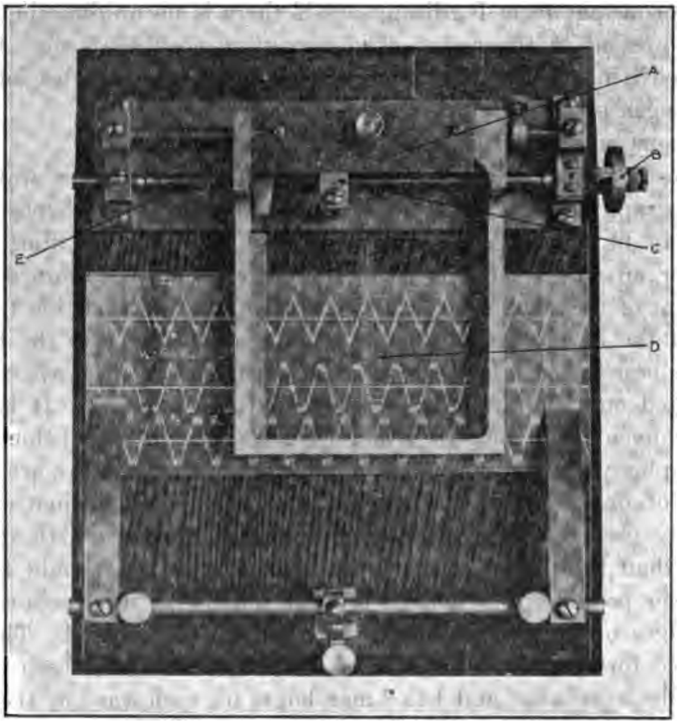
The wave micrometer, illustrated below, is a convenient measuring engine for determining the values of the ordinates and for dividing the half wave length into as many equal parts as may be desired. The scale D, for measuring the ordinates, can be moved in a direction at right angles to its length by the screw E, the amount of the transverse motion being indicated by the scale A, whose index is c. The unit of scales A and D is $\frac{1}{40}$ inch and the micrometer head B divides the unit of scale A directly into tenths, and hundredths may easily be estimated. This allows one to measure the abscissæ in units of $\frac{1}{4000}$ inch.

*Forms for 6 and 12 ordinates were developed later. See T. R. No. 63.

The oscillogram is clamped on the base of the instrument so that the axis of the wave is perpendicular to scale D at its zero point. The oscillogram is now properly oriented so that abscissæ may be measured on scale A and the ordinates of the curve, corresponding to any abscissæ, on scale D.

3—Computation.

The numerical operations involved in analyzing a wave, after the ordinates Y_0 ... Y_n are measured, are arranged on forms similar to



Wave micrometer.

Analysis Sheets No. 1 and No. 2, presented in T. R. No. 63. The various steps in an eighteen ordinate analysis are as follows:

a—Tabulate the measured ordinates in the columns headed "Original Data," Sheet No. 1.

b—Compute $S_n = Y_n + Y_{18-n}$, where $S_9 = Y_9$: $d_n = Y_n - Y_{18-n}$, where $d_0 = Y_0$.

If the values of Y_n are so large as to be cumbersome in the later computations, a large part of the fundamental component may be temporarily removed. A column headed .... sin x is provided for setting down the 0°, 10°, 20° .... 90° ordinates of

this component, and they are subtracted from the respective Y_n's before forming S_n and d_n.

c—Compute S'_1, S'_2, S'_3, S''_1, and d'_0, d'_1, d'_2, d''_0 as indicated.

d—Multiply the S terms indicated in various rows and columns on the upper half of Sheet No. 2 by the sine of the angle given at the left of the particular row in which they appear and tabulate each product in the row and column of the corresponding S term. In a similar manner perform the indicated operations with the d terms and record the results in their respective places on the lower half of Sheet No. 2.

e—Compute I, the algebraic sums of the first columns (headed 1, 3, 5, 7, and 9); and II, the algebraic sum of the second columns (headed 17, 15, 13, and 11).

f—Compute $(I + II)$ and $(I - II)$.

g—Compute $\frac{1}{9}(I + II)$ and $\frac{1}{9}(I - II)$.

If a part of the fundamental component was temporarily removed, the maximum ordinate of this fundamental component must be added to A_1 at this point.

The results of the last computation give the coefficients of the Fourier series A_1 .... A_{17}, B_1 .... B_{17}. These coefficients are checked and the analysis completed as indicated below.

h—Transfer the terms A and B to the columns headed A_n and B_n respectively on the lower half of Sheet No. 1. Carry these across to the columns indicated and add up these columns to give A, B, F, H, J, and K. Compute Y_0, Y_3, Y_6, Y_9, Y_{12}, Y_{15},

and Y_{18} from the equations given at the bottom of the sheet and tabulate in column "Ord. Comp.". By comparing these computed ordinates with the ordinates from the original data, an indication of the accuracy of the analysis is obtained, and if these seven ordinates check, all the computed coefficients are correct.

i—Compute $C_n = \sqrt{A^2_n + B^2_n}$

The result of this computation is the maximum value of the nth harmonic component of the complex wave. It is to be noted that values of C_n less than $\frac{1}{40}$ inch are considered unreliable and are, therefore, discarded.

It is sometimes desirable to compute F, the maximum value of the equivalent sine wave; G, its effective value, and 100 Cn/F, the value of the nth harmonic as a percentage of the equivalent sine wave.

4—Reduction to Milliamperes.

The values of C_n expressed in units of the wave micrometer are reduced to milliamperes by means of the calibration curve. From the calibration curve is computed the amount of current necessary to

produce unit deflection of the vibrator. Let this value be K milliamperes. Then the effective value of the current through the vibrator corresponding to the nth harmonic is—

$$I_v = \frac{K\,C_n}{\sqrt{2}}$$

Having thus obtained I_v the further computations are as indicated on P. I. C. No. 124. These have been noted previously in connection with the discussion of the circuit arrangements for oscillograms.

III. Resonant Shunt.

(a) *Description of Instrument and Its Operation.*

The resonant shunt consists of a low-impedance telephone receiver, an inductance coil and a variable condenser in series. By adjusting the variable condenser until a maximum of sound is heard in the receiver the shunt is made to resonate for the harmonic whose magnitude it is desired to measure. When the shunt is thus resonated, the sound in the receiver will be largely of the resonating frequency since the circuit will present a very much higher impedance for currents of all other frequencies.

To determine the magnitude of the current in the resonant shunt a throw-over mercury switch is provided so that the receiver may be alternately connected into the resonant shunt and into an auxiliary comparison circuit. This auxiliary circuit consists of a potentiometer arrangement and an alternating current generator by means of which different amounts of current of various frequencies are caused to flow through the telephone receiver. For an alternating current generator, a Vreeland mercury-arc sine-wave oscillator is generally used, together with an adjustable wave-filter. The wave-filter is used to purify the wave form of the current supplied by the oscillator.

With the oscillator and wave-filter set for the frequency of a particular harmonic, a measure of the magnitude of this harmonic is obtained by adjusting the series and shunt resistances of the comparison circuit until the volume of sound heard in the receiver is the same whether connected into the resonant shunt or into the comparison circuit. When a "balance" is thus obtained, the current through the resonant shunt is equal to the current through the receiver as a part of the comparison circuit. This current is readily computed as indicated below. By carrying through this process for all frequencies present in the complex wave, a complete analysis is obtained.

The connections of the resonant shunt and auxiliary comparison circuit are shown by P. I. C. No. 178.

For the necessary inductance, coils or groups of coils, giving an inductance of from 15 to 20 henrys, are generally used. These coils should have a large inductance and low effective resistance. Several types of retardation coils designed for use in the telephone plant, have been tried for this purpose. The American Telephone and Telegraph Company has developed a special coil having several advantages over existing coils designed for other purposes. All coils which have been used are of the toroidal type.

The variable capacity consists of air and mica condensers in parallel with a range of capacities suitable for resonating with the inductance of the circuit at frequencies ranging from approximately 180 to 1800 cycles per second. The Vreeland Oscillator will not operate at frequencies below 180.

The series resistance R_1 in the comparison circuit is adjustable in steps of 100 ohms up to 11000 ohms. This resistance is usually made large compared to the resistance of the receiver and shunt resistance R_h. R_h is the resistance between contact points on a drop-wire rheostat, variable from zero to 110 ohms in steps of 1 ohm. The resistance in the oscillator circuit remains constant. When a balance is obtained, the current in the receiver and, hence, the current through the shunt is—

$$I_s = I_o \frac{R_h}{R_1 + R_r} \qquad \text{(Approx.)}$$

This approximate formula is the one commonly used and is sufficiently accurate when the resistance R_1 in series with the receiver is so large that the inductance of the receiver may be neglected and the resistance R_h is small compared to R_1. R_r is the effective resistance of the receiver which is approximately 200 ohms. I_o is the total current supplied, which is generally 5 or 10 milliamperes.

The drop-wire rheostat is supplied from the secondary of a repeating coil, the mid-point of whose primary winding is grounded. By thus isolating this portion of the circuit from direct connection to the oscillator, unbalanced circuit conditions introduced by the regulating rheostat and wave-filter do not give rise to any appreciable residual tone in the receiver when R_h is zero. Unless such precautions are observed, serious errors are apt to be introduced. The milliammeter, indicating the total current supplied, is connected between the two equal secondary windings of the repeating coil, thereby avoiding unbalance in the secondary circuit.

The wave-filter consists of a variable condenser and an inductance coil in series. By adjusting the wave-filter for resonance at the particular frequency in use, currents of other frequencies are largely excluded.

The regulating rheostat, wave-filter, repeating coil and milliammeter are enclosed in a grounded metallic shield to obviate electrostatic induc-

tion from this apparatus. The leads to the oscillator and the leads from the secondary of the repeating coil are enclosed in cable with the sheath grounded.

(b) *Circuit Arrangements for Resonant Analyses.*

The circuit arrangements used in connection with the resonant shunt in making resonant analyses are shown on P. I. C. No. 178 (Figs. 2 and 3) and discussed below.

1—Voltage Analysis.

Fig. 2 of P. I. C. No. 178 shows the scheme of connections employed in analyzing voltage waves. The supply circuit, generally from a potential transformer, is closed through a capacity and a resistance connected in series and the resonant shunt is connected across a portion of this resistance. The magnitudes of the currents in the receiver of the resonant shunt are determined by comparison as described above. From these and the known circuit constants the magnitudes of the component harmonic voltages are computed. This computation is indicated below the diagram of connections. If a reasonably permanent calibration of the resonant shunt can be obtained, the labor involved in such recurrent computations may be greatly reduced by standardizing the circuit constants and the amount of current supplied by the oscillator. Curves can then be prepared giving directly the voltage corresponding to any harmonic for each setting of R_h.

2—Current Analysis.

The arrangement of circuits for use in current analysis is shown in Fig. 3 of P. I. C. No. 178. A known small resistance R, is inserted in the circuit whose current wave it is desired to analyze. The drop in potential across this resistance R, energizes the resonant shunt. The current in the resonant shunt at each frequency is determined as described above. The corresponding line current is then computed from the known values of the circuit constants. As in the case of voltage analyses a permanent calibration of the shunt saves much labor as curves may be prepared which give the results directly for different settings of the control rheostat R_h.

3—Use on Communication Circuits.

To analyze the induction in telephone circuits the resonant shunt is connected either in series with the circuit or across a noninductive resistance which is bridged across the terminals of the telephone circuit.

In the first case the telephone circuit is effectively a part of the resonant shunt inasmuch as the two are in series. The setting of the variable condenser for resonance at any particular frequency is, therefore,

dependent, in some degree, upon the reactance of the line. A line having inductive reactance requires less capacity and lines of capacity reactance require more capacity for resonance than in the second case described below. The effect of lines having inductive reactance is usually small as the added inductance is in most cases small compared to that in the shunt. The effect of lines of capacity reactance is particularly noticeable at low frequencies. If the equivalent capacity of the lines is very small it may be impossible to obtain resonance for the lower frequencies. With the resonant shunt connected to a circuit in this manner and adjusted for resonance at a particular frequency the reactance of the line is neutralized by that of the shunt and the current flows through an impedance equal to the effective resistance of the shunt plus the equivalent resistance of the line at that frequency. The open circuit voltage follows very simply.

In the second case the terminals of the circuit are bridged by a resistance and the resonant shunt connected across the whole or a portion of this resistance. If the resistance across which the resonant shunt is connected is small compared with the effective resistance of the shunt so that the reactance of the line has no influence upon the resonant settings, the analysis is exactly similar to the analysis of current in the power circuit. The open circuit voltage is the product of the observed line current into the impedance of the line plus the resistance bridged across the terminals of the circuit.

If the resistance across which the resonant shunt is connected is not small compared to the effective resistance of the shunt, the determination of the open circuit voltage is less simple. It is generally possible to avoid this condition and use the shunt either directly in series with the circuit or across a small resistance which closes the circuit. The direct connection is generally required for transverse measurements owing to the smallness of the induction. The other method is applicable sometimes for longitudinal measurements.

The short circuit current is obtained in all cases from the open circuit voltage and the line impedance.

4—Other Uses.

Two uses of the resonant shunt other than for the analysis of waves may be described here. By such uses a determination may be made of the ratio between two quantities, the absolute magnitudes of both being unknown. Such an application is the determination of the ratio of transformation of harmonics by current transformers. A second application of this nature is the determination of the ratio of the inducing current or voltage in a power circuit to that induced in neighboring circuits. This method is useful when the power circuit is energized from a special source for experimental purposes. In both instances above mentioned,

adjustable noninductive resistances are inserted in series with each of the two circuits involved. A throw-over mercury switch is provided for connecting the resonant shunt alternately across one and the other resistance. One or both of these resistances is varied until the sound in the receiver of the resonant shunt is the same for both positions of the switch. The drop of potential, corresponding to the particular harmonic for which the shunt is in resonance, is then the same across the two resistances. This affords a basis for computing the desired ratio of the two unknown quantities.

It should be noted that the accuracy of such determinations is entirely independent of the calibration of the shunt, which does not enter into the computations. The accuracy is controlled by the ability of the ear to detect small differences in volumes of sound of the same frequency. With care an accuracy of 5% should be obtained for a single observation. It is necessary to take the precaution to keep the resistances inserted in the two circuits as near ground potential as possible in order to avoid a residual tone in the receiver at zero settings of the resistances.

(c) *Calibration.*

Calibration of the resonant shunt consists in the determination of its effective resistance at various frequencies and current densities. This calibration is made at three or more current densities for each odd multiple of the fundamental frequency over the range encountered, so that a curve may be obtained for each frequency showing the effective resistance as a function of the current through the shunt. A modified form of impedance bridge is used for this calibration.

The shunts in use have an effective resistance of approximately 1000 ohms at 180 cycles and from 5500 to 14000 ohms at 1500 cycles.

(d) *General Considerations.*

1—Sensitiveness.

The sensitiveness of the instrument depends upon the amount of current which passes through the receiver for a given potential difference at the terminals and, hence, upon the effective resistance of the shunt. It is, therefore, desirable to have the effective resistance of the shunt as low as possible.

2—Separation.

The separation of the shunt is its effectiveness in excluding currents of frequencies other than that for which the circuit resonates. It is desirable, of course, that this exclusion shall be as great as possible. The separation factor depends upon the time constant, which is the ratio of the inductance of the circuit to its effective resistance. A high time constant will give a high degree of separation.

The separation factor for adjacent harmonics is defined as the ratio of the nth harmonic current to the $(n \pm 2)^{th}$ harmonic current where n is the order of the harmonic for which the shunt is resonated. It is obvious that this depends upon the relative values of the several harmonic voltages impressed on the shunt. If, however, the limitation of equal impressed voltage is assumed, the separation factor becomes a definite quantity, being the ratio of the impedance for the $(n \pm 2)^{th}$ harmonic to the effective resistance for the nth harmonic at which the shunt is adjusted for resonance; *i. e.*,

$$S_{(n \pm 2)} = \frac{Z_{(n \pm 2)}}{R_n}$$

Assuming that $R_{(n \pm 2)} = R_n$ this becomes,

$$S_{(n \pm 2)} = \sqrt{1 + 632\, T^2_n f^2 \left(\frac{n \pm 1}{n \pm 2}\right)^2} \quad \text{(Approx.)}$$

Where T_n is the time constant for the nth harmonic and f is the frequency of the fundamental.

The separation factor affords a basis for the comparison of different resonant shunts at various frequencies. The following separation factors are given by way of illustration.

Computed Values of Separation Factor.

	Shunt No. 3		Shunt No. 6	
n	n—2	n+2	n—2	n+2
5	29	19	21	14
9	15	12	13	10
15	7.3	6.4	8.5	7.5

The exact and approximate solutions are in very close agreement, within 1% or better. These results show that shunt No. 3 has a better separation than shunt No. 6 at low frequencies, the reverse being true at high frequencies. Experimental determinations check these computed values very closely.

When using the resonant shunt connected directly in series with a telephone circuit a high time constant is not the only requirement for good separation. It is likewise necessary that the inductance be large so that the telephone circuit, which becomes effectively a part of the resonant shunt, may not greatly reduce the time constant of the circuit as a whole below that of the resonant shunt, and thereby impair the separation.

3—Permanence.

Variations in the effective resistance of the resonant shunts have been noted from time to time and frequent recalibrations have been necessary. These changes in effective resistance are ascribed to two causes: (1) low and variable insulation of condensers, (2) changes in the magnetization of the cores of the inductance coils.

Calculations indicate that due to the first cause, leaky condensers, the effective resistance of shunt No. 6, for example, will be increased by aproximately 1000 ohms for the 19th harmonic due to a leakage of 0.1 microhm. For a leakage of 0.01 microhm (insulation resistance 100 megohms) this increase in effective resistance amounts to approximately 100 ohms, and for a smaller leakage the increase in effective resistance is negligible. The figures given are with reference to the effective resistance with zero leakage. It is to be noted that the effect of a given leakage increases with the frequency. Also, assuming a given total leakage, or a given value of g/c for the condenser, its effect is more marked for shunts having coils of high inductance. In practice, it has been found that most of the leakage is external and is, therefore, independent of the condenser setting. Leakage sufficient to account for the greater part of the observed variations has been found in the condensers in use. The trouble from this cause may be avoided by careful drying and repeated tests of the condensers. A better method, however, is to use condensers of proper construction which will be done when those already authorized are obtained.

Variations due to the second cause, changes in the magnetization of the cores of the inductance coils, are probably much smaller in magnitude than those due to variations in the leakage of the condensers. Such an effect is, to some extent, inherent in any coil constructed with an iron core. In those coils designed especially for the resonant shunts, an endeavor was made to eliminate, as far as possible, this cause for variations in the effective resistance of the resonant shunts.

With the elimination of leakage in the condensers, it is expected that permanent calibrations will be obtained.

A possible cause of apparent variations in effective resistance is the use of an unshielded impedance bridge in calibrating the instrument.

IV. Comparison of Oscillograph and Resonant Shunt.

(a) *Single Frequency Test.*

For a test of the accuracy of two resonant shunts and the oscillograph, the circuit was arranged as shown on P. I. C. No. 157.

To test the resonant shunts, settings of R_h were made at various frequencies from 180 up to 1380 cycles and for various amounts of current in the receiver, obtained by varying R. Several observers

were used in each case. The currents as computed from the resonant shunt measurements were found to be uniformly low, the error averaging about 10%. To determine the cause of this unidirectional error a further investigation was made. This investigation located the causes of the error and showed that by observing proper precautions an accuracy of 5% could be obtained. The four sources of error are as follows:

1—Inexact resonance for the harmonic which is being measured. Hence, the impedance of the circuit is higher than the value of effective resistance given by the calibration curves. This causes the computed current to be lower than the true value.

2—Variations in the insulation resistance of the condensers, causing the effective resistance of the resonant shunt to vary. As the calibration was made with high insulation (effective resistance minimum), the error introduced by this cause will make the computed current less than its true value.

3—Unbalanced admittance to ground, with respect to the receiver, of portions of the resonant shunt, causing a current to flow through the receiver even though there is no difference of potential between the two terminals of the shunt. This current is superimposed on the circulating current due to the difference of potential between the terminals of the shunt, introducing an error in the measured current, dependent on how the resonant shunt is connected into the circuit.

4—Inability of the human ear to detect small differences in sound. This error may be in either direction, depending in amount upon the observer.

To obtain results to an accuracy of 5%, which is about the accuracy of the ear in detecting differences in sound, the following precautions should be taken:

1—The resonant shunt should be very carefully resonated, so that its impedance will be equal to its effective resistance.

2—Care should be taken to see that in all cases the insulation of the condensers is maintained at a high value.

3—The terminals of the shunt should be as near ground potential as possible, to avoid the current flow to ground, mentioned above. When this current is appreciable in comparison with the circulating current in the shunt two measurements should be taken, reversing the terminals of the shunt with respect to the external circuit.

Single frequency tests on the oscillograph have been made to determine the variation, with frequency, of the current per unit deflection of the vibrators.

In the following table is given a sample of such data:

Oscillogram No.	Frequency f	Milliamperes per unit * deflection K	$\frac{K_f}{K_o}$
451 to 456	0	0.83	1.00
452	180	0.86	1.04
453	300	0.88	1.06
454	540	0.95	1.15
455	900	1.00	1.21
456	1,260	1.10	1.32

*1/40 inch.

The adjustment of the vibrators has been changed so that, under present conditions, the variations in K are much smaller, being within 10% of the direct current value at 1260 cycles. The precision of this calibration is dependent on the accuracy of the measurement of the oscillogram and errors do not exceed 3%.

Results obtained with the oscillograph are subject to error principally in the measurement of the waves. The accuracy with which any given harmonic may be determined depends on the amount present in the wave. The harmonic currents through the vibrators can be determined to the nearest milliampere.

(b) *Complex Wave Test.*

A comparison of two resonant shunts and the oscillograph, in determining the harmonics of a complex wave, was carried out, using connections shown on P. I. C. No. 157, attached. The oscillograph circuit arrangement used was that commonly employed for line current measurement (P. I. C. No. 124, Fig. 4). With the resonant shunts balanced for each of several harmonics an oscillogram was made and analyzed for the one harmonic, only. The table below shows the results obtained.

Current in Milliamperes.

Frequency	Shunt No. 3	Shunt No. 6	Average of shunts	Oscillograph	Osc. No.
180	88	90	89	101	447
300	33	37	35	32	449
420	4.1	4.5	4.3	—	—
540	1.4	1.2	1.3	1.6	448
660	5.3	6.2	5.8	—	—
780	9.8	9.7	9.8	9.6	450
900	0.9	0.6	0.8	—	—
1020	*	*	*	—	—
1140	1.5	0.8	1.2	—	—

*Too small to measure.

The two shunts are seen to be in fair agreement with each other and with the oscillograph.

(c) *Adaptability of the Methods.*

This oscillograph is not suited for measuring currents less than several milliamperes, hence, its use is limited to those cases where the values are at least of this magnitude. The resonant shunt, however, can be used for measuring currents of but a few microamperes provided it may be connected directly in series with the circuit whose current is to be measured, but its accuracy is questionable if the current through the shunt is below 5 microamperes, especially at low frequencies. This is true also for low current values at high frequencies if the shunt as a whole is much above ground potential.

For currents above 5 milliamperes, the oscillograph is at least as accurate as the resonant shunt, with the great advantage that three complete waves may be taken simultaneously and instantaneously. A disadvantage of the resonant shunt is the lack of simultaneity in the measurements of different harmonics of the same wave, the time required for a complete set of observations for a resonant analysis being approximately twenty minutes.

For measurements of the transverse induction in communication circuits the resonant shunt is better adapted than this oscillograph owing to its greater sensitivity and this is often times the case in longitudinal measurements.

For values of the lower harmonics in power circuits the oscillograph is better adapted than the resonant shunt, which gives no indication at all at 60 cycles. For high harmonics of small magnitude the resonant shunt is probably better, its desirability being inversely proportional to the magnitude of the harmonic.

The time required for analyzing an oscillograph wave and expressing the results in electrical units is in general greater than that required for the computations involved in a resonant analysis. This is particularly true when curves can be prepared for the resonant shunt giving the final results directly from the observations.

Respectfully submitted.

(Signed) J. E. WOODBRIDGE,

Chairman Subcommittee on Tests No. 1 and No. 2.

ATTACHED: P. I. C's. 124, 157 and 178.

IN FILES OF JOINT COMMITTEE: Oscillograms: Nos. 447, 448, 449, 450, 452, 453, 454, 455 and 456.

15—20080

P.I.C. #124
2-16-14.
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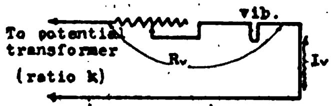
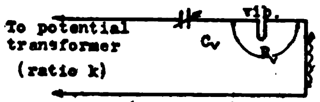
CIRCUIT DIAGRAMS FOR OSCILLOGRAMS

Voltage ———— Power Circuits.



(undistorted)
Fig. 1.

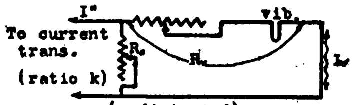
Primary Voltage. $E'_n = k(I_v)_n R_v$



(distorted)
Fig. 2

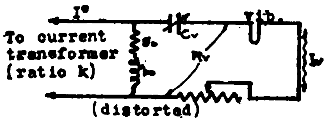
Primary Voltage. $E'_n = k(I_v)_n \sqrt{R_v^2 + \left(\frac{10^6}{2\pi f_n C_v}\right)^2}$

Current ———— Power Circuits.



(undistorted)
Fig. 3.

Primary Current. $I'_n = k(I_v)_n \left(1+\frac{R_v}{R_s}\right)$



(distorted)
Fig. 4

Primary Current. $I'_n = k(I_v)_n Z_v \sqrt{g_o^2 + b_o^2}$

$$g_v = \frac{R_v}{R_v^2 + \left(\frac{10^6}{2\pi f_n C_v}\right)^2} \qquad b_v = \frac{\frac{10^6}{2\pi f_n C_v}}{R_v^2 + \left(\frac{10^6}{2\pi f_n C_v}\right)^2}$$

$$g_o = g_v + g_s \qquad b_o = b_v - b_s$$

Induction ———— Telephone Circuits.

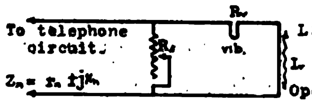


Fig. 5

Line Current $I_n = (I_v)_n \left(1+\frac{R_v}{R_s}\right)$

Open Circuit Voltage $(E_{oc})_n = I_n \sqrt{\left(r_n + \frac{R_v R_s}{R_v + R_s}\right)^2 + X_n^2}$

Short Circuit Current $(I_{sc})_n = \frac{(E_{oc})_n}{\sqrt{r_n^2 + X_n^2}}$

NOTE: In all cases R, X and Z are in ohms, g and b in mhos, C in microfarads, L in henrys, f_n is the frequency in cycles per second, and n is the order of the harmonic.

Dec. T.R. #41.

P.I.C.No.157
1-3-14
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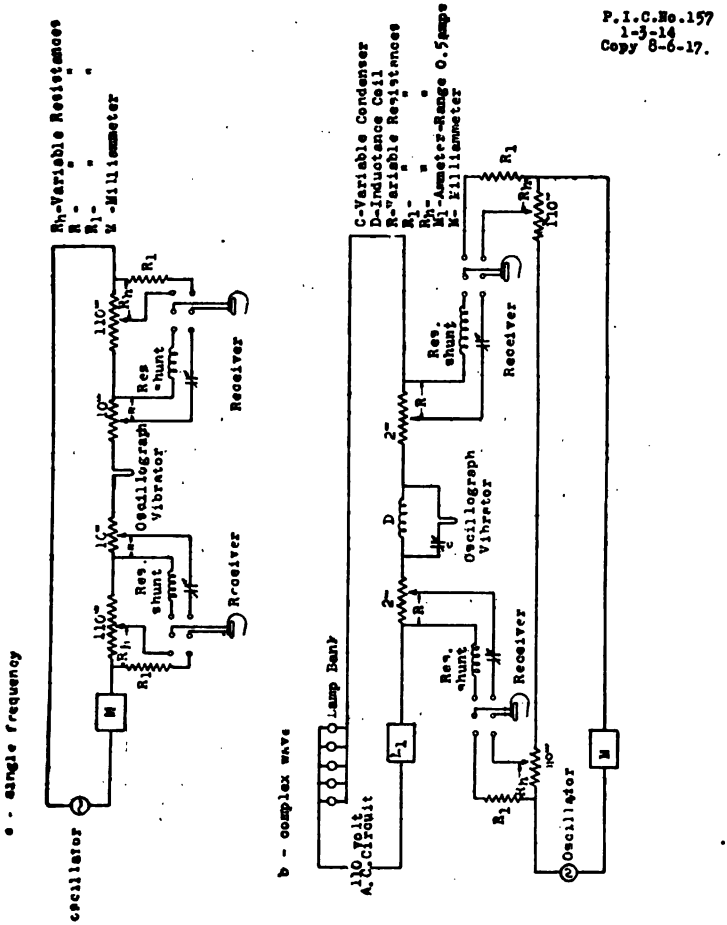
HARMONIC ANALYSIS OF ALTERNATING CURRENT WAVES
COMPARISON OF OSCILLOGRAPH AND RESONANT SHUNT
CIRCUIT DIAGRAM



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CIRCUIT DIAGRAMS FOR RESONANT ANALYSES

P.I.C.#178
March 5, '14
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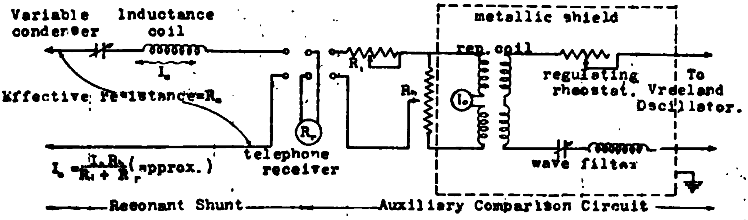
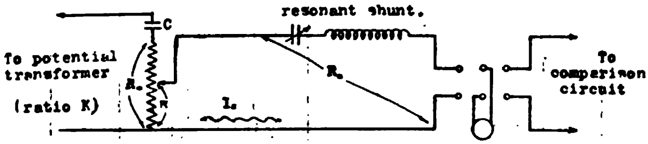
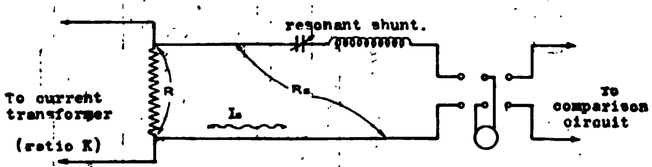


Fig.#1



Primary Voltage, $E_n = K(I_n)(\frac{R_s}{R})\sqrt{R_s^2 + \frac{1}{(2\pi n C)^2}}$ (approx.)

Fig.#2-Voltage Analysis.



Primary Current, $I_n' = K(I_n)\frac{R_s}{R}$ (approx.)

Fig. #3-Current Analysis.

T.R.#41.

g.f.f.

Technical Report No. 42.

January 26, 1914.

INVESTIGATION OF CURRENT TRANSFORMERS IN USE AT SANTA CRUZ.

1. Purpose.

The tests which are reported below were made on the three current transformers, in use at Santa Cruz, in order to obtain information as to the suitability of these transformers for the purposes of our work. Primarily, it was desired to know:

a—The ratio of transformation for the fundamental and higher harmonics under various loads and throughout the range of secondary impedance required for purposes of other tests.

b—The suitability of these transformers for the measurement of residual current and the limiting conditions under which this may be done.

2. Description of Current Transformers.

The current transformers in use at Santa Cruz and upon which these tests were made, are three similar transformers manufactured by the General Electric Company. The name plate data are as follows:

Serial numbers—967008, 998385, 998387.

Specification No. 144686, Type S, Form K-9.

Amperes 20/40. Ratio 4/8 to 1.

Cycles 25—125. Voltage 27,000.

3. Discussion of Tests and Results.

The different tests which were made in the course of the investigation, together with the results obtained, are discussed in accordance with the following outline:

A—Ratio of Transformation for Effective Value of Current.

B—Suitability for the Measurement of Residual Current.

C—Ratio of Transformation for Harmonics.

D—Exciting Current.

E—Primary and Secondary Impedance.

A—Ratio of Transformation for Effective Values of Current.

Tests to determine the ratio of transformation for the effective value of the complex current were made by taking simultaneous meter read-

ings of the primary and secondary currents. The results of these tests are given in the following table:

TABLE I.

Ratio of Transformation for Effective Values—Meter Readings.

Current transformer number	Primary current, amperes	Secondary current, amperes	Ratio Ip/Is	Average ratio
967008	12.0	1.50	8.0	
	21.0	2.63	7.98	7.96
	29.7	3.74	7.94	
	39.4	4.98	7.91	
998385	11.0	1.30	7.97	
	19.7	2.45	8.04	7.95
	30.2	3.81	7.93	
	38.3	4.86	7.88	
998387	11.7	1.49	7.85	
	20.0	2.51	7.97	7.91
	30.1	3.80	7.92	
	38.6	4.88	7.91	

The above results indicate that the average ratio of transformation is within 1% of the rating. For purposes of our work the rated ratio for the fundamental is, therefore, sufficiently accurate. This test was made only as a rough check upon the transformers. The precision of the results is not such that they can be said to show any inequality in ratio among the three transformers.

B—Suitability for the Measurement of Residual Current.

Residual current is defined as the vector sum of the line currents. The residual current of a three-phase system is measured by a method involving the use of three current transformers. The primaries of the transformers are connected in series with the line conductors. The secondary windings are connected in star grounded. Ammeters indicating the line currents are connected in series with each secondary and grounded through a meter indicating the vector sum of the secondary currents. Disregarding errors introduced by the current transformers the secondary neutral current is a true measure of the vector sum of the primary currents or the residual current of the line.

A consideration of the problem indicates the following possible sources of error in measurements of residual current:

a—Unequal ratios of transformation for the three transformers under equal loads.
b—Variations in the ratio of transformation with load.
c—Unequal impedances of the secondary windings of the three transformers.

d—Unbalanced external impedances in the secondary circuits. (Meters, drop-wire, rheostats, etc.)
e—The presence in the secondary neutral of the 3d harmonic and its multiple arising from hysteresis in the current transformers.

An apparent residual current due to the combined effect of the causes enumerated above will appear in the secondary neutral when no true residual current exists in the primary. Such errors are, therefore, in addition to errors in the ratios of transformation of the primary residual current. Errors in ratio of transformation apply equally to the line currents and are not considered as a factor in determining the suitability of the transformers for measurements of residual current.

Errors due to the first four causes will introduce into the current in the secondary neutral harmonics characteristic of balanced three-phase currents, that is, fundamental, 5, 7, 11, harmonics, etc. Variations in the ratio of transformation with load will give the effect of unequal ratios of transformation when the load currents on the three transformers are unequal.

The errors introduced by the third harmonic and its multiples arising from hysteresis in the current transformers are inherent with the type of connection necessary for measuring residual current. These harmonics are in phase in all three secondaries and, therefore, flow through the secondary neutral circuit. Their presence is independent of the presence or absence of corresponding harmonics induced by the primary windings and with which they combine vectorially in the case of actual residual current.

The tests described below were made to determine the magnitude of the errors arising from the causes above discussed, to be expected in measurements of residual current made with these particular transformers. The diagram of connections used in testing is given on the attached blue print (P. I. C. No. 132); the secondary connections there shown are those in normal use. In addition to an ammeter, a two ohm drop wire resistance unit and an air core inductance coil of 0.0075 henry are inserted in the secondary circuit of each transformer. The are used in the operation of resonant shunts and oscillograph. A 10 ohm drop wire resistance unit and a milliammeter are placed in the secondary neutral for the purpose of analyzing and measuring the effective value of the residual current. Short-circuiting switches are provided in the secondary circuits for cutting out apparatus not in use.

In order to determine the magnitude of the apparent residual current the transformers were excited and the primary neutral opened, thus eliminating all true residual current. Measurements, meter readings, oscillograms and resonant analyses were taken of the apparent residual current in the secondary neutral under different load conditions of the

primary and under different conditions of the secondary circuits as regards the included apparatus. In particular, the measurements were made for practically the full range of primary load current and the effect of unequal load was also noted. As regards the secondary conditions, the measurements were made with different amounts of balanced and unbalanced impedances, represented by different pieces of apparatus in the secondary circuits. The results of all these measurements are summarized in tables below. Following the tables some discussion of the results is given.

A complete set of meter readings of the effective value of the apparent residual current for different loads and secondary conditions are plotted on P. I. C. No. 160, attached. These results are not tabulated.

TABLE II.

Apparent Residual Current—Meter Readings.

Effect of Unbalanced Impedances—Equal Load Currents.

Secondary load current, amperes	Apparent residual current, milli-amperes	Conditions					
		2 ohm resistances			Inductance coils		
		No. 1	No. 2	No. 3	No. 1	No. 2	No. 3
2.95	8.12		All in			All in	
2.95	3.39		All out			All out	
2.95	3.49		All in			All out	
2.90	12.1	In	Out	Out		All out	
2.93	11.2	In	In	Out		All out	
2.87	16.0		All out		In	Out	Out
2.87	17.1		All out		In	In	Out
2.87	7.74		All out			All in	
4.84	14.0		All in			All in	
4.76	6.36		All out			All out	
4.60	4.92		All in			All out	
4.5	15.5	In	Out	Out		All out	
4.5	16.0	In	In	Out		All out	
4.4	*		All out		In	Out	Out
4.4	*		All out		In	In	Out
4.4	13.6		All out			All in	

NOTE.—Ammeters in secondary leads during all measurements.
*Off scale. Full scale deflection 20 milliamperes.

TABLE III.

Apparent Residual Current—Meter Readings.

Balanced Impedances—Unequal Load Currents.

Secondary load current—amperes			Apparent residual current, milliamperes	Conditions	
I_1	I_2	I_3		3 ohm resistances	Inductance coils
2.05	3.17	3.47	4.41	All out	All out
2.03	3.18	3.44	12.1	All in	All in
2.03	3.27	3.55	7.18	All in	All out
1.97	3.30	3.55	10.9	All out	All in
4.86	4.45	1.60	4.54*	All out	All out
4.75	4.32	1.61	13.0	All in	All in
4.80	4.40	1.70	6.08	All in	All out
4.75	4.35	1.70	11.3	All out	All in

NOTE—Ammeters in secondary leads during all measurements.
*See oscillogram No. 413.

TABLE IV.

Apparent Residual Current—Resonant Analyses.

Balanced Impedances—Equal Load Currents.

Secondary load current—amperes	Harmonic	Apparent residual current—milliamperes Condition of secondary leads			
		Resistances out Coils out	Resistances in Coils in	Resistances out Coils in	Resistances in Coils out
2.90	3	3.8	6.8	6.8	3.8
2.95	5	0.09	0.13	0.13	0.09
3.10	7	0.11	0.16	0.16	0.05
2.85	9	0.21	0.45	0.45	0.21
2.90	11	*	0.08	0.08	*
2.95	13	*	*	*	*
3.0	15	*	0.10	*	*
2.95	17	*	*	*	*
—	19	*	*	*	*
2.90	21	*	*	*	*
—	23	0	*	*	0
2.85	25	0	*	*	0
4.6	3	5.8	15	15	5.1
4.25	5	0.31	0.62	0.62	0.47
4.20	7	0.37	0.20	0.59	0.20
4.00	9	0.24	1.4	1.2	0.36
3.95	11	*	*	*	*

NOTE—Ammeters in secondary leads during all measurements. 10 ohm drop wire resistance in secondary neutral.
*Denotes values too small to measure.

TABLE V.

Apparent Residual Current—Resonant Analyses.

Balanced Impedances—Equal Load Currents.

Resistance out—Coils out			Resistance in—Coils in		
Secondary load current, amperes	Apparent residual current		Secondary load current, amperes	Apparent residual current	
	Harmonic	Milliamperes		Harmonic	Milliamperes
1.55	3	2.4	1.49	3	3.4
	5	0.0	1.50	5	0.09
1.64	7	0.54	1.50	7	0.08
1.64	9	0.22	1.53	9	0.23
1.68	11	0.12	1.53	11	*
1.62	13	*	1.49	13	*
1.60	15	*	1.48	15	*
	17	0	1.48	17	*
	19	0		19	0
1.51	21	*	1.50	21	*
	23	0		23	0
	25	0		25	0

NOTE.—Ammeters in secondary leads during all measurements. 10 ohm drop wire resistance in secondary neutral.

*Denotes value too small to measure.

Apparent Residual Current—Resonant Analyses. Unbalanced Impedances—Equal Load Currents.

Cutting in inductance coils in one, two, or all three secondary leads produced an increase in the third harmonic and its odd multiples up to the 21st harmonic, the highest one examined. The increase appears to be approximately proportional to the number of coils added.

The 2 ohm resistance units have little effect on the 3d and 9th harmonics. With heavy secondary load current the addition of one, two, or three units of resistance produced a slight decrease, the inductance coils being out of circuit. These observations are not in accord with those noted above for the effect of the inductance coils. It is to be expected that the effect of the resistance units would be of the same character, but less marked, than the effect of the coils.

For the 5th, 7th and 11th harmonics one inductance coil or resistance produced an increase; the addition of a second coil or resistance produced no effect; and the addition of a third coil or resistance produced a decrease. The resistance units produced less effect than the inductance coils.

TABLE VI.

Apparent Residual Current—Oscillogram Analyses.

Balanced Impedances—Equal Load Currents.

Oscillogram No.	Secondary load current, amperes	Harmonic	Apparent residual current—milliamperes	
			Resistances in Coils in	Resistances in Coils out
			Vibrator No. 1	Vibrator No. 2
410	3.1	1	2.0	2.5
		3	8.9	3.6
		5	*	*
		7	*	*
		9	*	*
		11	*	*

Balanced Impedances—Equal Load Currents.

Oscillogram No.	Secondary load current, amperes	Harmonic	Apparent residual current—milliamperes	
			Resistances in Coils in	Resistances out Coils out
			Vibrator No. 2	Vibrator No. 1
411	4.07	1	No exposure on film	2.7
		3		4.5
		5		*
		7		*
		9		*
		11		*

NOTE—Ammeters in secondary leads during all measurements.
*Denotes values which are too small to measure.

TABLE VII.

Apparent Residual Current—Oscillogram Analyses.

Balanced Impedances—Equal Load Currents.

Oscillogram No.	Secondary load current, amperes	Harmonic	Apparent residual current—milliamperes: Resistances in Coils in, Vibrator No. 3	Apparent residual current—milliamperes: Resistances out Coils out, Vibrator No. 1
412	4.9	1	2.9	1.8
		3	13.4	5.5
		5	*	*
		7	*	*
		9	*	*
		11	*	*

Balanced Impedances—Unequal Load Currents.

Oscillogram No.	Secondary load current, amperes I_1 I_2 I_3	Harmonic	Apparent residual current—milliamperes: Resistances in Coils in, Vibrator No. 3	Apparent residual current—milliamperes: Resistances out Coils out, Vibrator No. 1
413	4.75 4.32 1.60	1	12.0	4.6
	(Vibrator No. 3)	3	6.0	2.6
		5	1.9	*
	4.86 4.45 1.60	7	*	*
	(Vibrator No. 1)	9	*	*
		11	*	*

NOTE.—Ammeters in secondary leads during all measurements.
*Denotes values which are too small to measure.

TABLE VIII.

Apparent Residual Current—Oscillogram Analyses.

Effect of Unbalanced Impedances—Equal Load Currents.

Oscillogram No.	Secondary load current, amperes	Harmonic	Apparent residual current—milliamperes: One inductance coil in—all else out, Vibrator No. 1	Apparent residual current—milliamperes: One resistance in—all else out, Vibrator No. 3
414	3.7	1	21.0	17.0
		3	6.6	3.9
		5	1.0	*
		7	*	*
		9	*	*
		11	*	*

NOTE.—Ammeters in secondary leads during all measurements.
*Denotes values which are too small to measure.

Apparent Residual Current—Discussion of Results. An inspection of the results obtained under conditions approaching equal loads and with similar apparatus in each secondary circuit shows an appreciable amount of apparent residual current. Oscillograms of the apparent residual current under these conditions show it to be composed principally of the third harmonic. The analyses of oscillograms No. 410 and No. 412 taken with the maximum impedance in each secondary circuit and with the transformers under as nearly as possible equal loads, are plotted on P. I. C. No. 162, attached. These oscillograms show a fundamental in the apparent residual current amounting to approximately one seventeen-hundredth part of the fundamental in the secondary load currents. The magnitude of the third harmonic in these oscillograms is 4.5 times that of the fundamental. Resonant analyses made under similar conditions are in reasonable accord with the oscillogram as regards the third harmonic and show also the presence of small amounts of fifth, seventh, and ninth harmonics.

The effect of unequal load currents is shown by the meter readings in Table III and oscillogram No. 413 in Table VII. Under the extreme conditions of load unbalance to which these measurements apply the effective value of the apparent residual current is practically the same as that corresponding to balanced load currents equal to the maximum of the unequal currents. Oscillogram No. 413 shows, however, a marked change in wave form, the fundamental is greatly increased and the third harmonic diminished as compared with results obtained for equal load currents. The increase in the fundamental is due probably to the difference in the ratios of transformation corresponding to the different loads on the individual transformers. Also, under these conditions the three line currents are not 120 degrees apart in phase and, therefore, the hysteretic third harmonics of the three transformers are somewhat out of phase, which accounts for the diminution observed in the third harmonic. The fundamental in the apparent residual current as shown by this oscillogram is approximately one four-hundredth part of the maximum secondary load current. All the results obtained with the so-called balanced impedance of the secondary windings and associated apparatus, given in a later section of this report, indicate that further refinements to eliminate from the apparent residual current those harmonics characteristic of the balanced currents, would be unwarranted.

The tests with balanced impedances show an increase in the third and ninth harmonics with an increase in the magnitude of the secondary impedance. This is accounted for by the increase in the primary impedance voltage. It is best, therefore, to make measurements of residual current with a minimum of impedance in the secondary windings.

Oscillogram No. 414 (Table VIII) shows the marked increase in the fundamental due to an unbalance in the secondary impedances caused by the introduction of an inductance coil or resistance unit in the circuit of one transformer. Observations with the resonant shunt confirm this for the 5th, 7th, and 11th harmonics. Residual current should never be measured under such conditions.

C. Ratio of Transformation of Harmonics.

In order to determine the ratio of transformation of harmonics under normal load conditions the transformers were excited from the three-phase supply. The Vreeland Oscillator was introduced into the primary neutral circuit adjusted for resonance at the test frequency. Resonating the primary neutral circuit effectually eliminated currents of other frequencies.

The chief advantage of this method of determining the ratio of transformation of harmonics lies in the fact that a single harmonic may be isolated for testing purposes and its ratio of transformation determined under full load conditions of the transformers. It has the advantage and accuracy, therefore, of a single frequency test and yet may be made under normal load conditions of the transformers as regards saturation and core loss. It was easily possible to measure the ratio of transformation of harmonics whose magnitudes were as small as one four-thousandth part of the magnitude of the fundamental.

Two methods of measuring the ratio of transformation of the currents introduced into the primary neutral were used—the resonant shunt and the oscillograph. The resonant shunt method is described in technical report No. 41. It is sufficient to repeat here that the accuracy of the method is independent of the calibration of the shunt and is determined solely by the ability of the ear to detect differences in volumes of sound of the same frequency.

The results of the tests are recorded in Tables IX, X, XI and XII.

TABLE IX.

Ratio of Transformation of Harmonics—By Resonant Shunt.
Ammeters Only in Secondary Circuits.

Secondary load current, amperes	Primary neutral		Ratio of transformation Observer				
	Frequency	Current milli-amperes	No. 1	No. 2	No. 3	No. 4	Average
No load	180	80.0	8.4	8.3	8.4		8.4
No load	1260	30.0	8.2	8.1	7.8	7.8	8.0
3.7	180	75	Fluctuating—Not possible to measure.				
4.3 to 5.0	540	75.0	8.0				
2.45 to 2.75	540	75.0		8.0			8.0
4.8 to 5.0	540	75.0			7.9		
4.6 to 4.8	540	75.0				7.9	
3.4 to 3.6	900	49.0	7.9				
3.6 to 3.7	900	50.0		7.85			7.9
3.0 to 3.1	900	50.0				8.05	
2.1 to 2.7	1260	30.0	7.7				
2.75 to 3.4	1260	31.0		7.8			7.8
3.7 to 4.4	1260	31.0			7.7		
4.1 to 4.6	1260	31.0				7.9	

NOTE.—Primary neutral circuit adjusted for resonance at test frequency and supplied by Vreeland Oscillator. Ten ohm drop wire resistance in secondary neutral.

TABLE X.

Ratio of Transformation of Harmonics—By Resonant Shunt.
Ammeters and Inductance Coils in Secondary Circuits.

Secondary load current, amperes	Primary neutral		Ratio of transformation Observer				
	Frequency	Current milli-amperes	No. 1	No. 2	No. 3	No. 4	Average
4.0 to 4.6	420	75.0	8.0				
4.5	420	75.0		7.9			8.0
4.4	420	75.0			7.9		
4.4	420	75.0				8.0	
4.0 approx.	540	75.0	Fluctuating—Not possible to measure.				
2.10	1260	31.0	7.8				
2.65	1260	30.0		7.9			
4.50	1260	30.0		7.8			
4.0	1260	31.0			7.9		7.8
4.0	1260	31.0				7.8	
4.6	1260	30.0				7.8	
4.5	1260	30.0				7.8	
4.0	1260	30.0		7.7			

NOTE.—Primary neutral circuit adjusted for resonance at test frequency and supplied by Vreeland Oscillator. Ten ohm drop wire resistance in secondary neutral.

TABLE XI.

Ratio of Transformation of Harmonics—By Oscillograph.

Ammeters Only in Secondary Leads.

Oscillogram No.	Secondary load current, amperes	Frequency	Primary neutral current, milliamperes	Secondary neutral current, milliamperes	Ratio of transformation
461	3.07	60	132.0	16.4	8.05
		180	85.0	11.45	7.4
		300	*	*	—
		420	*	*	—
		540	*	*	—
		660	*	*	—
		Effective	157.0	20.1	7.82

NOTE.—Unbalanced load current through primary neutral. Oscillator not in circuit.

*Denotes values which are too small to measure.

TABLE XII.

Ratio of Transformation of Harmonics—By Oscillograph.

Ammeters Only in Secondary Leads.

Oscillogram No.	Secondary load current, amperes	Primary neutral		Ratio of transformation
		Frequency	Current milliamperes	
466	4.7	180	75.0	6.0 to 10
468*	4.7	180	75.0	‡
471	No load	180	80.0	8.5
469	4.0 to 5.0	540	167.0	8.3
465	4.8	900	72.0	8.1
463†	4.7	1260	52.0	8.4

NOTE.—Primary neutral circuit adjusted for resonance at test frequency and supplied by Vreeland Oscillator.

*Inductance coils in circuit also.

†10 ohm drop wire resistance in secondary neutral.

‡See discussion of this table.

Ratio of Transformation of Harmonics—Discussion of Results. The ratio of transformation, as determined by the resonant shunt method, shows a slight decrease with an increase in the frequency. Most of the tests were made with the transformers under practically full load with three-phase excitation. Over a considerable range of load, however, the observations show no change in ratio of transformation of the harmonics. With no three-phase load on the transformers the ratio shows a slight increase above the ratio as obtained under load conditions. The results show no change in the ratio due to an increase in the secondary impedance caused by cutting the inductance coils in circuit. The ratios of transformation as determined by these tests were unaffected by the introduction of harmonics in the secondary neutral by hysteretic action as such harmonics were inappreciable as compared with currents

of the same frequency supplied by the oscillator. Owing to the fluctuations caused by the hysteretic third harmonic it was impossible to make determinations at 180 cycles. For a similar reason the ratio for the ninth could not be determined when the inductance coils were in circuit. In future investigations of this nature it will be better to make determinations for frequencies corresponding to the 5th, 7th, and 11th harmonics, etc., which are not introduced in the secondary neutral by hysteresis. It is estimated that the average results obtained by the resonant shunt method are accurate within 3%.

Oscillogram No. 461 (Table XI) shows the primary and secondary neutral currents caused by a load unbalance. The ratio for the third harmonic is low (7.4) due to the effect of the hysteretic third harmonic.

Both oscillograms No. 466 and No. 468 (Table XII) show apparent variations in the ratio of transformation. This is due to the fact that the oscillator supply was not in exact synchronism with the third harmonic of the power system whose frequency varied slightly. This was evident from the fluctuations or beats observed in the primary neutral current. Under these conditions the third harmonic in the secondary neutral due to hysteresis, will add vectorially with varying phase angle to the third harmonic supplied by the oscillator. An amount of hysteretic third harmonic sufficient to account for the extreme variations in ratio shown by oscillogram No. 466, is shown by measurements made of the apparent residual current under conditions of load and secondary impedance the same as for this oscillogram.

The secondary wave form in oscillogram No. 468 is so distorted by the third and ninth harmonics due to hysteresis that it is impossible to determine the ratio of transformation with any degree of accuracy. The inductance coils were in circuit when this oscillogram was taken.

All oscillograms listed in Table XII show the presence of the hysteretic third harmonic in the secondary wave. This is a source of some error in determining the ratios of transformation for the particular harmonic supplied by the oscillator. It is estimated that these results are accurate within 5%.

Though all determinations of the ratio of transformation of harmonics were made for the primary and secondary neutral currents, the conditions of the tests were such as to make the results apply equally to harmonics of the balanced three-phase currents.

D. Impedance Voltage and Exciting Current.

Measurements of the impedance voltage and exciting current were made with the transformers connected for 8:1 ratio, at 60 cycles. Most of the measurements were made on transformer No. 998387. A few

readings were taken on the other two transformers and they checked within 1% the values obtained for transformer No. 998387.

The primary impedance voltage was measured over the range of primary load with various amounts of impedance connected on the secondary side. The values obtained are tabulated below and plotted on P. I. C. No. 158 together with a diagram of connections.

TABLE XIII.
Primary Impedance Voltage.

Primary load current, amperes	Primary impedance voltage, volts	Apparatus in secondary
12.8	2.8	
12.9	2.9	
20.1	4.4	
30.1	6.6	Short circuited.
39.1	8.7	
39.3	8.8	
39.7	8.8	
39.8	8.9	
13.9	3.2	
39.1	8.8	Ammeter
12.5	3.0	Ammeter and 2 ohm resistance unit.
39.8	9.5	
12.9	3.8	Ammeter, 2 ohm resistance unit and inductance coil.
39.8	11.5	

The exciting current was determined for the range of voltage indicated by the above table. The results are tabulated below and are plotted on P. I. C. No. 159, together with a diagram of connections.

TABLE XIV.

Primary volts	Exciting current, amperes
2.8	0.37
3.6	0.42
4.3	0.45
5.4	0.49
5.7	0.50
7.1	0.55
7.2	0.56
8.6	0.60
8.9	0.61
9.0	0.62
10.0	0.64
10.8	0.67
11.1	0.68

Referring to the diagram, the ammeter A will indicate the vector sum of (1) the exciting current, (2) the current taken by milliameter M in series with the noninductive resistance R_c, and (3) the current through the noninductive resistance R_s. For the same value of the current I_m through the meter M.

Let I_a = current through A with switch S open
I_b = current through A with switch S closed
I_x = exciting current

Then

$$\left.\begin{aligned} I_a &= I_x + I_m \\ I_b &= I_x + I_m + I_{R_s} \end{aligned}\right\} \text{(Vectorially)}$$

since the impedance of M is known, the last two equations are sufficient to solve for the exciting current I_x.

On P. I. C. No. 161 the exciting current is plotted as a function of the secondary current for three values of the external secondary impedance represented by different pieces of apparatus. These curves show that the addition of the 2-ohm resistance unit increases the exciting current about 3% above that required on short circuit. The addition of the 2-ohm resistance unit and the inductance coil increases the exciting current about 13%.

E. Primary and Secondary Impedance.

Preliminary to the determination of primary and secondary impedance of the current transformers, direct current measurements of the resistances were made of the primary and secondary windings of each transformer, together with the resistance of the secondary leads and associated apparatus. The results are given in the table below.

TABLE XV.

Current Transformer No.	Primary winding. ohms	Secondary winding. ohms	Secondary leads. ohms	Ammeters. ohms	Resistances. ohms	Coils. ohms	Total (including secondary winding) ohms
967008	0.049	0.999	0.368	0.095	1.989	1.058	4.509
998385	0.048	0.975	0.371	0.093	1.994	1.087	4.520
998387	0.048	0.967	0.378	0.094	1.990	1.108	4.537

Measurements of the inductance of the primary and secondary windings were made on transformer No. 967008. These measurements were made at several frequencies ranging up to 1260 cycles per second. The effective resistance was too small to measure with the impedance bridge. The measurements indicate no appreciable change in the inductance

with frequency. Taking the direct current resistance as the effective resistance, the results are as follows:

Secondary Winding—Primary Short-Circuited.

Inductance = 0.014 henrys.
Reactance at 60 cycles = 5.28 ohms.
Impedance at 60 cycles = $0.97 + j5.28 = 5.37$ ohms.
Approximate impedance at any frequency = 5.28 n ohms where n = order of harmonic.

Primary Winding—Secondary Short-Circuited.

Inductance = 0.0005 henrys.
Reactance at 60 cycles = 0.19 ohms.
Impedance at 60 cycles = $0.048 + j0.19 = 0.20$ ohms.
Approximate impedance at any frequency = 0.19 n ohms where n = order of harmonic.

4. Conclusions.

The ratio of transformation for the fundamental and higher harmonics under various loads and throughout the range of secondary impedance required for purposes of tests may be taken as the rated ratio, 8 to 1, within an accuracy of about 3%. This accuracy is sufficient for purposes of our work.

The value for the ratio of transformation given above does not take into account the effect of harmonics in the secondary circuits introduced by hysteresis, errors arising from the presence of which must be separately considered. The magnitude of the third harmonic and its multiples arising from this cause is shown by the measurements of apparent residual current. The values of the hysteretic third harmonic and its multiples, in each secondary circuit, will be one-third their value in the apparent residual current for the same condition of load and secondary impedance. The magnitude of harmonics other than the third and multiples which are introduced by hysteresis were not determined by these tests. It is reasonable to suppose, however, that the magnitudes of the fifth and other harmonics arising from hysteresis is less than that of the third.

The investigation of these transformers to determine their suitability for the measurement of residual current, showed the presence of an apparent residual current on the secondary side when no true residual current existed on the primary side. The magnitude of this apparent residual current increases with the load on the transformers and with the amount of impedance in the secondary circuits. Under conditions of equal load currents and balanced secondary impedances this apparent residual current varies from 0.1% of the secondary load current for meters only in secondary circuits to 0.3% for meters, 2-ohm resistance units and 0.0075 henry inductance coils in secondary circuits. It is

composed principally of the third harmonic together with some fundamental. The fundamental under the latter conditions amounts to approximately 0.06% of the secondary load current.

Unequal load currents give approximately an effective value of apparent residual current corresponding to the maximum of the unequal load currents. Under these conditions, however, the wave form is considerably altered, the fundamental being increased and the third harmonic decreased.

The harmonics characteristic of the balanced components of current (fundamental, 5th, 7th and 11th, etc.) present in the apparent residual current, are small with similar apparatus in each secondary circuit. The results indicate that it would be unwarranted to attempt further refinements of balance to eliminate these harmonics.

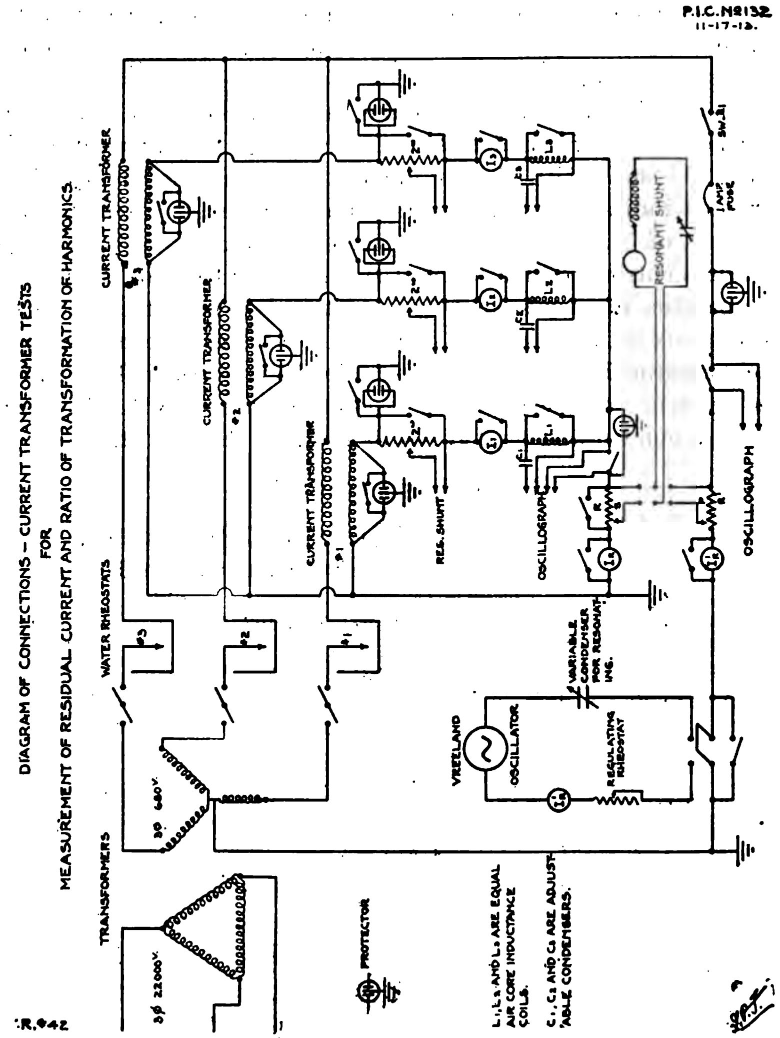
Measurements of residual current should never be made without similar apparatus in each secondary circuit and it is best to make such measurements with a minimum of impedance in each secondary circuit.

The magnitude of the possible errors in the true residual current due to the existence of the apparent residual current may be determined from the tabulated and plotted results given in this report.

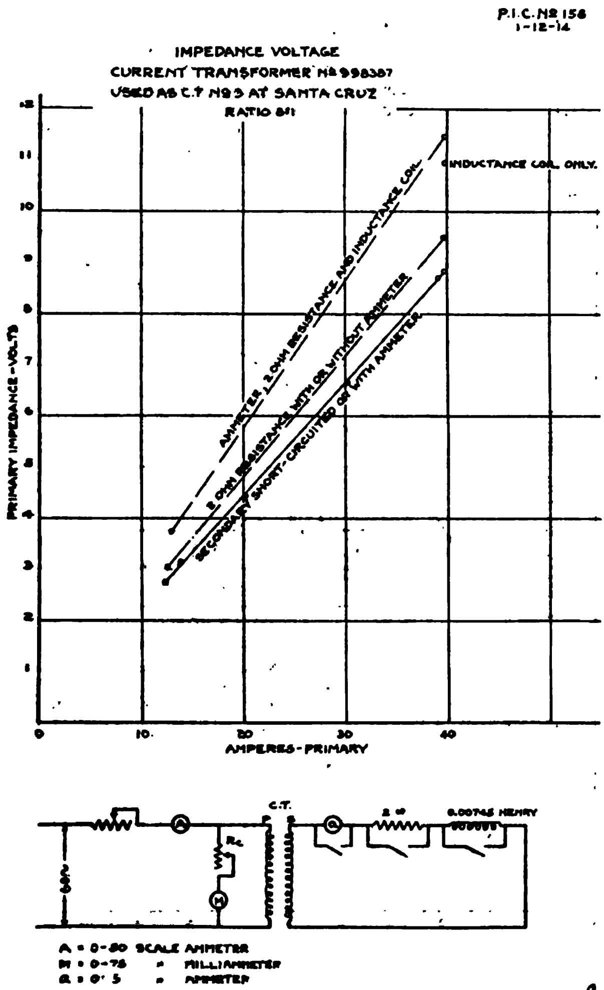
Respectfully submitted.

(Signed) J. E. Woodbridge,
Chairman Subcommittee on Test No. 3.

Attached: P. I. C. Nos. 132, 158, 159, 160, 161, 162.

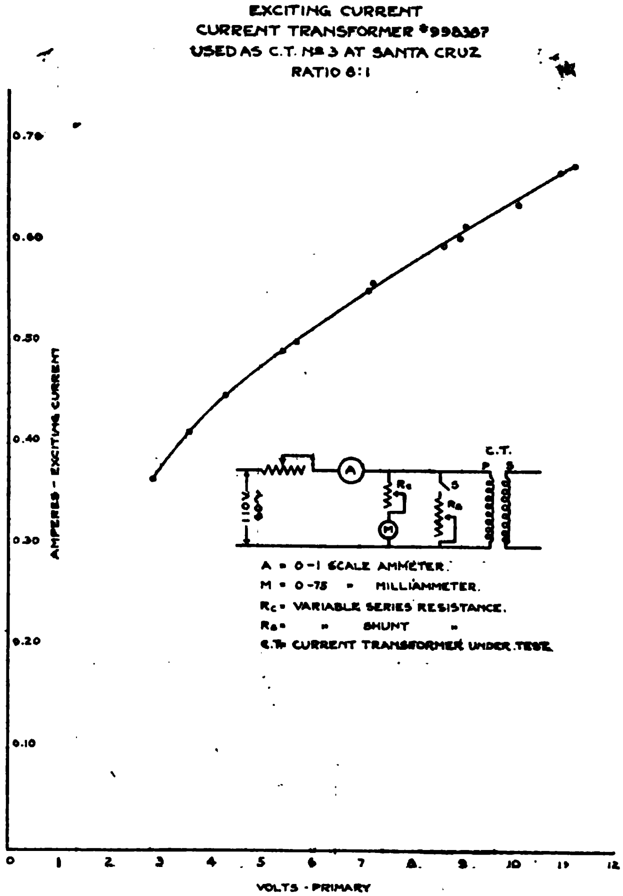


P.I.C. No 132
11-17-13.
DIAGRAM OF CONNECTIONS – CURRENT TRANSFORMER TESTS
FOR
MEASUREMENT OF RESIDUAL CURRENT AND RATIO OF TRANSFORMATION OF HARMONICS
TRANSFORMERS
3φ 22000 V.
3φ 680 V.
WATER RHEOSTATS
CURRENT TRANSFORMER
RES. SHUNT
OSCILLOGRAPH
VARIABLE CONDENSER FOR RESONATING.
VREELAND OSCILLATOR
REGULATING RHEOSTAT
RESONANT SHUNT
PROTECTOR
L1, L2 AND L3 ARE EQUAL AIR CORE INDUCTANCE COILS.
C1, C2 AND C3 ARE ADJUSTABLE CONDENSERS.
R.942


P.I.C. № 156
1-12-14
IMPEDANCE VOLTAGE
CURRENT TRANSFORMER № 998387
USED AS C.T. № 3 AT SANTA CRUZ
RATIO 8/1
INDUCTANCE COIL ONLY.
AMMETER, 2 OHM RESISTANCE AND INDUCTANCE COIL
2 OHM RESISTANCE WITH OR WITHOUT AMMETER
SECONDARY SHORT-CIRCUITED OR WITH AMMETER
PRIMARY IMPEDANCE - VOLTS
AMPERES - PRIMARY
0
10
20
30
40
C.T.
0.00745 HENRY
A = 0-50 SCALE AMMETER
M = 0-75 " MILLIAMMETER
G = 0-5 " AMMETER

TR #42

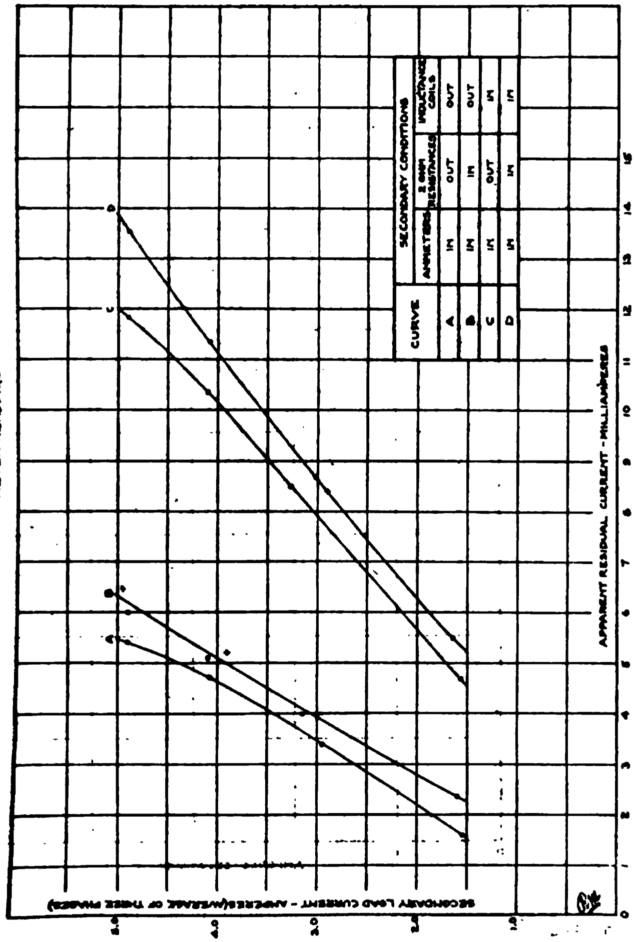
P.I.C. No 159
1-12-14



T.R.#42

P.I.C. No 160
12-12-13.

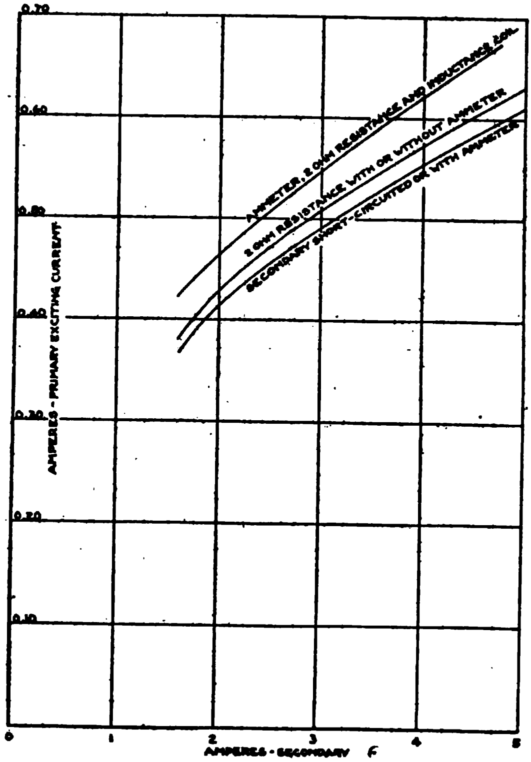
INVESTIGATION OF CURRENT TRANSFORMERS
IN USE AT SANTA CRUZ.
EFFECTIVE VALUE OF APPARENT RESIDUAL CURRENT
METER READINGS



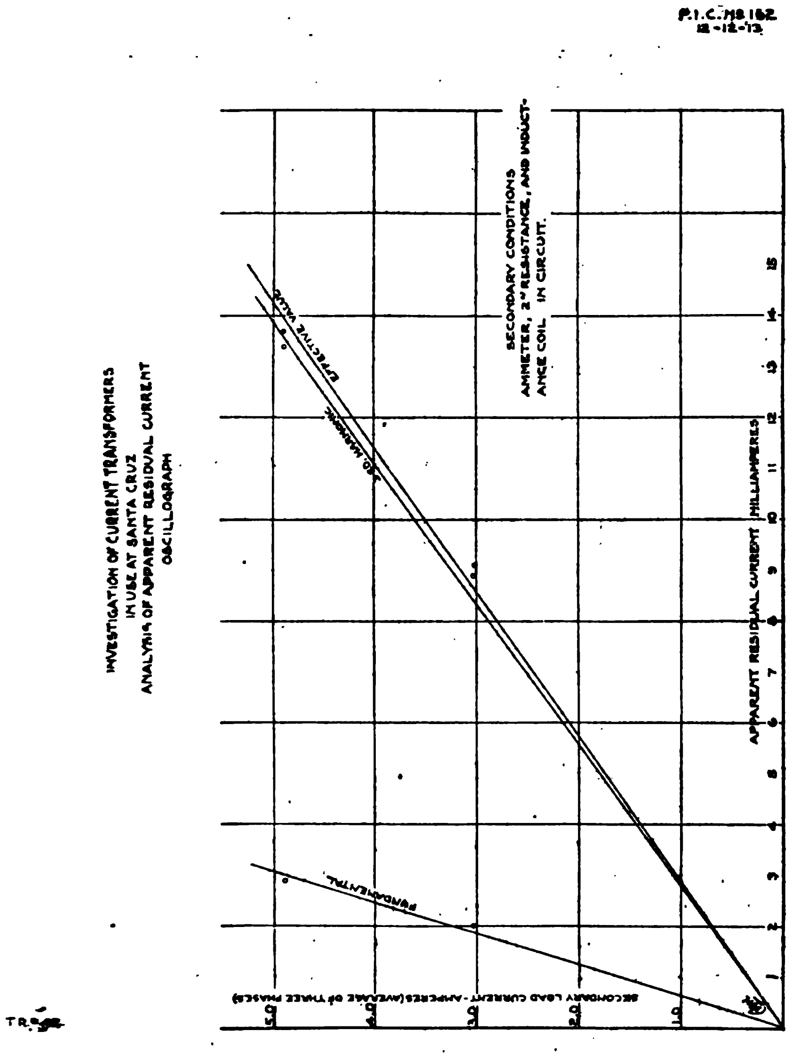
TR #42

P.I.C. No. 161
1-15-14

EXCITING CURRENT
CURRENT TRANSFORMER #906307
USED AS C.T. No. 3 AT SANTA CRUZ.
RATIO 5:1



T.R. 942


P.I.C. MS 162
12-12-13
INVESTIGATION OF CURRENT TRANSFORMERS
IN USE AT SANTA CRUZ
ANALYSIS OF APPARENT RESIDUAL CURRENT
OSCILLOGRAPH
SECONDARY CONDITIONS
AMMETER, 2" RESISTANCE, AND INDUCTANCE COIL IN CIRCUIT.
EFFECTIVE VALUE
3RD. HARMONIC
FUNDAMENTAL
APPARENT RESIDUAL CURRENT MILLIAMPERES
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
SECONDARY LOAD CURRENT - AMPERES (AVERAGE OF THREE PHASES)
1.0 2.0 3.0 4.0 5.0

Technical Report No. 50.

October 9, 1914.

TESTS RELATING TO THE THIRD HARMONIC AND ITS SUPPRESSION IN A THREE-PHASE BANK OF TRANSFORMERS.*

May 18, 21, 25, 26, June 22, 1914.

Introduction.

This report presents the results of tests made on a three-phase bank of single-phase transformers to determine the relative magnitudes of third harmonic produced therein when the magnetic densities of the transformer cores are varied; and a study of the efficacy of different methods of connection in suppressing the third harmonic and its odd multiples so as to provide an energy source free from these harmonics. The types of connections were:

Primary	Secondary	Abbreviation
Star	Star	Y-Y
Star	Interconnected-Star	Y-IY
Delta	Star	D-Y
Delta	Interconnected-Star	D-IY
Star	Delta	Y-D
Star	Interconnected-Delta	Y-ID
Delta	Delta	D-D
Delta	Interconnected-Delta	D-ID

Description and Results of Tests.

In these tests three G. E. 50 kv–a, Type H, Form R. P., 22000/19800–2400/480–volt transformers were used. They were energized from the 22000–volt three-phase buses of the Coast Counties Gas and Electric Company's plant at Santa Cruz. During part of the tests the Santa Cruz-Watsonville line of the above company was used as a balanced three-phase load. At Watsonville, switches were installed so that the line could be opened, short-circuited or grounded.

The variation of magnetic density was accomplished by applying to the primaries of the transformers, 22000 volts at the different coil terminals, namely, at the 100% and 90% taps, the halves of the primary windings being in series or in parallel. The terms "45% connection" and "50% connection" are used to designate the cases with halves of primary windings in parallel. The 22000-volt supply circuit was at all times isolated from ground.

*See T. R. Nos. 59 and 60.

At Santa Cruz, current and potential transformers were so arranged that readings could be made on the secondary side of these transformers. No facilities were available for making current and voltage observations on the primary. It has been assumed that the primary voltage was constant. The voltage from the secondary neutral to ground, E_N, was measured with as high an impedance as practicable between the neutral and ground. The neutral current, I_N, and residual current, I_R, were measured with a very low impedance between the secondary neutral and ground. The residual voltage was measured, both with the secondary neutral of the transformers isolated and with it grounded. The measuring instrument was inserted between the neutral of three star-connected potential transformers and ground, the secondaries of the potential transformers being delta-connected. The measurement thus obtained is one-third the residual voltage. The values are given in the tables as measured, for convenience in comparing with the neutral voltage.

In the tests with the Santa Cruz-Watsonville line connected to the transformers, there was induction from the 2200-volt distribution circuits, which caused errors in the observations of residual and neutral voltages and currents. This was, in some cases, of such magnitude as to mask the effects of changing the transformer connections and varying the magnetic density.

P. I. C. No. 212 shows the schematic diagrams of interconnected-star and interconnected-delta windings and the corresponding voltage diagrams. There is also shown the arrangement of secondary coils used for each transformer when "interconnected." It should be noted that this arrangement is not the best possible as regards the elimination of magnetic leakage between the two halves of the windings, the best arrangement probably having the diagonally opposite coils together as one unit.

In order to determine the variation in magnetic density the maximum value of the induced e.m.f. in the transformer should be known. As that was not obtained the effective value of the secondary line voltage is used as an approximate index of the magnetic density.

A—Tests with Transformer Secondaries Connected to Line.

In Table I are shown the meter readings of current and voltage for the several tests with the primaries of the transformers connected delta and Y, secondaries Y and IY. The following discussion is based on Table I and oscillograms shown in Tables II, III and IV.

TABLE I.

Meter Readings.

Per cent winding	Connection	E_{2-3} volts	I_L amperes	$\frac{1}{3}E_R$ volts, Neutral, Isolated	$\frac{1}{3}E_R$ volts, Neutral, Grounded	E_N volts	I_R amperes	I_N amperes
				Far end of line open.				
45	Y-Y	2200	0.16	—	—	690	—	0.56
50	Y-Y	2100	—	50*	610	670	0.20°	0.52
90	Y-Y	1140	—	—	350	340	0.20	0.25
100	Y-Y	1060	—	50*	400	340	0.18	0.25
45	Y-IY	1970	—	—	—	—	—	0.013
90	Y-IY	960	0.07	66	10*	57	—	0.012
						(76) (68)		(0.015) (0.014)
90	D-Y	2050	—	—	*	78		0.017
100	D-Y	1800	—	—	*	73	*	0.015
90	D-IY	1710	—	—	—	—	—	0.014 (0.013)
100	D-IY	1550	—	—	—	—	—	0.014 (0.013)
				Far end of line short-circuited.				
45	Y-Y	2120	40	—	—	680	—	0.52
50	Y-Y	2020	38	50	590	650	0.24°	0.51
90	Y-Y	1120	21	—	340	—	0.24	0.25
100	Y-Y	1030	19	50	330	—	0.22 0.10°	0.23 —
45	Y-IY	1880	36	—	—	—	—	0.012
90	Y-IY	900	17	63	10*	63	—	0.014
				Far end of line grounded.				
45	Y-Y	—	40+	—	—	690	5.2	5.2
50	Y-Y	2020	—	*	180	650	4.2	4.3
90	Y-Y	1140	21	*	10*	340	0.26°	0.57
100	Y-Y	1010	19	*	10*	320	0.19°	0.44
45	Y-IY	1900	36	—	—	*	—	0.038 0.017°
90	Y-IY	900	16	—	*	4	—	0.083
90	D-Y	1870	35	—	*	4	0.061 —	(0.020°) (0.040) (0.006)
100	D-Y	1750	33	—	—	—	0.024	0.016
90	D-IY	1540	29	*	—	19	—	0.24 (0.006)
100	D-IY	1370	26	—	—	13 (4)	—	0.19 (0.006)

Values within parentheses show values due to induction from low-voltage circuits, with the transformers under test unenergized.

E_{2-3}, voltage between lines.

I_L, average amperes per phase.

E_N, I_N, voltage and current from neutral to ground.

E_R, I_R, residual voltage, current.

*Denotes a value too small to measure accurately.

—Denotes a measurement not made.

°Reading taken on ammeter of high impedance.

TABLE II.

1/3 Residual Voltage—Volts—Y-Y Connection.

Secondary Neutral Grounded.

Condition of line / Connection / n	Far end isolated			Far end grounded
	50%	90%	100%	50%
1	*	*	*	9
3	550	340	350	170
5	*	*	*	*
7	*	*	*	*
9	120	*	*	*
Oscillogram No.	604	610	607	603

Neutral Current†—Amperes—Y-Y Connection.

Connection / n	45%	50%	90%	100%
	Far end of line isolated.			
1	0.02	*	0.02	0.03
3	0.35	0.86	0.22	0.22
9	0.20	0.22	0.02	0.02
15	0.14	0.05	*	*
Oscillogram No.	633	604	610	607
	Far end of line grounded.			
1	0.50	0.4	0.02	*
3	5.0	4.1	0.50	0.41
9	*	0.1	0.02	0.02
15	*	0.1	*	*
Oscillogram No.	634	603	609	608

NOTE.—With the far end of the line isolated, the residual voltage and neutral current were not affected by opening or short-circuiting the far end of the line.

*Value too small to measure accurately.

†The neutral and residual currents are practically identical.

TABLE III.

Neutral Current—Amperes—Various Connections.

n \ Connection	45% Y-Y	45% Y-IY	90% D-Y	90% D-IY
	Far end of line isolated.			
1	*	0.009	0.015	0.012
3	0.37	0.007	0.001	*
5	*	*	*	0.001
7	*	*	*	*
9	0.20	0.003	*	0.001
11	*	*	*	*
13	*	*	0.002	0.001
15	0.16	0.001	*	*
17	*	*	*	*
Oscillogram No.	632	†638	†612	†623
	Far end of line grounded.			
1	0.5	0.028	0.050	0.04
2	*	*	0.012	*
3	5.0	0.003	0.029	0.23
5	*	0.005	0.003	*
7	*	0.004	*	*
9	*	0.005	*	*
11	*	0.005	*	*
13	*	0.004	*	*
15	*	0.005	*	*
17	*	0.004	*	*
Oscillogram No.	634	†635	627	622

*Value too small to measure accurately.
†Mostly induction from low-voltage circuits.

Y-Y Connection. The residual and neutral voltages and currents when the far end of the line is isolated, appear to be directly proportional to the secondary line voltage. With the far end of the line grounded, the residual voltage is lowered and the residual current greatly increased, due to the low-impedance path offered to the triple harmonics. Comparison of values with the far end of the line open and with it short-circuited indicates that the balanced three-phase load has very little or no effect on the residuals. With increase in magnetic density, the neutral current increases very rapidly, being approximately proportional to the cube of the secondary line voltage. The variation of residual and neutral voltage and current with secondary line voltage and the effect of grounding the line are shown on P. I. C. No. 211.

An examination of oscillographic analyses (Table II) shows that with the far end of the line isolated the third harmonic residual voltage increases almost in direct proportion with the secondary line voltage. The ninth appears in appreciable magnitude with the 50% connection.

In the neutral current (or residual current) with the far end of the line isolated, there is practically only the third harmonic, with the 100% connection, but with the 50% connection (approximately doubled mag-

netic density) the ninth and fifteenth harmonics appear, the ninth being about half as large as the third.

The impedance of the line to ground for the third harmonic as shown by the relation of residual voltage to residual current, $\left(\frac{E_R}{3\,I_R}=Z\right)$ is about 1500 ohms with the far end of the line isolated. With the far end of the line grounded this impedance is about 40 ohms. For the ninth harmonic, with the far end of the line isolated, the impedance is about one-third that given for the third harmonic, as in this case the line is approximately equivalent to a condenser. On the other hand, with the line grounded the impedance for the ninth harmonic is approximately three times that for the third. These facts partially account for the prominence of the ninth as compared with the third harmonic in the neutral current when the line is isolated.

The residual current and the neutral current should, with the arrangement used, be practically identical. In a number of cases (see Table I) where the residual current value is much less than the neutral current value the discrepancy is apparently due to the high-impedance ammeter used.

One-third the residual voltage with grounded secondary neutral is practically identical with the neutral voltage when the far end of the line is isolated. Grounding the far end of the line does not affect the neutral voltage, for the admittance of the line to ground is large compared to that of the transformer secondary neutral point to ground whether the line be itself grounded or not. One-third the residual voltage with the secondary neutral and the far end of the line grounded is much less than the neutral voltage. In the above condition of measurement of the residual voltage there is opportunity for triple harmonics of magnetizing current to flow, which cut down the induced triple harmonic voltages. Also, the impedance drop in the transformers is probably of appreciable magnitude compared to the impedance drop in the line and ground, which latter determines the residual voltage under the conditions of this test.

In the foregoing discussion the residual voltage considered is that observed when the secondary neutral was grounded. When the secondary neutral is isolated, the residual voltage is very much less, too small to measure accurately with instruments used, and its true value is masked by induction from low-voltage circuits. That its value should be small is evident from the fact that the external circuit for the triple harmonic voltages induced in the transformers (in phase in the three transformers) consists of the impedance from the line to ground (all three conductors in parallel) in series with the impedance between the

secondary neutral point and ground. (The distributed constants of the transformer windings enter into a determination of equivalent impedance of neutral to ground.) The residual voltage is that portion of the total impedance drop, in the external circuit occurring between the line and ground, which is small compared to the drop between the secondary neutral point and ground.

Y-IY Connection. The impedance of the transformers to currents due to induction from low-voltage circuits, is low. The readings of neutral voltage and current are apparently mostly induction from low voltage circuits, except that there is appreciable neutral current due to the transformers, with the far end of the line grounded.

D-Y Connection. With the far end of the line isolated the current in the neutral caused by induction from low-voltage circuits is nearly equal to the neutral current with the transformers energized. The neutral current, mostly third harmonic, is appreciable with the far end of the line grounded, and the induced current from low-voltage circuits, which is apparently chiefly due to electrostatic induction, is halved.

D-IY Connection. There is a large neutral current with the far end of the line grounded. The neutral voltage is appreciable. With the far end of the line open, only the induction from low-voltage circuits is observable, which is halved by grounding the far end of the line.

Comparison of the Different Methods of Connection.

The Y-Y connection affords no path for triple frequency magnetizing current except through the admittance of neutral and line to ground, so the large triple harmonic neutral voltage, and large neutral current with line and secondary neutral grounded, are to be expected.

The Y-IY connection eliminates the triples in the secondary line to neutral e. m. fs. by causing neutralizing triple harmonic e. m. fs. to be generated in the two halves of each interconnected winding. With equal impressed voltages 120° apart in phase, identical transformers, no magnetic leakage between the halves of the secondary winding of each transformer and balanced load, this neutralization would be perfect; that is, the e. m. f. from line to neutral would have no triple harmonics, and no triple harmonic residual current would flow with the secondary neutral grounded. The effect, therefore, is the same as would be obtained with a primary delta, of zero impedance to the triple harmonic circulating current.

The D-Y connection allows the triple frequency magnetizing current to circulate in the primary, hence the induced triple harmonic e. m. fs. between secondary line and neutral are only as large as required to force the current through the impedance of the primary windings. Therefore, if, as in the present tests, the delta comprises the whole primary winding

of the transformer, the circulating current largely eliminates residual voltage and current on the secondary side. When the line and secondary neutral are grounded a second path for the triple frequency current is provided in parallel with the primary delta, and the current therefore divides between them.

The D-IY winding is intended to neutralize in the interconnected secondary windings the triple harmonic voltage required by the primary delta circulating current, thus eliminating the triple harmonic potential between lines and ground on the secondary side. It should be noted that the interconnection has the effect of open-circuiting the secondary winding of each transformer to the triple harmonics originating in the transformer. In the tests here reported a larger third harmonic neutral current was observed with D-IY connection than with the Y-IY connection, when the secondary neutral and the far end of the line were grounded. This is contrary to expectation and is a point which needs further investigation.

In Table III are shown oscillograms of neutral current for 45% Y-Y and 45% Y-IY connections. The effectiveness of the interconnected-star secondary is shown by a comparison of the 3d, 9th and 15th harmonics, under the two conditions.

The D-Y and D-IY connections with far end of the line open, can not be compared from the data of these tests as the induction from low-voltage circuits masks the other effects. With the far end of the line grounded the neutral voltage and neutral current are apparently very much greater with the D-IY connection as has been noted above.

In order to compare the Y-Y and D-Y connections it is necessary to have the voltage across the transformers the same. This is approximately true when the 50% Y-Y connection is compared with the 90% D-Y connection since $\frac{90\%}{\sqrt{3}} = 52\%$. The very great effect on the neutral current of changing the primary winding from star to delta is shown by the oscillograms in Table IV.

TABLE IV.

n	Neutral current—amperes		Residual current†—amperes	
	Far end of line isolated		Far end of line grounded	
	50% Y-Y	90% D-Y	50% Y-Y	90% D-Y
1	*	0.015	0.4	0.06
3	0.36	0.001	4.1	0.05
5	*	*	0.1	*
7	*	*	*	*
9	0.22	*	*	*
11	*	*	*	*
13	*	0.002	*	*
15	0.05	*	0.1	*
17	*	*	*	*
Oscillogram No.	605	‡612	603	613

†Residual and neutral currents are practically identical.
‡Mostly induction from low-voltage circuits.

The neutral current is small when the Y-IY, D-Y and D-IY connections are used (see Table III), particularly with the far end of the line isolated. Accurate comparisons, therefore, can not be made, on account of the masking effects of induction from low-voltage circuits, and possible changes in the impressed voltage.

B. Tests With Transformer Secondaries Disconnected From Line.

In order to further determine the extent to which the third harmonic is suppressed, oscillograms were made of the delta circulating current, the transformers being connected in D-D, D-ID, Y-D, Y-ID.

Both 90% and 100% connections were used, on the primary side.

In this series of tests the circulating current in the secondary delta is used to indicate the effectiveness of interconnected secondary windings and delta primary windings in neutralizing or suppressing induced triple harmonic voltages. The circulating current thus corresponds to the neutral current in the tests with star-connected secondaries and load. The measurements are free from the induction experienced in the tests with the Santa Cruz-Watsonville line connected to the secondary of the transformer bank. It should be noted that the mid-points of the several phases of the interconnected-delta are at a common potential, when the impressed voltages are equal and 120° apart in phase and the transformers identical. Connecting the midpoints together, however, results in a star winding with the two halves of the secondary winding of each transformer in parallel. So, while there is with this connection a "neutral" the triple harmonic voltage across one-half the secondary winding is impressed between each line and this neutral.

In Table V are given oscillograms of circulating current for the several connections, with 100% and 90% high-voltage windings. The effectiveness of the interconnection of windings in suppressing the third harmonic circulating current is evident. With the interconnection the circulating current is 5 to 10 % of the current with straight delta connection.

TABLE V.
Delta Circulating Current—Amperes.

n	Y-D	Y-ID	D-D	D-ID
100% winding on primary side of transformers.				
1	0.014	0.005	0.053	0.014
2	*	*	*	0.007
3	0.17	0.016	0.17	0.010
5	0.003	*	0.009	0.008
7	*	*	*	*
9	0.004	*	*	*
Osc. No	659	658	662	656
90% winding on primary side of transformers.				
1	0.016	—	0.05	0.022
2	*	—	*	0.006
3	0.15	—	0.44	0.020
5	0.003	—	0.02	0.003
7	*	—	*	*
9	0.005	—	*	*
Osc. No.	660		661	657

The 90% connection shows considerable increase in circulating current, compared to the 100% connection, with the high-voltage windings in delta.

In Table VI are given some values of the voltage from the line connection to the neutral and from the neutral to ground on the secondary side of the transformers. The transformers were not connected to any line or load on the secondary side. In the Y-Y connection there is a large third harmonic voltage from line to neutral, which is practically eliminated when the Y-IY, D-Y, D-IY or D-D connection is used. The effectiveness of the delta connection of the primary in eliminating triple harmonics in the secondary neutral voltage to ground is strikingly shown. The fundamental is also much reduced. With Y-Y connection, the third harmonic of the voltage from neutral to ground is approximately the same with no connection to the secondaries of the transformers as when the Watsonville line is connected.

TABLE VI.

100% winding on primary side of transformers.

n	Volts—line to neutral					Volts—neutral to ground			
	Y-Y	Y-IY	D-Y	D-IY	D-ID	Y-Y	Y-IY	D-Y	D-IY
1	550	510	960	840	440	40	50	10	2
2	*	*	*	*	*	*	†	†	*
3	280	*	10	10	*	280	280	1	*
4	*	*	*	*	*	20	†	†	*
5	10	*	10	*	*	10	20	*	*
6	*	*	*	*	*	10	†	†	*
7	5	2	*	5	*	*	10	*	*
9	10	*	*	*	*	20	25	*	*
11	5	3	10	7	7	*	5	*	*
13	10	5	20	9	13	*	*	1	*
15	*	*	*	*	2	*	5	*	*
17	*	*	*	*	*	*	*	*	*
Osc. No.	664	669	663	670	651	664	669	663	670

*Indicates a value too small to measure.
†Analysis not made for even harmonics.

Even harmonics were found in several of the oscillograms of the delta circulating current. Their source is not understood. Even harmonics were also noticed in the voltage between neutral and ground in tests made with Y-Y and Y-IY connections.

Conclusions.

1. With Y-Y connection the triple harmonics in residual voltage and current increase as the magnetic density in the transformers is increased. The increase in the ninth harmonic, when the transformers, connected in Y-Y, are over-excited, is very rapid.

2. Under the conditions of these tests with the line side of the transformers connected in grounded star, grounding the far end of the line increases the flow of triple harmonic neutral current, particularly when the primary is star-connected. The triple harmonic residual voltage is decreased. In general, these effects depend upon the character of the load and the length of the line.

3. Isolating the neutral on the line side of a Y-Y connected bank of transformers practically eliminates the third harmonic residual voltage impressed on the line by such transformers with this neutral grounded; assuming the admittance to ground of the transmission line to be very large compared to the admittance to ground of the transformer neutral, which may be said to be true, in general, of practical cases. When isolated, the neutral pulsates at a third harmonic voltage above ground approximately equal to the full third harmonic e. m. f. induced in the windings.

4. The use of a straight delta winding on the station side, or an interconnected-star winding on the line side of a transformer bank, nearly eliminates the triple harmonic residuals due to such transformers, observed with the Y-Y connection with grounded neutral on the line side.

5. The triple harmonic circulating current in a delta winding can be nearly eliminated by using the interconnected-delta. Over-excitation markedly increases the circulating current with both delta and interconnected-delta windings. The use of an interconnected-delta winding is approximately equivalent to an isolated star connection in so far as the triple harmonics are concerned, and therefore tends to produce relatively large triple harmonic residuals, if the line side of the transformer bank is connected in grounded star.

6. The magnitude of the balanced isolated load on the transformers has no appreciable effect on the triple harmonic residuals arising within such transformers.

7. The effective use of interconnected windings requires that the impedances of all the half-windings on the interconnected sides of the transformers be equal, and that magnetic leakage between halves of transformer windings on the interconnected side be small.

8. The results of this test indicate that further investigation is necessary* in order to determine the relative merits of the D-Y, Y-IY and D-IY connections as means of suppressing the triple harmonic residuals arising from a grounded neutral bank of transformers.

Respectfully submitted.

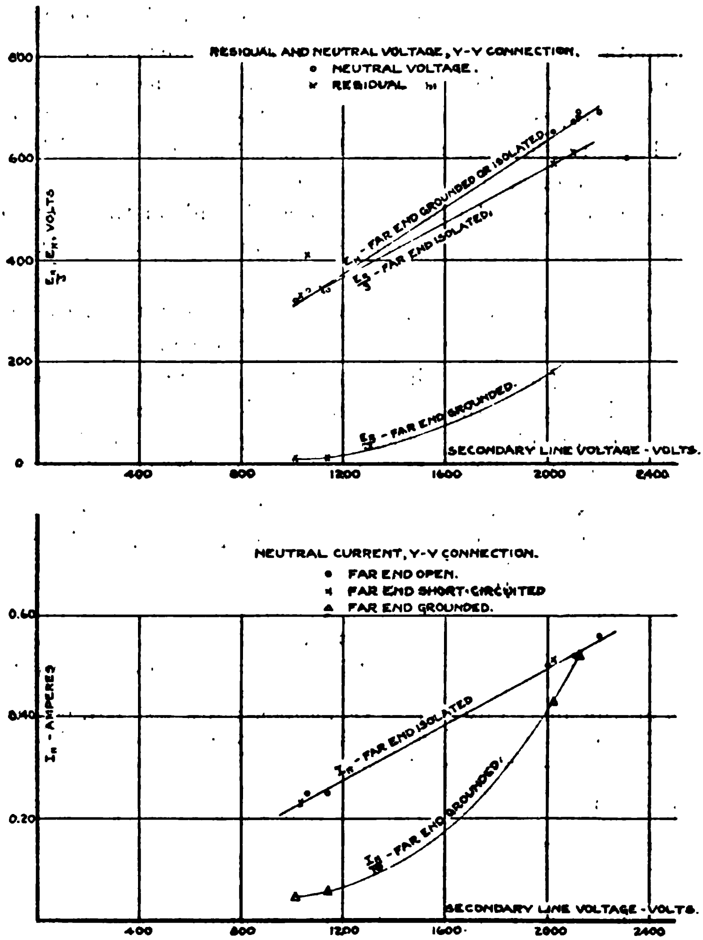
(Signed) J. E. WOODBRIDGE,
Chairman Subcommittee on Tests.

IN FILES OF THE JOINT COMMITTEE: Oscillograms: Nos. 603, 604, 605, 607, 608, 609, 610, 612, 613, 622, 623, 627, 652, 633, 634, 635, 638, 651, 656, 657, 658, 659, 660, 661, 662, 663, 664, 669, 670.

ATTACHED: P. I. C. Nos. 211 and 212.

*See Technical Report No. 60.

P.I.C. #211
10-6-14.



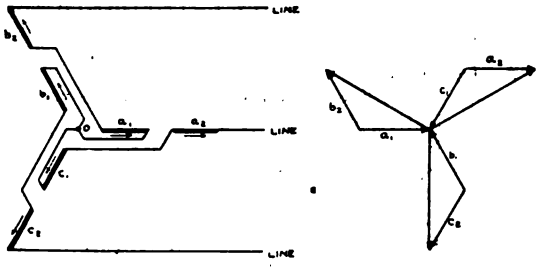
T.R. #50

F.A.C.

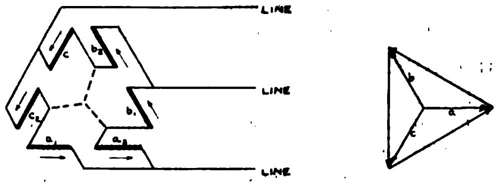
P.I.C. #212
9-28-14

INTER CONNECTED THREE-PHASE TRANSFORMER WINDINGS

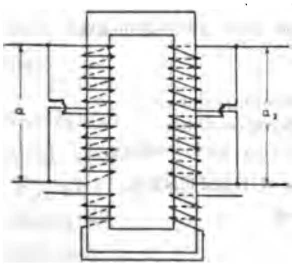
INTERCONNECTED-STAR



INTER CONNECTED DELTA



SCHEMATIC DIAGRAM OF TRANSFORMER WINDING
LOW VOLTAGE SIDE



T R #50

Technical Report No. 51.

September 30, 1916.

RESIDUAL VOLTAGE DUE TO THE LINE UNBALANCE OF POWER CIRCUITS ISOLATED FROM GROUND.

EFFECT OF CIRCUIT CONFIGURATION, TRANSPOSITIONS AND FREQUENCY.

OUTLINE.

Introduction.

The voltages to ground of the several conductors of power circuits having no metallic connection to ground are determined by the voltages between conductors and by the self and mutual admittances of the conductors. Unbalances among the corresponding admittances of the several conductors, in general, cause their voltages to ground to be unbalanced, even though the voltages between conductors are balanced. A residual voltage, defined as the vector sum of the voltages to ground of the several conductors, is thereby created. The residual voltage of power circuits isolated from ground is thus determined by line unbalances as distinguished from unbalanced loads and transformer connections. Presented herein are the results of theoretical and experimental investigations of different factors which affect the line unbalance of power circuits, including circuit configuration, transpositions, frequency and leakage. Formulas and data of general applicability are given.

SECTION I.

THEORETICAL.

1. Short Lines.

A—DEFINITIONS.

The term "short line" is here applied to a line which is short compared to a wave-length for a given frequency; say one twentieth wavelength or less. For such a line the effects of phase-change and attentation may be neglected and the line treated as if the capacitances were lumped at the sending end; conductances are neglected.

The residual voltage of a short uniform nontransposed power circuit when isolated from ground, and energized with balanced three-phase voltages between conductors, is termed herein its "characteristic residual voltage" (E_{RC}).

B—DERIVATION OF FORMULAS.

Consider an electrical system of three conductors isolated from ground, and let E_{12}, E_{23} and E_{31} be the potential differences between pairs of conductors designated by the subscripts. Let V_1. V_2 and V_3 be the potentials of conductors 1, 2 and 3, respectively, referred to ground. Then:

$$\left.\begin{aligned} E_{12} &= -V_1 + V_2 \\ E_{23} &= - V_2 + V_3 \\ E_{31} &= +V_1 - V_3 \end{aligned}\right\} \qquad (1)$$

The following equations* express the electric charges of the conductors in terms of the potentials:

$$\left.\begin{aligned} Q_1 &= K_{11}V_1 + K_{12}V_2 + K_{13}V_3 \\ Q_2 &= K_{21}V_1 + K_{22}V_2 + K_{23}V_3 \\ Q_3 &= K_{31}V_1 + K_{32}V_2 + K_{33}V_3 \end{aligned}\right\} \qquad (2)$$

Summing the charges

$$Q_1 + Q_2 + Q_3 = C_{10}V_1 + C_{20}V_2 + C_{30}V_3 \qquad (3)$$

where

$$\left.\begin{aligned} C_{10} &= K_{11} + K_{21} + K_{31} \\ C_{20} &= K_{12} + K_{22} + K_{32} \\ C_{30} &= K_{13} + K_{23} + K_{33} \end{aligned}\right\}$$

C_{10}, C_{20} and C_{30} are the direct capacitances to ground of conductors 1, 2 and 3, respectively. Since the system is isolated from ground:

$$Q_1 + Q_2 + Q_3 = 0 \qquad (4)$$

Solving for V_1, V_2 and V_3 from equations (1), (3) and (4) the following expressions are obtained:

$$\left.\begin{aligned} V_1 &= \frac{-C_{20}E_{12} - C_{30}(E_{12} + E_{23})}{C_{10} + C_{20} + C_{30}} \\ V_2 &= \frac{C_{10}E_{12} - C_{30}E_{23}}{C_{10} + C_{20} + C_{30}} \\ V_3 &= \frac{C_{10}(E_{12} + E_{23}) + C_{20}E_{23}}{C_{10} + C_{20} + C_{30}} \end{aligned}\right\} \qquad (5)$$

The residual voltage E_R is the sum of the three voltages to ground, or

$$E_R = V_1 + V_2 + V_3 = \frac{C_{10}(2E_{12} + E_{23}) + C_{20}(E_{23} - E_{12}) + C_{30}(-E_{12} - 2E_{23})}{C_{10} + C_{20} + C_{30}} \qquad (6)$$

For balanced three-phase voltages between conductors, let

$$E_{23} = E_{12}(-\tfrac{1}{2} + j\tfrac{1}{2}\sqrt{3}) \qquad (7)$$

Substituting this value in equation (6) the residual voltage is

$$E_R = E_{12}\frac{3(C_{10} - C_{20}) + j\sqrt{3}(C_{10} + C_{20} - 2C_{30})}{2(C_{10} + C_{20} + C_{30})} \qquad (8)$$

which is the characteristic residual voltage of a uniform nontransposed line.

*Following Clerk-Maxwell.

The residual voltage may also be obtained in terms of the balanced voltages to ground and the ratio of "residual capacitance" (gotten by combining the three direct capacitances in 120° relation) to the capacitance to ground of the three in parallel.

Let the balanced component of the voltage of conductor No. 1 to ground be

$$E = je \frac{E_{12}}{\sqrt{3}} \tag{9}$$

where e is an operator denoting rotation counter-clockwise through 120° or

$$e = -\tfrac{1}{2} + j\tfrac{1}{2}\sqrt{3}$$

Since balanced voltages between conductors are assumed the voltages to ground consist of balanced and residual components only and

$$V_1 = E + \frac{E_R}{3}, \quad V_2 = eE + \frac{E_R}{3}, \quad V_3 = e^2E + \frac{E_R}{3} \tag{11}$$

Substituting the values of V_1, V_2 and V_3 as above defined in equation (3):

$$Q_1+Q_2+Q_3=(C_{10}+C_{20}+C_{30})\frac{E_R}{3}+(C_{10}+eC_{20}+e^2C_{30})E \tag{12}$$

Since the system is isolated from ground

$$Q_1 + Q_2 + Q_3 = 0$$

and

$$E_R = -3E \frac{C_{10} + eC_{20} + e^2C_{30}}{C_{10} + C_{20} + C_{30}} \tag{13}$$

which is the characteristic residual voltage of a uniform nontransposed line.

If the value of E in terms of E_{12}, and that of e as defined by equation (10) be substituted in equation (13), equation (8) may be obtained.

To calculate the direct capacitances, C_{10}, C_{20} and C_{30}, it is necessary to express them in terms of potential coefficients, which may be readily computed in terms of the dimensions. The potential coefficients are the coefficients of the charges in a system of equations expressing the voltages in terms of the charges. They are designated by the letter P with two subscripts, one corresponding to the charge and the other to the potential. The methods of computation of the direct capacitances in terms of the potential coefficients and of the potential coefficients in terms of the dimensions are given in Appendix I.

When conductors 1 and 3 are in the same horizontal plane, conductor 2 equidistant from them and all are the same size, a considerable simplification results. The residual voltage may then be expressed directly in terms of the potential coefficients as follows:

$$E_R = \frac{(P_{22} + P_{12}) - (P_{11} + P_{13})}{P_{11} + 2P_{22} + P_{13} - 4P_{12}} (E_{12} - E_{23}) \tag{14}$$

and for balanced three-phase voltages between conductors the characteristic residual voltage is

$$E_{RC} = E_{12} \frac{(P_{22} + P_{12}) - (P_{11} + P_{13})}{P_{11} + 2P_{22} + P_{13} - 4P_{12}} \left(\frac{3}{2} - j\tfrac{1}{2}\sqrt{3}\right) \tag{15}$$

If the ground is at an infinite distance, formula (15) becomes

$$E_{RC} = E_{12}\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right) \frac{\log \dfrac{r_{13}}{r_{12}}}{\log \dfrac{r^4_{12}}{r_{13}r^3_{11}}} \tag{16}$$

where r_{12} and r_{13} are the distances between the conductors designated by the subscripts and r_{11} is the radius of the conductors. (See Appendix I.)

If the three conductors are in the same plane, No. 2 being equidistant from Nos. 1 and 3, formula (16) reduces to

$$E_{RC} = E_{12} \frac{0.1738}{\log_{10} \dfrac{r_{12}}{r_{11}} - 0.1003} \tag{17}$$

or very closely

$$E_{RC} = E_{12} \frac{0.1\sqrt{3}}{\log_{10} \dfrac{r_{13}}{r_{11}} - 0.1} \tag{18}$$

This is the limiting value of the characteristic residual voltage of symmetrical horizontal or vertical lines as the height of the conductors is increased. For symmetrical horizontal lines when the height above ground exceeds three times the width of the circuit the error involved by using formula (18) is less than 2 per cent.

Formulas for computing the residual voltage of a three-conductor system from the physical dimensions are given on drawing No. 279, attached.

Formulas (6) and (8), as given above for single-circuit lines, may be applied also to twin-circuit lines operated in parallel if instead of C_{10}, C_{20} and C_{30} (the direct capacitances of the three conductors) the sums of the direct capacitances of the paralleled conductors are used. The residual voltage so derived is the sum of the three and not of the six voltages and thus is the residual voltage of each circuit. When the two circuits are symmetrically placed with respect to an intermediate plane perpendicular to the earth's surface the direct capacitances of corresponding conductors are equal. If corresponding conductors are connected together, the formula for the residual voltages of a twin-circuit line is the same as for a single-circuit line, except that account must be taken of the presence of all six wires in computing the direct capacitances. The method of computing the capacitance coefficients of twin-circuit lines is given in Appendix I.

Formulas (6) and (8) may be applied to the computation of residual voltage of lines with ground wires, provided that in place of the direct capacitance to ground, the sum for each conductor of the direct capacitance to ground and the direct capacitance to the ground wire is used. The method of computing these sums is given in Appendix I.

For a line having conductors 1 and 3 of the same diameter, in the same horizontal plane; having conductor 2 and a ground wire (4) in a vertical plane midway between 1 and 3, the residual voltage is given in terms of the potential coefficients by the following expression:

$$E_R = \frac{P_{44}(P_{22} - P_{11} - P_{13} + P_{12}) + (P_{14} - P_{24})(2P_{14} + P_{24})}{P_{44}(P_{11} + 2P_{22} + P_{13} - 4P_{12}) - 2(P_{14} - P_{24})^2}(E_{12} - E_{23}) \qquad (19)$$

or for balanced three-phase voltages between conductors.

$$E_{RC} = \frac{P_{44}(P_{22} - P_{11} - P_{13} + P_{12}) + (P_{14} - P_{24})(2P_{14} + P_{24})}{P_{44}(P_{11} + 2P_{22} + P_{13} - 4P_{12}) - 2(P_{14} - P_{24})^2}\left(\tfrac{3}{2} - j\tfrac{1}{2}\sqrt{3}\right)E_{12} \qquad (20)$$

For a two-conductor line the expression for residual voltage, in terms of the direct capacitances, is,

$$E_R = E_{12}\frac{C_{10} - C_{20}}{C_{10} + C_{20}} \qquad (21)$$

or in terms of the potential coefficients;

$$E_R = E_{12}\frac{P_{11} - P_{22}}{P_{11} + P_{22} - 2P_{12}} \qquad (22)$$

In general, the residual voltage is equal to zero when the direct capacitances to ground of the conductors are equal. With such a condition the residual voltage will be zero irrespective of the relative magnitudes or phases of the voltages between conductors. In terms of the potential coefficients for a three-conductor system and ground the residual voltage is

$$E_R = (P_{11}+P_{12}+P_{13})Q_1 + (P_{12}+P_{22}+P_{23})Q_2 + (P_{13}+P_{23}+P_{33})Q_3 \qquad (23)$$

$$Q_1 + Q_2 + Q_3 = 0 \qquad (24)$$

Since the potential coefficients are all positive numbers and not vector quantities, E_R can not be equal to zero except that the coefficients of the Q's be equal; that is,

$$(P_{11} + P_{12} + P_{13}) = (P_{12} + P_{22} + P_{23}) = (P_{13} + P_{23} + P_{33}) \qquad (25)$$

For conductors whose spacings are large compared to their diameters and whose lengths are large compared to their spacings this formula reduces to

$$\left.\begin{aligned} \frac{S_{12}S_{22}}{r_{12}r_{22}} &= \frac{S_{13}S_{33}}{r_{13}r_{33}} \\ \frac{S_{23}S_{33}}{r_{23}r_{33}} &= \frac{S_{12}S_{11}}{r_{12}r_{11}} \\ \frac{S_{13}S_{11}}{r_{13}r_{11}} &= \frac{S_{23}S_{22}}{r_{23}r_{22}} \end{aligned}\right\} \qquad (26)$$

where S_{11}, S_{12}, etc., are the distances from the conductors denoted by the first subscripts to the images of the conductors denoted by the second; r_{12}, r_{13}, etc., are the distances between the conductors denoted by the subscripts; and r_{11}, r_{22}, r_{33} are the radii of the conductors. If all the conductors have the same diameter

$$r_{11} = r_{22} = r_{33}$$

and the condition for zero residual voltage is therefore independent of the actual diameter.

For a line having three conductors of equal size, 1 and 3 in the same horizontal plane, and conductor 2 equidistant from them, the condition for zero residual voltage reduces to

$$P_{12} + P_{22} = P_{11} + P_{13} \qquad (27)$$

or

$$\frac{S_{12}S_{22}}{r_{12}} = \frac{S_{11}S_{13}}{r_{13}} \qquad (28)$$

The condition for zero residual voltage, that the direct capacitances of the several conductors be equal, may be attained by transposing the line so that each conductor occupies each of the several conductor positions for equal lengths.

When the voltages between conductors are equal in magnitude and 120° apart in phase, if the transpositions are not spaced so as to fulfill the above condition, the residual voltage as compared to the characteristic residual voltage is the ratio of the unbalanced length to the total length of the line. In computing the unbalanced length, the lengths of line between successive transpositions are added vectorially in 120° relation in the same manner that the length of unbalanced exposure for induction from balanced components in a uniform parallel of power and communication circuits is computed.

Consider a line of length $3 + x + y$ having two transpositions dividing the line into lengths 1, $1 + x$ and $1 + y$. Let the direct capacitances for each conductor per unit length be C_{10}, C_{20} and C_{30}, respectively. Then the total direct capacitances of the several conductors are:

$$\left.\begin{aligned} C_{10} + (1 + x)C_{20} + (1 + y)C_{30} &= C'_{10} \\ C_{20} + (1 + x)C_{30} + (1 + y)C_{10} &= C'_{20} \\ C_{30} + (1 + x)C_{10} + (1 + y)C_{20} &= C'_{30} \end{aligned}\right\} \qquad (29)$$

Substituting these values of C'_{10}, C'_{20} and C'_{30} for C_{10}, C_{20} and C_{30}, respectively, in equation (13)

$$E_R = -3E \frac{C_{10} + eC_{20} + e^2C_{30}}{C_{10} + C_{20} + C_{30}} \cdot \frac{1 + e^2(1 + x) + e(1 + y)}{3 + x + y} \qquad (30)$$

Hence

$$E_R = E_{RC} \frac{1 + e^2(1 + x) + e(1 + y)}{3 + x + y} \qquad (31)$$

or

$$E_R = E_{RC} \frac{-\frac{1}{2}(x + y) + j\frac{1}{2}\sqrt{3}(y - x)}{3 + x + y} \qquad (32)$$

When the line is transposed a residual current is set up between sections separated by transpositions. The current required for a section of length L is equal to the product of one-third the difference between the actual residual voltage and the characteristic residual voltage, and the admittance to ground, for the length L, of the several conductors of the circuit in parallel.

From equation (12)

$$Q_1+Q_2+Q_3=\tfrac{1}{3}\left(3E\frac{C_{10}+eC_{20}+e^2C_{30}}{C_{10}+C_{20}+C_{30}}+E_R\right)(C_{10}+C_{20}+C_{30}) \qquad (33)$$

or substituting E_{RC} from equation (13)

$$Q_1+Q_2+Q_3=\tfrac{1}{3}(E_R-E_{RC})(C_{10}+C_{20}+C_{30}) \qquad (34)$$

hence

$$I_R=\tfrac{1}{3}(E_R-E_{RC})Y_{123-0} \qquad (35)$$

where Y_{123-0} is the admittance to ground of the conductors of the circuit in parallel and is determined by the equation

$$Y_{123-0}=j2\pi f(C_{10}+C_{20}+C_{30})L \qquad (36)$$

where L is the length of the section and f is the frequency.

If the line is symmetrically transposed so that the residual voltage is zero, the residual current supplied to one section is

$$I_R=\tfrac{1}{3}E_{RC}Y_{123-0} \qquad (37)$$

For successive sections separated by transpositions the term E_R of equation (35) is constant and E_{RC} changes in phase by 120°. Commencing at one of the terminals of the circuit where the residual current is zero, the residual current at any point may be found by adding vectorially the currents required by the various sections of line up to that point. Thus, if the line is symmetrically transposed, the current at the intermediate transposition points of the barrels is given by equation (37), Y_{123-0} being the admittance per one-third barrel. At junction points of successive barrel lengths the residual current is zero.

In general, if balanced voltages to ground be supplied, by grounding the neutral of the generator or supply bank of transformers, a residual current due to line unbalance is required. For a nontransposed line this current is given by the equation (37), where Y_{123-0} is the admittance to ground for the whole length of the line. If the line is transposed this residual current is the product of that required for the nontransposed line and the fractional length of unbalance, or the product of one-third the residual voltage, which would exist were the line isolated from ground, and the admittance to ground of the conductors of the circuit in parallel.

The conditions for zero residual voltage of double-circuit lines are fulfilled if each circuit is so transposed that it would be balanced were the other circuit not present. Thus the two circuits may be transposed at the same or different points and operated independently or in parallel. In case of branch lines where one circuit is continued and not the other or if the spacing of the two circuits is changed, the transpo-

sitions should be so spaced that balance is obtained in each section independently of the others. In general, when the two circuits are both transposed as above indicated, one circuit induces balanced three-phase and single-phase voltages in the other and, when not transposed, residual voltages as well. If the two are transposed at the same points, balanced voltages only are induced. The magnitude of the induced balanced voltages is the same irrespective of whether the two circuits are transposed in the same or opposite directions. It may be noted that by placing complete barrels in one circuit opposite thirds of barrels in the other, the induced balanced and single-phase voltages are reduced to zero.

To most effectively reduce the residual voltages and currents of twin isolated circuits which are operated in parallel, they should be transposed at the same points and care taken to secure the condition for minimum characteristic residual voltage in each section of line between transposition points. This involves the treatment of pairs of conductors of the same phase as units and transposing so that each pair occupies the three pairs of conductor positions for equal distances. To accomplish this in case the sequence of phases of the two circuits is different, on going around the configurations in the same direction, it is necessary that the two be transposed in opposite directions.

C—Application of Formulas.

The formulas above derived have been used to study the variation of characteristic residual voltage of single circuits with the relative position, spacing, size and height of conductors and position of ground wire. Several cases of twin circuits are considered and for each the characteristic residual voltages for six methods of connection for parallel operation are given.

1. *Single-Circuit Lines.*

The computations were made at two different times, the methods of attack being somewhat different for the two series. The first series of computations was made using actual dimensions and is partly covered by the second series. In the second series, which is more complete, the dimensions are expressed in terms of the base of the triangle or of the spacing of outside conductors when all are in the same plane. Since the residual voltage is dependent upon the relative and not upon the actual dimensions, this method is the more advantageous.

The results of the computations are shown graphically by curves on the drawings listed below:

FIRST SERIES.

Configuration	Variable	Drawing number
Isosceles triangle Base horizontal.	Altitude of triangle	276
Flat horizontal Isosceles triangle Base vertical.	Position of inside conductor Altitude of triangle	277
Triangle, base vertical Altitude constant. Equilateral triangle Base horizontal. Vertex upward.	Position of vertex in a vertical plane Position of ground wire	278
Equilateral triangle Base horizontal, Vertex upward.	Spacing of conductors Height of conductors. Size of conductors.	279
Vertical Flat horizontal Symmetrical. Equilateral triangle Base horizontal.	Height of conductors	300

In addition, drawing No. 274 shows the determination of conditions for zero residual voltage of isosceles-triangle configurations and drawing No. 275 gives relative values of base, altitude and height of isosceles-triangle configurations for which the characteristic residual voltage is zero.

SECOND SERIES.

Configuration	Variable	Drawing number
Equilateral triangle Base horizontal, Vertex upward.	Height and size of conductors	348
Isosceles triangle Base horizontal = d. Vertex upward. Altitude = 0.4 d.	Same	349
Equilateral triangle Base vertical.	Same	350
Vertical	Same	351
Flat horizontal, symmetrical	Same	
Flat horizontal, unsymmetrical Conductor spacings, d, d/4, 3d/4.	Same	352
Flat horizontal, unsymmetrical Conductor spacings, d, d/3, 2d/3.	Same	
Flat horizontal	Position of inside conductor	344[1]
Isosceles triangle Base horizontal. Vertex upward.	Altitude of triangle	345[1]
Isosceles triangle Base vertical.	Altitude of triangle	346
Isosceles triangle Base horizontal = d. Vertex upward. Altitude = 1.25 d.	Height of conductors	347

Of the several configurations studied the triangular arrangement in general gives the lowest characteristic residual voltage. The configurations in which all three conductors are in the same plane give the largest values. With the same ratios of height and size to spacing of outside conductors, the vertical configuration has a larger characteristic residual voltage than the symmetrical horizontal and the two approach the same value as the height of conductors is increased without limit. When the spacing of adjacent conductors in the horizontal configuration is made unequal the residual voltage is increased over that of the vertical and this arrangement gives rise to the maximum characteristic residual voltage found. The ranges of values to be expected with various

configurations are shown in the following table, taken from the curve sheets above listed. In two cases single values only are available:

TABLE I.

Range of Characteristic Residual Voltage—Single-Circuit Lines.

Configuration	Characteristic residual voltage; per cent of balanced three-phase voltage between conductors
Equilateral triangle	0.5–4
Vertical	6–11
Horizontal—	
Symmetrical	5–9
Unsymmetrical	7–13
Isosceles triangle—	
Base horizontal	0–8
Base vertical	0.5–9
"L"	2
Inverted "L"	5.5

It is stated, in the Report of the Joint Committee to the Railroad Commission dated July 7, 1914, page 28,* last paragraph, in discussing residuals due to line unbalance, that with a horizontal arrangement of conductors the capacitances are more nearly balanced than with the triangular or vertical arrangements. This was based on the assumption that equality of distances between conductors and ground is of more importance than equality of distances between conductors, whereas the opposite is shown by the results of these studies.

For all the configurations, increase in the size of the conductor is accompanied by an increase in the characteristic residual voltage, the latter being approximately proportional to the $\frac{1}{8}$ power of the diameter of the conductors.

For the equilateral triangular configuration, the characteristic residual voltage is practically the same whether the base of the triangle is horizontal, vertical or otherwise; assuming a constant ratio of size of conductors and height of lowest conductor to the distance between conductors. The characteristic residual voltage decreases approximately inversely as the height of the conductors above ground and approaches zero as the height is increased without limit.

For the flattened triangle (base horizontal, ratio of altitude to base = 0.4) the characteristic residual voltage is greater than that of the equilateral triangle. It increases as the height of the conductors is decreased, not so rapidly as for the equilateral triangle, but much more rapidly than for a flat horizontal line.

For the peaked triangle (base horizontal and ratio of altitude to base = 1.25) there is a value of height of conductors within the practical

*See p. 126 of this volume.

range for which the characteristic residual voltage is zero. For changes in height either way from this critical value the residual voltage increases rather rapidly. This configuration was studied only for one size of conductor, but it is reasonable to suppose that the variation with size of conductor follows practically the same law as for the other configurations, which is approximately as the $\frac{1}{8}$ power of the conductor-diameter. The value of height of conductors above ground for zero residual voltage is independent of the size of the conductors.

Drawings Nos. 276 and 345 show the characteristic residual voltage of an isosceles triangle with the base horizontal, as the altitude is varied. The residual voltage is a maximum when the middle conductor is a little above the plane of the others. For the two cases shown, the characteristic residual voltage is zero when the ratio of altitude to height is 1.46 and 1.34, with the middle conductor above, and 0.60 for one case (drawing No. 276) with the middle conductor below the plane of the others. For increased height of conductors the configuration for zero residual voltage approaches more nearly the equilateral triangle. This is also shown by drawing No. 275 upon which are plotted the relationships of height, base, and altitude of isosceles triangles, the bases being horizontal, for which the characteristic residual voltage is zero. For the vertex upward the triangles are peaked and for vertex downward are flattened as compared to the equilateral configuration. The degree of departure from the equilateral decreases as the ratio of height of the conductors to the base of the triangle increases.

On drawing No. 278 is given a curve showing the effect of a ground wire on the residual voltage of an equilateral triangle with base horizontal and vertex upward. It is seen that there are two positions for which the characteristic residual voltage is zero and that it varies rapidly as the position of the ground wire is changed. The positions for zero residual voltage will vary as the ratios of height and size to spacing of conductors and the size of the ground wire are varied. In most cases they are apt to be impractical. For the particular case studied the two positions are (1) within the triangle just above the centroid and, (2) above the vertex 3.4 times the distance between conductors. The computations were made only for this one set of dimensions.

For the horizontal configurations the height of conductors is comparatively an unimportant factor, particularly for ratios of height to spacing of outside conductors equal to three or greater, which includes most practical cases. The characteristic residual voltage increases slightly as the height of the conductors increases and approaches a definite value if the height of conductors is increased without limit.

The characteristic residual voltage of the vertical line decreases as the height above ground is increased, the limiting value approached being

the same as that of the symmetrical horizontal line for the same ratio of size to spacing of conductors. Drawing No. 300 affords a graphical comparison of the vertical, symmetrical horizontal, and equilateral triangular configurations for varying height of conductors.

On drawings Nos. 277 and 346 are shown curves of the variation of characteristic residual voltage for an isosceles triangle, with base vertical, as the altitude is varied. This corresponds to the "wishbone" configuration. The residual voltage decreases from the value for vertical configuration to a minimum when the triangle is equilateral and then increases gradually.

On drawing No. 278 is shown a curve of variation of characteristic residual voltage for a circuit of triangular configuration with vertical base, as the vertex of the triangle is shifted vertically from the level of the lower conductor to the level of the upper conductor. The altitude of the triangle is 0.866 of the base, making the triangle equilateral when the vertex conductor is midway between the others. The two points of special interest are the upper and the lower limits, corresponding to "L" and inverted "L" configurations. The residual voltage passes through a minimum when the vertex conductor is midway between the equilateral and the lower positions. The residual voltage of the "L" configuration is 0.8, and that of the inverted "L" is 2.3 that of the equilateral triangle.

In comparing the various configurations with respect to induction from the characteristic residual voltage into a neighboring circuit, the magnitude of the characteristic residual voltage can not be used as an exact criterion. For a given magnitude of characteristic residual voltage and with neighboring circuits in a given position with respect to the power line the resulting induction is approximately proportional to the capacitance to ground of the conductors of the power circuit in parallel. This varies with the configuration, the value decreasing as the ratio of the height to spacings of the conductors is increased, and increasing as the size of the conductors is increased. This variation of the capacitance of the conductors is small compared to the variation of the characteristic residual voltage with different configurations, the ratio of maximum to minimum values likely to be encountered being about 2. Most cases will agree much more closely. The variation of induction from residual voltage with the configuration is treated in detail in Technical Report No. 65.

2. *Twin-Circuit Lines.*

For twin-circuit lines, only a few representative cases have been considered, five of vertical configuration, in one of which the middle conductors are displaced outward a small amount, and two cases of equilateral triangles, one having vertices upward, and the other, vertices

downward.* For each the effects of six different methods of parallel operation were determined. Diagrams of the configurations and parallel arrangements, together with the corresponding values of the characteristic residual voltages and capacitances to ground of all conductors in parallel, are given on drawing No. 365.

For the vertical configurations the characteristic residual voltage is increased by decreasing the separation of the circuits except for the method of parallel operation giving the minimum values (No. 4) in which the opposite effect occurs. For three of the methods of parallel operation (1, 2, 3) the characteristic residual voltage is greater than that of one of the circuits were the other not present, and for the other three, is less. The minimum value of residual voltage is given by method No. 4, in which the top conductors are interconnected and the middle conductor of one circuit connected to the lowest conductor of the other. This is true for the three heights of conductors and both separations of circuits.

When the middle conductor is displaced outward a small amount method No. 4 is still the best, but its superiority over the other methods of parallel operation is less than when the three conductors of each circuit are in the same vertical plane. Four of the twin-circuit arrangements give lower values of characteristic residual voltage than a single circuit. It should be noted that a regular hexagonal arrangement at great height would have zero residual voltage for any method of operation.

For the two triangular configurations the characteristic residual voltage is, except for method No. 4, less with the vertices of the triangles upward than with them downward. This is particularly noticeable in the case of method No. 2, which is the best for both configurations. With the vertices downward the residual voltage is greater, for all methods of parallel operation, than for one of the circuits by itself; with the vertices upward method No. 2, only, gives a value less than for the single-circuit line.

For all configurations, the residual voltages with methods Nos. 5 and 6 are exactly one-half those of method No. 1, which latter gives the largest values of any. The ratio of this largest value to the corresponding single-circuit value is about 1.5 for the vertical cases and about 10 for the equilateral triangles.

The largest value of characteristic residual voltage found is about 13 per cent, and is nearly the same for vertical and triangular configurations.

For twin-circuit lines the characteristic residual voltage exists on each of the two circuits and hence the induction in a neighboring cir-

*See page 349 for Twin Horizontal Circuits.

cuit will be greater than if one of the power circuits only were present with the same value of characteristic residual voltage. The induction from residual voltages is approximately proportional to the product of the residual voltage per circuit and the capacitance to ground of the conductors of both power circuits in parallel. For the configurations studied the capacitance to ground of the six conductors in parallel of a twin-circuit line is from 35 to 46 per cent greater than that of the three conductors of the corresponding single circuit. The induction in a neighboring circuit for a given magnitude of residual voltage is therefore 35 to 46 per cent greater for the twin-circuit line than for one of the single circuits were the other not present. Weighting the inductive effects of the single and twin-circuit lines in proportion to the product of the characteristic residual voltage and the total direct capacitance to ground, the following ratios show the best and worst (dependent upon method of parallel connection) which may be expected from the twin circuit lines in comparison with the corresponding single-circuit lines:

Configuration	Best	Worst
Vertical	0.15–0.40	1.9–2.2
Vertical, middle conductor slightly outward	0.75	1.9
Triangle, vertex up	0.55	14.0
Vertex down	4.7	14.6

These values show that of the cases studied, with one exception, twin circuits have a very material advantage over the corresponding single circuit, provided care is taken to interconnect them so as to secure minimum characteristic residual voltage. On the other hand, the result may be very much worse if such care is not taken.

II. Long Lines.

A—Definition.

The term "long line" is here applied to lines which are not short compared to a wave-length. For such lines it is necessary to consider the series impedances, as well as the capacitances between conductors and between conductors and ground, in modifying the voltages between conductors and ground. The effects of phase-change and attenuation can not be neglected.

B—Formulas Involving Equivalent Admittances.

Equations (2) given in the preceding section, dealing with short lines, may be extended to apply to long lines, if instead of the charges on the

conductors the charging currents are considered, and in place of the capacitance coefficients the equivalent admittance coefficients are used. Thus for a system of three conductors and ground;

$$\left.\begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 \\ I_2 &= Y_{12}V_1 + Y_{22}V_2 + Y_{23}V_3 \\ I_3 &= Y_{13}V_1 + Y_{23}V_2 + Y_{33}V_3 \end{aligned}\right\} \qquad (38)$$

Since the system is isolated from ground the sum of the currents is equal to zero and hence

$$0 = Y_{10}V_1 + Y_{20}V_2 + Y_{30}V_3 \qquad (39)$$

where

$$\left.\begin{aligned} Y_{10} &= Y_{11} + Y_{12} + Y_{13} \\ Y_{20} &= Y_{12} + Y_{22} + Y_{23} \\ Y_{30} &= Y_{13} + Y_{23} + Y_{33} \end{aligned}\right\} \qquad (40)$$

Y_{10}, Y_{20} and Y_{30} are direct admittances to ground of the respective conductors. They may be substituted in equations (6) and (8) for C_{10}, C_{20} and C_{30}, respectively, and equations obtained for the residual voltage in terms of the direct admittances. These equations may be applied to a line of any length.

$$E_R = v_1 + v_2 + v_3 = \frac{Y_{10}(2E_{12} + E_{23}) + Y_{20}(E_{23} - E_{12}) + Y_{30}(-E_{12} - 2E_{23})}{Y_{10} + Y_{20} + Y_{30}} \qquad (41)$$

For balanced three-phase voltages between conductors

$$E_R = E_{12}\frac{3(Y_{10} - Y_{20}) + j\sqrt{3}(Y_{10} + Y_{20} - 2Y_{30})}{2(Y_{10} + Y_{20} + Y_{30})} \qquad (42)$$

This may be rewritten in terms of direct-admittance unbalances:

$$E_R = E_{12}\frac{3(Y_{10} - Y_{20}) + j\sqrt{3}[(Y_{10} - Y_{20}) + 2(Y_{20} - Y_{30})]}{2(Y_{10} + Y_{20} + Y_{30})} \qquad (43)$$

For application to any given case it is necessary to determine the equivalent admittances at the point at which the value of residual voltage is desired. In general, for frequencies higher than fifty or sixty cycles this may be done by measurements with greater facility and accuracy than the residual voltage itself may be measured by any method heretofore used. A detailed discussion of the method is given in Appendix II.

C—General Equations Involving Primary Constants.

The preceding discussion does not include any method of determining the residual voltage at any point along a line from its physical dimensions. The purpose of this section is to outline a method of computing

the residual voltage from the primary constants, which in turn may be expressed in terms of the physical dimensions of the line.

For a system of three linear conductors and ground, consider an elementary length of line dx. The changes in voltages along this small element of line may be expressed in terms of the currents and series impedances as determined by the resistances of the conductors and the self and mutual inductances per unit length, thus:

$$\left.\begin{aligned}\frac{dV_1}{dx} &= Z_{11}I_1 + Z_{12}I_2 + Z_{13}I_3 \\ \frac{dV_2}{dx} &= Z_{12}I_1 + Z_{22}I_2 + Z_{23}I_3 \\ \frac{dV_3}{dx} &= Z_{13}I_1 + Z_{23}I_2 + Z_{33}I_3\end{aligned}\right\} \qquad (44)$$

Also the changes in the currents in the same elementary length may be expressed in terms of the voltages and the shunt admittance coefficients determined by the capacitance and the conductance coefficients per unit length, thus:

$$\left.\begin{aligned}\frac{dI_1}{dx} &= Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 \\ \frac{dI_2}{dx} &= Y_{12}V_1 + Y_{22}V_2 + Y_{23}V_3 \\ \frac{dI_3}{dx} &= Y_{13}V_1 + Y_{23}V_2 + Y_{33}V_3\end{aligned}\right\} \qquad (45)$$

By differentiating equations (44) with respect to x and substituting the values of $\frac{dI_1}{dx}, \frac{dI_2}{dx}, \frac{dI_3}{dx}$ from equations (45), a set of three simultaneous equations is obtained involving the second derivatives of the three voltages and the admittances and impedances per unit length. By differentiating equations (45) and substituting the first derivatives of the voltages from equations (44) another set of three simultaneous equations involving the currents and their second derivatives is obtained. From the solution of these the voltages from the three conductors to ground at any point along the circuit may be obtained, and thus the

residual voltage. The integral expressions involve six independent constants of integration. The detailed method of solution is given in Appendix III.

The expression for E_R at any point x is of the form

$$\left.\begin{aligned} E_R = & (1 + h_1 + p_1)(A_1 \cosh m_1x + B_1 \sinh m_1x) \\ & + (1 + h_2 + p_2)(A_2 \cosh m_2x + B_2 \sinh m_2x) \\ & + (1 + h_3 + p_3)(A_3 \cosh m_3x + B_3 \sinh m_3x) \end{aligned}\right\} \quad (46)$$

The coefficients in the above equation are all complicated functions of the impedances and admittances per unit length of the line.

To apply the integral equation to the computation of residual voltage it is necessary to evaluate the six constants of integration in terms of the given terminal conditions. In general, the solution of a system of six simultaneous equations is involved. When the constants are determined the residual voltage at any point distant x from the sending end may be obtained by substituting the proper value of x in the integral equation. Thus, the exact determination of the residual voltage of a long line requires the general solution of a three-phase circuit with uniformly distributed constants. The numerical work for any given case is exceedingly laborious and has not been undertaken.

SECTION II.

EXPERIMENTAL.

I. Description of Transmission Line.

The tests described herein were made on the San Fernando-Somis single-circuit 15-kv. line of the Pacific Light and Power Corporation which extended a distance of 36.7 miles beyond the Joint Committee's laboratory at San Fernando. The following is a tabulated description:

Length, 36.7 miles.

Type of support—Poles 90.2%
A-Frames 9.8%

Configuration—Vertical, five-foot spacing, on poles.
Vertical, six-foot spacing, on A-frames.

Height of lower conductor—average 40 feet.

Conductors—No. 4 B&S solid copper 78.6%
No. 2 B&S solid copper 7.8%
No. 2 B&S stranded copper 13.6%

The measurements were made with the line nontransposed, and when transposed for two different lengths of barrel. The transpositions were located so as to divide the line into sections of as nearly as practicable equal lengths, first into three equal sections by two transpositions (one barrel) and then into six equal sections by three additional transpositions (2 barrels). No effort was made to modify the location of trans-

positions to compensate for irregularities in the line, such as differences of size and spacing of conductors. The distances between transpositions and the computed lengths of unbalance for each of the two lengths of barrel are given below.

TABLE II.

Transposition Spacings and Lengths of Unbalance.

Transposition number		Spacing, feet		Total distance, feet
		One barrel	Two barrels	
Laboratory				0
			33,599	
1		64,261		33,599
			30,662	
2				64,261
			32,885	
3		64,001		97,146
			31,116	
4				128,262
			33,568	
5		65,756		161,830
			32,188	
Somis				194,018
Unbalanced length	feet	1,898	1,164	194,018
	per cent	0.98	0.60	100

II. 50-Cycle Measurements.

A—Description of Method and Arrangements.

The line was energized from the 15-kv. system of the Pacific Light and Power Corporation through two banks of transformers. The step-down bank was connected delta-delta, 15,000-220 volts and the step-up bank delta-star, 220-14,250 volts per transformer, thus giving 26-kv. between pairs of conductors. The neutral of the step-up bank was isolated. At Somis the transformer bank was connected to the line in interconnected star-delta, 13,000-110 volts per transformer, with neutral isolated.

Two methods of measuring residual voltage were employed: (1) the delta method, by grounding the primary neutral of the potential transformer bank and connecting the measuring instrument in the open corner of the secondary delta; and (2) the Y method, by closing the secondary delta of the potential transformer bank and connecting the measuring instrument between the primary neutral and ground.

The residual voltage was measured simultaneously with the line voltages to ground at San Fernando for the three conditions of line as regards transpositions, that is, no transpositions, one barrel and two

barrels. The voltage to ground of the neutral at Somis was also measured in order to obtain the residual voltage at that point.

B—Results.

The average values of the residual voltages, in terms of voltage between conductors, as measured at San Fernando and Somis, are given in the following table. The values given summarize the results of a large number of meter readings and oscillograms made both by the delta and Y methods:

TABLE III.

Transposition system	Residual voltage—Per cent of voltage between conductors				
	Y-method		Delta method		Average
	Meter	Osc.	Meter	Osc.	
No transpositions	6.6	6.8	7.1	7.2	6.9
One barrel	0.6—		0.6—		0.6—
Two barrels	0.6—		0.6—		0.6—

C—Discussion.

The measurements of residual voltage on the nontransposed line give a value of 6.9 per cent of the voltage between conductors. The precision of the measurement is about 5 per cent, hence the residual voltage lies between the limits 6.6 and 7.2 per cent. The residual voltage measured at Somis was practically the same as that measured at San Fernando. For this short line it was to be expected that the residual voltage of fundamental frequency would be constant throughout the length of the line. The value of residual voltage as obtained by calculation is 7.2 per cent of the voltage between wires. This compares favorably with the measured value as given above.

The residual voltage measured with one barrel and two barrels in the transmission line is of about the same magnitude as the possible errors due to the potential transformers (see technical report No. 58), hence it is possible to determine only the upper limit of the residual voltage produced under these conditions. The residual voltage of the transposed line is less than 0.6 per cent of the voltage between wires or less than 8 per cent of the residual voltage with no transpositions. The lengths of unbalance as computed from the distances between transpositions are respectively 100 per cent, 0.98 per cent and 0.60 per cent of the total length of the line for no transpositions, one barrel and two barrels.

When the line was transposed for one barrel some measurements were made under wet weather conditions, thus increasing the effect of leakage. The measurements do not, however, show any appreciable effect on the residual voltage which may be attributed to the low insulation. It may be concluded, therefore, that unbalanced leakage is of relatively small importance, as compared to unbalanced capacitance, in determining the residual voltage of a well maintained isolated system.

III. High-Frequency Measurements.

A—Description of Method.

No direct method of measuring the residual voltage of the line at frequencies higher than 50 cycles was available, so the method outlined under Section I and Appendix II was used. The direct admittances and direct admittance unbalances were determined. No method of measuring these quantities directly was available but the unbalances of pairs of conductors with the third both isolated and grounded were measured with the capacity and conductance unbalance set. (See technical report No. 40 for description of this apparatus and the method of its use and technical report No. 63 for a description of forms used in recording the data). In addition, it was necessary to determine the admittance to ground of one of the conductors with the other two grounded and the admittance to ground of two of the conductors in parallel with the third grounded. These were determined by impedance bridge measurements. All of the measurements were made at several frequencies in the range between 200 and 1000 cycles per second.

The measurements of admittance of one conductor to ground with the others grounded, and the admittance of two conductors in parallel with the third grounded were made only for the case when there were two barrels in the line. The unbalance measurements were made for all three cases. The method of computation of the residual voltage from the unbalance measurements was not developed until after the transpositions had been placed in the line. The necessity for the additional measurements was not realized when the first were made. To compute the direct admittance unbalances and thus the residual voltage for the two cases in which these measurements were omitted it was necessary to assume that the admittance to ground of all three conductors in parallel was the same irrespective of the transpositions. The magnitude of the error to which this assumption leads is not definitely known but it is believed to be small.

The detailed method of computing the direct admittance unbalances and the residual voltage is described in Appendix II.

B—DATA.

The data of the tests have been plotted and are given on attached curve sheets as follows:

No Transpositions. Subject.	Drawing No.
Capacitance unbalance—Pairs of conductors. Third conductor isolated.	280
Capacitance unbalance—Pairs of conductors. Third conductor grounded at San Fernando.	281
Conductance unbalance—Pairs of conductors. Third conductor isolated.	282
Conductance unbalance—Pairs of conductors. Third conductor grounded at San Fernando.	283
One Barrel.	
Capacitance unbalance—Pairs of conductors. Third conductor isolated.	284
Capacitance unbalance—Pairs of conductors. Third conductor grounded at San Fernando.	285
Conductance unbalance—Pairs of conductors. Third conductor isolated.	286
Conductance unbalance—Pairs of conductors. Third conductor grounded at San Fernando.	287
Two Barrels.	
Capacitance unbalance—Pairs of conductors. Third conductor isolated.	288
Capacitance unbalance—Pairs of conductors. Third conductor grounded at San Fernando.	289
Conductance unbalance—Pairs of conductors. Third conductor isolated.	290
Conductance unbalance—Pairs of conductors. Third conductor grounded at San Fernando.	291
Admittances to ground. One conductor—Other two grounded. Two conductors in parallel—Third grounded.	292
Telephone Circuit.	
Admittance unbalance (attached to T. R. No. 55)	297
Admittance to ground—Two conductors in parallel	298

C—RESULTS.

The results of the measurements have been plotted and are given on curve sheets as follows:

Subject.	Drawing No.
Direct conductance unbalances—Pairs of conductors. Three lengths of barrel.	293
Direct susceptance unbalances—Pairs of conductors. Three lengths of barrel.	294
Variation of residual voltage with frequency. Three lengths of barrel. Abscissæ in cycles per second.	295
Variation of residual voltage with frequency. Three lengths of barrel. Abscissæ in per cent of resonating frequency.	296
Variation of residual voltage with frequency. Telephone circuit considered as single-phase power circuit.	299

The value of residual voltage for 50 cycles and no transpositions as determined by extrapolation of the curve of drawing 295 is 6.6 per cent of the voltage between conductors as compared with 6.9 per cent determined by the 50-cycle measurements and 7.2 per cent obtained by computations. This is regarded as a favorable agreement considering the sources of error involved in both 50-cycle and high-frequency measurements.

For no transpositions the residual voltage first increases slightly with increasing frequency reaching a maximum at 600 cycles and then decreases rather rapidly for the remainder of the range covered. For one barrel the values of residual voltage in the range of frequencies between 200 and 1000 cycles per second follow quite closely the law:

$$E_R = 0.16e^{0.0043f} \qquad (47)$$

where E_R is in per cent of balanced three-phase voltage between conductors, e is the base of natural logarithms and f is the frequency in cycles per second. For two barrels the values follow approximately the law:

$$E_R = 0.074e^{0.0041f} \qquad (48)$$

The degree of departure of the values from this law is greater than in the case of one barrel. In neither case is it to be expected that the laws will hold even approximately for frequencies much above 1000 cycles.

At low frequencies the transpositions are very effective but as the frequency increases the efficacy of the transpositions decreases until at a frequency of 830 cycles the residual voltage is the same in magnitude both for no transpositions and for one barrel. At approximately 1000 cycles the value is the same both for no transpositions and for two barrels.

For frequencies below 400 cycles and a 37-mile barrel, and for frequencies below 650 cycles and an 18-mile barrel, the residual voltage is 1 per cent or less of the balanced voltage between conductors. These lengths of barrel correspond approximately to a spacing of 12 barrels per wave-length. Since length of the barrel is of the same order to magnitude as the length of line, reflection effects are of importance. For longer lines or shorter barrels these results would be somewhat modified due to differences in the reflection effects.

Section III.

RESUME.

The voltages to ground of power circuits having no metallic connection to ground are determined by the voltages between conductors and by the direct admittances between conductors and ground. If there are unbalances among the direct admittances a residual voltage is caused.

The term "characteristic residual voltage" is used to denote the residual voltage of a short uniform nontransposed power circuit when isolated from ground and energized with balanced three-phase voltages between conductors; it is a measure of the line unbalance.

Formulas have been developed whereby the residual voltage of lines short as compared to a wave length may be determined from the physical dimensions of the lines. These formulas are applicable to both single and twin-circuit lines with and without ground wires. They have been applied in a study of the effects of varying the relative position, spacing, size and height of conductors and position of ground wire, on the characteristic residual voltage of single-circuit lines.

It was found that in causing direct capacitance unbalance and thus residual voltage, inequalities in the distances between conductors are of more importance than inequalities in the distances between conductors and ground. Thus the characteristic residual voltage of triangular lines is less than that of horizontal lines. This is at variance with the statement made in the report of the Joint Committee to the Railroad Commission, dated July 7, 1914, page 28,* last paragraph. The horizontal configuration with unequal spacing of adjacent conductors gives rise to the maximum characteristic residual voltage found in practical cases. Drawing No. 300 affords a graphical comparison of vertical, symmetrical horizontal and equilateral triangular configurations for varying height of conductors. The characteristic residual voltages of the vertical and horizontal lines approach a common limit as the height of the conductors is increased. The characteristic residual voltage of the vertical type of line is somewhat greater than that of the symmetrical horizontal line. Within the practical range the height of conductors is comparatively an unimportant factor for the horizontal line. For the equilateral triangle, the characteristic residual voltage decreases approximately inversely as the height of conductors and approaches zero as a limit as the height of the conductors is indefinitely increased.

The characteristic residual voltage for the equilateral triangle is practically the same whether the base of the triangle is horizontal, vertical or otherwise; assuming constant ratios of size of conductors and height of lowest conductor to the distance between conductors.

*See page 126 of this volume.

For an isosceles triangular configuration with base horizontal, two positions of the vertex conductor give zero characteristic residual voltage. With the vertex upward the triangles are peaked and with the vertex downward they are flattened, as compared to the equilateral position. The degree of departure from the equilateral decreases as the ratio of height of conductors to base of triangle increases. The relative positions of conductors for zero characteristic residual voltage are independent of the size of the conductor provided all three are the same in size.

For all configurations the characteristic residual voltage is approximately proportional to the one-sixth power of the diameter of the conductors.

The characteristic residual voltage of a triangular circuit was found to be very materially affected by a ground wire, the effect varying greatly with its position. Two impractical positions gave zero characteristic residual voltage.

In general, for triangular circuits the range of characteristic residual voltage is 0 to 5 per cent, while for circuits having all conductors in one plane or nearly so, the value ranges from 5 to 13 per cent of the balanced voltages between conductors. From the numerous curves which are attached, the characteristic residual voltages of most practical configurations of single-circuit lines can be determined quite accurately.

For twin-circuit lines only a few representative cases have been considered, five of vertical configuration, in one of which the middle conductors are displaced outward by a small amount, and two cases of equilateral triangles, one having vertices upward and the other vertices downward. For each, the effects of six different methods of parallel operation were determined. The maximum value of characteristic residual voltage obtains when symmetrically located conductors of the two circuits are interconnected. This value may be halved if, in interconnecting the two circuits, the phases of one circuit are transposed with respect to the other. Another method of interconnection gives still better results. For vertical lines the most effective arrangement is to connect the two top conductors, and cross-connect the middle and lowest conductors. To most effectively reduce the residual voltages and currents of twin-circuit lines operated in parallel, the two circuits should be transposed at the same points and care taken to secure the condition for minimum characteristic residual voltage in each section of line between transposition points. Of the cases studied, with one exception, twin circuits have a very material advantage over the corresponding single circuit, providing care is taken to interconnect them so as to secure minimum characteristic residual voltage. On the other hand, the result may be very much worse if such care is not taken.

Few power circuits in practical operation fulfill the conditions which will make their actual residual voltage equal to their characteristic residual voltage. Transpositions, irregularities and length are factors which materially modify the residual voltage of most circuits. For short transposed lines the residual voltage, as compared to the characteristic residual voltage, is the ratio of unbalanced length to the total length of the line. Thus, if the line is symmetrically transposed the residual voltage is zero, irregularities being neglected.

When a line is transposed a residual current is set up between sections separated by transpositions. This current is zero at the terminals, is maximum at the transposition points within each barrel length, and minimum at the junction points of the barrels whether there are transpositions at these junction points or not.

If a line be supplied with balanced voltages to ground, a residual current due to line unbalance is required, whose magnitude is equal to the product of one-third the residual voltage which would exist were the line isolated from ground, and the admittance to ground of the conductors of the circuit in parallel.

The formulas developed for the residual voltage of short lines have been extended so as to apply to lines of any length, and methods are outlined whereby the required direct admittances may be determined from measurements.

The determination of the residual voltage of a long line in terms of its physical dimensions requires the general solution of a three-phase circuit with distributed constants. The numerical work of solution for any given case is so laborious that it has not been undertaken.

Measurements of residual voltage were made on a 37-mile vertical line having successively, no transpositions, one barrel and two barrels; by direct measurement at 50 cycles and by determining equivalent admittances for frequencies of 200 to 1000 cycles. Comparison of the measurements at 50 cycles on the nontransposed line with the value obtained by computations based on the physical dimensions, shows very satisfactory agreement. Measurements at 50 cycles made under wet weather conditions, when the line was transposed for one barrel, do not show any appreciable effect on the residual voltage which may be attributed to low insulation. It may be concluded, therefore, that unbalanced leakage is of relatively small importance as compared to unbalanced capacitance in determining the residual voltage of a well maintained isolated system. It was found that a single barrel was highly effective in reducing the residual voltage of fundamental frequency, 50 cycles per second. The single barrel at this frequency corresponds approximately to 78 barrels per wave length. For higher frequencies this length of barrel and also two barrels of half this length

were found quite ineffective, the residual voltage at 1000 cycles being approximately double that of the nontransposed line for one barrel and about equal to that of the nontransposed line for two barrels. Within the range from 200 to 1000 cycles it was found that the residual voltages with one and two barrels were, approximately, simple exponential functions of the frequency. For frequencies up to 400 cycles per second with one barrel, and for frequencies up to 600 cycles for two barrels, the residual voltage was less than 1 per cent of the voltage of the same frequency between conductors. These lengths of barrel correspond approximately to the spacing of 12 barrels per wave-length for the limiting frequency mentioned in each case. For the nontransposed condition, the residual voltage ranged from 6.5 to 7 per cent of the voltage between conductors for frequencies up to 600 cycles per second and for the higher frequencies it decreased with increase of frequency within the range investigated, reaching a value of 5 per cent at 900 cycles. Since the length of barrel is of the same order of magnitude as the length of line, reflection effects are of importance, and for longer lines or shorter barrels these results would be somewhat modified. It would be interesting but very laborious to apply the general equation for the residual voltage of long lines to the study of its variation with frequency and length of barrel for lines of different lengths, configuration and different methods of transposition.

In determining the length of barrels required to limit the residual voltage of power circuits isolated from ground to a given magnitude, the characteristic residual voltage of the circuit is an important factor, shorter barrels being required with larger characteristic residual voltages. The exact relationship of these two factors for a constant residual voltage has not been determined. Under average conditions of waveform it should be expected that 9-mile barrels will effectually balance triangular lines (or twin-circuit lines so interconnected that their characteristic residual voltages are of the same order of magnitude as that of a single-circuit triangular line) and that 6-mile barrels will balance lines of other configurations, so as to limit the residual voltages of fundamental and higher frequencies to relatively small magnitudes as compared with those of a nontransposed line. It is probable that more effective results will be secured by omitting the transpositions at the junctions of successive barrels, since thereby the unbalance due to

phase change may be materially reduced. Thus for 6 and 9-mile barrels the average distances between transpositions would be 3 and 4.5 miles.

Respectfully submitted.

(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

ATTACHMENTS: Appendices I, II and III. Drawings Nos. 273 to 296, 298 to 300, 344 to 352, and 365.

APPROVED: October 2, 1916.

(Signed) R. W. MASTICK,
Field Engineer.

APPROVED: January 3, 1917.

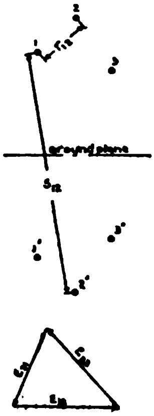
SUBCOMMITTEE ON TESTS,
(Signed) J. E. WOODBRIDGE,
Chairman.

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE,
(Signed) ARTHUR F. BRIDGE,
Secretary.

January 8, 1917.

P.I.C. No 279
8-11-18

FORMULAS FOR THE COMPUTATION OF RESIDUAL VOLTAGE
DUE TO THE
CAPACITANCE UNBALANCE OF AN ISOLATED. THREE-CONDUCTOR LINE.



Let E_{mn} = voltage, conductor m to n.
E_R = residual voltage.
C_{mo} = direct capacitance, conductor m to ground
p_{mn} = potential difference, m to ground, due to unit charge on n.
S_{mn} = distance from conductor m to image of n.
r_{mn} = distance from conductor m to n.
r_{mm} = radius of conductor m.

$$\text{Then } E_R = \frac{A(2E_{12}+E_{23})+B(E_{23}-E_{12})+C(-E_{12}-2E_{23})}{A+B+C} \qquad (1)$$

If the impressed voltages between conductors are equal in magnitude and 120° apart in phase, (1) becomes

$$E_R = E_{12}\,\frac{3/2(A-B)+j\sqrt{3}/2(A+B-2C)}{A+B+C} \qquad (2)$$

$$A = C_{10}D = p_{23}(p_{12}+p_{13}-p_{23})+p_{22}(p_{33}-p_{13})-p_{12}p_{33} \qquad (3)$$

$$B = C_{20}D = p_{13}(p_{23}+p_{12}-p_{13})+p_{33}(p_{11}-p_{12})-p_{23}p_{11} \qquad (4)$$

$$C = C_{30}D = p_{12}(p_{13}+p_{23}-p_{12})+p_{11}(p_{22}-p_{23})-p_{13}p_{22} \qquad (5)$$

$$D = p_{11}p_{22}p_{33}+2p_{12}p_{23}p_{13}-p_{22}p_{13}^2-p_{11}p_{23}^2-p_{33}p_{12}^2 \qquad (6)$$

$$p_{mm} = \log_{10}\frac{S_{mm}}{r_{mm}} \qquad p_{mn} = \log_{10}\frac{S_{mn}}{r_{mn}} \qquad (7)$$

It is not necessary to compute the value of D. Equations (7) give $\log_{10}\frac{S}{r}$ in place of $2\log_e\frac{S}{r}$ in the expressions for p, for convenience in computation, since the factor $2\log_e 10$ cancels out.

Formulas (1) and (2) are based on the assumption that the line is a small fraction of a wave length. Formula (7) is based on the assumptions: (1) that the conductors are round; (2) that the distances between conductors and to ground are large compared to their radii; (3) that the length of line is large compared to the distance between conductors. If $r_{mn} > 10r_{mm}$, (7) is correct within 0.1 percent. No account is taken of conductance unbalance.

CONDITION FOR ZERO RESIDUAL VOLTAGE -

$$p_{11}+p_{23}+p_{23} = p_{22}+p_{13}+p_{13} = p_{33}+p_{12}+p_{12} \qquad (8)$$

FORMULA FOR "HORIZONTAL" LINES:-

When conductors 1 and 3 are of same height, and equidistant from conductor 2, all three having the same diameter,

$$E_R = E_{12}\left(\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)\frac{(p_{22}+p_{13})-(p_{11}+p_{12})}{p_{11}+2p_{22}+p_{13}-4p_{12}} \qquad (9)$$

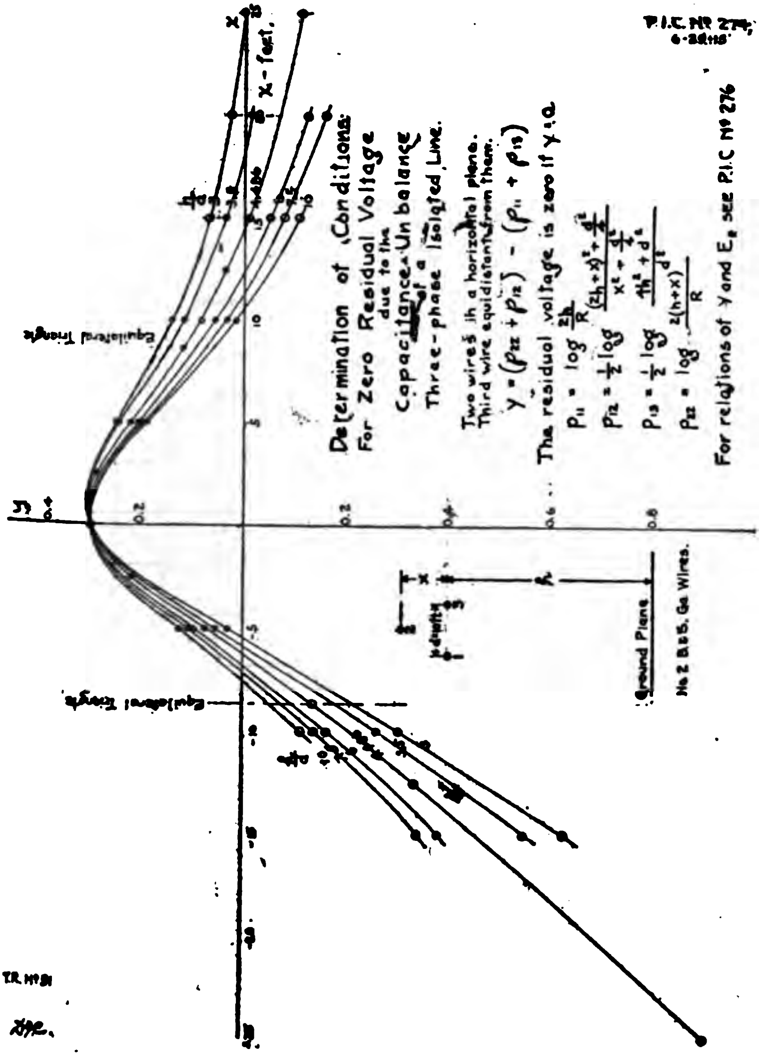
APPROXIMATE FORMULA, for three wires in one plane, one being midway between the other two:-

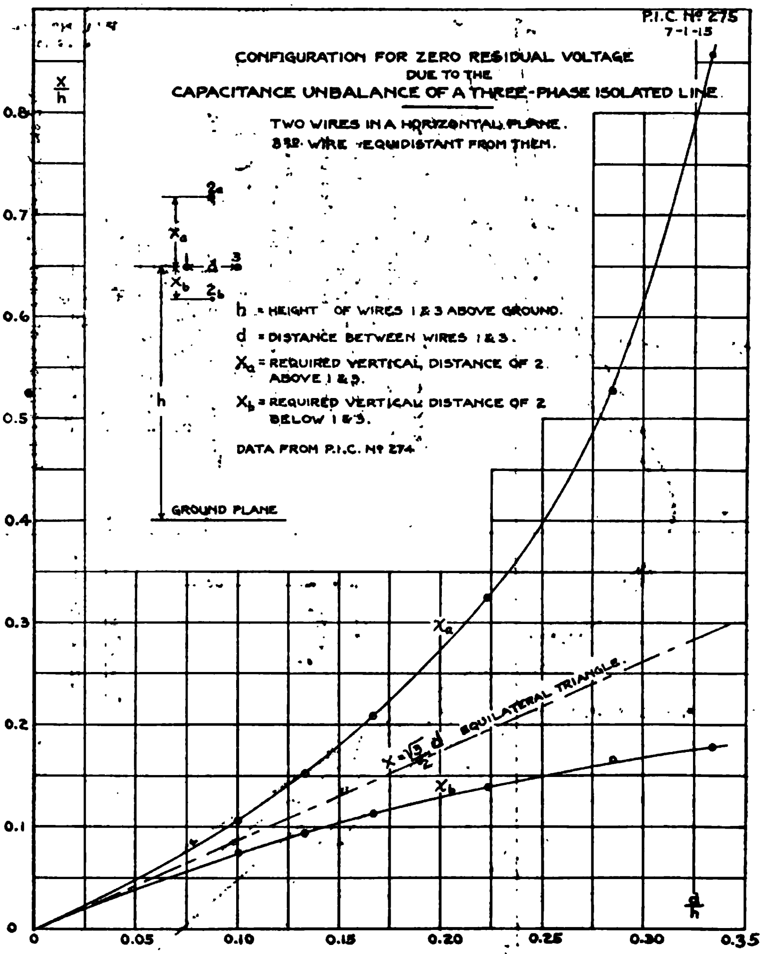
$$E_R = E_{12}\,\frac{0.1\sqrt{3}}{\log_{10}\frac{d}{R}-0.1} \qquad (10)$$

d = distance between adjacent wires.
R = radius of wire.

T.R. No 51

elf
JAC


Determination of Conditions
For Zero Residual Voltage
due to the
Capacitance-Unbalance
of a
Three-phase Isolated Line.
Two wires in a horizontal plane.
Third wire equidistant from them.
Y = (p22 + p12) - (p11 + p13)
The residual voltage is zero if Y = 0.
p11 = log 2h/R
p12 = 1/2 log ((2h+x)^2 + d^2/4)/(x^2 + d^2/4)
p13 = 1/2 log (4h^2 + d^2)/d^2
p22 = log 2(h+x)/R
For relations of Y and E0 see P.I.C No 276
P.I.C. No 274
6-28-18
X - feet.
Equilateral Triangle
Equilateral Triangle
Ground Plane
No 2 B&S. Ga Wires.
TR No 51

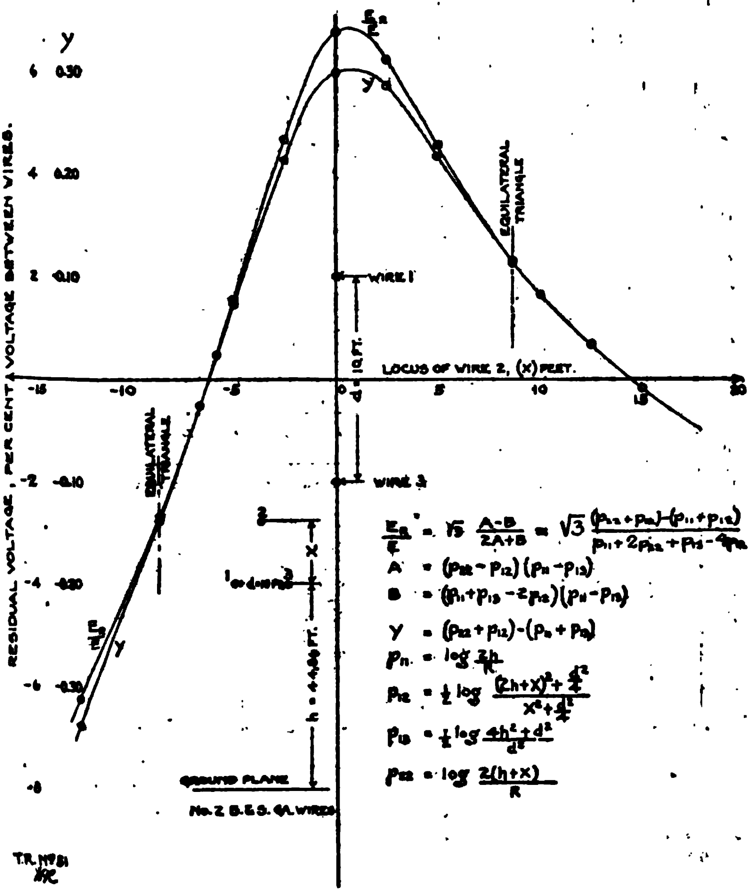

P.I.C. N° 275
7-1-15
CONFIGURATION FOR ZERO RESIDUAL VOLTAGE
DUE TO THE
CAPACITANCE UNBALANCE OF A THREE-PHASE ISOLATED LINE.
TWO WIRES IN A HORIZONTAL PLANE.
3RD WIRE EQUIDISTANT FROM THEM.
h = HEIGHT OF WIRES 1 & 3 ABOVE GROUND.
d = DISTANCE BETWEEN WIRES 1 & 3.
X_a = REQUIRED VERTICAL DISTANCE OF 2 ABOVE 1 & 3.
X_b = REQUIRED VERTICAL DISTANCE OF 2 BELOW 1 & 3.
DATA FROM P.I.C. N° 274
GROUND PLANE
$\frac{x}{h}$
$\frac{d}{h}$
X_a
X_b
$X = \frac{\sqrt{3}}{2} d$
EQUILATERAL TRIANGLE
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.05
0.10
0.15
0.20
0.25
0.30
0.35

T.R. N° 51
3.9C

P.I.C. Nº 276
7-2-18

EFFECT OF CONFIGURATION ON RESIDUAL VOLTAGE
DUE TO THE
CAPACITANCE UNBALANCE OF A THREE-PHASE ISOLATED LINE.

CASE I - TWO WIRES IN HORIZONTAL PLANE; THIRD WIRE EQUIDISTANT, MOVED VERTICALLY.



$$\frac{E_R}{E} = \sqrt{3}\,\frac{A-B}{2A+B} = \sqrt{3}\,\frac{(p_{22}+p_{12})-(p_{11}+p_{13})}{p_{11}+2p_{22}+p_{13}-4p_{12}}$$

$$A = (p_{22}-p_{12})(p_{11}-p_{13})$$

$$B = (p_{11}+p_{13}-2p_{12})(p_{11}-p_{13})$$

$$Y = (p_{22}+p_{12})-(p_{11}+p_{13})$$

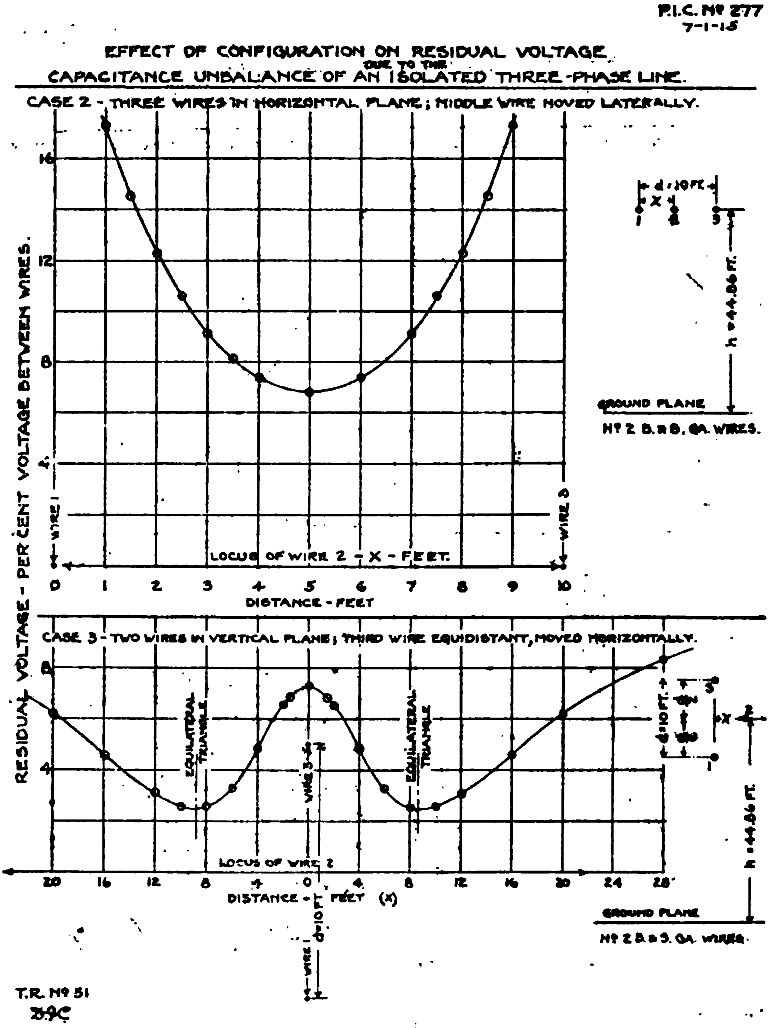
$$p_{11} = \log\frac{2h}{R}$$

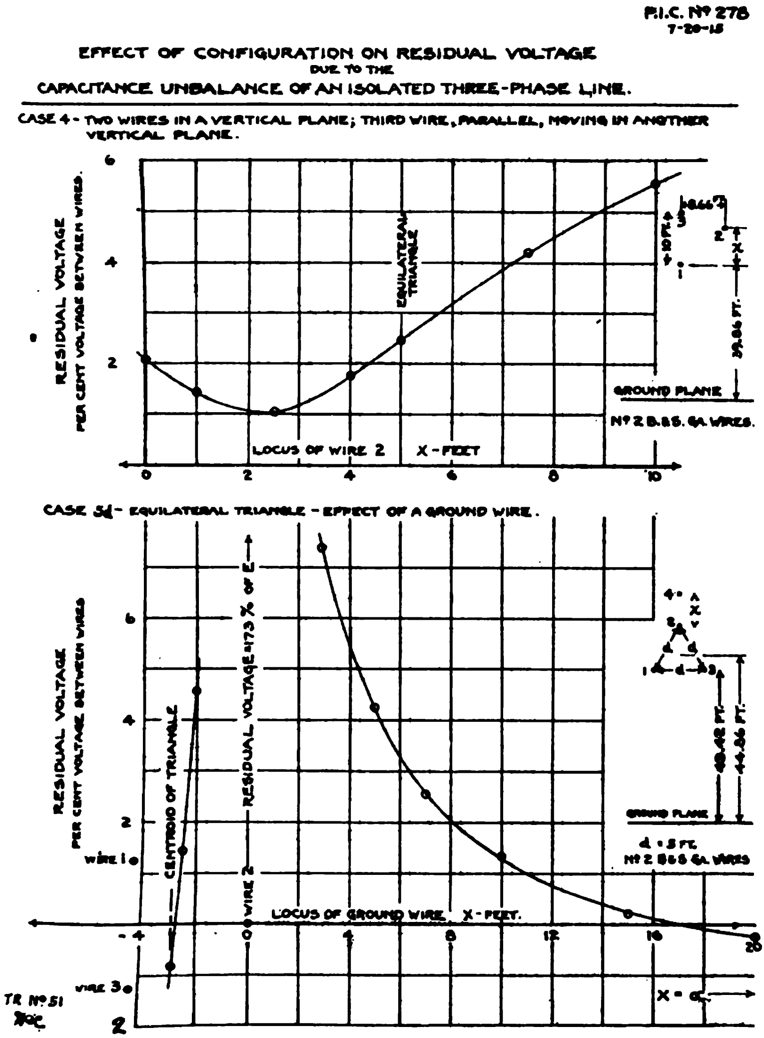
$$p_{12} = \frac{1}{2}\log\frac{(2h+X)^2+\frac{d^2}{4}}{X^2+\frac{d^2}{4}}$$

$$p_{13} = \frac{1}{2}\log\frac{4h^2+d^2}{d^2}$$

$$p_{22} = \log\frac{2(h+X)}{R}$$

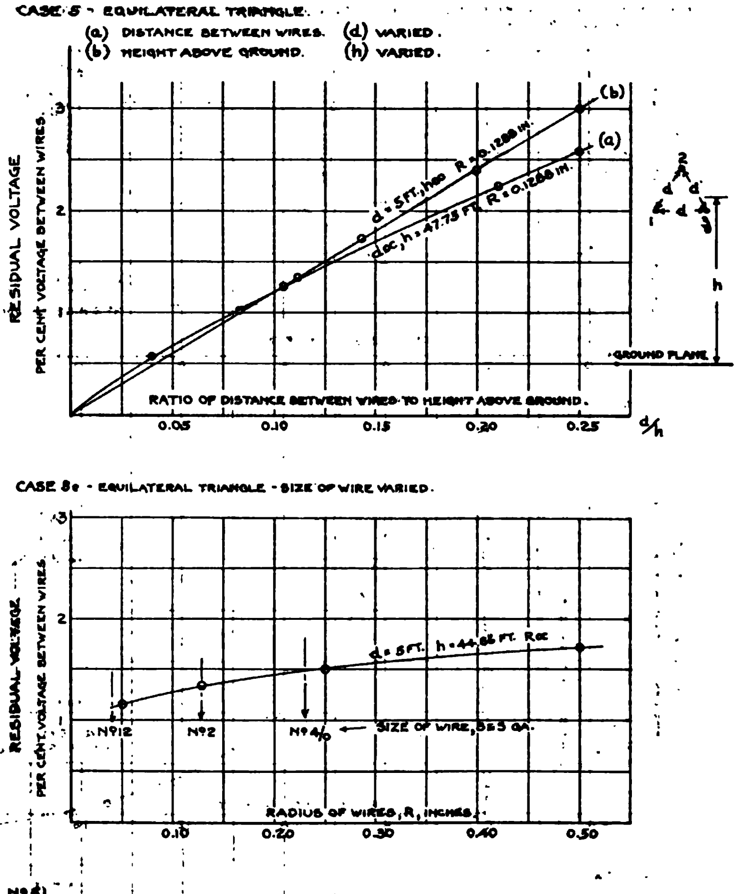
T.R. Nº 81


P.I.C. Nº 277
7-1-15
EFFECT OF CONFIGURATION ON RESIDUAL VOLTAGE
DUE TO THE
CAPACITANCE UNBALANCE OF AN ISOLATED THREE-PHASE LINE.
CASE 2 - THREE WIRES IN HORIZONTAL PLANE; MIDDLE WIRE MOVED LATERALLY.
d = 10 FT.
X
h = 44.86 FT.
GROUND PLANE
Nº 2 B. & S. GA. WIRES.
LOCUS OF WIRE 2 - X - FEET.
WIRE 1
WIRE 3
DISTANCE - FEET
RESIDUAL VOLTAGE - PER CENT VOLTAGE BETWEEN WIRES.
CASE 3 - TWO WIRES IN VERTICAL PLANE; THIRD WIRE EQUIDISTANT, MOVED HORIZONTALLY.
EQUILATERAL TRIANGLE
EQUILATERAL TRIANGLE
WIRE 3-5
LOCUS OF WIRE 2
d = 10 FT.
DISTANCE - FEET (X)
h = 44.86 FT.
GROUND PLANE
Nº 2 B. & S. GA. WIRES.
WIRE 1
T.R. Nº 51

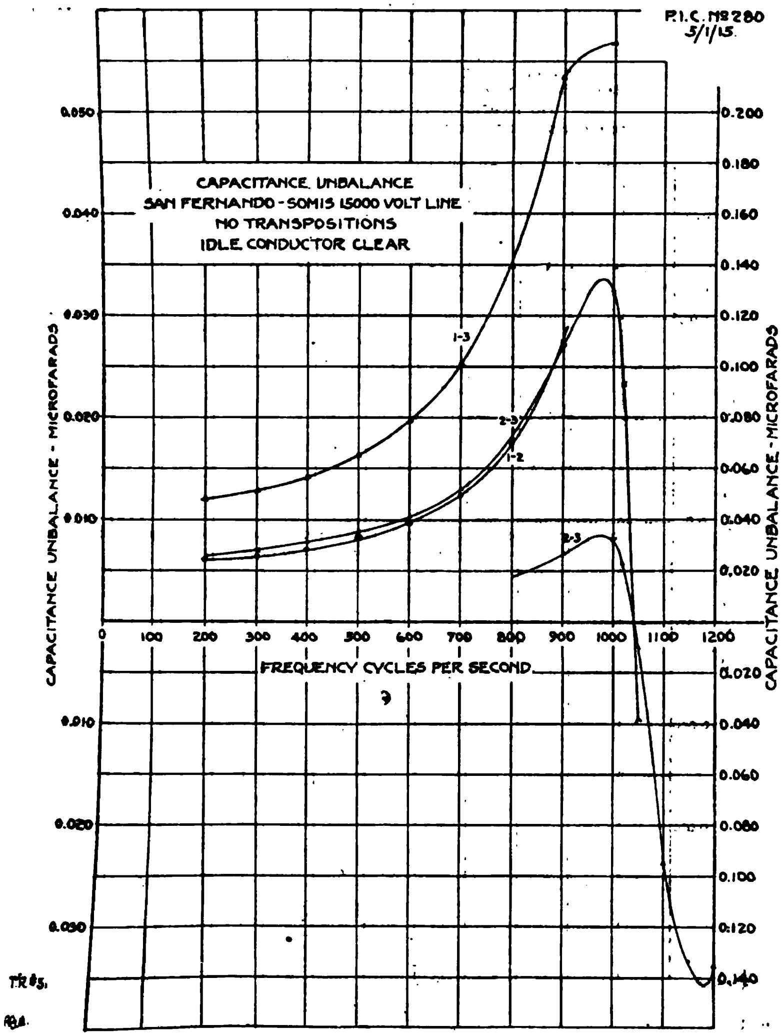

P.I.C. Nº 278
7-20-15
EFFECT OF CONFIGURATION ON RESIDUAL VOLTAGE
DUE TO THE
CAPACITANCE UNBALANCE OF AN ISOLATED THREE-PHASE LINE.
CASE 4 - TWO WIRES IN A VERTICAL PLANE; THIRD WIRE, PARALLEL, MOVING IN ANOTHER VERTICAL PLANE.
RESIDUAL VOLTAGE
PER CENT VOLTAGE BETWEEN WIRES
EQUILATERAL TRIANGLE
39.86 FT.
GROUND PLANE
Nº 2 B.&S. GA. WIRES.
LOCUS OF WIRE 2
X - FEET
CASE 5d - EQUILATERAL TRIANGLE - EFFECT OF A GROUND WIRE.
CENTROID OF TRIANGLE
RESIDUAL VOLTAGE = 173 % OF E
WIRE 2
wire 1
wire 3
40.42 FT.
44.86 FT.
GROUND PLANE
d = 5 FT.
Nº 2 B&S GA. WIRES
LOCUS OF GROUND WIRE. X - FEET.
X = ∞
TR Nº 51
2

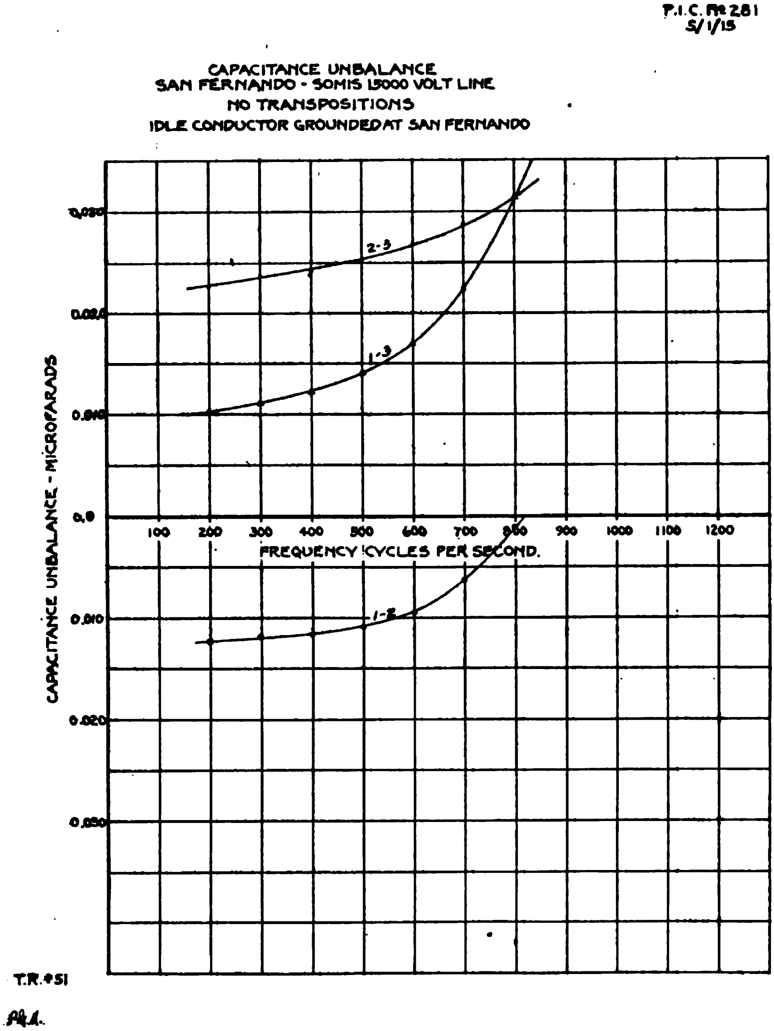
P.I.C. Nº 279
7-21-15

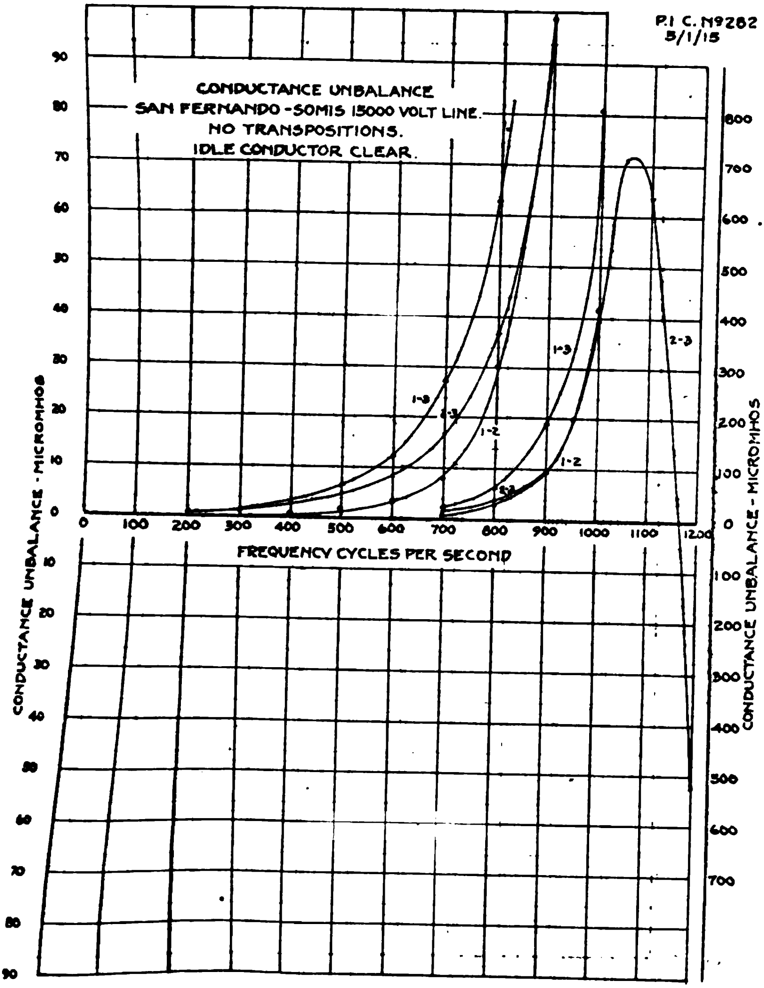
EFFECT OF CONFIGURATION ON RESIDUAL VOLTAGE
DUE TO THE
CAPACITANCE UNBALANCE OF AN ISOLATED THREE-PHASE LINE.



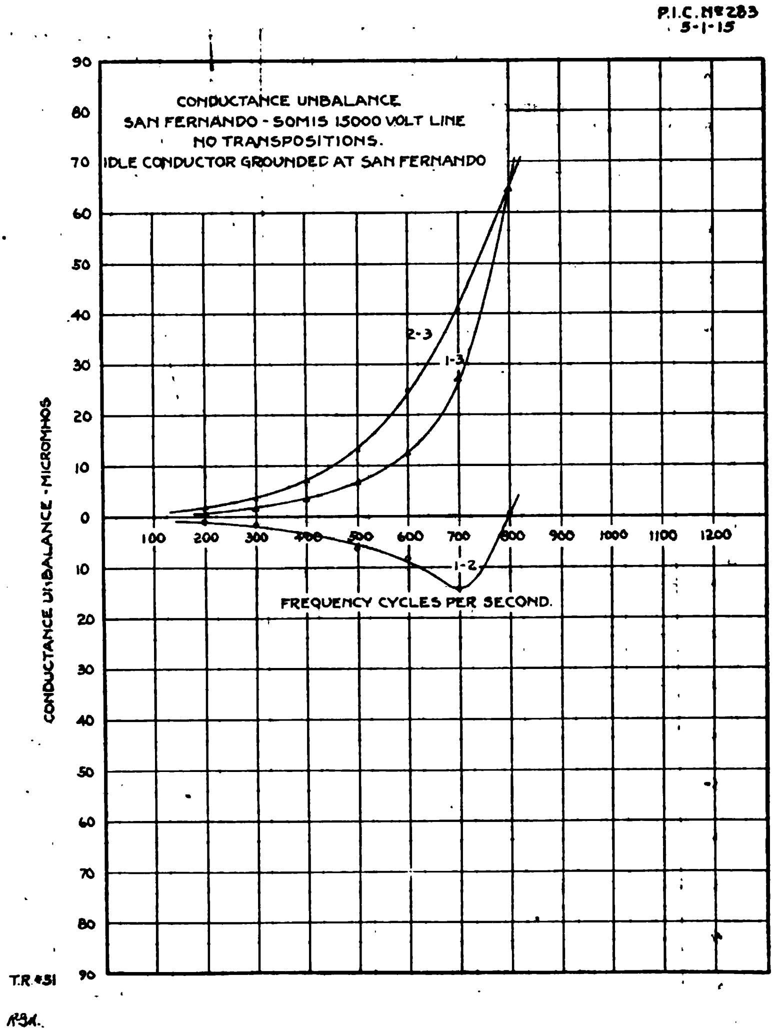
T.R. Nº 31
89e.

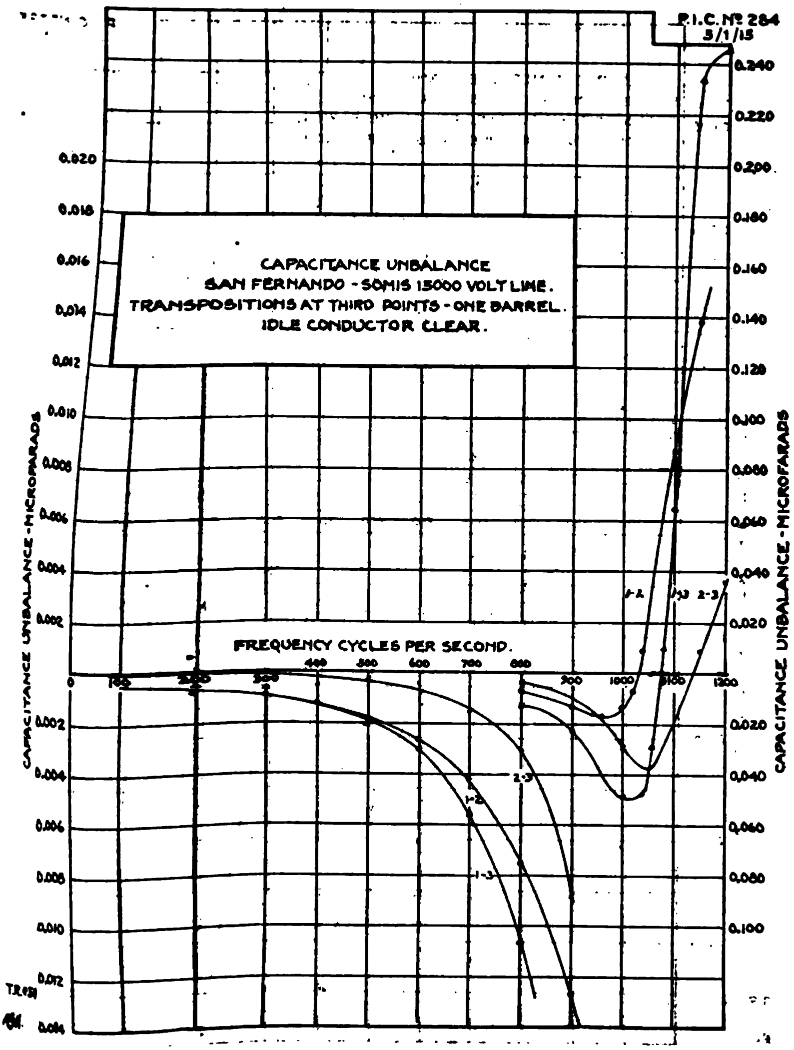

P.I.C. No 280
5/1/15
CAPACITANCE UNBALANCE
SAN FERNANDO - SOMIS 15000 VOLT LINE
NO TRANSPOSITIONS
IDLE CONDUCTOR CLEAR
CAPACITANCE UNBALANCE - MICROFARADS
FREQUENCY CYCLES PER SECOND
1-3
2-3
1-2
2-3
0.050
0.040
0.030
0.020
0.010
0.010
0.020
0.030
0.200
0.180
0.160
0.140
0.120
0.100
0.080
0.060
0.040
0.020
0.020
0.040
0.060
0.080
0.100
0.120
0.140
0 100 200 300 400 500 600 700 800 900 1000 1100 1200
T.R #51

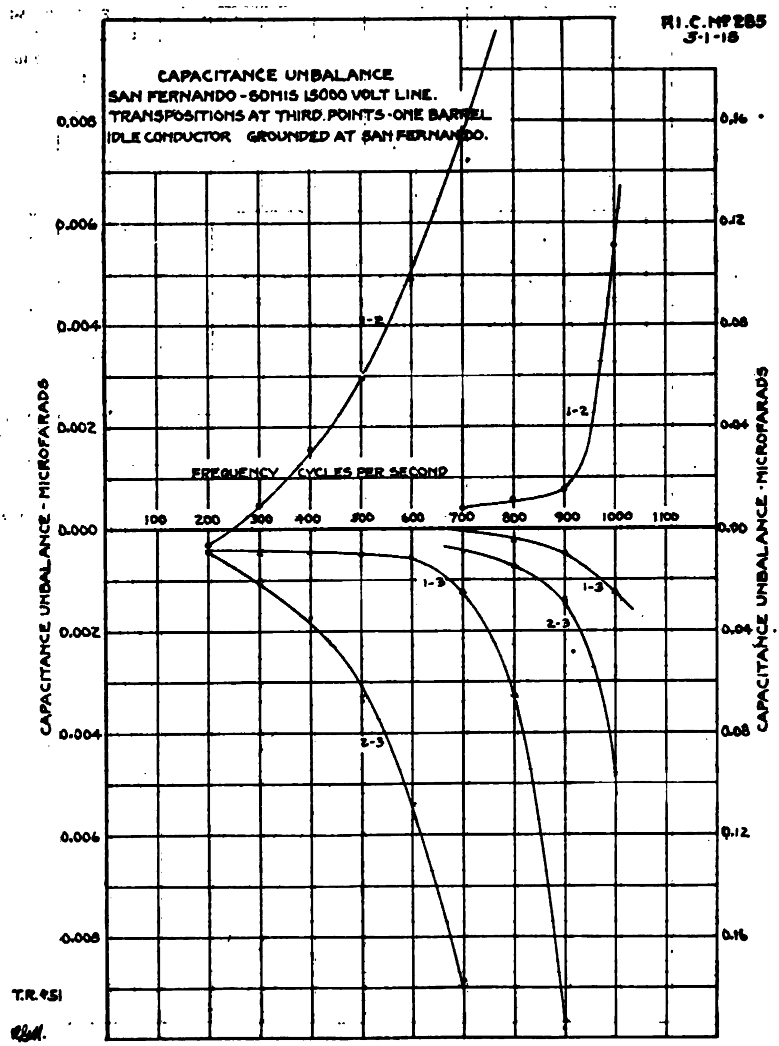

P.I.C. № 281
5/1/15
CAPACITANCE UNBALANCE
SAN FERNANDO - SOMIS 15000 VOLT LINE
NO TRANSPOSITIONS
IDLE CONDUCTOR GROUNDED AT SAN FERNANDO
CAPACITANCE UNBALANCE - MICROFARADS
0.030
0.020
0.010
0.0
0.010
0.020
0.030
100
200
300
400
500
600
700
800
900
1000
1100
1200
FREQUENCY CYCLES PER SECOND.
2-3
1-3
1-2
T.R.#51


P.I C. N9282
5/1/15
CONDUCTANCE UNBALANCE
SAN FERNANDO - SOMIS 15000 VOLT LINE.
NO TRANSPOSITIONS.
IDLE CONDUCTOR CLEAR.
FREQUENCY CYCLES PER SECOND
CONDUCTANCE UNBALANCE - MICROMHOS
CONDUCTANCE UNBALANCE - MICROMHOS
1-3
2-3
1-2
1-3
1-2
2-3
2-3
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800

TR #51

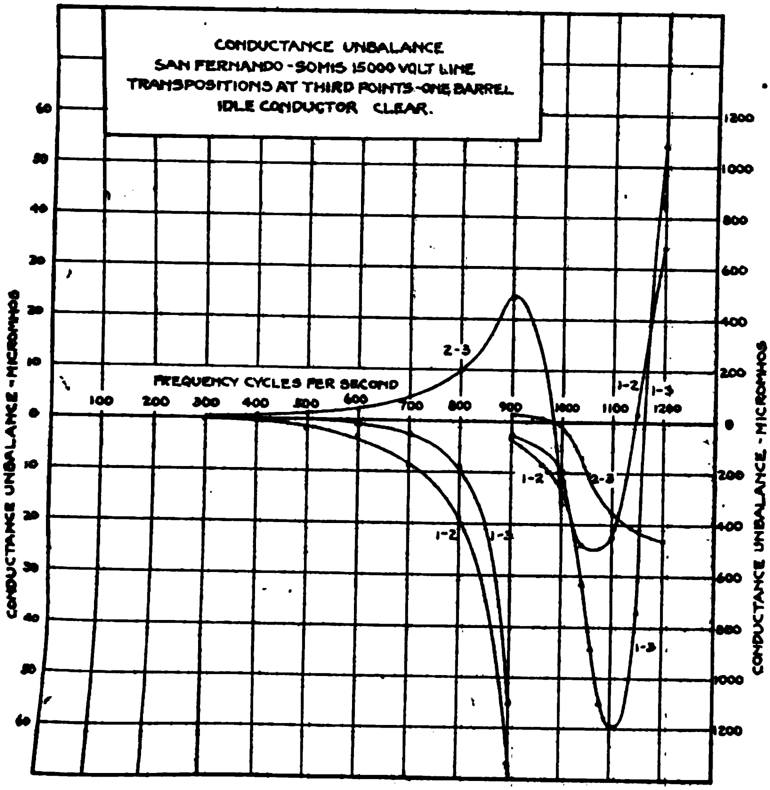

P.I.C. Nº 283
5-1-15
CONDUCTANCE UNBALANCE
SAN FERNANDO - SOMIS 15000 VOLT LINE
NO TRANSPOSITIONS.
IDLE CONDUCTOR GROUNDED AT SAN FERNANDO
2-3
1-3
1-2
CONDUCTANCE UNBALANCE - MICROMHOS
FREQUENCY CYCLES PER SECOND.
90
80
70
60
50
40
30
20
10
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
T.R. #31


P.I.C. Nº 284
3/1/15
CAPACITANCE UNBALANCE
SAN FERNANDO - SOMIS 15000 VOLT LINE.
TRANSPOSITIONS AT THIRD POINTS - ONE BARREL.
IDLE CONDUCTOR CLEAR.
FREQUENCY CYCLES PER SECOND.
CAPACITANCE UNBALANCE - MICROFARADS
CAPACITANCE UNBALANCE - MICROFARADS
1-2
1-3
2-3
T.R.#51

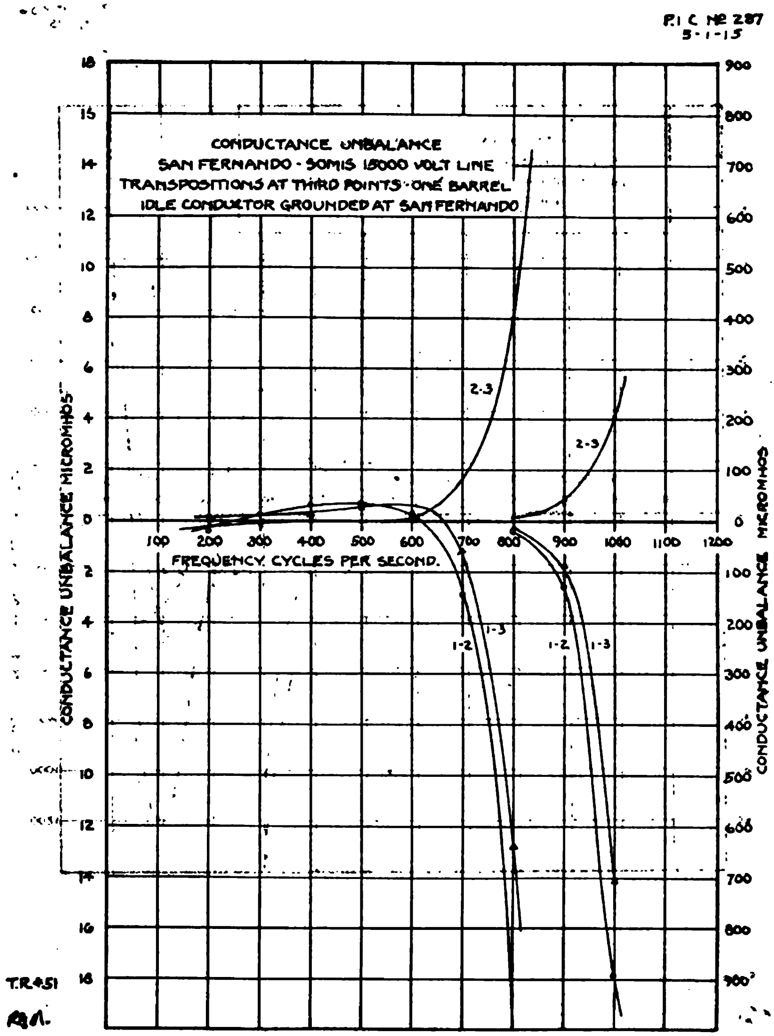


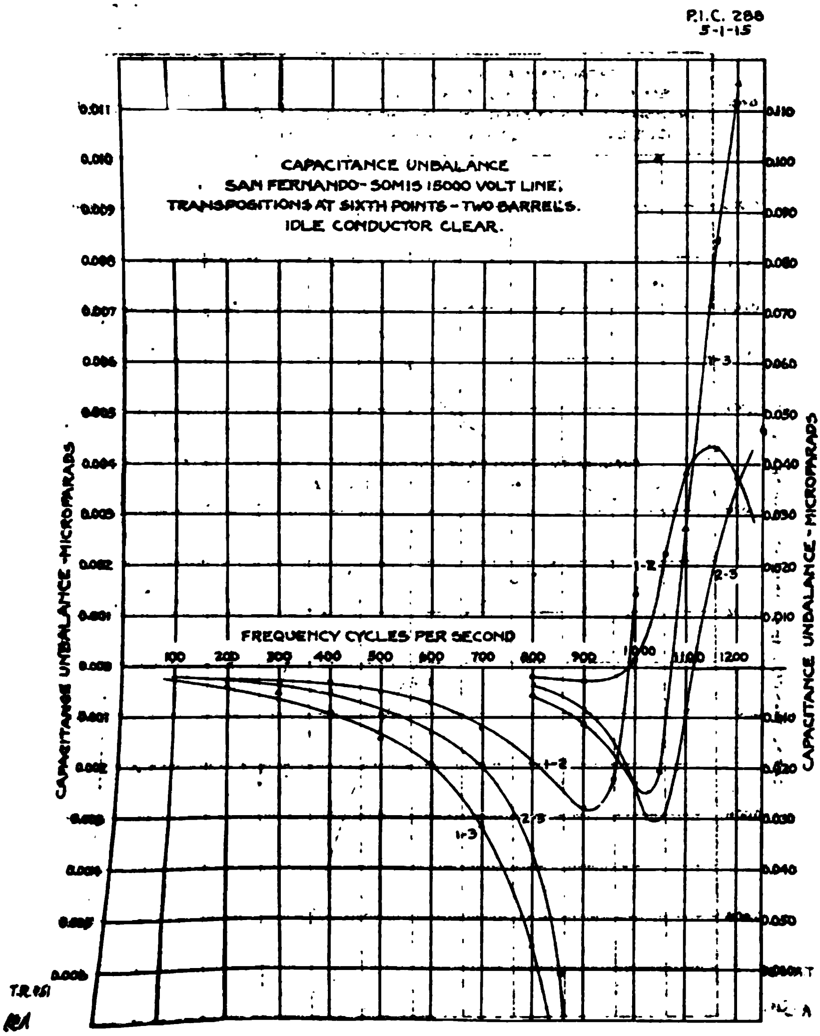
R.I.C. Nº 285
3-1-18
CAPACITANCE UNBALANCE
SAN FERNANDO-SOMIS 15000 VOLT LINE.
TRANSPOSITIONS AT THIRD POINTS-ONE BARREL
IDLE CONDUCTOR GROUNDED AT SAN FERNANDO.
FREQUENCY CYCLES PER SECOND
100
200
300
400
500
600
700
800
900
1000
1100
1-2
1-3
2-3
0.008
0.006
0.004
0.002
0.000
0.16
0.12
0.08
0.04
0.00
CAPACITANCE UNBALANCE - MICROFARADS
T.R. 451

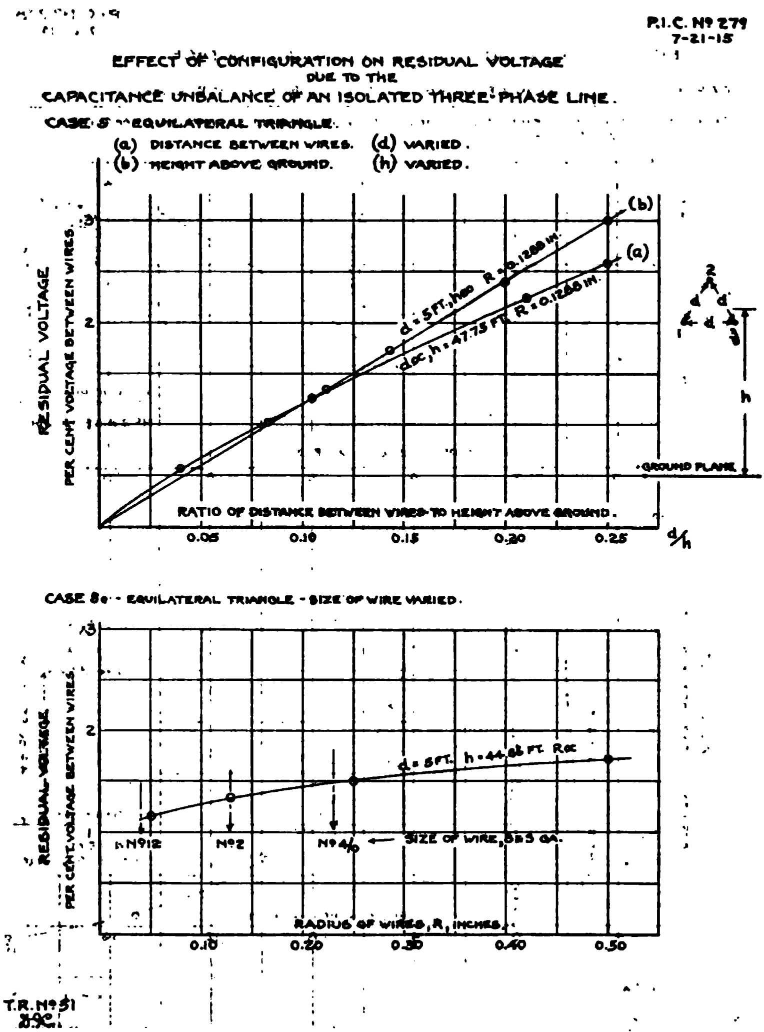
P.I.C. No 286
5/1/15

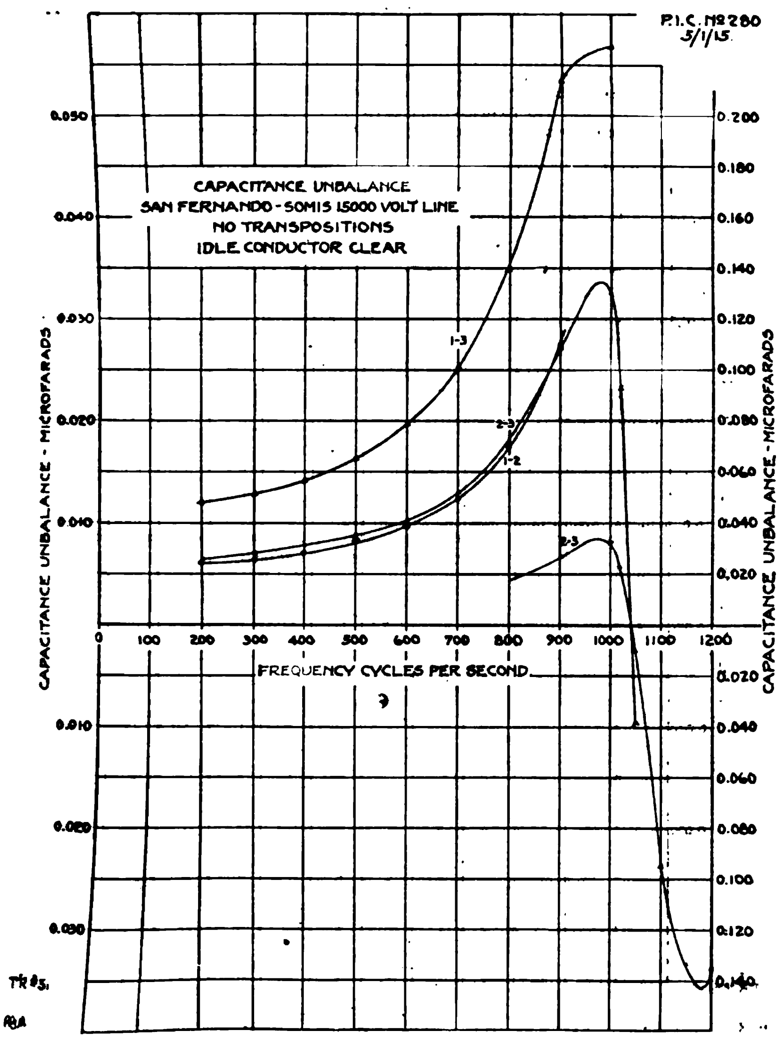


T.R. # 51

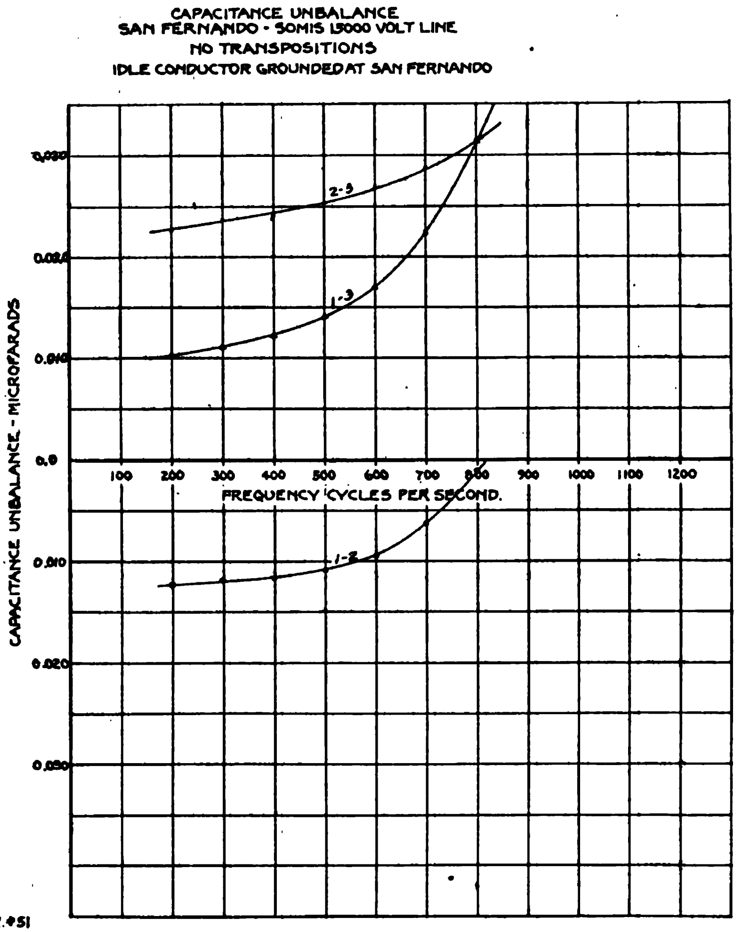

P.I.C. Nº 287
5-1-15
CONDUCTANCE UNBALANCE
SAN FERNANDO - SOMIS 15000 VOLT LINE
TRANSPOSITIONS AT THIRD POINTS - ONE BARREL
IDLE CONDUCTOR GROUNDED AT SAN FERNANDO
FREQUENCY CYCLES PER SECOND.
CONDUCTANCE UNBALANCE MICROMHOS
CONDUCTANCE UNBALANCE MICROMHOS
2-3
2-3
1-2
1-3
1-2
1-3
T.R.451


P.I.C. 288
5-1-15
CAPACITANCE UNBALANCE
SAN FERNANDO - SOMIS 15000 VOLT LINE.
TRANSPOSITIONS AT SIXTH POINTS - TWO BARRELS.
IDLE CONDUCTOR CLEAR.
FREQUENCY CYCLES PER SECOND
CAPACITANCE UNBALANCE - MICROFARADS
100
200
300
400
500
600
700
800
900
1000
1100
1200
1-2
1-3
2-3
T.R. 81

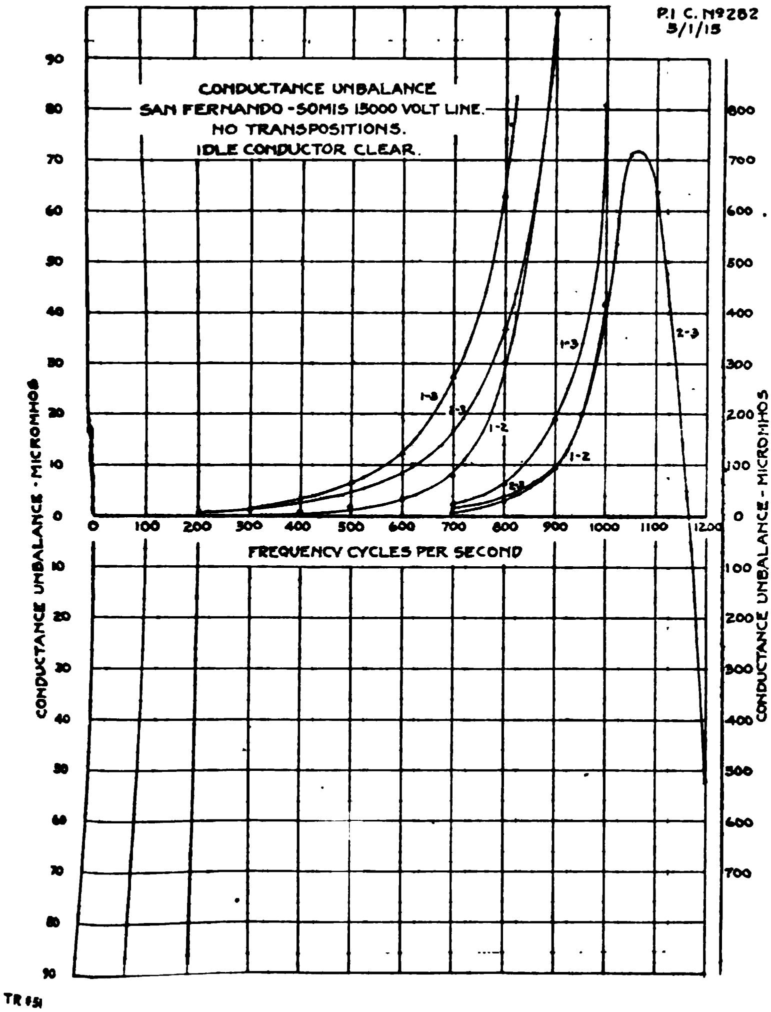

P.I.C. No 279
7-21-15
EFFECT OF CONFIGURATION ON RESIDUAL VOLTAGE
DUE TO THE
CAPACITANCE UNBALANCE OF AN ISOLATED THREE-PHASE LINE.
CASE 5 – EQUILATERAL TRIANGLE.
(a) DISTANCE BETWEEN WIRES. (d) VARIED.
(b) HEIGHT ABOVE GROUND. (h) VARIED.
RESIDUAL VOLTAGE
PER CENT VOLTAGE BETWEEN WIRES.
(b)
(a)
d = 5 FT, h
R = 0.1280 IN.
d
h = 47.75 FT.
R = 0.1280 IN.
GROUND PLANE
RATIO OF DISTANCE BETWEEN WIRES TO HEIGHT ABOVE GROUND.
0.05
0.10
0.15
0.20
0.25
d/h
CASE 8 – EQUILATERAL TRIANGLE – SIZE OF WIRE VARIED.
RESIDUAL VOLTAGE
PER CENT VOLTAGE BETWEEN WIRES.
d = 5 FT. h = 44.65 FT. R
No 12
No 2
No 4/0
SIZE OF WIRE, B&S GA.
RADIUS OF WIRES, R, INCHES.
0.10
0.20
0.30
0.40
0.50
T.R. No 51

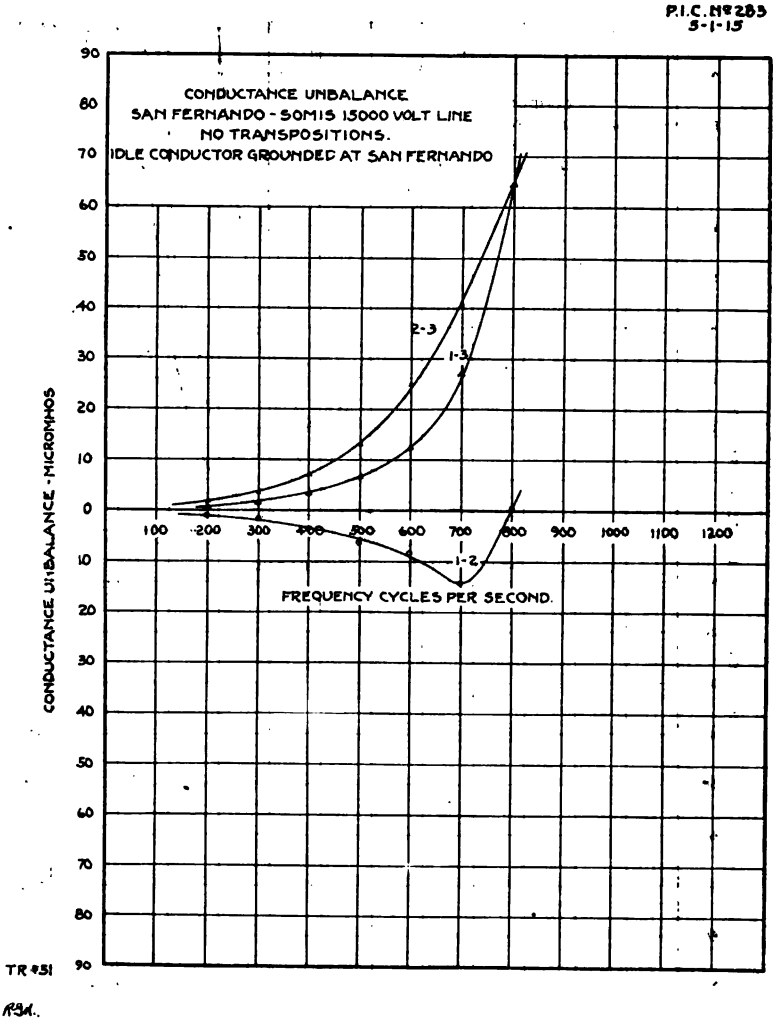

P.I.C. No 280
5/1/15
CAPACITANCE UNBALANCE
SAN FERNANDO - SOMIS 15000 VOLT LINE
NO TRANSPOSITIONS
IDLE CONDUCTOR CLEAR
1-3
2-3
1-2
2-3
FREQUENCY CYCLES PER SECOND
CAPACITANCE UNBALANCE - MICROFARADS
CAPACITANCE UNBALANCE - MICROFARADS
0.050
0.040
0.030
0.020
0.010
0.010
0.020
0.030
0.200
0.180
0.160
0.140
0.120
0.100
0.080
0.060
0.040
0.020
0.020
0.040
0.060
0.080
0.100
0.120
0.140
0 100 200 300 400 500 600 700 800 900 1000 1100 1200

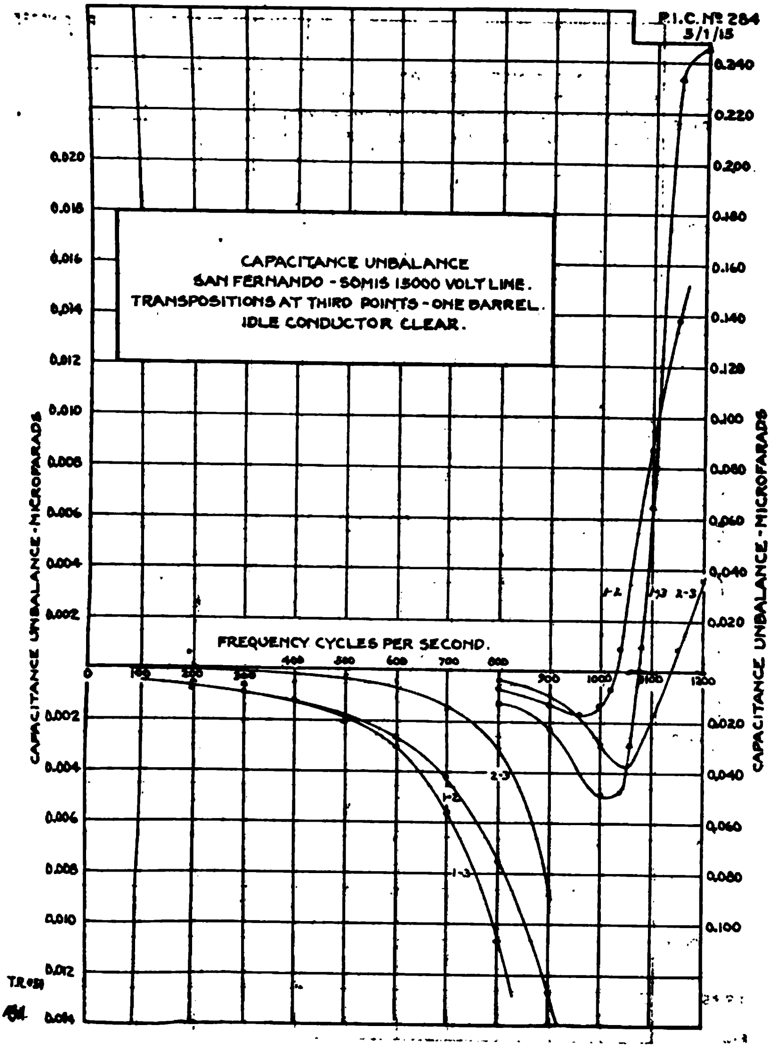
P.I.C. Fig. 281
5/1/15

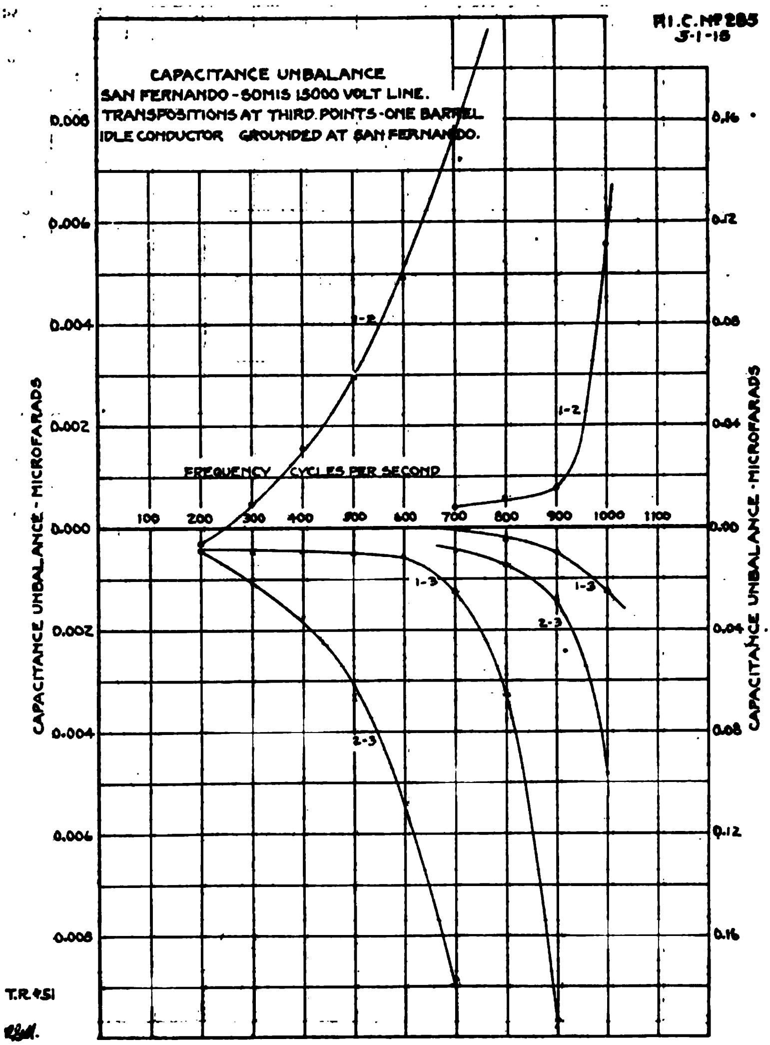


T.R. #51


P.I C. Nº282
3/1/15
CONDUCTANCE UNBALANCE
SAN FERNANDO - SOMIS 15000 VOLT LINE.
NO TRANSPOSITIONS.
IDLE CONDUCTOR CLEAR.
FREQUENCY CYCLES PER SECOND
CONDUCTANCE UNBALANCE - MICROMHOS
CONDUCTANCE UNBALANCE - MICROMHOS
1-3
2-3
1-2
1-3
1-2
2-3
2-3
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
TR 51

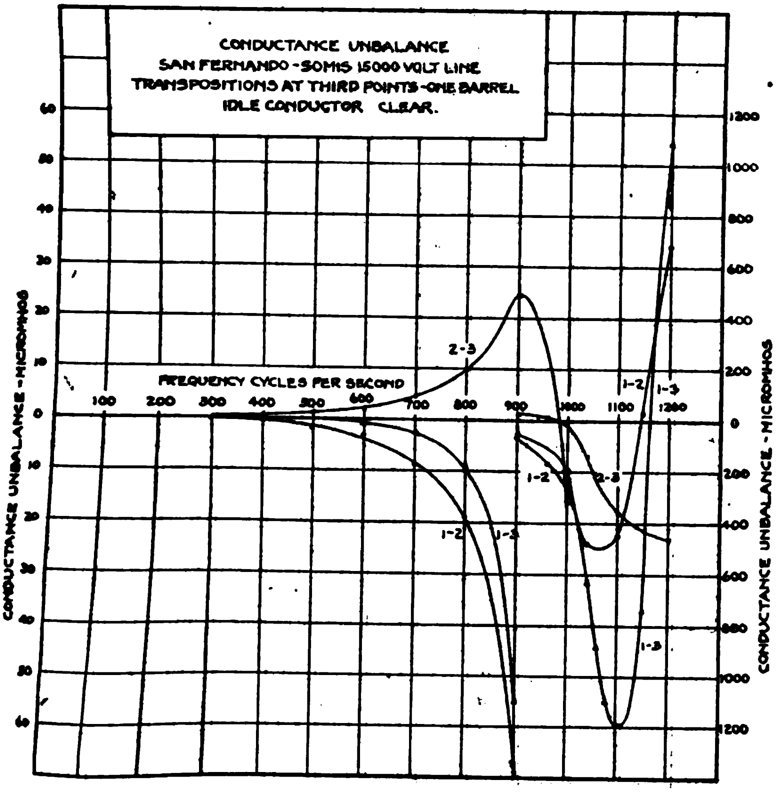

P.I.C. Nº 283
5-1-15
CONDUCTANCE UNBALANCE
SAN FERNANDO - SOMIS 15000 VOLT LINE
NO TRANSPOSITIONS.
IDLE CONDUCTOR GROUNDED AT SAN FERNANDO
CONDUCTANCE UNBALANCE - MICROMHOS
FREQUENCY CYCLES PER SECOND.
2-3
1-3
1-2
90
80
70
60
50
40
30
20
10
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
TR 451


P.I.C. No 284
5/1/15
CAPACITANCE UNBALANCE
SAN FERNANDO - SOMIS 15000 VOLT LINE.
TRANSPOSITIONS AT THIRD POINTS - ONE BARREL.
IDLE CONDUCTOR CLEAR.
FREQUENCY CYCLES PER SECOND.
CAPACITANCE UNBALANCE - MICROFARADS
CAPACITANCE UNBALANCE - MICROFARADS
1-2
1-3
2-3
T.R.#51



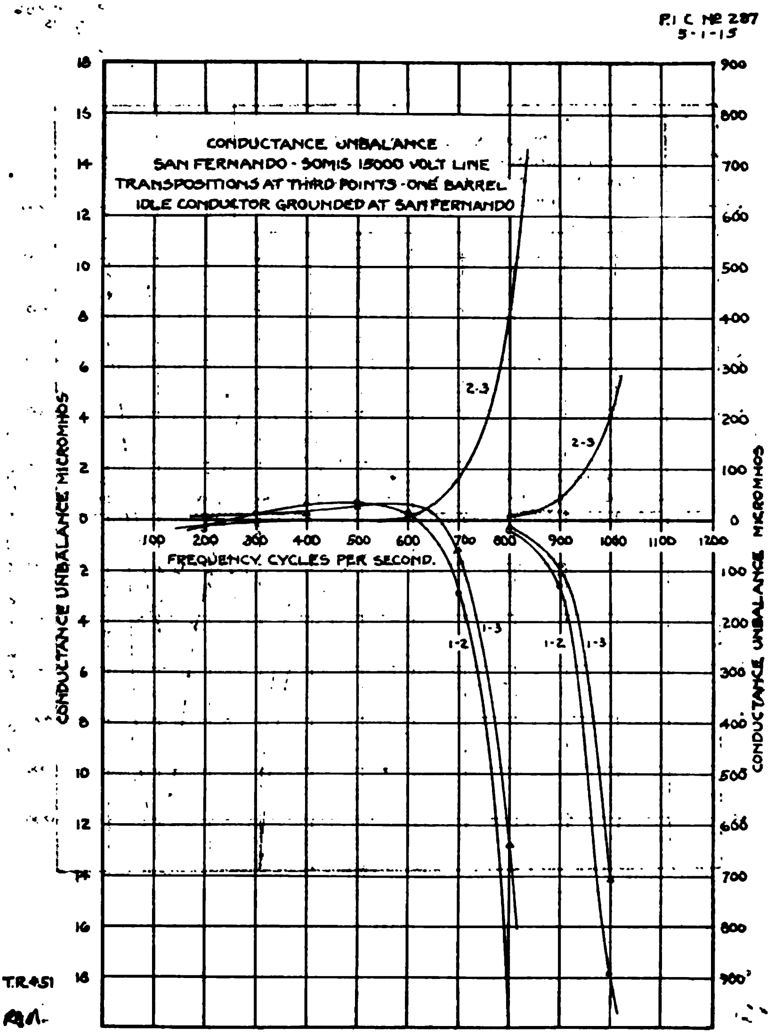
R.I.C. Nº 285
5-1-15
CAPACITANCE UNBALANCE
SAN FERNANDO - SOMIS 15000 VOLT LINE.
TRANSPOSITIONS AT THIRD POINTS - ONE BARREL
IDLE CONDUCTOR GROUNDED AT SAN FERNANDO.
FREQUENCY CYCLES PER SECOND
100 200 300 400 500 600 700 800 900 1000 1100
CAPACITANCE UNBALANCE - MICROFARADS
0.008
0.006
0.004
0.002
0.000
0.16
0.12
0.08
0.04
0.00
1-2
1-3
2-3
T.R.451

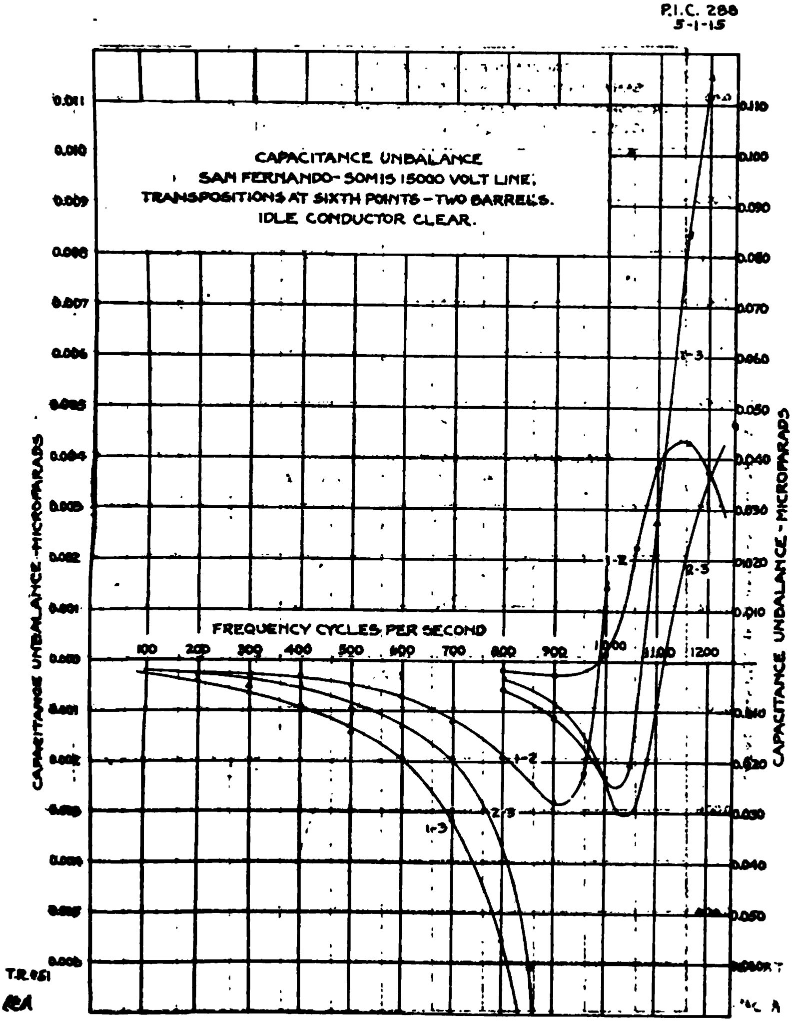
F.I.C. No 286
5/1/15

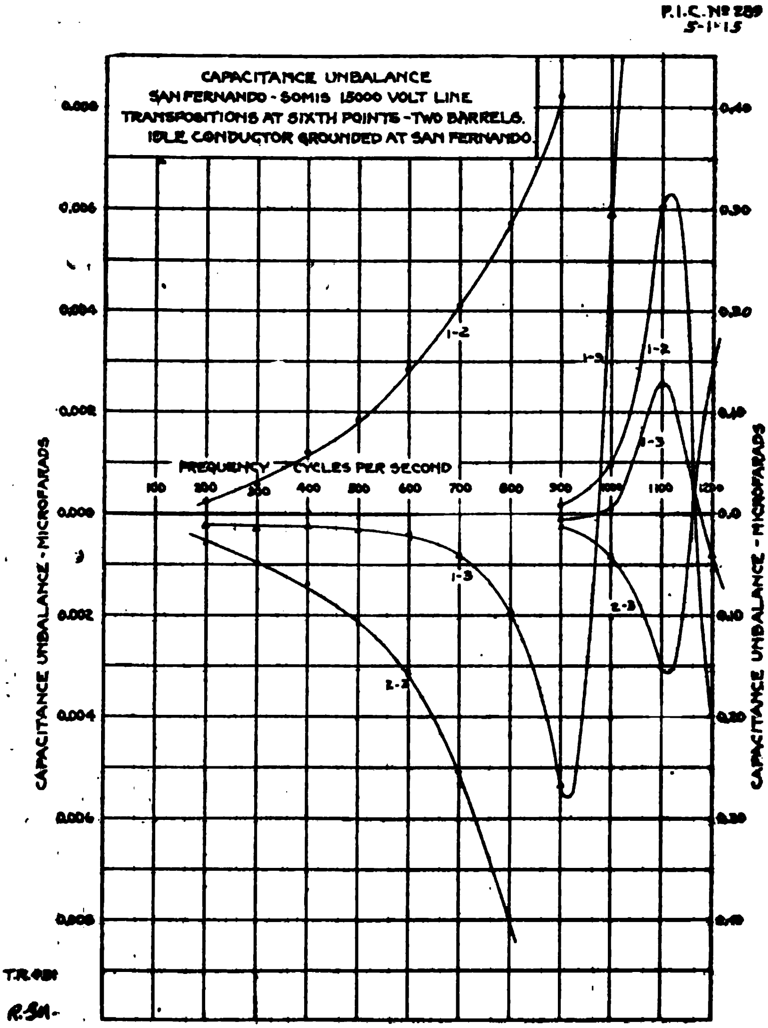


T.R.51

451.

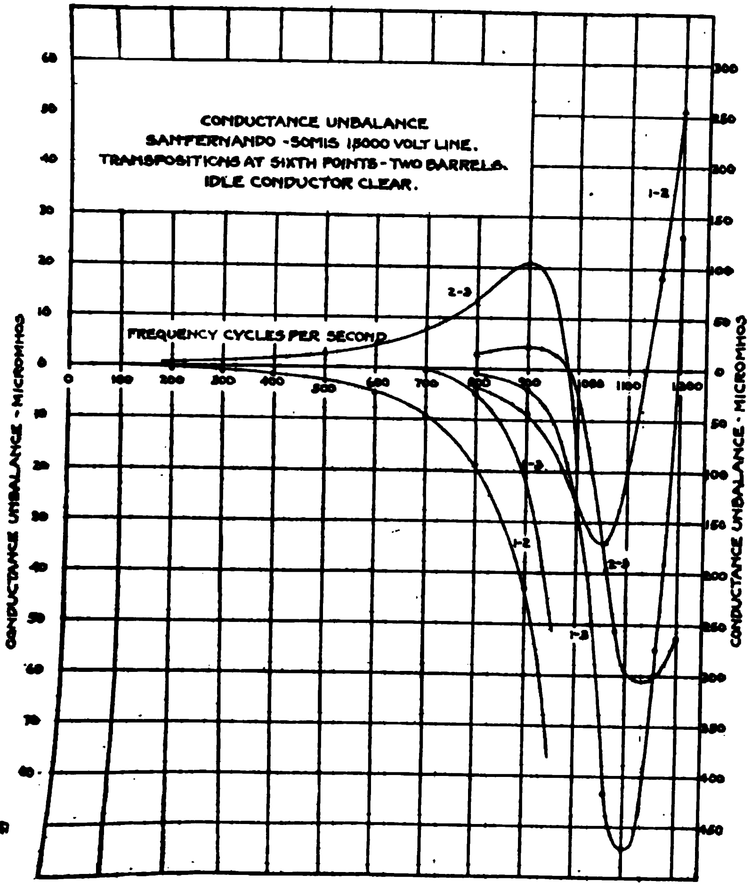

P.I.C. № 287
5-1-15
CONDUCTANCE UNBALANCE
SAN FERNANDO - SOMIS 15000 VOLT LINE
TRANSPOSITIONS AT THIRD POINTS - ONE BARREL
IDLE CONDUCTOR GROUNDED AT SAN FERNANDO
FREQUENCY CYCLES PER SECOND.
CONDUCTANCE UNBALANCE MICROMHOS
CONDUCTANCE UNBALANCE MICROMHOS
2-3
1-2
1-3
T.R.451


P.I.C. 288
5-1-15
CAPACITANCE UNBALANCE
SAN FERNANDO-SOMIS 15000 VOLT LINE.
TRANSPOSITIONS AT SIXTH POINTS - TWO BARRELS.
IDLE CONDUCTOR CLEAR.
FREQUENCY CYCLES PER SECOND
100 200 300 400 500 600 700 800 900 1000 1100 1200
CAPACITANCE UNBALANCE - MICROFARADS
CAPACITANCE UNBALANCE - MICROFARADS
1-2
1-3
2-3
T.R.#51

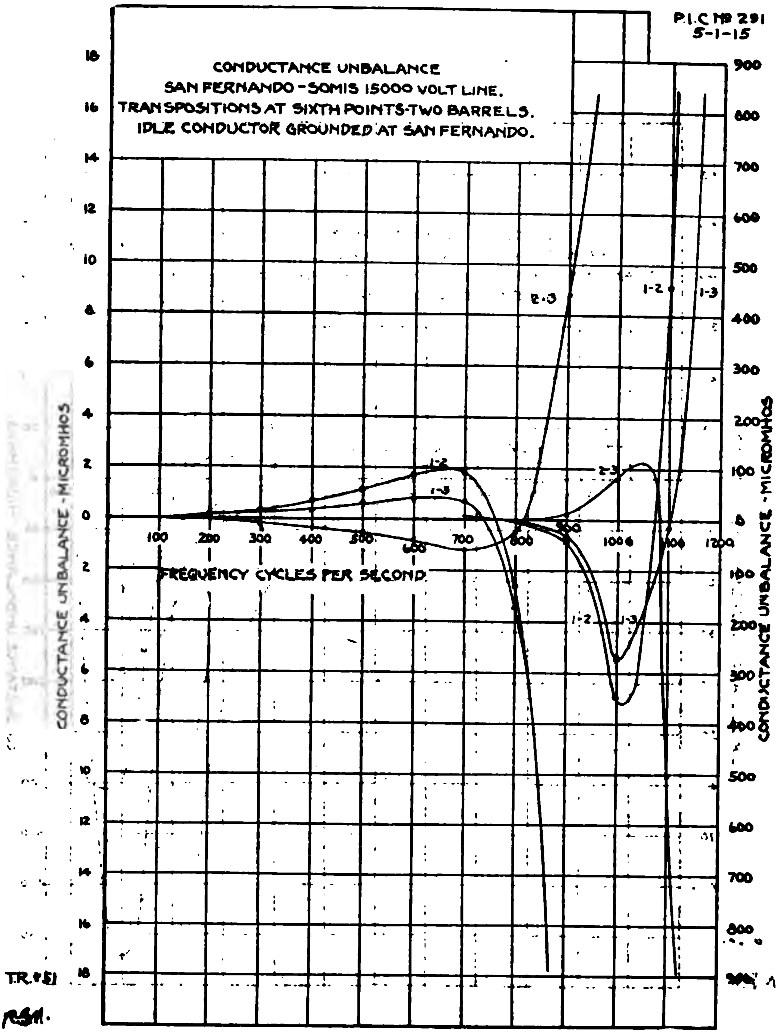


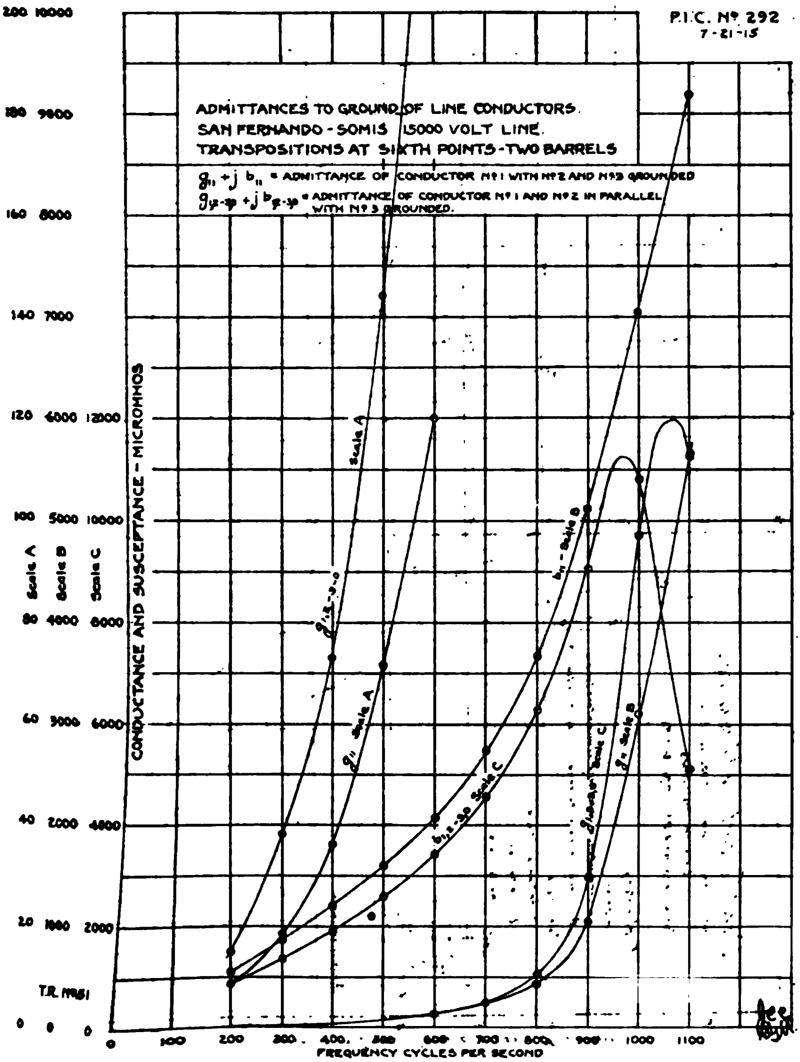
P.I.C. № 289
5-1-15
CAPACITANCE UNBALANCE
SAN FERNANDO - SOMIS 15000 VOLT LINE
TRANSPOSITIONS AT SIXTH POINTS - TWO BARRELS.
IDLE CONDUCTOR GROUNDED AT SAN FERNANDO
FREQUENCY - CYCLES PER SECOND
100
200
300
400
500
600
700
800
900
1000
1100
1200
0.008
0.006
0.004
0.002
0.000
0.002
0.004
0.006
0.008
0.40
0.30
0.20
0.10
0.0
0.10
0.20
0.30
0.40
1-2
1-3
2-3
CAPACITANCE UNBALANCE - MICROFARADS
T.R. 481
R.511

P.I.C. No 290
5-1-15

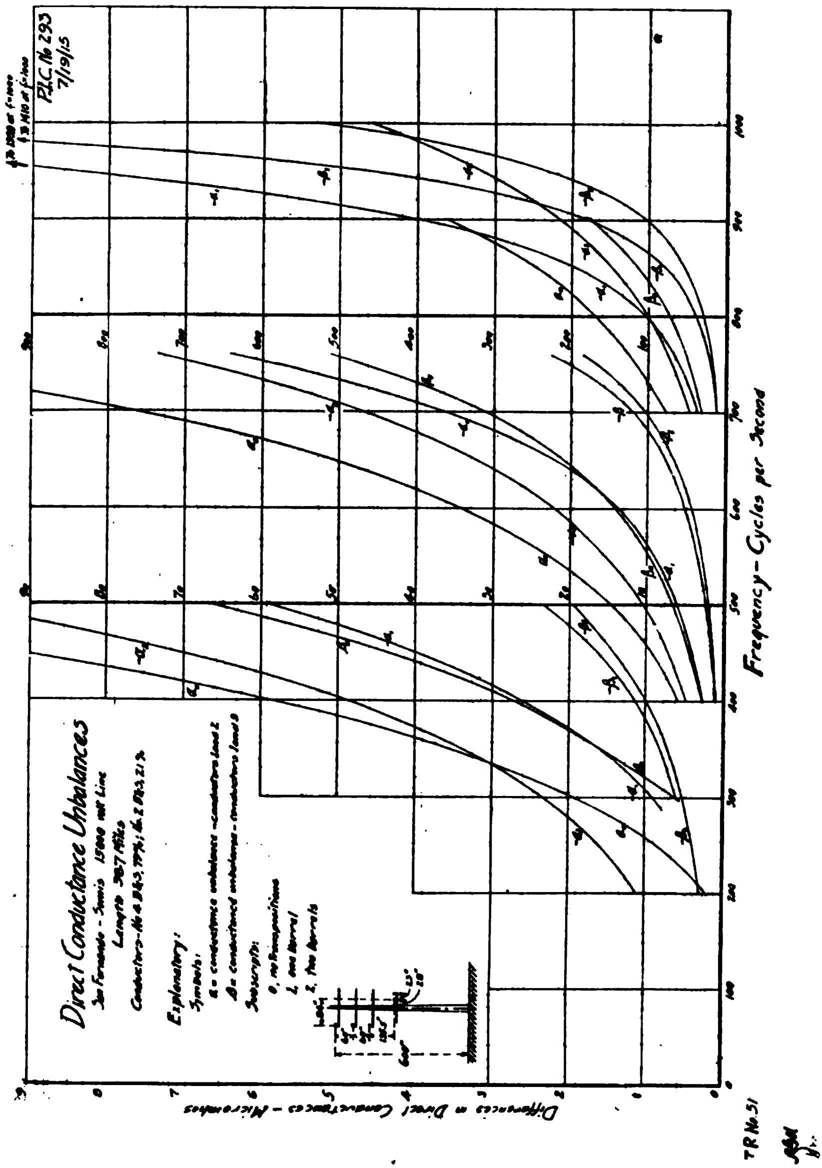


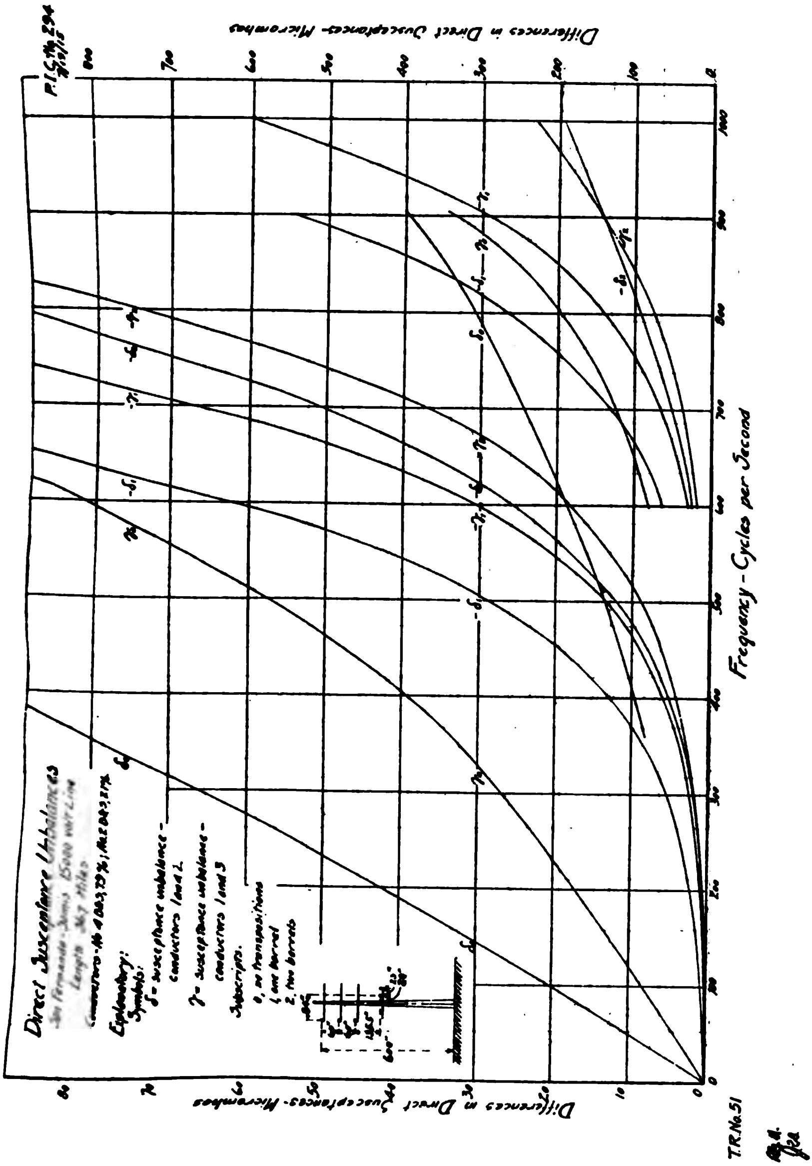
T.R. 51
A.H.

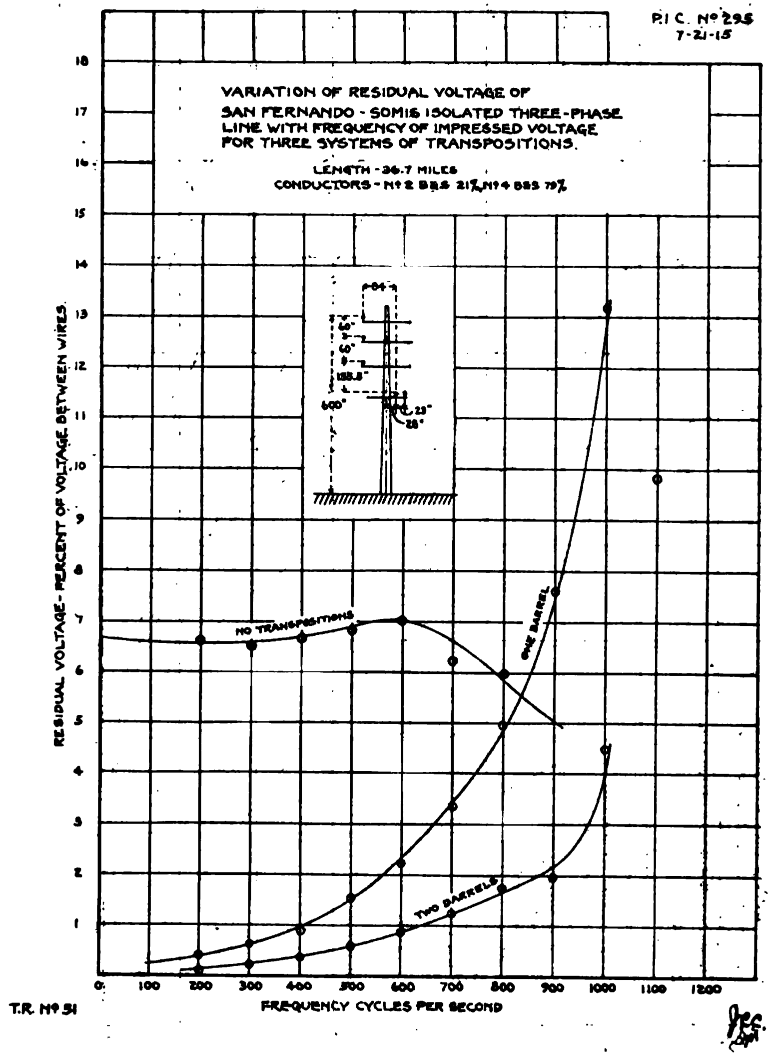

P.I.C № 291
5-1-15
CONDUCTANCE UNBALANCE
SAN FERNANDO - SOMIS 15000 VOLT LINE.
TRANSPOSITIONS AT SIXTH POINTS-TWO BARRELS.
IDLE CONDUCTOR GROUNDED AT SAN FERNANDO.
FREQUENCY CYCLES PER SECOND
CONDUCTANCE UNBALANCE - MICROMHOS
CONDUCTANCE UNBALANCE - MICROMHOS
1-2
1-3
2-3
100
200
300
400
500
600
700
800
900
1000
1100
1200
T.R. #51

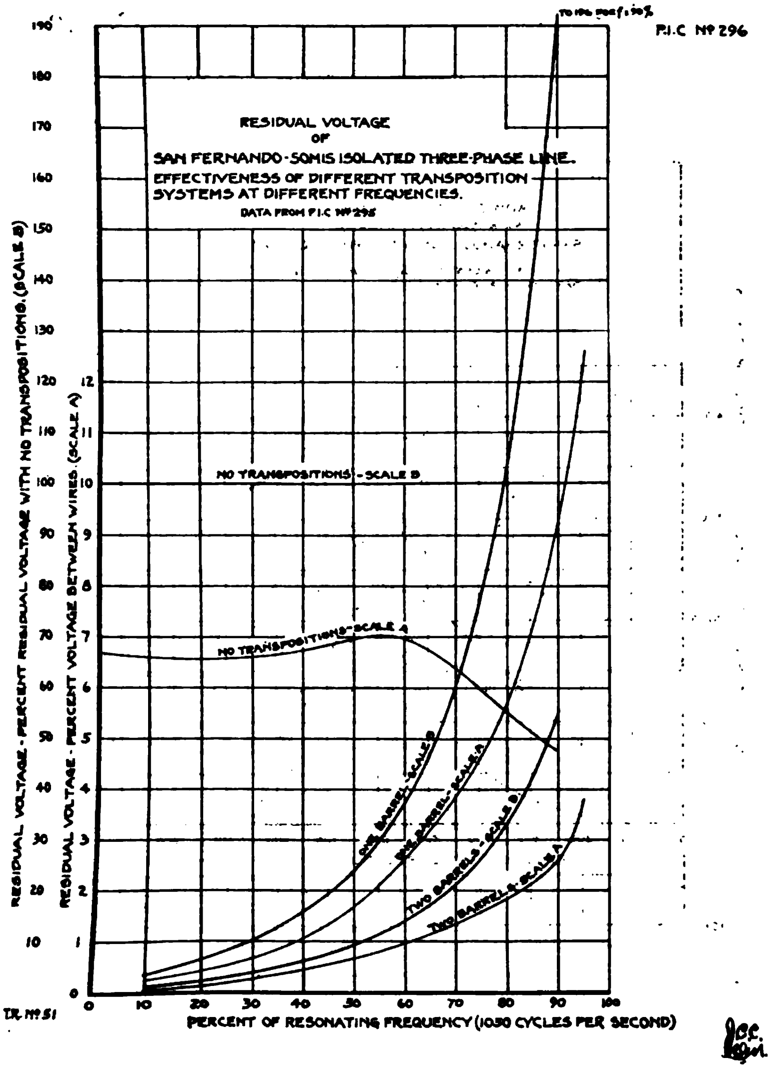


P.I.C. Nº 292
7-21-15
ADMITTANCES TO GROUND OF LINE CONDUCTORS.
SAN FERNANDO - SOMIS 15000 VOLT LINE.
TRANSPOSITIONS AT SIXTH POINTS - TWO BARRELS
$g_{11} + j\, b_{11}$ = ADMITTANCE OF CONDUCTOR Nº 1 WITH Nº 2 AND Nº 3 GROUNDED
$g_{12-30} + j\, b_{12-30}$ = ADMITTANCE OF CONDUCTOR Nº 1 AND Nº 2 IN PARALLEL WITH Nº 3 GROUNDED.
CONDUCTANCE AND SUSCEPTANCE - MICROMHOS
Scale A
Scale B
Scale C
FREQUENCY CYCLES PER SECOND
T.R. Nº 51


Direct Conductance Unbalances
Explanatory:
Symbols:
Subscripts:
0, no transpositions
Frequency - Cycles per Second
Differences in Direct Conductances - Micromhos
P.L.C. No 293
7/9/15
T R No. 51

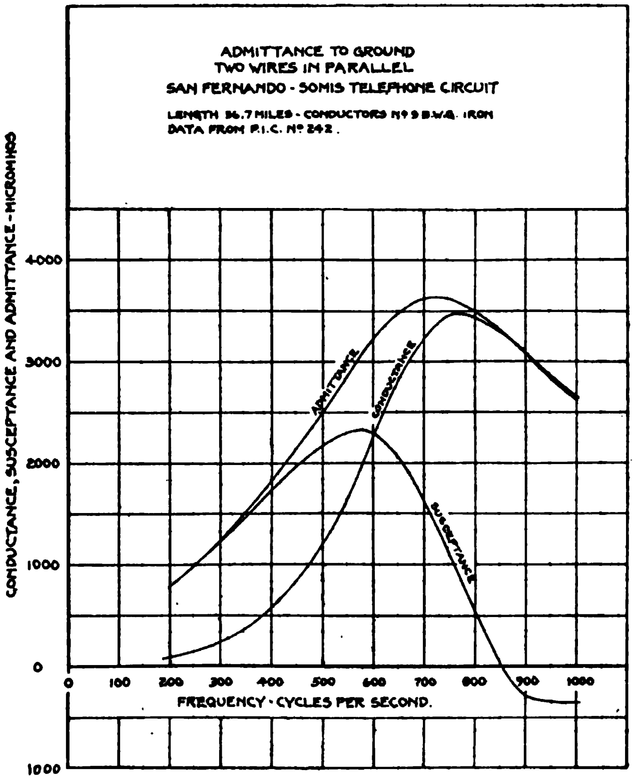

Differences in Direct Susceptances - Micromhos
Frequency - Cycles per Second
Direct Susceptance Unbalances
Explanatory Symbols:
Subscripts:
T.R. No. 51


P.I.C. Nº 295
7-21-15
VARIATION OF RESIDUAL VOLTAGE OF
SAN FERNANDO - SOMIS ISOLATED THREE-PHASE
LINE WITH FREQUENCY OF IMPRESSED VOLTAGE
FOR THREE SYSTEMS OF TRANSPOSITIONS.
LENGTH - 36.7 MILES
CONDUCTORS - Nº 2 B&S 21%, Nº 4 B&S 79%
NO TRANSPOSITIONS
ONE BARREL
TWO BARRELS
RESIDUAL VOLTAGE - PERCENT OF VOLTAGE BETWEEN WIRES
FREQUENCY CYCLES PER SECOND
T.R. Nº 51



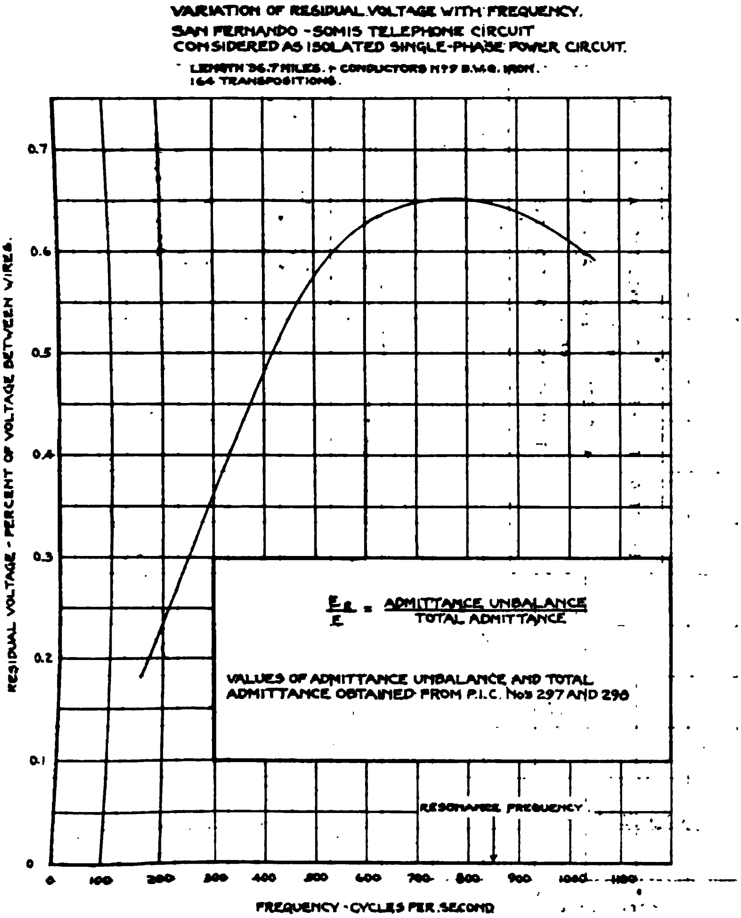
P.I.C Nº 296
RESIDUAL VOLTAGE
OF
SAN FERNANDO-SOMIS ISOLATED THREE-PHASE LINE.
EFFECTIVENESS OF DIFFERENT TRANSPOSITION SYSTEMS AT DIFFERENT FREQUENCIES.
DATA FROM P.I.C Nº 295
NO TRANSPOSITIONS - SCALE B
NO TRANSPOSITIONS - SCALE A
ONE BARREL - SCALE B
ONE BARREL - SCALE A
TWO BARRELS - SCALE B
TWO BARRELS - SCALE A
RESIDUAL VOLTAGE - PERCENT RESIDUAL VOLTAGE WITH NO TRANSPOSITIONS. (SCALE B)
RESIDUAL VOLTAGE - PERCENT VOLTAGE BETWEEN WIRES. (SCALE A)
PERCENT OF RESONATING FREQUENCY (1030 CYCLES PER SECOND)
0 10 20 30 40 50 60 70 80 90 100
TR. Nº 51

P.I.C. No 298
7-31-15

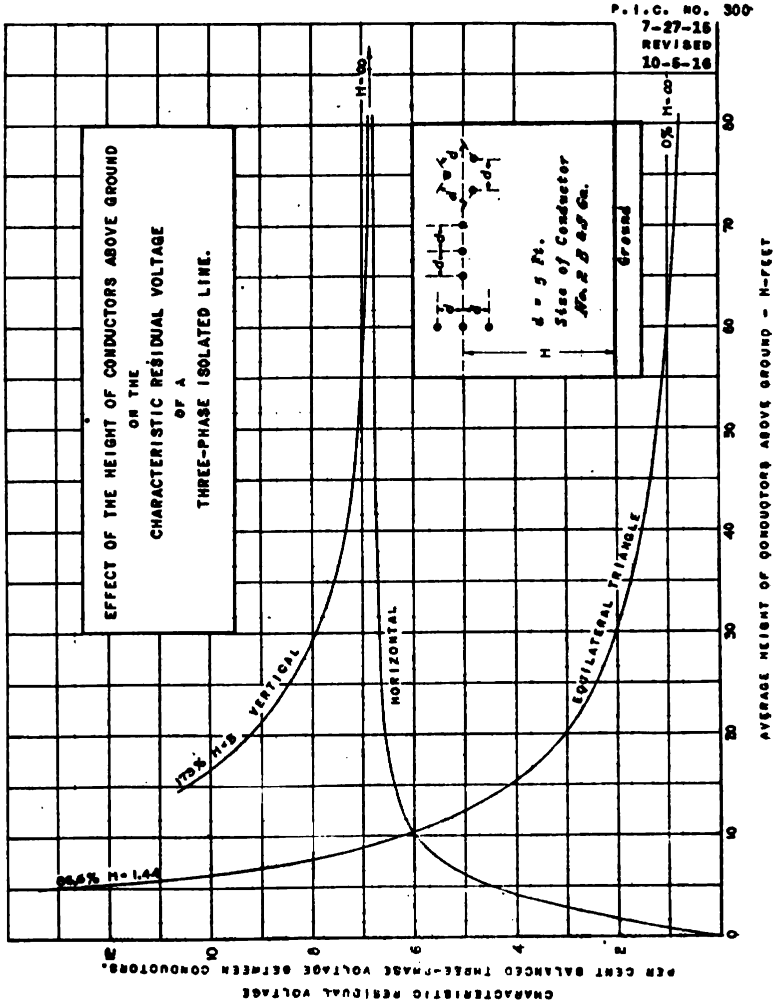


T.R. No 51

P.I.C. No 299
7-13-15



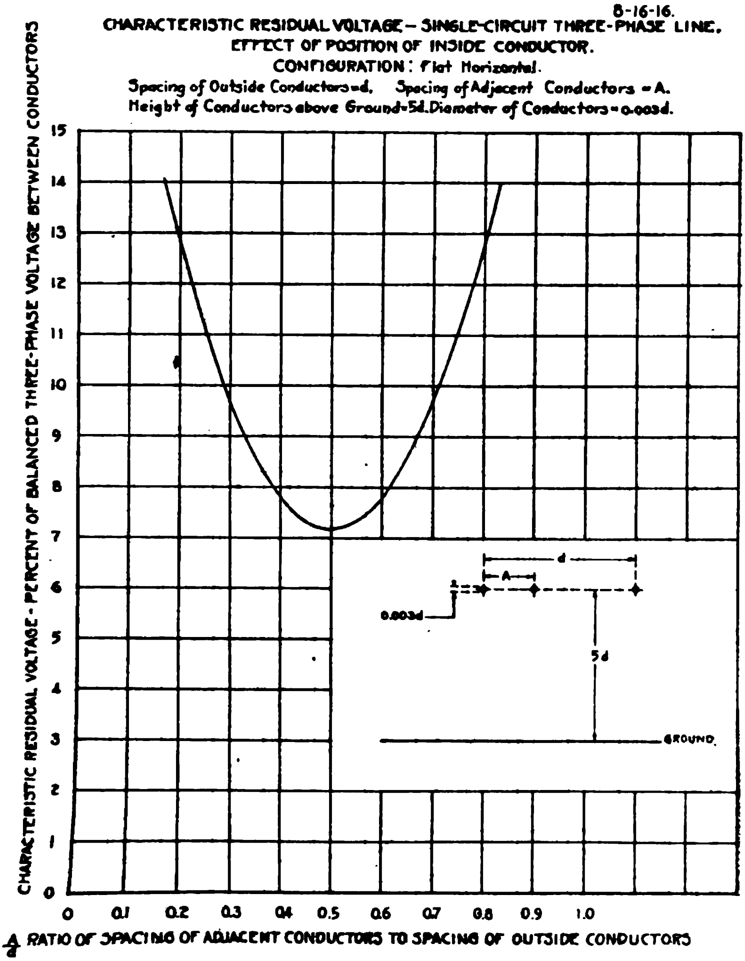
T.R. No 51


P.I.C. NO. 300
7-27-16
REVISED
10-6-16
EFFECT OF THE HEIGHT OF CONDUCTORS ABOVE GROUND
ON THE
CHARACTERISTIC RESIDUAL VOLTAGE
OF A
THREE-PHASE ISOLATED LINE.
d = 5 Ft.
Size of Conductor
No. 2 B & S Ga.
Ground
H
VERTICAL
HORIZONTAL
EQUILATERAL TRIANGLE
0%
H=∞
86.6% H=1.44
AVERAGE HEIGHT OF CONDUCTORS ABOVE GROUND - H-FEET
PER CENT BALANCED THREE-PHASE VOLTAGE BETWEEN CONDUCTORS.
CHARACTERISTIC RESIDUAL VOLTAGE

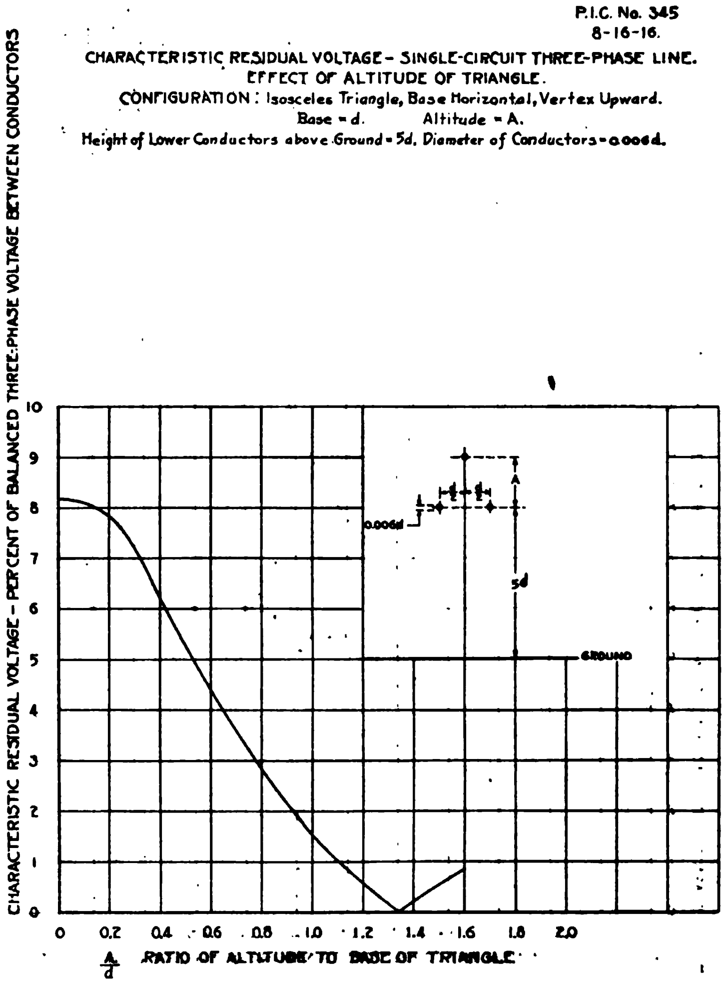
T.R. NO.51.

P.I.C. No. 344
8-16-16.

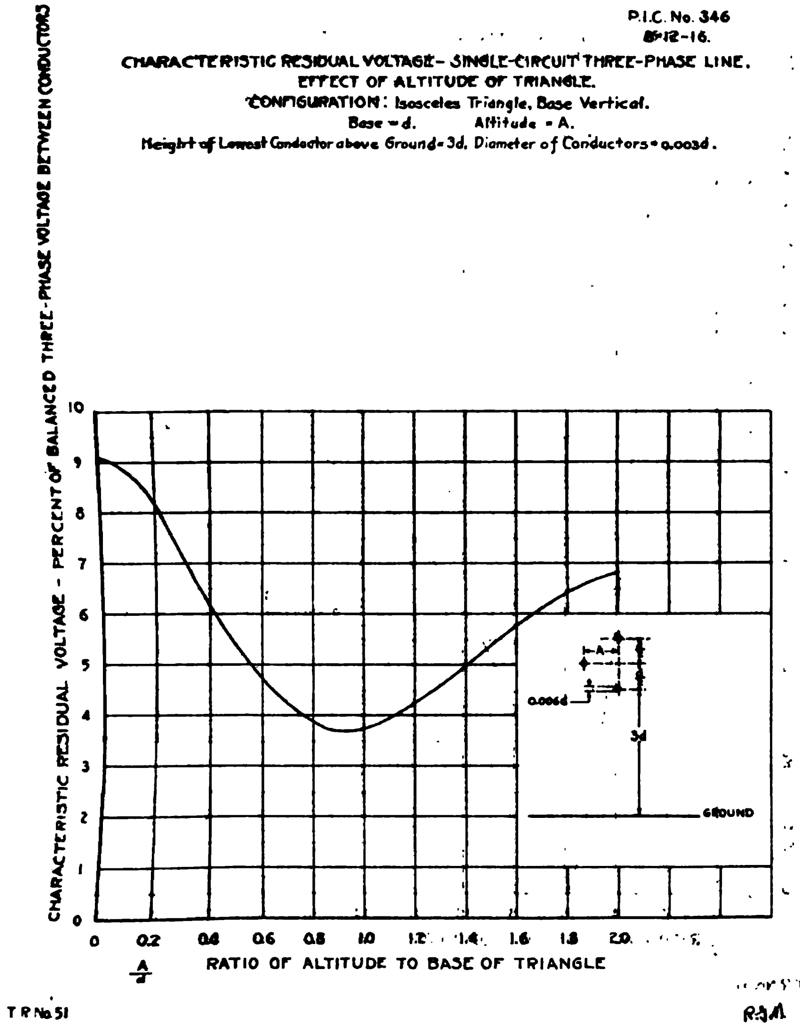
CHARACTERISTIC RESIDUAL VOLTAGE – SINGLE-CIRCUIT THREE-PHASE LINE.
EFFECT OF POSITION OF INSIDE CONDUCTOR.
CONFIGURATION: Flat Horizontal.
Spacing of Outside Conductors = d. Spacing of Adjacent Conductors = A.
Height of Conductors above Ground = 5d. Diameter of Conductors = 0.003d.



T.R. No. 51

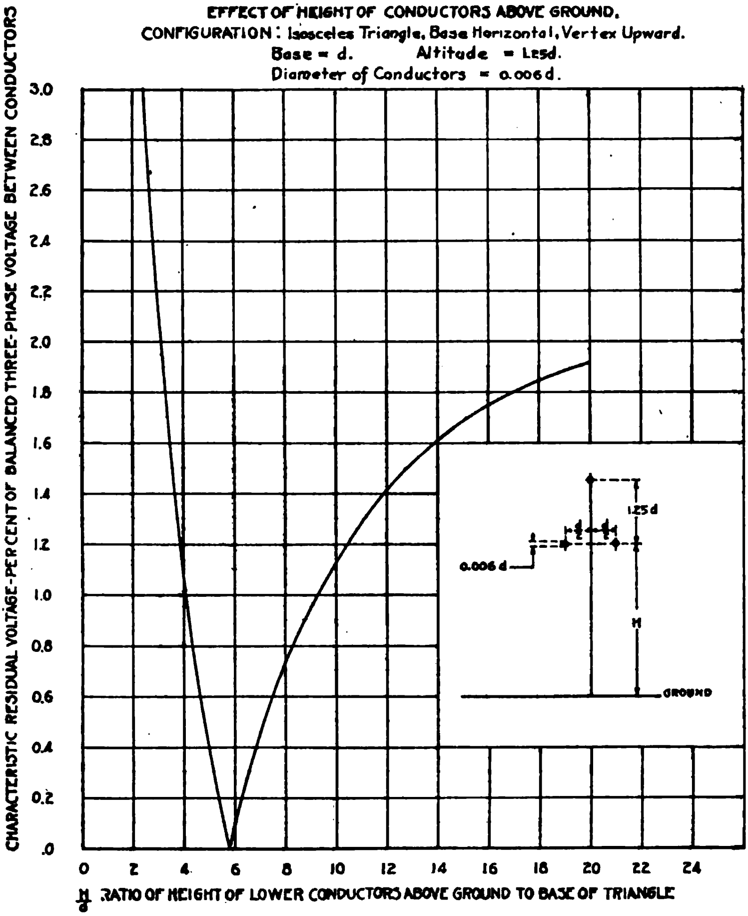

P.I.C. No. 345
8-16-16.
CHARACTERISTIC RESIDUAL VOLTAGE - SINGLE-CIRCUIT THREE-PHASE LINE.
EFFECT OF ALTITUDE OF TRIANGLE.
CONFIGURATION: Isosceles Triangle, Base Horizontal, Vertex Upward.
Base = d. Altitude = A.
Height of Lower Conductors above Ground = 5d. Diameter of Conductors = 0.006d.
CHARACTERISTIC RESIDUAL VOLTAGE - PERCENT OF BALANCED THREE-PHASE VOLTAGE BETWEEN CONDUCTORS
0 1 2 3 4 5 6 7 8 9 10
0.006d
A
5d
GROUND
0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0
A/d RATIO OF ALTITUDE TO BASE OF TRIANGLE

T.R. No 51

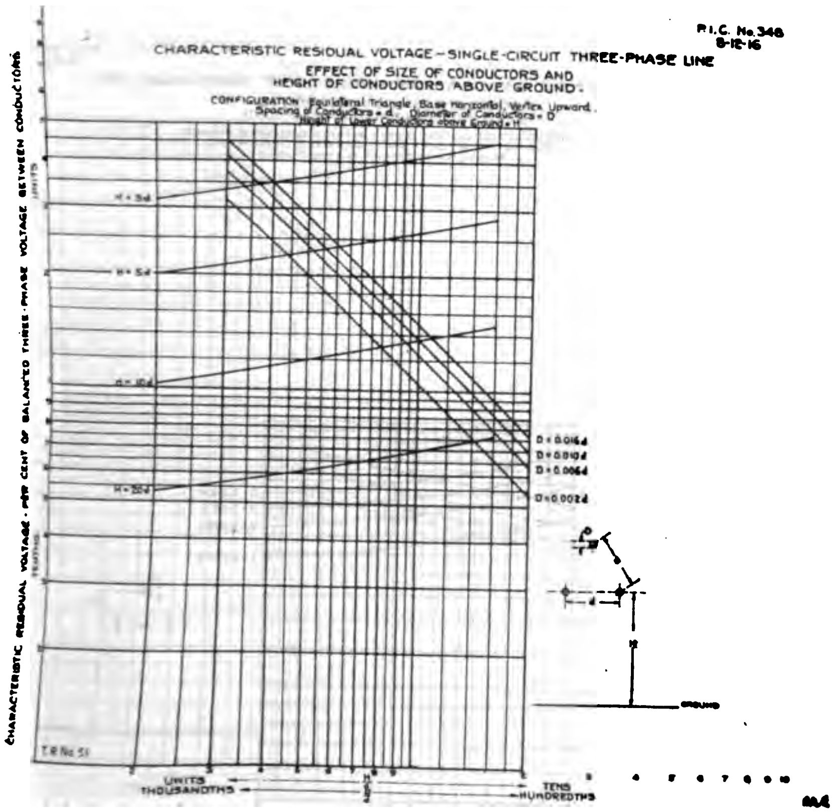

P.I.C. No. 346
8-12-16.
CHARACTERISTIC RESIDUAL VOLTAGE - SINGLE-CIRCUIT THREE-PHASE LINE.
EFFECT OF ALTITUDE OF TRIANGLE.
CONFIGURATION: Isosceles Triangle, Base Vertical.
Base = d. Altitude = A.
Height of Lowest Conductor above Ground = 3d. Diameter of Conductors = 0.003d.
CHARACTERISTIC RESIDUAL VOLTAGE - PERCENT OF BALANCED THREE-PHASE VOLTAGE BETWEEN CONDUCTORS
RATIO OF ALTITUDE TO BASE OF TRIANGLE
A/d
0.006d
3d
GROUND
T R No 51
R.J.M.

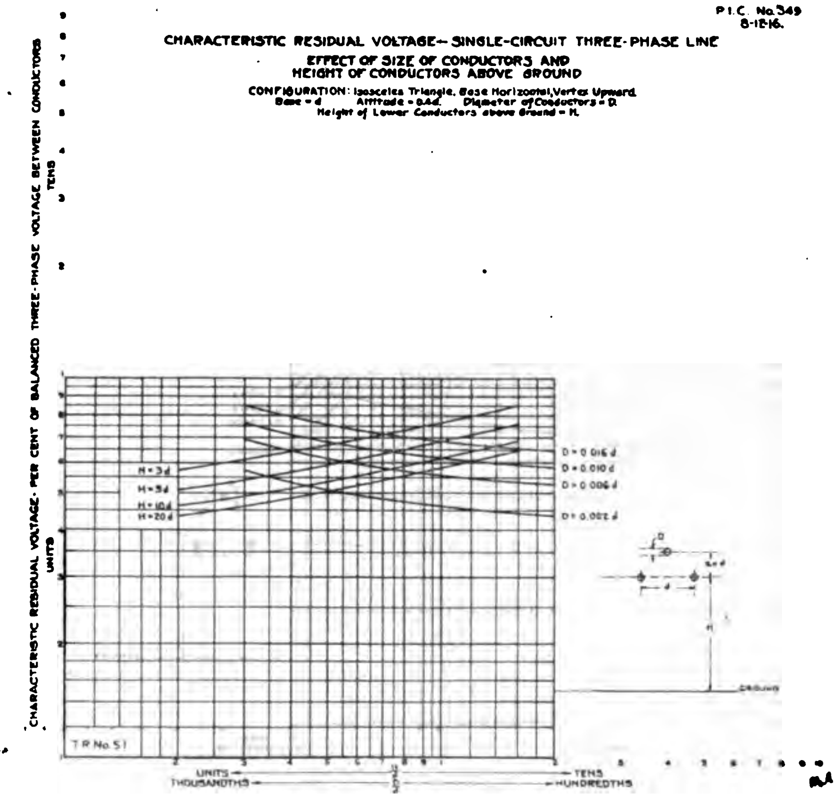
P.I.C. No 347
8-16-16.

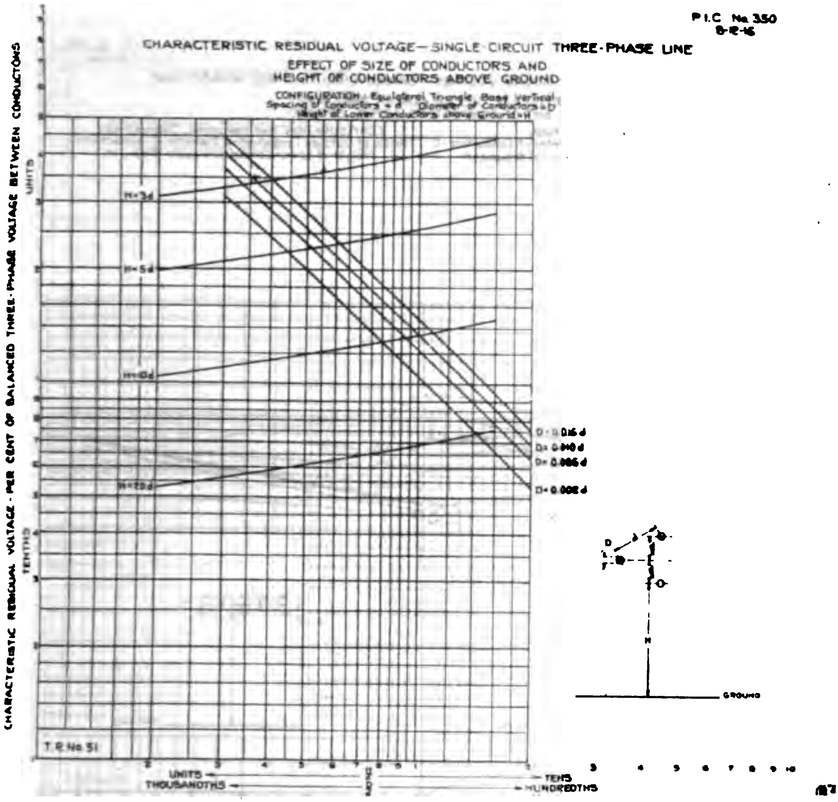
CHARACTERISTIC RESIDUAL VOLTAGE— SINGLE-CIRCUIT THREE-PHASE LINE.
EFFECT OF HEIGHT OF CONDUCTORS ABOVE GROUND.
CONFIGURATION: Isosceles Triangle, Base Horizontal, Vertex Upward.
Base = d. Altitude = 1.25d.
Diameter of Conductors = 0.006d.

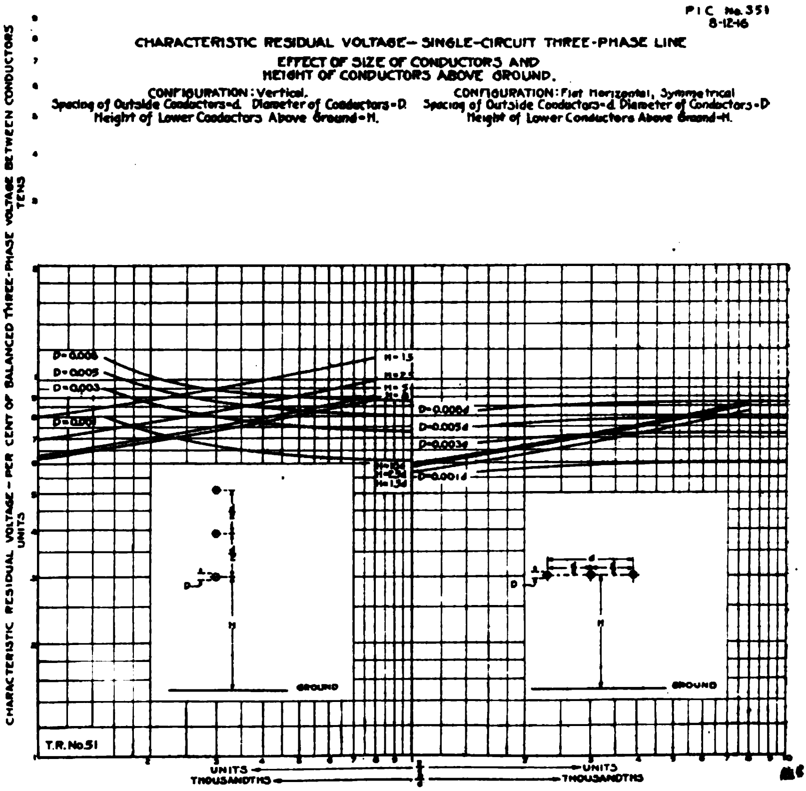


T.R.No.31

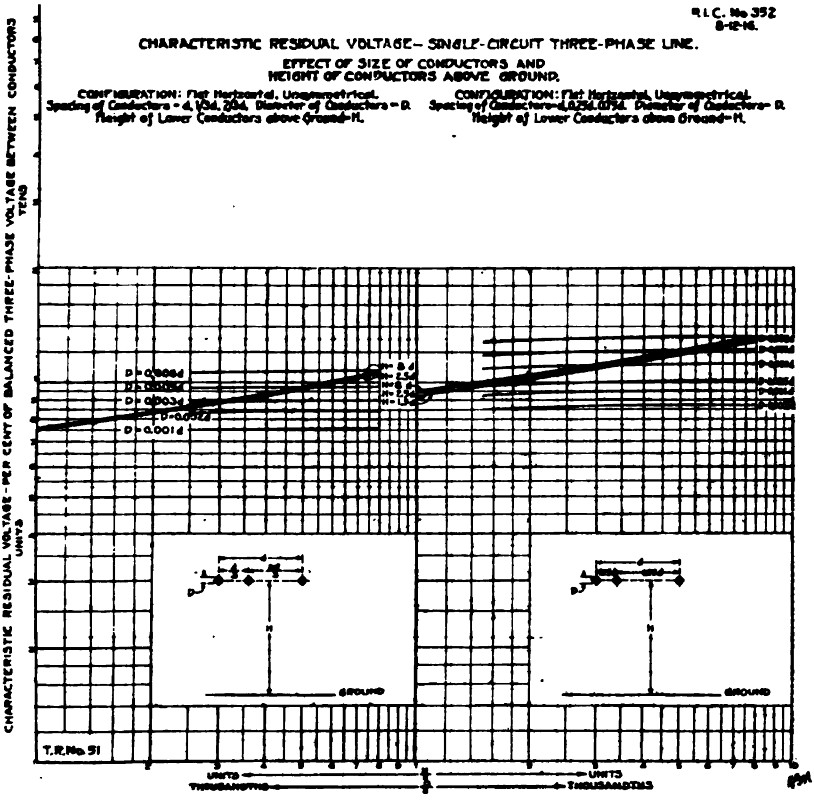

P.I.C. No. 348
8-12-16
CHARACTERISTIC RESIDUAL VOLTAGE—SINGLE-CIRCUIT THREE-PHASE LINE
EFFECT OF SIZE OF CONDUCTORS AND
HEIGHT OF CONDUCTORS ABOVE GROUND.
CHARACTERISTIC RESIDUAL VOLTAGE - PER CENT OF BALANCED THREE-PHASE VOLTAGE BETWEEN CONDUCTORS
UNITS
THOUSANDTHS
TENS
HUNDREDTHS
GROUND


P.I.C. No. 349
8-12-16.
CHARACTERISTIC RESIDUAL VOLTAGE—SINGLE-CIRCUIT THREE-PHASE LINE
EFFECT OF SIZE OF CONDUCTORS AND
HEIGHT OF CONDUCTORS ABOVE GROUND
CONFIGURATION: Isosceles Triangle, Base Horizontal, Vertex Upward
Base = d Altitude = 0.4d. Diameter of Conductors = D.
Height of Lower Conductors above Ground = H.
CHARACTERISTIC RESIDUAL VOLTAGE - PER CENT OF BALANCED THREE-PHASE VOLTAGE BETWEEN CONDUCTORS
TENS
UNITS
H = 3d
H = 5d
H = 10d
H = 20d
D = 0.016 d
D = 0.010 d
D = 0.006 d
D = 0.002 d
GROUND
T R No. 51
UNITS
THOUSANDTHS
TENS
HUNDREDTHS


P.I.C. No. 350
8-12-16
CHARACTERISTIC RESIDUAL VOLTAGE—SINGLE CIRCUIT THREE-PHASE LINE
EFFECT OF SIZE OF CONDUCTORS AND
HEIGHT OF CONDUCTORS ABOVE GROUND
CONFIGURATION: Equilateral Triangle, Base Vertical
Spacing of Conductors = d Diameter of Conductors = D
Height of Lower Conductors above Ground = H
CHARACTERISTIC RESIDUAL VOLTAGE - PER CENT OF BALANCED THREE-PHASE VOLTAGE BETWEEN CONDUCTORS
UNITS
TENTHS
H=3d
H=5d
H=10d
H=20d
D=0.016 d
D=0.010 d
D=0.006 d
D=0.002 d
T.R. No. 51
UNITS
THOUSANDTHS
TENS
HUNDREDTHS
GROUND



P I C No. 351
8-12-16
CHARACTERISTIC RESIDUAL VOLTAGE— SINGLE-CIRCUIT THREE-PHASE LINE
EFFECT OF SIZE OF CONDUCTORS AND
HEIGHT OF CONDUCTORS ABOVE GROUND.
CONFIGURATION: Vertical.
Spacing of Outside Conductors-d. Diameter of Conductors-D.
Height of Lower Conductors Above Ground-H.
CONFIGURATION: Flat Horizontal, Symmetrical
Spacing of Outside Conductors-d. Diameter of Conductors-D
Height of Lower Conductors Above Ground-H.
CHARACTERISTIC RESIDUAL VOLTAGE - PER CENT OF BALANCED THREE-PHASE VOLTAGE BETWEEN CONDUCTORS
TENS
UNITS
D=0.008
D=0.005
D=0.003
D=0.001
H=1.5
H=2.5
H=3.5
D=0.008d
D=0.005d
D=0.003d
D=0.001d
GROUND
GROUND
T.R. No.51
UNITS
THOUSANDTHS
UNITS
THOUSANDTHS


CHARACTERISTIC RESIDUAL VOLTAGE– SINGLE-CIRCUIT THREE-PHASE LINE.
EFFECT OF SIZE OF CONDUCTORS AND HEIGHT OF CONDUCTORS ABOVE GROUND.
GROUND
T.R. No. 51
UNITS
THOUSANDTHS

P.I.C. No. 385
8-23-16.

CHARACTERISTIC RESIDUAL VOLTAGES AND CAPACITANCE TO GROUND OF CONDUCTORS IN PARALLEL
SINGLE AND TWIN-CIRCUIT LINES

E_{RC} = Characteristic Residual Voltage in per cent of Balanced Three-phase Voltage Between Conductors
C_o = Capacitance to Ground, Conductors in Parallel, Millimicrofarads per 1000 feet

Single Circuit	Twin Circuits—Method of Parallel Operation					
	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
E_{RC} 8.25 C_o 3.98	E_{RC} 12.41 C_o 5.65	E_{RC} 10.08 C_o 5.65	E_{RC} 11.31 C_o 5.65	E_{RC} 1.23 C_o 5.65	E_{RC} 6.20 C_o 5.65	E_{RC} 6.20 C_o 5.65
E_{RC} 8.25 C_o 3.98	E_{RC} 12.81 C_o 5.53	E_{RC} 10.63 C_o 5.53	E_{RC} 11.51 C_o 5.53	E_{RC} 0.88 C_o 5.53	E_{RC} 6.41 C_o 5.53	E_{RC} 6.41 C_o 5.53
E_{RC} 7.51 C_o 3.48	E_{RC} 10.68 C_o 4.70	E_{RC} 9.93 C_o 4.70	E_{RC} 8.38 C_o 4.70	E_{RC} 1.54 C_o 4.70	E_{RC} 5.34 C_o 4.70	E_{RC} 5.34 C_o 4.70
E_{RC} 8.66 C_o 4.17	E_{RC} 13.37 C_o 6.02	E_{RC} 10.17 C_o 6.02	E_{RC} 12.61 C_o 6.02	E_{RC} 2.44 C_o 6.02	E_{RC} 6.69 C_o 6.02	E_{RC} 6.69 C_o 6.02
E_{RC} 8.40 C_o 4.18	E_{RC} 10.80 C_o 6.10	E_{RC} 6.43 C_o 6.10	E_{RC} 10.72 C_o 6.10	E_{RC} 4.30 C_o 6.10	E_{RC} 5.40 C_o 6.10	E_{RC} 5.40 C_o 6.10
E_{RC} 1.24 C_o 3.40	E_{RC} 13.34 C_o 4.62	E_{RC} 4.31 C_o 4.62	E_{RC} 13.09 C_o 4.62	E_{RC} 8.78 C_o 4.62	E_{RC} 6.67 C_o 4.62	E_{RC} 6.67 C_o 4.62
E_{RC} 1.24 C_o 3.41	E_{RC} 12.68 C_o 4.65	E_{RC} 0.50 C_o 4.65	E_{RC} 10.72 C_o 4.65	E_{RC} 11.23 C_o 4.65	E_{RC} 6.34 C_o 4.65	E_{RC} 6.34 C_o 4.65

APPENDIX I.

CALCULATION OF DIRECT CAPACITANCES IN TERMS OF DIMENSIONS.

This appendix supplements Section I, Subdivision I–B, giving in detail the method of computing the direct capacitances of a system of conductors in terms of the physical dimensions.

Relationship of Charges and Potentials.

The relationship of the charges and potentials of a system of m conductors and ground is expressed by the following system of equations*:

$$\left.\begin{array}{l}
V_1 = P_{11}Q_1 + P_{21}Q_2 + P_{31}Q_3 + \quad . \quad . \quad . \quad + P_{m1}Q_m \\
V_2 = P_{12}Q_1 + P_{22}Q_2 + P_{32}Q_3 + \quad . \quad . \quad . \quad + P_{m2}Q_m \\
V_3 = P_{13}Q_1 + P_{23}Q_2 + P_{33}Q_3 + \quad . \quad . \quad . \quad + P_{m3}Q_m \\
\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\
\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\
V_m = P_{1m}Q_1 + P_{2m}Q_2 + P_{3m}Q_3 + \quad . \quad . \quad . \quad + P_{mm}Q_m
\end{array}\right\} \qquad (49)$$

V_1, V_2, V_3 . . . V_m are the potentials referred to ground of the conductors denoted by the subscripts; Q_1, Q_2, Q_3 . . . Q_m are the corresponding charges.

The coefficients P_{11}, P_{12}, P_{13}, etc., termed "potential coefficients," are constants depending upon the dimensions of the system and the nature of the dielectric medium in which the conductors are placed (air in this case), and are independent of how the conductors may be charged or how they may be intereonnected (provided the surfaces of the connecting wires are small as compared to the surfaces of the conductors). Each coefficient has two subscripts, the first corresponding to the charge and the second to the potential. A coefficient in which the two subscripts are the same denotes the potential of the conductor designated thereby when its charge is unity, and the charges of other conductors are zero. A coefficient in which the two subscripts are different denotes the potential of the conductor designated by the second subscript when the conductor designated by the first subscript receives a unit charge, the charges of all the other conductors being zero. Reciprocal relations exist such that the order in which the subscripts appear is immaterial, *i. e.*, $P_{mn} = P_{nm}$. All the potential coefficients are positive and those with like subscripts are not less than those with unlike subscripts.

*Following Clerk-Maxwell.

The relationship of the charges and potentials may also be expressed in the following form:

$$\left.\begin{array}{l} Q_1 = K_{11}V_1 + K_{12}V_2 + K_{13}V_3 + \quad . \; . \; . \quad + K_{1m}V_m \\ Q_2 = K_{21}V_1 + K_{22}V_2 + K_{23}V_3 + \quad . \; . \; . \quad + K_{2m}V_m \\ Q_3 = K_{31}V_1 + K_{32}V_2 + K_{33}V_3 + \quad . \; . \; . \quad + K_{3m}V_m \\ . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ Q_m = K_{m1}V_1 + K_{m2}V_2 + K_{m3}V_3 + \quad . \; . \; . \quad + K_{mm}V_m \end{array}\right\} \qquad (50)$$

The coefficients K_{11}, K_{12}, K_{13}, etc., will be termed "capacitance coefficients" and correspond to Maxwell's coefficients of capacity and induction; termed by Russell the coefficients of self and mutual induction for electrostatic charges. Like the potential coefficients the capacitance coefficients are constants independent of the manner in which the conductors may be charged or interconnected. Each coefficient has two subscripts, the first conventionally denoting the charge and the second the potential. As with the potential coefficients reciprocal relations make the order of the subscripts immaterial, *i. e.*, $K_{mn} = K_{nm}$. A coefficient in which the two subscripts are the same denotes the charge on the correponding conductor when its potential is unity, and that of all the other conductors zero (grounded). This quantity is variously termed the "capacity," the "total capacity" and the "grounded capacity" of the conductor, also its "coefficient of self-induction for electrostatic charges." A coefficient in which the two subscripts are different denotes the charge induced on the conductor designated by the first when the conductor designated by the second is raised to potential unity, the potential of all the other conductors being zero (grounded). This quantity is variously termed "coefficient of induction" and "coefficient of mutual induction," and is equal in magnitude but opposite in sign to the "direct" or "normal" capacitance between corresponding conductors. The direct capacitance between conductors m and n is defined as the ratio of the charge on m to the difference of potential between m and n when all conductors of the system except n are at the same potential as m. All the coefficients with like subscripts are positive and all those with unlike subscripts are negative and the algebraic sum of all the coefficients belonging to a single conductor is equal to the direct capacitance to ground of that conductor. The direct capacitance of conductor m to ground is the ratio of the charge on conductor m to its potential when all other conductors except the reference conductor, ground, are at the potential of m. Thus, substituting in equations (50) $V_1 = V_2 = V_3 = \; . \; . \; . \; . \; . \; . \; V_m$.

$$C_{m0} = \frac{Q_m}{V_m} = K_{m1} + K_{m2} + K_{m3} + \quad . \; . \; . \quad K_{mm} \qquad (51)$$

Capacitance Coefficients in Terms of Potential Coefficients.

In order that both sets of equations (49) and (50), connecting the charges and potentials, may hold simultaneously, the potential and capacitance coefficients must be so related that any capacitance coefficient K_{mn} in equation (50) is the minor of P_{mn} in the determinant of the potential coefficients in equations (49), divided by the value of that determinant. This relation affords a means of computing the capacitance coefficients when the potential coefficients are known. If the number of conductors exceeds four, the computations become very laborious unless symmetrical relations simplify the operations.

For a three-conductor system the following solution results:

$$\left.\begin{aligned}
K_{11} &= \frac{P_{22}P_{33} - P^2_{23}}{\sigma} & K_{12} &= \frac{P_{23}P_{13} - P_{12}P_{33}}{\sigma} \\
K_{22} &= \frac{P_{11}P_{33} - P^2_{13}}{\sigma} & K_{13} &= \frac{P_{12}P_{23} - P_{13}P_{22}}{\sigma} \\
K_{33} &= \frac{P_{11}P_{22} - P^2_{12}}{\sigma} & K_{23} &= \frac{P_{13}P_{12} - P_{23}P_{11}}{\sigma} \\
\sigma &= P_{11}P_{22}P_{33} + 2P_{12}P_{13}P_{23} - P_{11}P^2_{23} - P_{22}P^2_{13} - P_{33}P^2_{12}
\end{aligned}\right\} \quad (52)$$

Since for the computation of residual voltage only the relative magnitudes of the direct capacitances are significant it is not necessary to evaluate σ in the above equations.

For a six-conductor system it is necessary in general to determine six charges, there being 21 potential coefficients and 21 capacitance coefficients involved. The evaluation of a sixth-order determinant is necessary for a complete solution. For twin circuits in which the conductors of the two circuits are symmetrically located with respect to an intermediate plane perpendicular to the earth's surface equalities among the potential coefficients reduce the number to 12. The method of solution is then as follows:

Summing the voltages of the symmetrically placed conductors, 1 and 4, 2 and 5, 3 and 6, the following group of three equations involving the sums of pairs of voltages in terms of the sums of pairs of charges is obtained from equations (49):

$$\left.\begin{aligned}
V_1+V_4 &= (P_{11}+P_{14})(Q_1+Q_4)+(P_{12}+P_{15})(Q_2+Q_5)+(P_{13}+P_{16})(Q_3+Q_6) \\
V_2+V_5 &= (P_{12}+P_{15})(Q_1+Q_4)+(P_{22}+P_{25})(Q_2+Q_5)+(P_{23}+P_{26})(Q_3+Q_6) \\
V_3+V_6 &= (P_{13}+P_{16})(Q_1+Q_4)+(P_{23}+P_{26})(Q_2+Q_5)+(P_{33}+P_{36})(Q_3+Q_6)
\end{aligned}\right\} \quad (53)$$

Let

$$\left.\begin{aligned}
P_{11} + P_{14} &= A_{11} & P_{12} + P_{15} &= A_{12} \\
P_{22} + P_{25} &= A_{22} & P_{13} + P_{16} &= A_{13} \\
P_{33} + P_{36} &= A_{33} & P_{23} + P_{26} &= A_{23}
\end{aligned}\right\} \quad (54)$$

These three simultaneous equations may be solved for the sums of pairs of charges in terms of the sums of pairs of voltages whereby the following group of equations results:

$$\left.\begin{aligned} Q_1 + Q_4 &= C_{11}(V_1 + V_4) + C_{12}(V_2 + V_5) + C_{13}(V_3 + V_6) \\ Q_2 + Q_5 &= C_{12}(V_1 + V_4) + C_{22}(V_2 + V_5) + C_{23}(V_3 + V_6) \\ Q_3 + Q_6 &= C_{13}(V_1 + V_4) + C_{23}(V_2 + V_5) + C_{33}(V_3 + V_6) \end{aligned}\right\} \quad (55)$$

The coefficient C_{mn} in the above equations is the minor of A_{mn} in the following determinant, divided by the value of the determinant (56).

$$\Delta = \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{12} & A_{22} & A_{23} \\ A_{13} & A_{23} & A_{33} \end{vmatrix} \quad (56)$$

The solution of the determinant gives expressions identical in form with equations (52), C_{11}, C_{12}, etc., replacing K_{11}, K_{12}, etc., and A_{11}, A_{12}, etc., replacing P_{11}, P_{12}, etc.

The direct capacitances to ground of the conductors are:

$$\left.\begin{aligned} C_{10} = C_{40} &= C_{11} + C_{12} + C_{13} \\ C_{20} = C_{50} &= C_{12} + C_{22} + C_{23} \\ C_{30} = C_{60} &= C_{13} + C_{23} + C_{33} \end{aligned}\right\} \quad (57)$$

For if in equation (55), $V_1 = V_2 = V_3 = V_4 = V_5 = V_6$

$$\left.\begin{aligned} Q_1 + Q_4 &= 2(C_{11} + C_{12} + C_{13})V_1 \\ Q_2 + Q_5 &= 2(C_{12} + C_{22} + C_{23})V_1 \\ Q_3 + Q_6 &= 2(C_{13} + C_{23} + C_{33})V_1 \end{aligned}\right\} \quad (58)$$

By symmetry

$$Q_1 = Q_4,\ Q_2 = Q_5,\ Q_3 = Q_6 \quad (59)$$

hence

$$\left.\begin{aligned} \frac{Q_1}{V_1} = \frac{Q_4}{V_4} &= C_{11} + C_{12} + C_{13} \\ \frac{Q_2}{V_2} = \frac{Q_5}{V_5} &= C_{12} + C_{22} + C_{23} \\ \frac{Q_3}{V_3} = \frac{Q_6}{V_6} &= C_{13} + C_{23} + C_{33} \end{aligned}\right\} \quad (60)$$

By definition $\frac{Q_1}{V_1}, \frac{Q_2}{V_2},$ etc., are the direct capacitances of the respective conductors since they are the ratios of the charges of the conductors to their potentials when all are at the same potential.

The relative and not the absolute magnitudes of the direct capacitances are required for the computation of residual voltage, hence the evaluation of Δ is unnecessary.

For a single-circuit line with a ground wire the sums of the direct capacitances to ground and to the ground wire are required in place of the direct capacitances to ground. These are obtained by summing all of the capacitance coefficients belonging to each conductor except that involving the ground wire. It is not necessary to evaluate any of the capacitance coefficients belonging to the ground wire. However, the terms involving the charge on the ground wire in the equations of group (49) must be considered in expressing the capacitance coefficients in terms of the potential coefficients.

For a twin-circuit line with one or two symmetrically placed ground wires, by summing the voltages of the symmetrically placed conductors the sums of pairs of voltages are expressed in terms of sums of pairs of corresponding charges, and the sums of the direct capacitances to ground and to the grounded conductor or conductors may be obtained as for the single-circuit line provided the sums of pairs of voltages and of pairs of charges are used in place of the individual voltages and charges.

Evaluation of Potential Coefficients.

To evaluate the potential coefficients it is necessary to compute, for each one, the work dône by an external agent against the repulsive force of a unit positive charge on one of the conductors in bringing a unit positive charge from an infinite distance, or from any place where the potential is zero, to the conductor whose potential is desired, which may or may not be the charged conductor. To evaluate P_{11} assume unit positive charge on conductor 1, charges on all other conductors being zero, and compute the work done in bringing another unit positive charge from ground to conductor 1; similarly for the other coefficients with like subscripts. To evaluate P_{21} assume unit positive charge on conductor 2, charges on all other conductors being zero, and compute work done in bringing unit positive charge from ground to conductor 1; similarly for the other coefficients with unlike subscripts. The computations involved in this process are simplified by considering the ground replaced by the images of the conductors at a distance below the ground surface equal to the height of the conductors above the surface and having charges of equal magnitude but opposite sign to the charges on the actual conductors. If the radii of the conductors are small as compared to the distances between them and these distances

small as compared to the length of the conductors the values of the potential coefficients in terms of dimensions of the system are as follows:

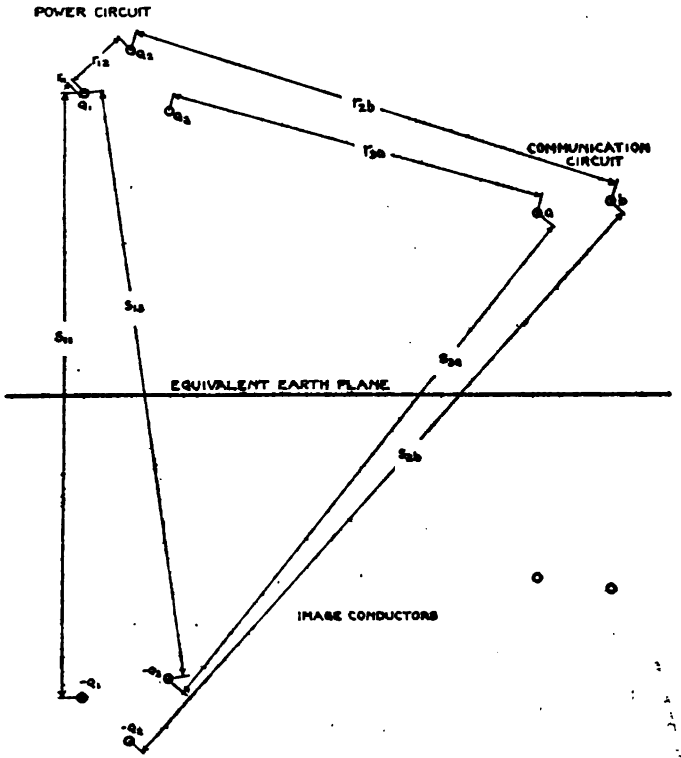
$$\left.\begin{array}{ll} P_{11} = 2\log\dfrac{S_{11}}{r_{11}} & P_{12} = 2\log\dfrac{S_{12}}{r_{12}} \\[2ex] P_{22} = 2\log\dfrac{S_{22}}{r_{22}} & P_{13} = 2\log\dfrac{S_{13}}{r_{13}} \\ \cdot \quad \cdot \quad \cdot & \cdot \quad \cdot \quad \cdot \\ P_{mm} = 2\log\dfrac{S_{mm}}{r_{mm}} & P_{mn} = 2\log\dfrac{S_{mn}}{r_{mn}} \end{array}\right\} \qquad (61)$$

S_{11}, S_{22} . . . S_{mm} are the distances from the conductors to their own images. S_{12}, S_{13} . . . S_{mn} are respectively the distances from the axis of the conductor denoted by one subscript to the axis of the image conductor denoted by the other subscript (for example, S_{12} is the distance from conductor 1 to the image of conductor 2 or vice versa). r_{11}, r_{22} . . . r_{mm} are the radii of the conductors. For stranded conductors the perimeters of the cross-sections divided by 2π may be taken as the radii with small error. r_{12}, r_{13} . . . r_{mn} are the distances between the axes of the conductors denoted by the subscripts. The notation used is illustrated on drawing No. 331, attached. If the distances between conductors are greater than ten diameters the above expressions for the potential coefficients are accurate to within less than $\frac{1}{10}$ per cent.

ATTACHED: Drawing No. 331.

P.I.C. № 331
2-21-16

DIAGRAM SHOWING NOTATION USED IN FORMULAS FOR COMPUTING COEFFICIENTS OF INDUCTION.



T.R. № 64

APPENDIX II.

RESIDUAL VOLTAGE OF AN ISOLATED SYSTEM IN TERMS OF MEASURED ADMITTANCE UNBALANCES.

This appendix supplements Section I, Subdivision II–B, and Section II, Subdivision III, giving, in greater detail, the method of measurement and computation of the residual voltage in terms of equivalent admittances.

Equations.

Equations (50) given in Appendix I, which express the relationship between the charges and potentials of an independent electrical system, may be extended to apply to lines of greater length by substituting the charging currents for the electric charges and equivalent admittances for the capacitance coefficients. Such a group of equations for a system of three conductors and ground is as follows:

$$\left.\begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 \\ I_2 &= Y_{12}V_1 + Y_{22}V_2 + Y_{23}V_3 \\ I_3 &= Y_{13}V_1 + Y_{23}V_2 + Y_{33}V_3 \end{aligned}\right\} \qquad (62)$$

Summing the currents

$$I_1 + I_2 + I_3 = Y_{10}V_1 + Y_{20}V_2 + Y_{30}V_3 \qquad (63)$$

where

$$\left.\begin{aligned} Y_{10} &= Y_{11} + Y_{12} + Y_{13} \\ Y_{20} &= Y_{12} + Y_{22} + Y_{23} \\ Y_{30} &= Y_{13} + Y_{23} + Y_{33} \end{aligned}\right\} \qquad (64)$$

Y_{10}, Y_{20} and $_{30}$ are the direct admittances to ground of the respective conductors. The direct admittance of conductor m to ground is defined as the ratio of the charging current of conductor m to its potential when all other conductors of the system except ground (the reference conductor) are connected to conductor m. It thus corresponds to the direct capacitance as defined in Appendix I except that the charging current is used instead of the charge.

The equation for the residual voltage may be obtained from equations (6) and (8) by substituting the direct admittances for the direct capacitances thus,

$$E_R = V_1 + V_2 + V_3 = \frac{Y_{10}(2E_{12}+E_{23}) + Y_{20}(E_{23}-E_{12}) + Y_{30}(-E_{12}-2E_{23})}{Y_{10}+Y_{20}+Y_{30}} \qquad (65)$$

or for balanced three-phase voltages between conductors

$$E_R = E_{12}\frac{3(Y_{10}-Y_{20})+j\sqrt{3}(Y_{10}+Y_{20}-2Y_{30})}{2(Y_{10}+Y_{20}+Y_{30})} \quad (66)$$

This may be rewritten in terms of the direct admittance unbalances

$$E_R = E_{12}\frac{3(Y_{10}-Y_{20})+j\sqrt{3}[(Y_{10}-Y_{20})+2(Y_{20}-Y_{30})]}{2(Y_{10}+Y_{20}+Y_{30})} \quad (67)$$

Determination of Direct Admittances.

For computing the residual voltage for any given case the equivalent direct admittances, at the point where the residual voltage is desired, must be determined. No direct method of measuring the direct admittances has been developed,* but the following indirect method may be followed, a modification of which was used in obtaining the experimental results recorded in Section II.

The admittance to ground of one of the conductors is measured with the other two grounded, and of two conductors in parallel to ground with the other grounded. If these measurements are made for each conductor and each pair of conductors all the coefficients of equations (64) may be determined.

The coefficients Y_{11}, Y_{22} and Y_{33} are the admittances to ground of the respective conductors when the other two are grounded, and may be measured directly. Thus if conductors 2 and 3 be grounded

$$V_2 = V_3 = 0$$

Then from equations (62)

$$I_1 = Y_{11}V_1 \text{ or } Y_{11} = \frac{I_1}{V_1}$$

which is measured directly. By repeating the measurements for the other two conductors Y_{22} and Y_{33} may be directly determined.

The coefficients Y_{12}, Y_{23} and Y_{13} are obtained from the preceding and from measurements of the admittance of two conductors in parallel when the third is grounded. Thus if conductor 3 be grounded and conductors 1 and 2 be paralleled

$$V_3 = 0 \text{ and } V_1 = V_2$$

Then from equations (62)

$$I_1 = Y_{11}V_1 + Y_{12}V_1$$
$$I_2 = Y_{12}V_1 + Y_{22}V_1$$
$$\frac{I_1 + I_2}{V_1} = Y_{11} + Y_{22} + 2Y_{12}$$

The quantity $\frac{I_1 + I_2}{V_1}$ is the admittance of conductors 1 and 2 in parallel

*See page 350, Memo. of July 16, 1917.

and is obtained by this measurement; Y_{11} and Y_{22} are determined by the previous measurements. Hence Y_{12} may be obtained from the following equation:

$$Y_{12} = \tfrac{1}{2}\left(\frac{I_1 + I_2}{V_1} - Y_{11} - Y_{22}\right)$$

Y_{13} and Y_{23} are obtained in similar fashion, using the other two combinations of conductors.

Since the residual voltage is a function of the differences of the direct admittances to ground it is desirable that the method of measurement obtain these differences as accurately as possible. The method given above is disadvantageous in this respect. The following additional measurements will determine the differences more accurately.

With the capacity and conductance unbalance set (see technical report No. 40) the difference of two admittances may be measured more accurately than either of the admittances. The components of the unbalance (the equivalent conductance and the equivalent capacitance unbalance) are obtained.

The unbalances of the admittances of pairs of conductors with the third grounded give directly $Y_{11} - Y_{22}$, $Y_{22} - Y_{33}$ and $Y_{33} - Y_{11}$. If conductor 3 be grounded and the admittance unbalance $\left(\frac{I_1 - I_2}{V_1}\right)$ of conductors 1 and 2 be measured,

$$V_1 = V_2,\ V_3 = 0$$

Then from equation (62)

$$I_1 = Y_{11}V_1 + Y_{12}V_1$$
$$I_2 = Y_{12}V_1 + Y_{22}V_1$$

whence

$$\frac{I_1 - I_2}{V_1} = Y_{11} - Y_{22}$$

Similarly for the two other combinations of conductors.

From the preceding and from the unbalances of the admittances of pairs of conductors with the third isolated $Y_{13} - Y_{23}$, $Y_{12} - Y_{13}$, $Y_{23} - Y_{12}$ may be obtained; thus if conductor 3 be isolated and the admittance unbalance $\left(\frac{I_1 - I_2}{V_1}\right)$ of conductors 1 and 2 be measured,

$$V_1 = V_2 \text{ and } I_3 = 0$$

and from equations (62)

$$\begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_1 + Y_{13}V_3 \\ I_2 &= Y_{12}V_1 + Y_{22}V_1 + Y_{23}V_3 \\ 0 &= Y_{13}V_1 + Y_{23}V_1 + Y_{33}V_3 \end{aligned}$$

$$V_3 = -V_1 \frac{Y_{13} + Y_{23}}{Y_{33}}$$

$$\frac{I_1 - I_2}{V_1} = (Y_{11} - Y_{22}) - \frac{(Y_{13} - Y_{23})(Y_{13} + Y_{23})}{Y_{33}}$$

or

$$Y_{13} - Y_{23} = -\left[\left(\frac{I_1 - I_2}{V_1}\right) - (Y_{11} - Y_{22})\right] \frac{Y_{33}}{Y_{13} + Y_{23}} .$$

Similarly for the other two pairs of conductors.

From the two sets of unbalances the direct admittance unbalances are obtained thus:

$$\left.\begin{aligned} (Y_{11} - Y_{22}) + (Y_{13} - Y_{23}) &= Y_{10} - Y_{20} \\ (Y_{22} - Y_{33}) + (Y_{12} - Y_{13}) &= Y_{20} - Y_{30} \\ (Y_{33} - Y_{11}) + (Y_{23} - Y_{12}) &= Y_{30} - Y_{10} \end{aligned}\right\} \qquad (68)$$

The complete set of measurements at each frequency for this method of determination is:

1. Admittance of one conductor to ground, others grounded.
2. Admittance unbalance of a pair of conductors, the third grounded.
3. Same measurement for second pair of conductors.
4. Same measurement for third pair of conductors.
5. Admittance of two conductors in parallel, the third grounded.
6. The same measurement for another pair of conductors.
7. The same measurement for the third pair of conductors.
8. Admittance unbalance of a pair of conductors with the third isolated.
9. Same measurement for second pair.
10. Same measurement for third pair.

Since the values of one of the direct admittances and of two of the unbalances are sufficient to determine the three admittances and their differences measurements (4) and (10) may be omitted but it is desirable to make them as a check.

The values of the direct admittances and their unbalances may now be substituted in the equation (67) for the residual voltage.

Modified Method.

The measurements made at San Fernando and described in Section II were made previous to the development of this method and some of the measurements above outlined were not made. For no transpositions and for one barrel, only the unbalance measurements Nos. 2, 3, 4, and 8, 9,

10, as above, were made. For two barrels, in addition, the admittances to ground of one of the conductors with the others grounded and of one pair of conductors in parallel to ground with the third grounded, were measured.

The direct admittances for the case of two barrels are all determinable from the measurements but the numerical solution is far more laborious since it is necessary to solve quadratic instead of linear equations. Since $Y_{13} + Y_{23}$ is not determined, the equation given for $Y_{13} - Y_{23}$ can not be solved for this difference but instead it must be solved as a quadratic for Y_{13} or Y_{23} in terms of the other admittances. Two such quadratic equations must be solved since only one of the quantities Y_{12}, Y_{23} and Y_{13} is determined by the admittance to ground of a pair of conductors with the third grounded.

For the other two cases in order to obtain the admittance unbalances it was necessary to assume that the averages of Y_{11}, Y_{22} and Y_{33} and of Y_{12}, Y_{23}, and Y_{13} were constant irrespective of the transposition system. Having the averages of Y_{11}, Y_{22} and Y_{33} and their differences it was possible to obtain their values directly. To obtain the values of Y_{12}, Y_{13}, Y_{23} and their differences a system of successive approximations was used. It was assumed for the first approximation that $(Y_{13} + Y_{23}) = (Y_{12} + Y_{13}) =$ twice the average of Y_{12}, Y_{23} and Y_{13}. $(Y_{13} - Y_{23})$ and $(Y_{12} - Y_{13})$ were determined and a new set of values of $Y_{13} + Y_{23}$ and $Y_{12} + Y_{13}$ computed from the average value of the three and the differences obtained from the first approximations. The new values of the sums were used in a second approximation and the process continued until a satisfactory result was obtained, the precision of which was judged by the percentage by which successive values of the sums differed. With the values of Y_{11}, Y_{22}, Y_{33}, and their differences, and Y_{12}, Y_{23} and Y_{13} determined, the direct admittance unbalances and the residual voltages were determined by equations (68) and (67).

Since the admittances are all complex quantities the numerical work involved in making this solution for a number of frequencies was very laborious. The method outlined in the first part of this appendix is to be recommended in preference.*

*See page 350, Memo. of July 16, 1917.

APPENDIX III.

GENERAL EQUATIONS FOR RESIDUAL VOLTAGE IN TERMS OF PRIMARY CONSTANTS.

To obtain a general expression for residual voltage of a three-phase circuit which will hold for all frequencies and lengths of line requires the exact solution of the circuit, assuming uniformly distributed constants. The discussion here given is in amplification of Section I, Subdivision II-C.

Derivation.

For a system of three linear conductors and ground, consider an elementary length of line dx. A set of equations may be written expressing the changes in voltages along this element of line in terms of the currents and the self and mutual impedances as determined by the resistances of the conductors and the self and mutual inductances per unit length, quantities that may be computed from the dimensions of the system. Thus:

$$\left.\begin{aligned}\frac{dV_1}{dx} &= Z_{11}I_1 + Z_{12}I_2 + Z_{13}I_3\\ \frac{dV_2}{dx} &= Z_{12}I_1 + Z_{22}I_2 + Z_{23}I_3\\ \frac{dV_3}{dx} &= Z_{13}I_1 + Z_{23}I_2 + Z_{33}I_3\end{aligned}\right\} \qquad (69)$$

Also a set of equations may be written expressing the changes in current along the element of line, in terms of the voltages and the self and mutual admittances as determined by the capacitance and conductance coefficients per unit length. The capacitance coefficients may be computed in terms of the dimensions as shown in Appendix I. Thus also

$$\left.\begin{aligned}\frac{dI_1}{dx} &= Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3\\ \frac{dI_2}{dx} &= Y_{12}V_1 + Y_{22}V_2 + Y_{23}V_3\\ \frac{dI_3}{dx} &= Y_{13}V_1 + Y_{23}V_2 + Y_{33}V_3\end{aligned}\right\} \qquad (70)$$

To solve these equations it is necessary to first separate the variables. By differentiating equations (69) with respect to x and substituting for $\frac{dI_1}{dx}$, $\frac{dI_2}{dx}$ and $\frac{dI_3}{dx}$ their values from equations (70), a set of three simultaneous equations is obtained involving the three voltages and their second derivatives. By differentiating equations (70) with respect to x and substituting for $\frac{dV_1}{dx}$, $\frac{dV_2}{dx}$ and $\frac{dV_3}{dx}$ their values from equations (69), a similar set of simultaneous equations is obtained involving the three currents and their second derivatives.

The two sets of equations are

$$\left.\begin{aligned} D^2V_1 &= a_1V_1 + a_2V_2 + a_3V_3 \\ D^2V_2 &= b_1V_1 + b_2V_2 + b_3V_3 \\ D^2V_3 &= c_1V_1 + c_2V_2 + c_3V_3 \end{aligned}\right\} \qquad (71)$$

$$\left.\begin{aligned} D^2I_1 &= a_1I_1 + b_1I_2 + c_1I_3 \\ D^2I_2 &= a_2I_1 + b_2I_2 + c_2I_3 \\ D^2I_3 &= a_3I_1 + b_3I_2 + c_3I_3 \end{aligned}\right\} \qquad (72)$$

D is the symbolic operator denoting derivative with respect to x, D^2 therefore denotes the second derivative with respect to x.

The coefficients a_1, a_2, a_3, etc., are combinations of the admittances and impedances per unit length as follows:

$$\left.\begin{aligned} a_1 &= Z_{11}Y_{11} + Z_{12}Y_{12} + Z_{13}Y_{13} \\ a_2 &= Z_{11}Y_{12} + Z_{12}Y_{22} + Z_{13}Y_{23} \\ a_3 &= Z_{11}Y_{13} + Z_{12}Y_{23} + Z_{13}Y_{33} \\ b_1 &= Z_{12}Y_{11} + Z_{22}Y_{12} + Z_{23}Y_{13} \\ b_2 &= Z_{12}Y_{12} + Z_{22}Y_{22} + Z_{23}Y_{23} \\ b_3 &= Z_{12}Y_{13} + Z_{22}Y_{23} + Z_{23}Y_{33} \\ c_1 &= Z_{13}Y_{11} + Z_{23}Y_{12} + Z_{33}Y_{13} \\ c_2 &= Z_{13}Y_{12} + Z_{23}Y_{22} + Z_{33}Y_{23} \\ c_3 &= Z_{13}Y_{13} + Z_{23}Y_{23} + Z_{33}Y_{33} \end{aligned}\right\} \qquad (73)$$

Eliminating V_2 and V_3 from equations (71) a sixth order differential equation in V_1 is obtained.

$$\begin{bmatrix} D^6 - D^4(a_1 + b_2 + c_3) + D^2(a_1b_2 + a_1c_3 + b_2c_3 - \\ a_3c_1 - b_1a_2 - b_3c_2) + a_3c_1b_2 + b_1a_2c_3 + b_3c_2a_1 - \\ a_1b_2c_3 - a_2b_3c_1 - a_3c_2b_1 \end{bmatrix} V_1 = 0 \qquad (74)$$

The differential equations for V_2, V_3 and also for the three currents are identical with (74) except that V_2, V_3, I_1, I_2 and I_3, respectively,

replace V_1. Hence it is necessary to solve only one differential equation. The auxiliary equation is

$$m^6 - Am^4 + Bm^2 + C = 0 \tag{75}$$

The coefficients of m in (75) are identical with those of D in equation (74). This equation is a cubic in m^2 and may be solved by Cardan's method. Let the roots of the equation be

$$m_1, -m_1, m_2, -m_2, m_3, -m_3$$

Then the general integral equations are:

$$\left.\begin{aligned} V_1 = {} & A_1 \cosh m_1x + B_1 \sinh m_1x + A_2 \cosh m_2x + \\ & B_2 \sinh m_2x + A_3 \cosh m_3x + B_3 \sinh m_3x \\ V_2 = {} & h_1(A_1 \cosh m_1x + B_1 \sinh m_1x) + h_2(A_2 \cosh m_2x + \\ & B_2 \sinh m_2x) + h_3(A_3 \cosh m_3x + B_3 \sinh m_3x) \\ V_3 = {} & p_1(A_1 \cosh m_1x + B_1 \sinh m_1x) + p_2(A_2 \cosh m_2x + \\ & B_2 \sinh m_2x) + p_3(A_3 \cosh m_3x + B_3 \sinh m_3x) \end{aligned}\right\} \tag{76}$$

$$\left.\begin{aligned} I_1 = {} & e_1(B_1 \cosh m_1x + A_1 \sinh m_1x) + e_2(B_2 \cosh m_2x + \\ & A_2 \sinh m_2x) + e_3(B_3 \cosh m_3x + A_3 \sinh m_3x) \\ I_2 = {} & f_1(B_1 \cosh m_1x + A_1 \sinh m_1x) + f_2(B_2 \cosh m_2x + \\ & A_2 \sinh m_2x) + f_3(B_3 \cosh m_3x + A_3 \sinh m_3x) \\ I_3 = {} & g_1(B_1 \cosh m_1x + A_1 \sinh m_1x) + g_2(B_2 \cosh m_2x + \\ & A_2 \sinh m_2x) + g_3(B_3 \cosh m_3x + A_3 \sinh m_3x) \end{aligned}\right\} \tag{77}$$

A_1, B_1, A_2, B_2, A_3, B_3 are constants of integration. The coefficients h_1, h_2, p_1, p_2, etc., are complicated functions of the impedances and admittances per unit length of the circuit.

The expression for the residual voltage is

$$\left.\begin{aligned} E_R = {} & (1 + h_1 + p_1)(A_1 \cosh m_1x + B_1 \sinh m_1x) \\ & + (1 + h_2 + p_2)(A_2 \cosh m_2x + B_2 \sinh m_2x) \\ & + (1 + h_3 + p_3)(A_3 \cosh m_3x + B_3 \sinh m_3x) \end{aligned}\right\} \tag{78}$$

Application.

To apply these equations for the computations of the residual voltage of any given case it is necessary to evaluate the constants of integration in terms of the given terminal conditions.

Consider a nontransposed line of length L isolated from ground throughout with the receiving end open, and energized at the sending end with voltages between pairs of conductors:

$$\begin{aligned} -V_1 + V_2 &= E_{12} \\ -V_2 + V_3 &= E_{23} \end{aligned} \tag{79}$$

At $\qquad x = 0 \quad \sinh x = 0 \quad \cosh x = 1 \qquad$ (80)

hence

$$\left.\begin{aligned} E_{12} &= (h_1 - 1)A_1 + (h_2 - 1)A_2 + (h_3 - 1)A_3 \\ E_{23} &= (p_1 - h_1)A_1 + (p_2 - h_2)A_2 + (p_3 - h_3)A_3 \end{aligned}\right\} \tag{81}$$

Also at $x = 0$, $I_1 + I_2 + I_3 = 0$ since the line is isolated;

therefore $0 = (e_1+f_1+g_1)B_1 + (e_2+f_2+g_2)B_2 + (e_3+f_3+g_3)B_3 \qquad$ (82)

At $\qquad x = L, \qquad I_1 = I_2 = I_3 = 0 \qquad$ (83)

since the line is open circuited.

Substituting L for x and the values of the currents and solving for B_1, B_2 and B_3

$$\left.\begin{aligned} B_1 &= -A_1 \tanh m_1L \\ B_2 &= -A_2 \tanh m_2L \\ B_3 &= -A_3 \tanh m_3L \end{aligned}\right\} \qquad (84)$$

Therefore (82) becomes:

$$0 = (e_1 + f_1 + g_1)\ A_1 \tanh m_1L + \left.\begin{aligned} &(e_2 + f_2 + g_2)\ A_2 \tanh m_2L \\ + &(e_3 + f_3 + g_3)\ A_3 \tanh m_3L \end{aligned}\right\} \quad (85)$$

Equations (81) and (85) may be solved for A_1, A_2 and A_3 and B_1, B_2, and B_3 obtained from (84).

When the values of the constants of integration thus determined are substituted in (78) the residual voltage may be obtained for any point on the line by substituting in the equation the value of x.

The solution for other terminal conditions may be carried out in a similar manner. However, the evaluation of the constants of integration is in general more difficult than in the case considered above.

Since all the coefficients above are complex quantities it is readily seen that the labor involved in a numerical solution of a given case is very great.

CHARACTERISTIC RESIDUAL VOLTAGES OF TWIN HORIZONTAL THREE-PHASE CIRCUITS.

February 28, 1917.

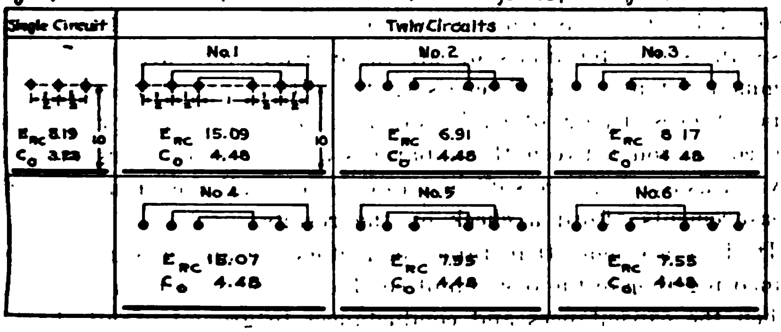
Memorandum—Supplementing Technical Report No. 51.

This memorandum presents the results of a study of the characteristic residual voltages of a representative case of twin horizontal three-phase circuits. Six different methods of interconnection of the two circuits for parallel operation were considered. The dimensions are shown on the accompanying drawing. This drawing which is similar to drawing No. 365 attached to technical report No. 51, gives the diagrams of the various interconnections for parallel operation with the capacitances to ground of the conductors of the circuits in parallel. For comparison these are also given for one of the circuits with the other absent.

CHARACTERISTIC RESIDUAL VOLTAGES AND CAPACITANCE TO GROUND OF CONDUCTORS IN PARALLEL SINGLE AND TWIN HORIZONTAL THREE-PHASE CIRCUITS

E_{RC}=Characteristic Residual Voltage in Per cent of Balanced Three-Phase Voltage between Conductors

C_0=Capacitance to Ground, Conductors in Parallel, Millimicrofarads per 1000 feet.



The method of interconnection giving the smallest residual voltages is No. 2, whereby similarly located (right, middle and left) conductors of the two circuits are at common potentials. The method giving the largest residual voltages is No. 1, whereby symmetrically located (outside, middle and inside) conductors of the two circuits are at common potentials. In this case the residual voltage of each circuit is 84 per cent greater than that of the corresponding single-circuit line. The residual voltages for method No. 4 are practically the same as for method No. 1. For method No. 3 the residual voltages are practically equal to that of the corresponding single-circuit line. Methods 5 and 6 give residual voltages one-half those of method No. 1.

The capacitance of the six conductors of the twin circuits in multiple to ground is about 40 per cent greater than that of the three conductors of a corresponding single-circuit line.

Respectfully submitted.

(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

MEASUREMENT OF DIRECT ADMITTANCE UNBALANCES.

July 16, 1917.

Memorandum supplementing Technical Report No. 51.

The method of obtaining the direct admittance unbalances of a system of three conductors and ground, described in technical report No. 51, involves the measurement of the unbalances of the conductors in pairs with the third conductor alternately grounded and isolated and the measurement of the total admittances to ground of various combinations of conductors. From these measurements the direct admittance unbalances may be derived by a tedious mathematical process.

Drawing No. 380 attached, shows a schematic diagram of connections of a method proposed for measuring these unbalances directly. This method is an extension of that described in technical report No. 40, auxiliary apparatus being added to bring the "idle" conductors of the system to the potential of the pair whose unbalances are desired. In general, by the adjustment of two variable condensers and two variable resistances the terminals of all three conductors are brought to a common potential. Differences of the direct capacitances and direct conductances of pairs of conductors are measured in turn by transposing the connection between the apparatus and the conductors under test. Two measurements are sufficient to determine all the unbalances, three measurements afford a check on the results.

If there are more than three conductors the idle conductors may be joined together in the position of conductor "a" shown on the diagram. When the capacitances and conductances to ground of all three conductors are approximately equal, then R_2 should be approximately equal to R_0 and R_3 approximately equal to R_1. If the capacitance and conductance of the idle conductor or group of conductors are greater, then R_2 should be proportionately less than R_0 and R_3 less than R_1. Fine adjustment of potential of the idle conductor or conductors should be made with R_x and C_x. Several trial adjustments will be necessary before obtaining silence with the receiver in both positions. The final adjustment should be made with the receiver connected between the two conductors whose unbalances are desired, as an error in setting of C and r affects the result directly and by the full amount of the incorrect settings, whereas an error in adjustment of the auxiliary rheostat (R_x) and condenser (C_x) controlling the potential of the idle conductor, has relatively less effect on the result.

If the conductances are negligible, the circuit may be simplified by eliminating R_3, R_x and C_x, the necessary control of the potential of the idle conductor being obtained by varying R_2.

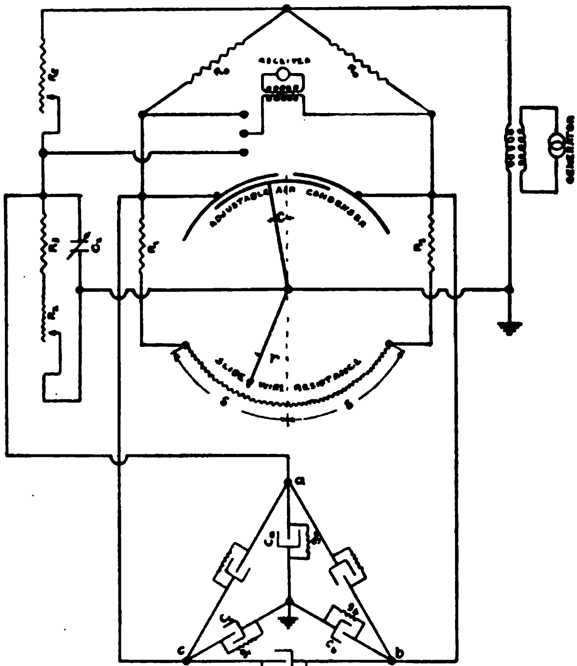
Respectfully submitted.

(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

ATTACHED: P. I. C. Drawing No. 380.

R.L.G. No. 280
7-18-17

MEASUREMENT OF DIRECT ADMITTANCE UNBALANCES
SYSTEM OF THREE CONDUCTORS AND GROUND
SCHEMATIC DIAGRAM



EQUIVALENT NETWORK
SYSTEM OF THREE CONDUCTORS AND GROUND

$$C_b - C_c = 2C \qquad g_b - g_c = \frac{2r}{(R+S)^2 - r^2}$$

T.R. No. 51

Technical Report No. 52.

July 28, 1915.

RESIDUALS PRODUCED BY A GROUND ON ONE PHASE OF A NORMALLY ISOLATED THREE-PHASE SYSTEM.

OUTLINE.

I. Introduction.

The object of this report is to consider the relationships between the residuals of both voltage and current, produced by the abnormal condition of a ground on one phase of a normally isolated three-phase system, and the voltage and line constants of the system.

The report contains the results of tests made to determine the residuals produced by deliberately grounding one phase of a line energized as an isolated system. The subject is also treated theoretically in order to present a rational explanation of the observed results and to make the conclusions more generally applicable to other cases. As a verification of the theoretical conclusions the computed values of the residuals and unbalanced charging currents, for conditions corresponding to the tests, are compared with the observed values. Published information on the subject is reviewed and compared with the results obtained in the tests and theoretical study.

23—39969

II. Experimental Investigation (May 26, 1915).

A—Description of Tests.

1. *Transmission Line.*

The transmission line involved in these tests was placed at the disposal of the Committee by the Pacific Light and Power Corporation. This line extends from the field laboratory in San Fernando to Somis. Data in regard to this line are as follows:

Length: 36.7 miles.

Conductors: No. 4 B&S gauge solid copper for 79% of the distance, No. 2 B&S gauge copper (some stranded and some solid) for the remaining 21% of the distance in 4 sections.

Configuration: vertical.

Supporting structures: poles 90%; A-frames 10%.

Spacing of conductors: on poles 5 feet, on A-frames, 6 feet.

Average height of top cross arm: 50 feet.

Transpositions: the line is divided into six equal sections by five transpositions, forming two barrels.

Insulators: Pin type (Locke 408-A) except at crossings, corners, dead ends and A-frames where double disk insulators are used.

Normal voltage between wires: 15 kV.

Telephone circuit: No. 9 BWG iron, carried on pins 1 and 3 of an 8 pin, 9 ft. cross arm placed 11 feet below lower cross arm of power circuit. The telephone circuit is on the opposite side of pole from power circuit.

2. *Energy Supply.*

The line was energized at the Committee's temporary substation through two banks of 37.5-kVA. transformers, from the 15-kV. network of the Pacific Light and Power Corporation. The connections of these two banks of transformers were as follows:

Step-down bank, delta-delta $^{15000}/_{220}$.

Step-up bank, delta-delta $^{220}/_{15000}$.

At the Somis end of the line there was a bank of 15-kVA. transformers, which was connected to the line at times during the course of the tests to determine the effect, if any, of tying the phases together by a transformer bank at the receiving end of the line, the normal condition for a line in service. The connection of the Somis bank of transformers was interconnected star-delta with high-tension neutral isolated. This type of connection was employed for other tests and it was not considered important enough for the purposes of these particular tests to change the connection to a type more commonly employed.

3. *Procedure for Tests.*

Banks of current and potential transformers were arranged for measuring the line currents and voltages and residual current and voltage. Means were provided for grounding one phase (phase 3) either on the line side or on the supply side of the current transformers. A 5:5 current transformer was placed in the ground connection to measure the ground current as a check upon the measurement of residual current by the usual method.

Observations of line and residual currents and voltages were made both with the line isolated and with one phase grounded, the location of the "ground" being changed with respect to the bank of current transformers. For the grounded condition the ground connection was always made prior to energizing the line. The switching was done on the low-tension bus-wires between banks of transformers.

Some observations were made on the telephone circuit under the abnormal condition of the power circuit.

The results of the tests together with a statement of the conditions applying in each case are given in the following section of this report.

B—Results of Test.

The results of the test are given in the following tables which summarize the data.

TABLE I.

Line voltages to ground kV.			Residual voltage, kV.	Line currents, amperes			Residual current, amperes	Ground current, amperes	Line conditions: San Fernando, phase 3			Somis*
									Normal	Grounded		
E_1	E_2	E_3	E_R	I_1	I_2	I_3	I_R	I_G		In.	Out.	
9.2	9.3	9.2	0	1.66	1.66	1.67	0	0	x	------	------	I
16.1	16.1	0	27.8	2.17	2.10	0.84	2.30	2.37	------	x	------	I
16.1	16.1	0	27.8	2.17	2.11	3.18	0	2.37	------	------	x	I
16.1	16.1	0	27.8	2.16	2.10	3.15	0	2.37	------	------	x	T
16.1	16.1	0	27.8	2.15	2.08	0.84	2.30	2.37	------	x	------	T
9.2	9.3	9.2	0	1.64	1.63	1.65	0	0	x	------	------	T
16.2	16.1	0	27.9	2.15	2.11	3.18	0	0	x	------	------	3G
16.1	16.2	0	27.9	2.17	2.11	0.92	2.30	2.37	------	x	------	3G
16.0	16.1	0	27.8	2.15	2.11	3.18	0	2.34	------	------	x	3G

I_G = current in ground connection; other symbols have usual meanings.
In. = supply side current transformers; Out. = line side current transformers.
*Condition at Somis indicated thus:
I = line open and isolated.
T = line connected to interconnected star-delta transformers, neutral isolated.
3G = phase 3 grounded, transformers disconnected.

TABLE II.
Effect on Telephone Circuit.

Residual voltage (E_R)	Ground current (I_G)	Longitudinal induction		Line conditions		
		Short circuit current	Open circuit* voltage	San Fernando phase 3	Somis	
					Phase 3	Telephone circuit
kV.	amp.	amp.	kV.			
27.8	2.49	0.50	2.6	G	G	I
27.8	2.43	0.32	0.1	G	G	G
27.8	2.52	0.50	2.6	G	I	I
27.8	2.50	0.31	0.1	G	I	G

*Computed from short-circuit current and line impedance.
G = grounded. I = isolated.

C—Discussion of Results.

An inspection of Table I shows that the result of grounding one phase of the power circuit at San Fernando was:

(1) to cause a residual voltage equal to three times the normal voltage between lines and ground;
(2) to cause a residual current equal to approximately 1.4 times the normal charging current per phase;
(3) to unbalance the line currents in such a manner as to increase the charging currents of the two non-grounded phases to 1.3 times normal, and the current of the grounded phases to 1.9 times normal;
(4) to decrease the current of the grounded phase on the line side of the fault, to 0.51 times normal.

An explanation of these effects in terms of the line constants is given below in section III, B, dealing with the "Application of Theory."

Inspection of Table I also shows that the transformer bank at Somis had practically no effect upon the normal charging currents of the line or upon the currents under the abnormal condition of a ground on one phase. As the transformers were of small capacity and under no load it is not to be expected that they would have much effect. A further point to be noted from the table is that grounding one phase at both ends of the line produced no appreciably different results from those obtained by grounding that phase at the supply end of the line only. Also as to the unbalanced charging currents supplied to the line it makes no difference whether the ground be at the supply end, far end, or at points intermediate.

The observations on the telephone circuit (see Table II) were merely incidental to the observations on the power circuit. They show a very large longitudinal electrostatically induced voltage on the telephone circuit due to the abnormal residual voltage produced by grounding one phase of the power circuit. This voltage of the telephone circuit to ground amounted to 2.6 kilovolts with the telephone circuit clear at

both ends. The current to ground, when the telephone circuit was grounded at one end, amounted to approximately 0.5 ampere. The fact that the effect is chiefly electrostatic is shown very clearly by the decrease in the short-circuit current to ground, due to grounding the far end of the telephone circuit at Somis (decrease from .50 to .31 ampere, 38 per cent). The ground at the far end offered a shunt path to part of the electrostatically induced charges. Grounding the circuit at the far end also had the effect of greatly increasing that component of the short-circuit current due to electromagnetic induction from the residual current, by affording a relatively low-impedance path for this component. The computed value of the current at San Fernando with the circuit grounded at that end only, is 0.476 amp. The computed values of the current to ground with the circuit grounded at both ends are given below. The electromagnetic component is chosen as the vector of reference since the phase of this component is practically the same at all points on the circuit.

Short-Circuit Current to Ground—Amperes.
Telephone Circuit Grounded at Both Ends.

	San Fernando	Somis
Electromagnetic	0.073 $\angle 0°$	0.073 $\angle 0°$
Electrostatic	0.238 $\angle -78°$	0.238 $\angle 102°$
Total	0.263 $\angle -52°$	0.235 $\angle 84°$

No measurements were made at Somis, so there are no experimental results to compare with the computed value of the current at that point. The computed values given above indicate a decrease of 45 per cent in the current at San Fernando due to grounding the circuit at Somis. This decrease is somewhat greater than the observed decrease. The discrepancy may be due to several causes, the most probable being: (a) greater induction per unit length on San Fernando end due to larger conductors; (b) irregularities in impedance with greater impedance on Somis end.

When, as in this case, the induction is chiefly electrostatic, the voltage to ground of the free end of the circuit when grounded at one end only, is very much less than the voltage with the circuit isolated from ground at all points—see values of open circuit voltages given in Table II.

III. Theoretical Treatment.

A—General Development.

Any transmission line may be represented by a network of equivalent admittances connected between points representing the several conductors of the system, earth included. For such a model the value of the admittance between any two points should equal the direct admittance between corresponding conductors of the system represented. The direct admittance between conductors m and n is defined as the ratio of the

charging current of conductor m to the difference of potential between m and n when all conductors of the system except n are at the same potential as m. "Direct admittance" is analagous to "direct capacitance." For proof of the statements of this paragraph see Alexander Russell's "Alternating Current Theory," Vol. 1, Chapter IV. Russell does not, however, make use of the terms "direct admittance" or "direct capacitance." Refer to technical report No. 51 for definition of direct capacitance.

As it is possible to represent any given transmission line by a network of admittances so it is possible to consider the charging currents of any such system as made up of two sets of components: (1) components flowing between wires through the direct admittances between wires; (2) components flowing between wires and ground through the direct admittances between wires and ground. In computing the charging current of a three-phase system the method almost universally employed is to determine the equivalent capacitance to "neutral" and then make the calculations on a single-phase basis. The equivalent capacitance to neutral takes account of direct capacitance to ground as well as direct capacitance between wires.

Figs. 1 and 2, below, represent, respectively, models of a three-phase system under normal conditions and under the condition of a ground on one phase. Fig. 3 represents the equivalent star admittances (or capacitances) to neutral, useful in determining the charging current under normal conditions.

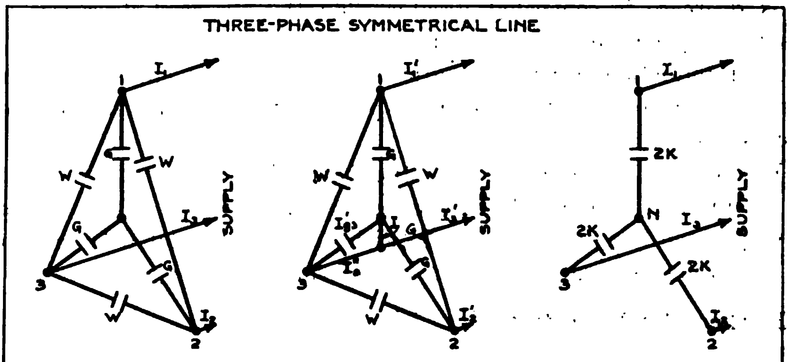


FIG. 1.- MODEL-NORMAL CONDITION.

FIG. 2.- MODEL-ONE PHASE GROUNDED.

FIG. 3.-EQUIVALENT STAR CONDENSERS-NORMAL CONDITION.

Let—

E = normal voltage between wires.

E_G = normal voltage between wires and ground $= \frac{E}{\sqrt{3}}$

I_G = component of normal charging current per phase represented by direct admittance to ground $= E_G G$.

G = direct admittance to ground per phase.

W = direct admittance between wires.

E_R = abnormal residual voltage due to a ground on one phase.

I_R = total abnormal residual current due to a ground on one phase.

Meanings of other symbols will be apparent from the figures.

All the above quantities are to be taken to apply to the line at the point where the "ground" occurs and for the fundamental frequency. For the purposes of this study the residuals of the system in normal operation may be neglected. In other words the system may be assumed to be, normally, perfectly balanced or symmetrical. This assumption is permissible as the unbalances of normal operation are small compared to those of the abnormal condition treated in this report. Also, since a ground may occur on any phase producing effects differing slightly in magnitude, it is well to compute the results for the average case of a symmetrical system.

When a "ground" occurs on one phase that phase immediately assumes ground potential while the other two phases assume a potential above ground equal to the normal voltage between wires. The voltage between wires will remain sensibly unchanged. The residual voltage, being the vector sum of the voltages between wires and ground, becomes the vector sum of two voltages equal in magnitude to the voltages between wires and 60° apart in phase. This sum is equal to $\sqrt{3}\, E$ (173 per cent of the voltage between wires) or 3 E_G. This relation is stated in the Joint Committee's report of July 7, 1914. It is to be noted that this residual voltage extends over the entire system, practically unchanged in magnitude.

The total residual current in the event of a "ground" becomes the vector sum of the charging currents to ground of the two nongrounded phases. As noted above the voltages to ground of these phases become $\sqrt{3}$ times normal, hence the ground components of charging currents are increased in the same proportion, equaling $\sqrt{3}\, I_G$. These currents are 60 degrees apart in phase; their sum is therefore $\sqrt{3}\ (\sqrt{3}\ I_G = 3$ I_G. This is equivalent to stating that the abnormal residual current is equal to the charging current of the three line conductors in parallel when energized single-phase at a voltage to ground equal to the normal voltage between conductors and ground.

From the above it is to be noted that the abnormal residual current bears to the normal ground component of charging current per phase the same relation that the abnormal residual voltage bears to the normal voltage to ground per phase, *i. e.,* in both cases the former is three times the latter in magnitude.

In order to determine the magnitude and phase relation of the currents and voltages under both normal and abnormal conditions the voltages applied to the system represented by Figs. 1, 2 and 3 are assumed to be as follows:

$$\left.\begin{aligned} E_{12} &= E \\ E_{23} &= E\left(-\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right) \\ E_{31} &= E\left(-\tfrac{1}{2} - j\frac{\sqrt{3}}{2}\right) \end{aligned}\right\} \qquad (1)$$

Referring to Fig. 1, representing a model of a line under normal conditions, the following relations hold for the charging currents.

$$I_1 = E\left(\sqrt{3}\,W + \frac{G}{\sqrt{3}}\right) \underline{/120^\circ} \qquad (2)$$

$$= \frac{E}{\sqrt{3}}(3W + G) \underline{/120^\circ} \qquad (3)$$

$$I_2 = \frac{E}{\sqrt{3}}(3W + G) \underline{/240^\circ} \qquad (4)$$

$$I_3 = \frac{E}{\sqrt{3}}(3W + G) \underline{/0^\circ} \qquad (5)$$

The value of $3W + G$ is the admittance of the equivalent condensers in terms of the direct admittances between wires and between wires and ground—see Fig. 3. This value of the admittance of each of the equivalent condensers is equal to double the admittance between any two conductors as measured with the other conductor grounded, or isolated.

Consider the case with one phase grounded which is represented by Fig. 2:

$$\text{Residual current, } I_R = E\sqrt{3}\,G \underline{/180^\circ} \qquad (6)$$

$$\text{Ground current, } I_G = E\sqrt{3}\,G \underline{/0^\circ} \qquad (7)$$

Current in nongrounded phases,

$$I'_1 = E(G^2 + 3\,GW + 3\,W^2)^{\frac{1}{2}} \underline{/\tan^{-1}\frac{G + 3W}{-\sqrt{3}\,(G + W)}} \qquad (8)$$

$$I'_2 = E(G^2 + 3\,GW + 3\,W^2)^{\frac{1}{2}} \underline{/\tan^{-1}\frac{-(G + 3W)}{-\sqrt{3}\,(G + W)}} \qquad (9)$$

Current in grounded phase, supply side of fault,

$$I'_s = E\sqrt{3}\,(W + G)\,\underline{/0^\circ} \tag{10}$$

Current in grounded phase, beyond fault,

$$I''_s = E\sqrt{3}\,W\,\underline{/0^\circ} \tag{11}$$

From (2) and (6) the ratio of the magnitude of the abnormal residual current and normal charging current per phase is,

$$\frac{I_R}{I} = \frac{3G}{3W + G} \tag{12}$$

The values of the susceptance components of W and G are each dependent differently upon the configuration, separation between conductors, height of conductors above ground, and size of conductors. Since in practice the factors separation, configuration, height and size of conductors all vary independently, there is no fixed relationship between W and G for three-phase lines, hence there can be no general simple mathematical relation between abnormal residual current and normal charging current. By considering a number of typical transmission lines it is possible to determine an empirical relation giving approximate limits for the abnormal residual current in terms of normal charging current.

Vector diagrams showing the effects of a ground on the voltages and charging currents of a normally isolated three-phase line are given on P. I. C. No. 301, illustrating graphically, the unbalancing effects which are indicated mathematically by the foregoing equations. These diagrams represent particularly the case of the San Fernando-Somis line, discussed below, but they may be taken as indicative of the general effects on any line. The diagrams illustrate how the actual unbalanced charging currents may be resolved into balanced, residual and single-phase components. The four current diagrams are drawn to the same scale.

Figs. 1 and 2 represent the effect of a ground at the supply end of the line. Fig. 1 shows the currents on the supply side of the ground and Fig. 2 shows the currents immediately beyond the ground. Fig. 1 also shows the currents at the supply end of the line, due to a ground at any point along the line. Figs. 4 and 5 show the effects of a ground at a point distant ⅔ of the full length of the line from the supply end. Fig. 4 shows the currents on the supply side of the ground immediately adjacent to the ground; Fig. 5 shows the currents immediately beyond the ground.

Fig. 1 shows that the unbalanced currents at the supply end may be resolved into two sets of components, balanced and single-phase. The

balanced components are identical in magnitude and phase with the charging currents of the line under normal conditions. The single-phase components in the two nongrounded phases are equal in magnitude to ⅓ of the total current to ground and opposite in time phase thereto. The circuit for these single-phase components is completed by the grounded phase in which the single-phase components sum up to ⅔ the total current in the ground connection and are in time phase with it.

In Fig. 4 the actual unbalanced currents are resolved into three sets of components, balanced, single-phase, and residual. The balanced components are equal in magnitude to the normal charging currents at a point distant ⅔ the length of the line from the supply end. The single-phase components are the same in magnitude and phase as the single-phase components at the supply end of the line. The residual components are equal in magnitude to ⅓ the residual current for the section of line between the supply end and the fault, which in turn is equal to ⅔ of the total current to ground. The phase relationships among the several components will be apparent from the figure.

Fig. 2 shows the currents on the line side of a ground when it occurs at the supply end of the line. The actual unbalanced currents may in this case be resolved into balanced and residual components. The balanced components are the same as the normal charging currents of the line. The residual components are equal to ⅓ the total residual current of the line and are opposite in time phase to the normal charging current of the grounded phase.

Fig. 5 shows the currents immediately beyond the ground when it occurs at a point distant ⅔ the length of the line from the supply end. The condition is exactly similar to that represented by Fig. 2, all the currents being reduced in magnitude in proportion to the distance from the supply end of the line.

Fig. 3 shows the voltage unbalance at all points on the line due to a ground at any point on the line. The actual unbalanced voltages to ground may be resolved into balanced and residual components. The balanced components are equal in magnitude to the balanced components under normal conditions at the point where the ground occurs. The residual components are equal in magnitude to the balanced components and opposite in phase to the balanced component of the grounded conductor. A comparison of the voltage and current diagrams will show the relative phase angles of the components of the voltages and currents.

In the current diagrams the components of the unbalanced currents on the supply side of the ground include in each case a single-phase component in the grounded conductor which divides equally in the

nongrounded conductors. Thus this method of analysis assigns single-phase components to all conductors. This is done, in this instance, in preference to the use of single-phase components in only two of the conductors, as it is simpler and permits the use of balanced components equal to the normally balanced charging currents at the point in question. Also, it follows directly from the use of equivalent admittances, Figs. 1 and 2.

The diagrams are drawn on the assumption of a symmetrical line, *i. e.*, a line which normally gives rise to no residuals. For an unsymmetrical line, therefore, the diagrams will not exactly apply as the magnitudes and phase relationships will vary slightly, dependent upon which conductor becomes grounded.

No consideration has been given in this discussion to load currents, as it has been assumed in all cases that the line was under no load, being open circuited at the receiving end. All the diagrams are drawn on the basis of this assumption. Except in so far as it affects the voltages between wires, the effect of a ground will not alter the strictly load components of the currents. Hence, in order to represent the unbalanced currents due to the occurrence of a ground under load conditions it is necessary simply to superpose (add vectorially) the load components of current upon the unbalanced charging currents. The existence of a heavy load would obviously tend to make less apparent the unbalancing effect of a ground.

Reference is made above to the total residual current by which is meant the current in the contact between line and ground. Should the fault be at one end of the line, the total residual current will all flow in the one direction from the fault. The magnitude of the residual current decreases to zero at the other end of the line. This decrease is very closely a linear function of the distance from the fault, assuming uniform line construction and a line whose length is a small fraction of a wave length for the fundamental frequency. Should the fault occur at a point intermediate between the ends of the line the total residual current will divide at the point of contact, part flowing in each direction.

The following relations hold, the primes (′) and seconds (″) referring to the sections of line in the two directions from the ground.

$$I_R = I'_R + I''_R \quad \text{(vectorially)}$$

$$\frac{I'_R}{I''_R} = \frac{G'}{G''} \qquad \text{(vectorially)}$$

It follows that the total residual current divides approximately in proportion to the lengths of line on the two sides of the fault. The degree of approximation in this statement is dependent upon the similarity of the line characteristics in the two directions.

No mention has been made heretofore of the effect of the contact and ground resistances. As an isolated system increases in extent its admittance to ground increases. For a very large system the contact and ground resistances may therefore appreciably affect the total admittance in the residual current circuit.

A more rigorous deduction of the relationship of abnormal residual current and line constants is given below:

The charging currents of a system of three conductors and earth for any condition of equilibrium with alternating potential differences are:

$$\left.\begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 \\ I_2 &= Y_{12}V_1 + Y_{22}V_2 + Y_{23}V_3 \\ I_3 &= Y_{13}V_1 + Y_{23}V_2 + Y_{33}V_3 \end{aligned}\right\} \qquad (1)$$

$$I_R = I_1 + I_2 + I_3 \qquad (2)$$

The Y's in the above equations are admittance coefficients and are analogous to Maxwell's coefficients of capacitance and electrostatic induction which enter into equations connecting the charges and potentials of a system of electrical conductors in a dielectric medium. The V's are the differences of potential between the three line conductors and ground.

From (1) and (2),

$$I_R = (Y_{11}+Y_{12}+Y_{13})V_1 + (Y_{12}+Y_{22}+Y_{23})V_2 + (Y_{13}+Y_{23}+Y_{33})V_3 \qquad (3)$$

By definition the direct admittance to ground of any conductor of this system is the ratio of its charging current to its potential to ground, *when all other conductors are at its potential.* In other words if we connect all three conductors in parallel, energize them at potential V above ground, and measure their several charging currents we will have from (1)

$$\left.\begin{aligned} Y_{10} &= \frac{I_1}{V} = Y_{11} + Y_{12} + Y_{13} \\ Y_{20} &= \frac{I_2}{V} = Y_{21} + Y_{22} + Y_{23} \\ Y_{30} &= \frac{I_3}{V} = Y_{31} + Y_{23} + Y_{33} \end{aligned}\right\} \qquad (4)$$

$$Y_{123\text{-}0} = Y_{10} + Y_{20} + Y_{30} \qquad (5)$$

Therefore, substituting in (3) the proper values from (4), the equation for I_R becomes,

$$I_R = Y_{10}V_1 + Y_{20}V_2 + Y_{30}V_3 \tag{6}$$

In the case of a symmetrical or balanced line,

$$Y_{10} = Y_{20} = Y_{30} = \frac{Y_{123-0}}{3} \tag{7}$$

As stated before in this section, it is sufficiently accurate for the purpose of this study to take the average direct admittance to ground for the unsymmetrical case. Making this approximation equation (6) becomes,

$$I_R = \frac{Y_{123-0}}{3}(V_1 + V_2 + V_3) \tag{8}$$

$$I_R = \frac{Y_{123-0}}{3} E_R \tag{9}$$

In the event of a ground on one phase that phase assumes ground potential (zero) and the other two phases assume a potential above ground equal to the normal potential between wires.

Let the potential between wires be

$$\left.\begin{aligned} E_{12} &= E \\ E_{23} &= E\left(-\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right) \\ E_{31} &= E\left(-\tfrac{1}{2} - j\frac{\sqrt{3}}{2}\right) \end{aligned}\right\} \tag{10}$$

Suppose a ground to occur on phase (3), then,

$$\left.\begin{aligned} V_1 &= E_{13} = E\left(+\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right) \\ V_2 &= E_{23} = E\left(-\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right) \\ V_3 &= 0 \end{aligned}\right\} \tag{11}$$

$$V_1 + V_2 + V_3 = E_R = j\,E\sqrt{3} \tag{12}$$

Therefore, from (9) and (12),

$$\left.\begin{aligned} I_R &= j\frac{Y_{123-0}}{3} E\sqrt{3} \\ &= j\,Y_{123-0}\frac{E}{\sqrt{3}} \end{aligned}\right\} \tag{13}$$

$$= j\,Y_{123-0}\,E_G \tag{14}$$

where $E_G = \dfrac{E}{\sqrt{3}}$ = normal voltage to ground.

Since Y_{123-0} is the admittance to ground of the three line conductors in parallel, the abnormal residual current is equal to the charging current of the three conductors in parallel when energized single-phase at a voltage above ground equal to the normal voltage between conductors and ground or "neutral." It is evident also that this is three times the magnitude of the average normal ground component of the charging current per phase.

In order to facilitate the application of the results of this study to cases in practice, formulas are given on P. I. C. No. 266, attached, for computing the capacitance to ground of three conductors in parallel, for the more common configurations of conductors which occur in practice.

B—Application to San Fernando-Somis Line.

The results of two separate sets of impedance bridge measurements give the following values for the direct admittances (in micromhos for 50 cycles) of the San Fernando-Somis line. The admittances are sensibly equal to the susceptances since the conductances at 50 cycles are negligible.

	March 12, 1915	June 20, 1915	Average
G	79.2	79.2	79.2
W	28.0	28.1	28.0

From measurements of the charging current of the three conductors in parallel the average value of G is 83.7 micromhos, 5.7 per cent higher than the value obtained by bridge measurements. These measurements of the charging current were made at the operating frequency of the power system which presumably is very close to 50 cycles per second.

From computations of the capacitance of the three conductors in parallel (formula for case of vertical line, P. I. C. No. 266) the value of 71.2 micromhos is obtained for the average G of this line.

Based on the above three values of the average direct admittance to ground per phase, and the line voltages, the following values of abnormal residual current are obtained for comparison with the value observed in the tests.

TABLE III.

Residual Current Due to Ground on One Phase—Amperes.

Observed value	Computed values		
	Measured constants		Computed constants
	By bridge	By 50 cycles	
2.34*	2.21	2.33	2.0

*Average of I_R and I_G from Table I.

The ratios of all the currents under the abnormal condition to the normal charging current, are given below; (a) from values observed during the test, (b) and from computations based on the measured values of the direct admittances.

Abnormal current	Ratio	
	Observed	Computed
Residual	1.42	1.45
Grounded phase, supply side of fault	1.92	1.97
Grounded phase, beyond fault	0.501	0.514
Nongrounded phases	1.28	1.31

The agreement between the ratios of the currents as observed in the test and the computed ratios is very satisfactory, the slight discrepancies being within the limits of experimental error.

Vector diagrams showing graphically the results for this line are given on P. I. C. No. 301, attached. These diagrams, though for this line in particular, are typical and were therefore discussed above in connection with the general treatment.

C—PUBLISHED DATA.

The results heretofore given in this report had been practically completed, when reference was made to an article* by George S. Humphrey, appearing in the Electrical World of November 25, 1911—Vol. 58, page 1300. This article gives an analysis of the variations of the charging currents of a three-phase system when one conductor is connected to earth. It is of special interest in connection with this study, for the author derives the fundamental equations for computing the charging currents under any condition and also gives several numerical examples illustrating the application of the theory. The development of the theory as given in this article is somewhat different from that used in this report, but fundamentally the two treatments are the same. The writer of the article does not use the term "residual current" but instead speaks of the "charging current supplied to the ground," which, with the exception of one numerical example, is equivalent to "residual current."

The results of the numerical examples are of such interest in connection with this study that they are given herewith:

Example I. Flat spacing, outside conductors 12 feet from middle, height of all 40 feet above ground. Conductors of 7 strands of No. 8 B&S gauge.

*Entitled "The Charging Currents of Three-phase Transmission Lines."

Two conditions of a grounded conductor are considered: (a) outside conductor grounded, and (b) middle conductor grounded. The ratios of the several currents to the normal charging current, under the abnormal conditions (a) and (b), are as follows:

Abnormal current	Ratio	
	Grounded conductor	
	(a) Outside	(b) Middle
Residual	1.64	1.73
Grounded phase:		
(1) Supply side of fault	2.02	2.23
(2) Beyond fault	0.38	0.50
Nongrounded phases:		
(1) Middle conductor	1.35	—
(2) Outside conductors	1.31	1.37

Example II. Equilateral triangular spacing, vertex upward, base of triangle 25 feet above ground. Conductors 6 feet apart, No. 0 B&S gauge.

The ratios of abnormal currents to the normal charging current for the two conditions, (a) outside conductor grounded, and (b) middle conductor grounded, are given as follows:

Abnormal	Ratio	
	Grounded conductor	
	(a) Outside	(b) Middle
Residual	1.43	1.47
Grounded phase:		
(1) Supply side of fault	1.95	2.00
(2) Beyond fault	0.52	0.53
Nongrounded phases:		
(1) Middle conductor	1.29	—
(2) Outside conductors	1.32	1.33

Example III. Same as example II with a 0.25-inch ground wire strung three feet vertically above middle conductor. The results given for this case are:

Abnormal current	Ratio	
	Grounded conductor	
	(a) Outside	(b) Middle
Residual:		
(1) Ground component	1.19	1.33
(2) Ground wire component	0.53	0.30
(3) Total	1.70	1.63
Grounded phase:		
(1) Supply side of fault	2.17	2.06
(2) Beyond fault	0.45	0.44
Nongrounded phases:		
(1) Middle conductor	1.44	—
(2) Outside conductors	1.35	1.35

Comparing example III with example II, it is to be noted that the effect of the ground wire is to reduce the component of the residual current flowing in the ground. The total residual current (vector sum of currents in ground and ground wire) is, however, increased. When the middle conductor is grounded the total residual current is equal to the numerical sum of the currents in the ground and in the ground wire, since these currents are in the same time phase; when one of the side conductors is grounded the condition is unsymmetrical with respect to the ground wire and in this case the current in the ground wire and in the ground differ in time phase by 18 degrees, hence the total residual current is slightly less than the numerical sum of the two components. On the whole, the effect of the ground wire is slightly beneficial from the standpoint of induction from abnormal residual current, since the decrease in the ground current overbalances the added effect of the current in the ground wire. (For a consideration of the effect of a ground wire on the residual voltage under normal conditions, see technical report No. 51.) The matter of inductive interference was not considered in the paper from which these results are taken.

Summarizing the results of his study, the author concludes as follows:

> "A grounded conductor produces about the same relative effect for all types of construction which are used in practice. In general, with one conductor grounded the charging current of the two ungrounded conductors becomes approximately 1.35 normal; and that of the grounded conductor on the side of the ground away from the generator, approximately 0.50 normal; the total charging current which must be supplied to the grounded conductor is approximately 2.10 normal."

IV. Conclusions.

The conclusions to be drawn from this study of the effects of a "ground" on one phase of a normally isolated three-phase system may be briefly stated as follows:

1. A residual voltage equal in magnitude to $\sqrt{3}$ times the normal voltage between wires or three times the normal voltage between wires and ground is created. This residual voltage extends over the entire system practically unchanged in magnitude.
2. The total residual current produced is equal to the charging current of the three line conductors energized in parallel, at a single-phase voltage equal to the normal voltage between conductors and ground. This, in general, is equal approximately to 1.5 the normal charging current per phase.
3. The current in the ground contact is equal in magnitude to the total residual current.
4. If the "ground" occurs at a point intermediate between the two ends of the line the residual currents in the two sections of line are opposite in phase. At the fault the residual currents in the two directions are approximately proportional to the lengths of line on the two sides of the fault, and their numerical sum is equal to the total residual current, or current in ground contact.
5. The residual currents diminish approximately uniformly to zero at the ends of the line.
6. The unbalanced charging currents supplied to the nongrounded phases are approximately 1.3 normal.
7. The unbalanced charging current supplied to the grounded phase is approximately double the normal charging current.
8. The charging current in the grounded phase immediately beyond the "ground" is approximately 0.5 of the normal charging current of the section of line beyond.
9. The existence of a load upon the line does not affect the residuals.
10. Under load conditions the unbalanced charging currents are superposed upon the load currents to give the actual unbalanced currents.
11. Published information indicates that the effect of a ground wire carried above a single circuit line is to increase somewhat the total residual current in the event of a "ground," but to decrease that component flowing in the ground (part flows in ground wire).

The above statements as to the magnitudes of the residual current and the unbalanced charging currents in terms of normal charging current of the line are average figures based upon data of several typical cases. They are, therefore, not exact in all cases but are considered to represent fairly closely the cases within practical range. No distinction has been made as to the slightly different effects of a ground on different phases of an unsymmetrical or nontransposed system. These figures are also given with particular reference to single

circuit lines; their applicability to the conditions of double circuit lines is accordingly limited.* Published data in regard to this subject is in accord with that obtained in this study.

Respectfully submitted.

(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

ATTACHED: P. I. C. 286 and 301.

APPROVED: August 4, 1915.

(Signed) R. W. MASTICK,
Field Engineer.

APPROVED: September 28, 1915.

SUBCOMMITTEE ON TESTS,
(Signed) J. E. WOODBRIDGE,
Chairman.

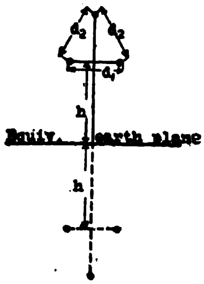
JOINT COMMITTEE ON INDUCTIVE INTERFERENCE,
(Signed) ARTHUR F. BRIDGE,
Secretary.

October 15, 1915.

*See supplementary memorandum immediately following.

CAPACITANCE TO EARTH
THREE CONDUCTORS IN PARALLEL.
r = radius of conductors.

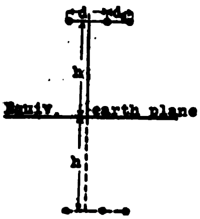
P.I.C. No.266
Revised
7/27/15



$$a = \log_{10} \frac{2\left[h+\sqrt{d_2^2 - \frac{d_1^2}{4}}\right]}{r} \qquad e = \log_{10} \frac{\left[\left(2h+\sqrt{d_2^2 - \frac{d_1^2}{4}}\right)^2 + \frac{d_1^2}{4}\right]^{\frac{1}{2}}}{d_2}$$

$$b = \log_{10} \frac{2h}{r} \qquad g = \log_{10} \frac{\left[(2h)^2 + d_1^2\right]^{\frac{1}{2}}}{d_1}$$

$$C = 0.0388\ \frac{b + g + 2a - 4e}{a(b + g) - 2e^2}$$



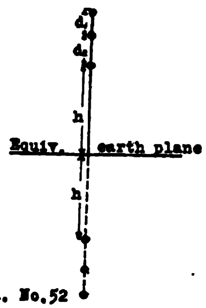
$$a = \log_{10} \frac{2h}{r} \qquad f = \log_{10} \frac{\left[(2h)^2 + (d_1 + d_2)^2\right]^{\frac{1}{2}}}{d_1 + d_2}$$

$$e = \log_{10} \frac{\left[(2h)^2 + d_1^2\right]^{\frac{1}{2}}}{d_1} \qquad g = \log_{10} \frac{\left[(2h)^2 + d_2^2\right]^{\frac{1}{2}}}{d_2}$$

$$C = 0.0388\ \frac{a(3a-2g-2f-2e)-g(g-2e-2f)-(e-f)^2}{a(a^2-e^2-f^2-g^2)+2efg}$$

if $d_1 = d_2$ $e = g$ and

$$C = 0.0388\ \frac{3a - 4e + f}{a(a + f) - 2e^2}$$



$$a = \log_{10} \frac{2(h+d_1 + d_2)}{r} \qquad e = \log_{10} \frac{2h+d_1 +2d_2}{d_1}$$

$$b = \log_{10} \frac{2(h+d_2)}{r} \qquad f = \log_{10} \frac{2h+d_1 +d_2}{d_1 + d_2}$$

$$c = \log_{10} \frac{2h}{r} \qquad g = \log_{10} \frac{2h+d_2}{d_2}$$

$$C = 0.0388\ \frac{a(b+c-2g)+b(c-2f)-2ec-g(g-2e-2f)-(e-f)^2}{abc+2efg-ag^2-bf^2-ce^2}$$

Values of C, in all cases, are in microfarads per mile.

T.R. No.52

P.I.C. No. 301
7-18-15.

VECTOR DIAGRAMS SHOWING EFFECTS OF A GROUND ON THE VOLTAGES AND CHARGING CURRENTS OF A NORMALLY ISOLATED THREE-PHASE LINE.

PHASE 3 GROUNDED IN ALL CASES.

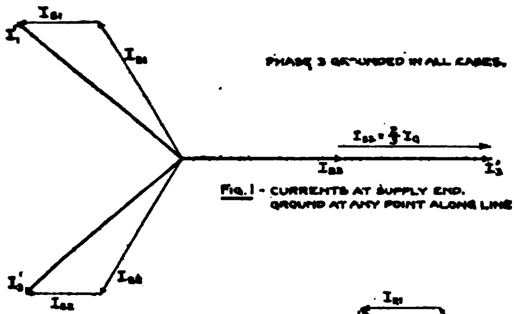


FIG. 1 - CURRENTS AT SUPPLY END. GROUND AT ANY POINT ALONG LINE

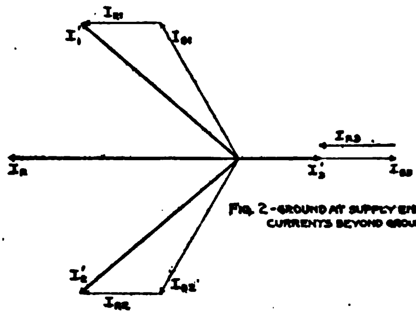


FIG. 2 - GROUND AT SUPPLY END. CURRENTS BEYOND GROUND.

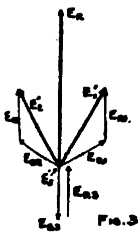
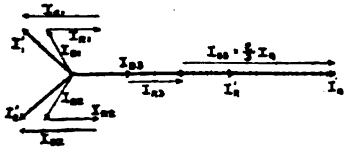
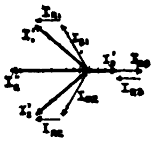


FIG. 3

VOLTAGES AT ANY POINT. GROUND AT ANY POINT.





T.R. No. 52

FIGS. 4 AND 5. - GROUND DISTANT TWO-THIRDS LENGTH OF LINE FROM SUPPLY END.

FIG. 4 - CURRENTS ON SUPPLY SIDE OF GROUND.

FIG. 5 - CURRENTS BEYOND GROUND

RESIDUALS PRODUCED BY A GROUND ON ONE CONDUCTOR OF TWIN THREE-PHASE CIRCUITS, NORMALLY WITHOUT METALLIC CONNECTIONS TO GROUND.

July 3, 1917.

Memorandum Supplementing Technical Report No. 52.

In connection with the study of twin circuits carried out for technical report No. 65, opportunity was given to determine, with small additional labor, the effect of grounds on twin-circuit lines normally without metallic connection to ground. Accordingly, such computations have been made and the results are presented herewith.

The theory involved has been adequately treated in technical report No. 52, hence it need not be discussed here. It should be noted, however, that the numerical results given in technical report No. 52 apply only to single-circuit lines.

There are two different cases of twin-circuit lines which require consideration and which will give different results in the event of an accidental ground. They are: (1) circuits metallically interconnected and (2) circuits not metallically interconnected. For convenience these two cases are treated separately.

1. *Circuits Metallically Interconnected.*

Circuits paralleled on the line side of connected transformer banks meet the condition imposed by this case. When a ground occurs on one conductor, this conductor assumes ground potential, as does also the conductor of the corresponding phase of the other circuit, to which the grounded conductor is metallically connected. The conductors of the other two phases of both circuits assume a potential above ground equal to the normal difference of potential between conductors. Thus the residual voltage of each circuit becomes $\sqrt{3}$ times the normal voltage between conductors or three times the normal voltage between conductors and ground. This value is the same as occurs in the case of the single-circuit line.* The total residual current is the product of one-third the residual voltage, *i. e.*, the normal voltage to ground, and the capacitance of the six wires in multiple to ground. This is approximately 40 per cent greater than the corresponding capacitance of a single-circuit line of the same conductor spacing, assuming the twin circuit construction to be uniform throughout the extent of the metallically connected network.

The numerical values of the residual current and abnormal charging currents in terms of the normal charging current per conductor will vary somewhat, depending upon the size and spacing of conductors, the

*See T. R. No. 52.

configuration and height above ground. The values given below apply to high-voltage lines of vertical configuration. They will also apply with close approximation to types of construction in which the top and lowest conductors are in the same vertical plane and the middle conductors displaced slightly outward.

	Twin-circuit	Single-circuit
Residual voltage (each circuit)	1.73 E	1.73 E
Residual current (total)	2.17 I	1.48 I
Charging currents—		
Nongrounded conductors	1.22 I	1.32 I
Grounded conductor beyond fault	0.64 I	0.51 I
Associated conductor of other circuit	0.64 I	
Total supplied to grounded conductor	2.81 I	1.99 I

E = Normal voltage between conductors.
I = Normal charging current per conductor.

It is assumed in this example that the twin-circuit construction is uniform and extends throughout the whole length of line. The results will be modified if there is considerable single-circuit construction, aproaching as a limit the results for single circuits when the length of twin-circuit construction is a small portion of the total.

The inductive effects on a parallel communication circuit of the abnormal residual voltage are approximately 40 per cent greater than those occurring in the case of a single-circuit line. When the twin-circuit construction is uniform throughout the total length of line, the inductive effects from the abnormal residual current are 40 per cent greater than those of a single-circuit line of the same length and character. The inductive effects of the abnormal residual current may be increased if there are considerable portions of single-circuit construction metallically connected.

2. *Circuits Not Metallically Interconnected.*

Circuits paralleled only on the station side of connected transformer banks, or not paralleled at all, meet this condition. When a ground occurs on one conductor of one of the circuits a residual voltage equal to the $\sqrt{3}$ times the normal voltage between conductors is caused in this circuit. A residual voltage is induced in the other circuit. The magnitude of this induced residual voltage will vary, depending upon the distance between the two circuits, their configuration and height above ground, the size of conductors and upon whether the construction is uniform throughout the metallically connected length of both circuits. The magnitudes of the residual current and abnormal charging currents in terms of the normal charging current per conductor are also dependent upon these factors. If the twin-circuit construction is uniform

throughout the metallically connected network the charging currents of the nongrounded circuit are not affected, while in the circuit having the grounded conductor the values are the same as in the case of the corresponding single-circuit line. For high-voltage twin vertical circuits the residual voltage induced in the nongrounded circuit is about 0.7 the normal voltage between conductors of the first circuit.

From the point of view of induction in parallel communication circuits the effects produced by a ground on one conductor of twin-circuit lines of this class (circuits *not* metallically interconnected) are approximately the same as in the case of corresponding single-circuit lines, provided the twin-circuit construction extends uniformly throughout the metallically connected network. Thus the presence of the nongrounded circuit may be neglected in obtaining the abnormal induction in the communication circuit. If there are considerable portions of single-circuit construction electrically connected to the nongrounded circuit the inductive effects on the communication circuit may be considerably lessened as compared to those occurring in the case of a single-circuit line, due to the shielding effect of the nongrounded circuit.

Respectfully submitted.

(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

Technical Report No. 54.

October 1, 1915.

San Fernando-Somis Parallel.

EXPERIMENTAL DETERMINATION OF COEFFICIENTS OF LONGITUDINAL INDUCTION AT 50 CYCLES—EFFECT OF TRANSPOSITIONS IN POWER CIRCUIT.

February 8-25, 1915.

OUTLINE.

I. Introduction.

The close and uniform association of the power and telephone circuits carried on the San Fernando-Somis transmission line of the Pacific Light and Power Corporation afforded an unusual opportunity for a comparison of experimental determinations and computed values of the coefficients of electric and magnetic induction, and for a test of the

effectiveness of transpositions in the power and telephone circuits. The tests described in this report were accordingly made to determine the induction in the telephone circuit from given voltages and currents in the power circuit and the effect thereon of transpositions in the power circuit. Tests were made with the power circuit energized in four different ways, so as to give in turn (A) balanced voltages (B) balanced currents (C) residual voltage (D) residual current, and under three conditions of the power circuit as regards transpositions (*a*) no transpositions (*b*) one barrel and (*c*) two barrels. The telephone transpositions remained the same throughout the tests.

It was not possible to make satisfactory measurements of the coefficients of transverse induction with the 50-cycle supply as used in these tests. This was due chiefly to the effectiveness of the telephone transpositions. Hence, this report is concerned only with the longitudinal induction. For a consideration of transverse induction and of the effect of telephone transpositions see technical reports Nos. 55 and 56, wherein are given, also, the computed coefficients of induction referred to in this report.

Energy supply at a frequency of 50 cycles was used for all the tests. Under the special conditions employed, the harmonics present were too small for satisfactory measurement and therefore the coefficients of induction are considered for 50 cycles only. The detailed arrangements and results are given in III, below. Except where otherwise stated, the induced voltages and currents were measured on the two wires of the telephone circuit in parallel, to ground.

II. Description of Parallel.

A—Physical Characteristics of Lines.

1. *Power Line.*

The power line involved is the San Fernando-Somis 15000-volt single-circuit three-phase line of the Pacific Light and Power Corporation, which extends a distance of 36.7 miles beyond the test pole at the Joint Committee's laboratory. Following is a tabulated description:

Nominal voltage—15000 volts.
Length—36.7 miles.
Type of support—Poles, 90.2%.
A-Frames, 9.8%.
Configuration—Vertical—five-foot spacing, on poles.
Vertical—six-foot spacing, on A-frames.
Height of upper conductor—average 50 feet.
Crossarms—Wooden normally;
Steel at crossings and corners.
Insulators—Pin type Locke No. 408-A normally,
Two-disc suspension type at crossings, corners and on A-frames.

Conductors—No. 4 B&S solid copper, 78.6%.
No. 2 B&S solid copper, 7.8%
No. 2 B&S stranded copper, 13.6%.

2. *Telephone Line.*

The telephone line is the power company's private telephone circuit, carried on the same poles as the power circuit. The telephone circuit is on the opposite side of the pole from the power circuit, on a cross-arm 11 feet below the lowest cross-arm of the power circuit.

Spacing—23 inches on poles,
90 inches on A-frames.
Cross-arms—9 ft. wooden on poles.
Conductors—No. 9 BWG iron normally; ¼-inch messenger cable for several long spans on A-frames.

P. I. C. No. 230, attached to technical report No. 56, gives a cross-sectional diagram of the line showing the locations of the conductors of both circuits, where supported on poles.

B—Transposition Systems.

1. *Power Circuit.*

The transposition systems employed were designed to divide the line into sections of equal length: first into three equal sections by two transpositions (one barrel) and then into six equal sections by three additional transpositions (two barrels). No effort was made to modify the location of transpositions to compensate for irregularities in the line, such as differences in sizes and spacings of conductors. The distances between transpositions for each of the two lengths of barrel are given below.

TABLE I.
Transposition Spacings.

Transposition number	Spacing: One barrel, feet	Spacing: Two barrels, feet	Total distance feet
Laboratory			0
		33,599	
1	64,261		33,599
		30,662	
2			64,261
		32,885	
3	64,001		97,146
		31,116	
4			128,262
		33,568	
5	65,756		161,830
		32,188	
Somis			194,018

2. *Telephone Circuit.*

The telephone circuit is transposed, on the average, every 1180 feet (about every fourth pole); a total of 164 transpositions between the laboratory and Somis. The resultant lengths of unbalanced exposure of the telephone circuit to induction from currents and voltages in the power circuit are as given in the table below:

TABLE II.

Resultant Lengths of Unbalanced Exposure in Feet. Telephone Circuit to Power Circuit. San Fernando-Somis Line.

Disturbing factor	Type of induction	Type of support	Unbalanced exposure		
			No transpositions	36-mile barrel	18-mile barrel
Balanced voltages and currents.	Transverse	Poles A-frames	2,970 353	950 257	1,750 257
	Longitudinal	Both	194,018	1,898	1,164
Residual voltages and currents.	Transverse	Poles A-frames	2,970 353	2,970 353	2,970 353
	Longitudinal	Both	194,018	194,018	194,018

III. Tests—Description and Results.

A—Balanced Voltages.

1. *Description of Method.*

For the balanced-voltage test the line was energized through two banks of transformers; the step-down bank being connected delta-delta and the step-up bank delta-star, with neutral isolated. At Somis the transformer bank was connected to the line in interconnected star-delta with neutral grounded. For the three systems of power circuit transpositions, measurements were made of induced voltage and current to ground in the telephone circuit both with the far end clear and with it grounded.

2. *Summary of Results.*

The detailed observations and results are given in a memorandum on file. The following table summarizes the results for the three transposition systems.

TABLE III.

Longitudinal Induction from Balanced Voltages. Telephone Circuit Isolated at Somis.

Transposition system	Ratio of induction to line voltage (to ground)					
	Induced voltage volts per kilovolt			Short-circuit current m A. per kilovolt		
	Meter	Osc.	Mean	Meter	Osc.	Mean
No transpositions	49.0	52.8	52.1	8.7	9.4	8.9
One barrel	1.31	1.36	1.34	0.29	0.30	0.30
Two barrels	1.01	0.92	0.96	0.20	0.18	0.19

3. *Discussion* (*Balanced Voltages*).

The results for the far end of the telephone circuit grounded are not given in the above table. They are of value only in determining the possible error due to electrostatic induction from balanced voltages in the balanced current tests. Similarly the measurements in the balanced current test of the induction with the far end of the telephone circuit clear are of value in determining the possible error due to electromagnetic induction from the charging currents in the balanced voltage tests. For balanced voltages the error due to this source is less than ½% in the case of no transpositions, but may amount to not more than 2% in the case of one and two barrels.

The voltage to ground induced in the telephone circuit from balanced voltages in the power circuit is 52 volts per kilovolt when there are no transpositions in the power circuit. For one barrel the induced voltage is reduced to 2.6% and for two barrels to 1.8% of the voltage obtained with no transpositions. The resultant lengths of unbalanced exposure are respectively 0.98% and 0.60% of the unbalance, with no transpositions. The ratio of induced voltage with two barrels to induced voltage with one barrel is 72%; the corresponding ratio of length of unbalanced exposure is 61%. The discrepancy between ratios of induced voltages and corresponding ratios of lengths of unbalanced exposure is due probably to irregularities in the power and telephone circuits, such as different sizes and spacings of conductors in the various transposition sections. The results are considered satisfactory as the quantities measured were small.

A comparison of the experimentally determined coefficient of induction, 52 volts per kilovolt, with a coefficient, 52.0 volts per kilovolt, computed from the dimensions of the systems, in the portion supported on poles, shows very satisfactory agreement between the two. The computed coefficients are discussed in technical report No. 56.

B—Balanced Currents.

1. *Description of Method.*

In order to obtain the maximum inductive effect from balanced currents with a minimum of electrostatic induction from the balanced voltages, the power circuit was short-circuited at the Somis end. The circuit was energized at low voltage at the laboratory, giving balanced currents of a suitable magnitude. The induced voltage and current to ground in the telephone circuit were measured simultaneously with the currents in the power circuit, for the three transposition systems.

2. *Summary of Results.*

The detailed observations and results are given in a memorandum on file. The following table summarizes the results for the three transposition systems.

TABLE IV.

Longitudinal Induction from Balanced Currents. Telephone Circuit Grounded at Somis.

Transposition system	Ratio of induction to balanced current					
	Induced voltage volts per ampere			Short-circuit current m A. per ampere		
	Meter	Osc.	Average	Meter	Osc.	Average
No transpositions	1.69	1.69	1.69	5.3	5.4	5.4
One barrel			0.04—			0.1—
Two barrels			0.02—			0.06—

3. *Discussion* (*Balanced Currents*).

With no transpositions in the power circuit, the longitudinal induced voltage in the telephone circuit is 1.69 volts for each ampere of balanced current in the power circuit. This corresponds to a coefficient of induction of 27.7 microhenrys per 1000 feet. A coefficient of 26.3 microhenrys has been computed from the dimensions of the system. The two results differ by only five per cent, which is considered a very satisfactory agreement. It may be noted in this connection that the telephone conductors are of iron, whereas the computations were based on a conductor of unit permeability.*

For one barrel and two barrels in the power circuit the induction into the telephone circuit was so small as to make precise measurement impossible, hence the table gives an upper limit only for the induction under these conditions. The induced voltages are respectively less than 2.4 per cent and less than 1.2 per cent of the induced voltage obtained with no transpositions.

*See T. R. No. 56.

C—Residual Voltage.

1. *Description of Method.*

With the Somis end isolated the power circuit was arranged at the laboratory so that the conductors might be energized singly, two in parallel or three in parallel. The voltage induced in the telephone circuit was measured under several conditions. In order to study the effects of shielding, arrangements were made for grounding the unenergized conductor or conductors at the laboratory, and measuring the induced voltage or current in the telephone circuit under these conditions. Moreover, the effect of grounding one of the telephone conductors upon the induced voltage in the other was determined in a similar manner.

2. *Summary of Results.*

The detailed observations and results are given in a memorandum on file. The following table summarizes the results obtained for the various conditions of the power and telephone circuits.

TABLE V.

Longitudinal Induction from Residual Voltage. Telephone Circuit Isolated at Somis. (One barrel in power circuit.)

Number of power conductors			Ratio of induction to residual voltage	
Energized	Grounded	Isolated	Induced voltage volts per kilovolt	Short-circuit current mA. per kilovolt
3	0	0	91	16.8
2	0	1	99	19.8
1	0	2	129	23.8
2	1	0	88	17.1
1	2	0	86	16.3
1	1	1	106	20.0

3. *Discussion* (*Residual Voltage*).

The term "residual voltage" is here applied to the energized conductors only. The coefficient for residual voltage of the three-phase circuit is given by the results for all three power conductors energized. A comparison of these results with the results given in Table III shows the relative effects of unit values of balanced and residual voltages. The results for other cases given in Table V are for special conditions not to be confused with the residual voltage for the three-phase circuit.

The power circuit was transposed for one barrel at the time these observations were made and hence the effects of different separations between disturbing and disturbed conductors could not be determined.

The coefficient of induction for the telephone circuit and one power conductor is approximately the average of the coefficients for the telephone circuit and the three power conductors energized singly and with no transpositions in the power circuit. This has been assumed in comparing the experimentally determined and computed coefficients for this case.

The experimentally determined coefficients are: for three power conductors energized 91 volts per kilovolt, two conductors energized 99 volts per kilovolt and for one conductor energized 129 volts per kilovolt residual. The coefficients have not been computed for two power conductors energized, but for three conductors energized and one conductor energized, they are respectively 99 and 157 volts per kilovolt. For both cases the measured coefficient is smaller than the computed coefficient; three conductors 8%, one conductor 18%. With three conductors energized, a much larger number of observations were made than was the case with one conductor energized (about twenty as compared to three). The individual observations varied over a range of 15%.

When three power conductors are energized, grounding one of the telephone conductors reduces the voltage induced in the other 33%. When two power conductors are energized grounding the third power conductor reduces the voltage induced in the telephone circuit 16%. With one power conductor energized, grounding one of the idle power conductors reduces the induced voltage in the telephone circuit 18%, grounding both idle power conductors causes a reduction of 33%. There is much closer association between the two telephone conductors than between the two telephone conductors and one power conductor. Hence a greater shielding effect on one telephone conductor is observed by grounding the other than by grounding one power conductor. When two power conductors are grounded, however, the shielding effect is about the same as that obtained by grounding one of the telephone conductors.

D—Residual Current.

1. *Description of Method.*

For the residual-current test both power and telephone circuits were grounded at Somis, connection being made to a common ground electrode. A common ground connection was likewise used at the laboratory end of the circuits. Facilities were provided for energizing the power conductors singly, two in parallel or three in parallel. The voltages in the telephone circuit and also in the unenergized power conductors were measured under the various conditions of energy supply. Some measurements were also made to determine the shielding effects obtained by

grounding the unenergized power conductor or conductors, and of grounding one of the telephone conductors.

2. *Summary of Results.*

The detailed observations and results are given in a memorandum on file. The following table summarizes the results obtained for the various conditions of the power and telephone circuits.

TABLE VI.

Longitudinal Induction from Residual Currents. Power and Telephone Circuits Grounded at Somis.

(One barrel in power circuit.)

Conditions at laboratory				Open circuit voltage, volts per ampere			Short circuit current, mA. per ampere		
Number of power conductors			Idle telephone conductor	Telephone		Power conductor	Telephone		Power conductor
Energ.	Isol.	Grd.		One conductor	Two conductors		One conductor	Two conductors	
3	0	0	Isol.				28.3		
3	0	0			18.6			56	
3	0	0	Grd.				27.6		
2	1	0			19.0			58	
2	0	1			13.6			42	
1	2	0	Isol.	18.7					
1	2	0			19.1			59	
1	1	1	Isol.	13.6	13.8			43	
1	1	0				21.9			311
1	0	1				15.9			234
1	0	2	Isol.	11.1					
1	2	0	Grd.	18.3					
1	0	2	Grd.	10.9					

3. *Discussion* (*Residual Current*).

As in the residual voltage test, the coefficient for residual current of the three-phase circuit is given by the results for all three power conductors energized. These results should be compared with those given in Table IV to show the relative effects of unit values of balanced and residual currents. The results for the other cases given in Table VI represent special conditions. The term "residual current" in Table VI applies to the energized conductors only.

The power circuit was transposed for one barrel at the time of this test and hence the longitudinal voltage in the telephone circuit per ampere of residual current in the power circuit is the same whether one, two or three of the power conductors are energized.

When two power conductors are energized the voltage in the telephone circuit is reduced 28% by grounding the third power conductor.

When one power conductor is energized, grounding one of the unenergized conductors reduces the voltage in the telephone circuit 28%, and grounding both unenergized conductors causes a reduction of 41%. The electromagnetic shielding effect of one telephone conductor on the other is very small, the reduction in voltage being about 2%. With one power conductor energized, the voltage in one of the unenergized power conductors is reduced 27% by grounding the third power conductor. These open circuit voltages of Table VI, upon which this discussion is based, include both induction and conduction effects as pointed out below.

In making these measurements disturbing and disturbed circuits were connected to ground through the same electrodes. Hence part of the observed open circuit voltage is due to a conduction effect, the IR drop of the residual current in the common ground resistance. The induction and conduction components of the voltage in the telephone circuit are in quadrature. If we denote the residual current by I, the mutual inductance by M, the common ground resistance by R, and the open circuit voltage of the disturbed circuit by e, we have:

$$e = I\sqrt{R^2 + (2\pi fM)^2}$$

hence

$$M = \frac{1}{2\pi f}\sqrt{\frac{e^2}{I^2} - R^2}$$

The direct measurements of ground resistance varied from 8 to 19 ohms, on different days. The impedance of the power circuit as determined by the ratio of impressed voltage to residual current indicates a ground resistance of 12 ohms at the time these measurements were made. Correcting for the conduction effect the value of 14.2 volts induced per ampere of residual current is obtained, compared with the value of 18.6 volts for the open circuit voltage. If we compute M from the equations above, using R = 12 ohms, we obtain a value of 233 microhenrys per 1000 feet for the mutual inductance between the power conductors energized singly or in parallel and the telephone circuit; and 301 microhenrys per 1000 feet for the mutual inductance between any two power conductors. The computed values are respectively 222 microhenrys and 290 microhenrys per 1000 feet, which differ from the experimental values by 5% and 4%, respectively, for the two cases. These computed values depend upon an experimental determination, by impedance measurements on the power circuit, of the depth of the equivalent ground plane. The location of this plane, as determined by the impedance measurements, is at a distance of 360 feet below the middle power conductor.

IV. Conclusions.

As was pointed out in the introduction these tests gave satisfactory results only for the fundamental frequency, fifty cycles per second, and hence the conclusions apply to that frequency only. Moreover, longitudinal induction only is considered, the transverse induction being considered in technical report No. 55.

Very satisfactory agreement was obtained in all cases between the observed and computed coefficients of induction. The comparisons have been made in the discussions at the end of each section. A general summary of the principal results for the nontransposed power circuit is given in the following table:

TABLE VII.
General Summary—Nontransposed Power Circuit.

Power circuit		Telephone circuit		
Disturbing factors		Longitudinal induction volts per volt or amp.		Experimental / Computed
		Experimental	Computed	
Voltage	Balanced	0.052	0.0520	1.00
	Residual	0.091	0.099	0.92
Current	Balanced	1.69	1.60	1.05
	Residual	14.2	13.5	1.05

Comparison of the experimental and computed results for electromagnetic induction indicate that, under the conditions of this test, the material of the disturbed conductors has negligible effect on the magnitude of the induced voltage, as no account was taken in the computations of the fact that the telephone conductors are of iron. Obviously, the material of the conductors has no effect upon the electrostatic induced voltages.

In the residual voltage and current tests a considerable reduction in the induction in the telephone circuit was observed upon grounding the unenergized power conductor or conductors. In the residual voltage test a large shielding effect on one telephone conductor was observed when the other was grounded. In the residual current tests, grounding one of the telephone conductors shielded the other only about two per cent. For effective electromagnetic shielding, the shielding circuit should be of low resistance and closely coupled with either the disturbed or disturbing circuits. As regards coupling, the power conductor is somewhat less effectively situated than the telephone conductor. The much greater shielding effect of the power conductor is due, therefore,

to its low resistance as compared with the smaller iron telephone conductor. Electrostatic shielding is not affected materially by the impedance of shielding conductors for the induced charging current is relatively small so that the potential of the shielding conductors is only very slightly modified by the impedance drop.

To compute the induction in the telephone circuit, under operating conditions of the power circuit, it is necessary to know the phase relationship between the components of the total induction due to the various components of current and voltage in the power circuit, as well as the magnitudes of the coefficients of induction. These phase relationships are not determined by the tests recorded in this report. With the computed coefficients discussed in technical report No. 56 is given the phase of the induction with respect to the particular component of voltage or current from which it arises. Hence if the magnitudes and phase relationships of the various components of current and voltage in the power circuit are known, the resultant induction in the telephone circuit may be determined.

A comparison of the results for the three conditions as regards power circuit transpositions shows that the use of either one 36 mile barrel or two 18 mile barrels practically neutralized the longitudinal induction from balanced currents and voltages at fifty cycles. At this frequency the 36 and 18 mile barrels give approximately 78 and 157 barrels per wave length, respectively. Roughly the same degree of neutralization may be expected for higher frequencies with a corresponding decrease in the length of barrel. This is not strictly so, due to the fact that the line is short and hence reflection effects from the terminals will differ at different frequencies.

Respectfully submitted.

(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

IN FILES OF SECRETARY: Memorandum of detailed results, including prints of oscillograms.

APPROVED: October 6, 1915.

(Signed) R. W. MASTICK,
Field Engineer.

APPROVED: October 7, 1915.

SUBCOMMITTEE ON TESTS,
(Signed) J. E. WOODBRIDGE,
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JOINT COMMITTEE ON INDUCTIVE INTERFERENCE,
(Signed) ARTHUR F. BRIDGE,
Secretary.

October 15, 1915.

Technical Report No. 55.

January 7, 1916.

EXPERIMENTAL DETERMINATION OF COEFFICIENTS OF INDUCTION FROM RESIDUALS AT TELEPHONIC FREQUENCIES—EFFECT OF ADMITTANCE UNBALANCE SAN FERNANDO SOMIS PARALLEL.

March 23-27, 1915.

OUTLINE.

I. Introduction.

The tests herein reported are supplementary to those described in technical report No. 54. As pointed out in that report, the results there given apply only to 50 cycles and do not include transverse measurements. The tests described in this report include measurements of both longitudinal and transverse induction, and cover a range of frequencies common to telephonic transmission and harmonics of power circuits—the "noise" frequencies. These tests were limited to residuals, owing to the lack of a three-phase source of energy at high frequency. The Vreeland Sine Wave Oscillator was used for the source of single-phase high-frequency energy. The coefficients of induction were obtained for four different conditions of the circuits at the distant end. For longitudinal induction two methods of testing are described and the results given and compared.

II. Description of Parallel.

A detailed description of the circuits involved in this parallel is given in technical report No. 54.

The telephone circuit was transposed at intervals of about 1180 feet (every fourth pole), there being 164 transpositions between the laboratory and Somis. The resultant lengths of unbalanced exposure to induction from residuals are 3050 feet in the section of line carried on poles and 350 feet in the section carried on A-frames. These two lengths combine in opposite phase. Weighting the two lengths in proportion to the separation between the telephone conductors in the two sections of line, the resultant length of unbalanced exposure is approximately 1670 feet, or 0.7% the total length of the line.

At the time of these tests the power circuit was transposed to form two barrels. This, however, is of no importance as regards induction from residual currents and voltages.

III. Description and Results of Tests.

A—Longitudinal Induction.

1. *Description of Methods.*

Letting V_1, V_2, I_1 and I_2 be the voltages and currents in circuits No. 1 and No. 2, respectively, at the sending end of a pair of coupled circuits the following equations may be written:

$$V_1 = Z_1 I_1 + Z_m I_2 \quad (1)$$
$$V_2 = Z_2 I_2 + Z_m I_1 \quad (2)$$

In these equations Z_1 and Z_2 are the self impedances of the circuits, each measured with the other circuit open at the sending end; and Z_m is the mutual impedance. These equations hold for all terminal conditions at the distant end, values of the self and mutual impedances being different for each different terminal condition.

Applied to the case of longitudinal induction from residuals, Z_1 is the impedance between the power-circuit conductors in parallel and ground (with telephone-circuit conductors isolated at sending end); and Z_2 is the impedance between the telephone-circuit conductors in parallel and ground (with power-circuit conductors isolated at sending end). Z_m is then the mutual impedance between the two ground-return single-phase circuits—the negative vector ratio of the open-circuit voltage at the sending end of the telephone circuit to the residual current at the sending end of the power circuit (Standardization Rules—A. I. E. E.—July 1, 1915—paragraph 916). Z_m may therefore be considered the coefficient of longitudinal induction from residual current, since it gives the relationship of the open-circuit voltage in the telephone circuit to the *residual current* of the power circuit under the given

conditions of both circuits. The induction, in general, is due to both voltage and current and the mutual impedance gives the resultant or the combined effect of both.

Two methods were employed in determining the coefficients of longitudinal induction: (1) *Direct Method* and (2) *Bridge Method.*

In the *Direct Method* the current I_2 in the disturbed circuit was measured, simultaneously with the current I_1 in the disturbing circuit, with a meter of known impedance, z. The impedance Z_2, having been separately determined by bridge measurements, the mutual impedance is therefore:

$$Z_m = \frac{-I_2 (Z_2 + z)}{I_1} \tag{3}$$

In order to obtain a check on the value of Z_m measurements of I_1 and I_2 were made with a number of values of meter impedance, z, ranging from approximately short-circuit (meter shunted by low resistance) to as high an impedance (resistance in series with meter) as possible consistent with a good deflection.

In the *Bridge Method* the self impedances of the two circuits were measured; also the impedance of the circuits connected in "series opposing." Under the conditions of these tests the latter impedance is that measured between the power-circuit conductors in parallel and the telephone-circuit conductors in parallel. Designating this impedance by Z_{1-2} the mutual impedance Z_m is obtained from the following equation:

$$Z_m = \frac{Z_1 + Z_2 - Z_{1-2}}{2} \tag{4}$$

For checking purposes measurements were made of the impedance of the circuits connected in parallel "aiding"; being in this case the impedance between all conductors of the power and telephone circuits in parallel and ground. Designating this impedance by Z_{12-0} the following equation holds:

$$Z_{12-0} = \frac{Z_1 Z_2 - Z^2_m}{Z_1 + Z_2 - 2Z_m} \tag{5}$$

then:

$$Z_m = Z_{12-0} \pm \sqrt{Z^2_{12-0} + Z_1 Z_2 - Z_{12-0} (Z_1 + Z_2)} \tag{6}$$

From this latter equation the value of Z_m may be computed and compared with the value first obtained. Equation (4) being linear and (6) quadratic, Z_m is much more readily obtained by a solution of the former. The results given in the following section were obtained from

equation (4) except for one frequency where Z_m was also computed from equation (6).

The accuracy of this method is dependent very largely upon the closeness of coupling between the two circuits involved. As the coefficient of coupling (Standardization Rules—A. I. E. E.—July 1, 1915—paragraph 962) is decreased Z_{1-2} approaches $(Z_1 + Z_2)$. Z_m is then determined by a small difference between relatively large quantities, thus greatly magnifying the effect of errors of observation. It is only for longitudinal induction from residuals that this method becomes at all practicable and then only for cases of small separation between lines.

It is essential either that the two circuits have a common ground connection at the sending end; or, if separate ground connections be used, that the two be sufficiently far apart so that they do not appreciably affect one another. In the latter event the measured values of Z_1 and Z_2 must be corrected by subtracting out the local ground resistances. In the case of these tests a common ground connection was used for both circuits.

The measurements were made by both methods for the four conditions of the circuits at the distant end and for a range of frequencies from 300 to 1100 cycles per second.

2. *Summary of Results.*

The summarized results are given below in tables. The results are expressed as impedances, for the most part mutual and self impedances, from which the open-circuit voltage, or current through any value of terminal impedance at the sending end of the telephone circuit, may be computed for any given value of residual current or voltage at the sending end of the power circuit; assuming necessarily one of the four combinations of distant-end conditions for which these tests apply.

TABLE I.

Distant-End Conditions—Longitudinal Induction.

Designation	Condition of circuit	
	Power	Telephone
a	Isolated	Isolated
b	Isolated	Grounded
c	Grounded	Isolated
d	Grounded	Grounded

TABLE II.

Coefficients of Longitudinal Induction from Residual Current. By Direct (D) and Bridge (B) Methods.

Frequency cycles per second	Mutual impedance—ohms							
	a		b		c		d	
	D	B	D	B	D	B	D	B
300	144	157	97.6		38.7	55.3	89.1	89.3
500	76.4	63.5	188	190	66.9	125	146	175
700	22.1	23.4	203	251	122	228	194	253
900	30.5	45.0	192	210	214	478	240	320
1,100	71.9		177		497	814	800	549

TABLE III.

Self Impedance of Power Circuit to Residual Current—(Z_1)—ohms.

Frequency cycles per second	a			b			c			d		
	R_1	X_1	Z_1	R_1	X_1	Z_1	R_1	X_1	Z_1	R_1	X_1	Z_1
300	25	—642	642	31	—610	610	42	+161	166	39	+160	165
500	25	—324	325	48	—298	302	77	+305	315	77	+300	310
700	26	—170	172	70	—166	180	172	+535	562	160	+508	533
900	31	—64	71	68	—87	110	629	+1010	1190	490	+947	1070
1100	41	+28	50	62	—7	62	1980	—777	2130	2630	—34	2630

TABLE IV.

Self Impedance of Telephone Circuit to Longitudinal Induced Current—(Z_2)—ohms.

Frequency cycles per second	a			b			c			d		
	R_2	X_2	Z_2	R_2	X_2	Z_2	R_2	X_2	Z_2	R_2	X_2	Z_2
300	157	—804	819	516	+370	635	155	—748	764	531	369	647
500	197	—346	398	987	+427	1080	206	—287	353	1010	418	1090
700	249	—127	279	1350	—92	1350	279	—60	284	1340	—109	1340
900	325	+22	326	948	—581	1110	424	+141	447	962	—549	1110
1100							757	—101	764	593	—565	820

TABLE V.

Coefficients of Longitudinal Induction from Residual Current. Mutual Impedance (Z_m) by Bridge Method—ohms.

Showing Resistance (R_m) and Reactance (X_m) Components.

Frequency cycles per second	a			b			c			d		
	R_m	X_m	Z_m	R_m	X_m	Z_m	R_m	X_m	Z_m	R_m	X_m	Z_m
300	16.6	—156	157				16	+52.9	55.3	31	+83.7	89.3
500	16.2	—61.4	63.5	151	+116	190	36	+120	125	97	+145	175
700	19.0	—13.7	23.4	247	+42.4	251	86	+211	228	206	+146	253
900	28.5	+34.8	45.0	197	—73.6	210	288	+375	478	298	+118	320
1100							667	—466	814	484	—260	549

26—88009

TABLE VI.

Comparative Values of Z_m—From Equations (4) and (6). Frequency 500 Cycles.

Equation	a			b			c			d		
	R_m	X_m	Z_m	R_m	X_m	Z_m	R_m	X_m	Z_m	R_m	X_m	Z_m
(4)	16.2	—61.4	63.5	151	+116	190	36	+120	125	97	+145	175
(6)	7	—88.8	89.1	165	+97.9	192	64	+144	158	93	+136	165

TABLE VII.

Comparative Values of Z_{12-0}—Measured, and Computed. Frequency 500 Cycles.

Method	a			b			c			d		
	R	X	Z_{12-0}	R	X	Z_{12-0}	R	X	Z_{12-0}	R	X	Z_{12-0}
Computed*	59	—207	215	247	—403	473	229	+331	402	100	+294	311
Measured	56	—223	230	261	—408	484	185	+342	389	103	+293	310

*From Equation (5) with value of Z_m obtained from Equation (4).

3. *Discussion of Results.*

The results obtained by the two methods are plotted as functions of the frequency, on drawings Nos. 309 and 310 which are attached to this report. Although the two methods give curves of about the same general shape, there is a rather large disagreement between corresponding values. Of the two methods the first (*Direct Method*) is the more reliable. In the *Bridge Method* the mutual impedance is determined as the small difference between two nearly equal quantities, thus greatly magnifying the effect of errors of observation. The results given in each instance for the *Direct Method* are average values obtained from five sets of observations. Table VI gives comparative values of Z_m derived from equations (4) and (6) for a frequency of 500 cycles per second. In Table VII are given values of Z_{12-0} as measured and as computed from equation (5), using the value of Z_m derived from equation (4). The check between corresponding values of Z_{12-0} is much better than between corresponding values of Z_m. In obtaining Z_m from equation (6) errors in the measurement of Z_{12-0} are magnified in Z_m. The agreement between corresponding values of Z_m is not in general very satisfactory.

By the *Bridge Method* it is possible to obtain the resistance and reactance components of the mutual impedance, as shown by the results given in Table V. By the *Direct Method*, only the absolute magnitude is obtained, which gives no information concerning the relative phase of the quantities in the two circuits.

From the signs of the mutual reactances (X_m), given in Table V, an indication may be obtained of the relative importance of electro-

static and electromagnetic induction at the lower frequencies, for the various conditions of the circuits at the distant end. When both circuits are isolated at the distant end electrostatic induction is favored and the sign of the mutual reactance is negative up to approximately 800 cycles. When the distant ends of both circuits are grounded, conditions are most favorable for electromagnetic induction and the signs of the mutual reactances given in Table V for this condition and for frequencies 900 cycles and less, is positive. When either circuit is grounded at the distant end, the other being isolated, the sign of the mutual reactance is positive for the lower range of frequencies and negative for the upper, indicating that electromagnetic induction is the more important for the lower frequencies.

In the following table there are given, for purposes of comparison, corresponding values of mutual impedance measured by the *Direct Method* and the computed total mutual-reactance. The latter is obtained by applying the computed unit-coefficients given in technical report No. 56, to the total length of line, neglecting leakage, attenuation, and phase-change along the circuits, and assuming the induction to be entirely electrostatic when both circuits are open at the distant end and entirely electromagnetic when grounded. These approximations obviously do not hold for the length of line and frequencies used in these tests. The comparisons are interesting, however, in showing the magnitude of the error to which such assumptions would lead in this particular case.

TABLE VIII.

Measured and Computed Coefficients of Longitudinal Induction.

Frequency cycles per second	Distant end open			Distant end closed		
	Measured mutual impedance, ohms	Computed total mutual reactance, ohms	Ratio	Measured mutual impedance, ohms	Computed total mutual reactance, ohms	Ratio
300	144	76	1.9	89	81	1.10
500	76	46	1.7	146	135	1.08
700	22	33	0.7	194	189	1.03
900	30	25	1.2	240	243	0.99
1,100	72	21	3.4	300	297	1.01

It will be seen from the table that the numerical magnitude of the measured mutual-impedance and computed total mutual-reactance are very nearly equal when the distant ends of the circuits are closed but differ greatly with the distant ends open. The close agreement for the case of distant ends closed is regarded as largely accidental, as there seem to be no rational grounds for expecting such a result to occur in general, particularly at the higher frequencies. It is to be expected in both cases that the agreement between the measured impedance and

computed total reactance would be best at the low frequencies because at such frequencies the attenuation and phase-change are less and the condition of the circuits at the distant end becomes of more importance in determining the character of the induction, whether largely electrostatic or electromagnetic. Comparisons corresponding to those will be found in technical report No. 54, for a frequency of 50 cycles at which frequency the approximations above mentioned are considered justifiable.

B—Transverse Induction.

1. *Description of Method.*

The method used to determine the coefficients of transverse induction is analogous to the first or *Direct Method* employed in the longitudinal induction test, that is, simultaneous measurement of the current supplied to the power circuit and the current induced in the telephone circuit through a measuring circuit of known impedance. Since the current in the telephone circuit was too small to be measured by any of the meters available, a comparison method was employed, using a telephone receiver as a current detector. A definite proportion, adjustable at will, of the current supplied to the power circuit was shunted through the telephone receiver. A switch was arranged so that the receiver could be alternately placed across the shunt in the power circuit and across the telephone-circuit terminals. Resistances in series and shunt with the receiver were adjusted until the same volume of sound was observed with the receiver in the two positions. Settings were made by three or four observers, for a range of frequencies, and for four conditions of the circuits at the distant end.

2. *Summary of Results.*

The following tables summarize the results obtained.

TABLE IX.

Distant-End Conditions—Transverse Induction.

Designation	Condition of circuit	
	Power	Telephone
a	Isolated	Open
b	Isolated	Short-circuited
c	Grounded	Open
d	Grounded	Short-circuited

TABLE X.

Coefficients of Transverse Induction from Residual Current.

Frequency cycles per second	Mutual impedance—ohms			
	a	b	c	d
300	0.20	0.25	0.08	0.10
500	0.09	0.29	0.11	0.18
700	0.08	0.17	0.16	0.30
900	0.11	0.05	0.40	0.60
1.100	0.18	0.04	0.91	1.16
1,300	0.27	0.19	0.57	0.77

TABLE XI.

Transverse Self-Impedance of Telephone Circuit—ohms.

Frequency cycles per second	Distant end open*			Distant end closed*		
	R	X	Z	R	X	Z
300	567	—1,440	1,550	2,120	+582	2,208
500	779	—541	949	2,740	—603	2,800
700	1,010	—176	1,020	1,820	—1,230	2,200
900	1,290	—86	1,290	1,210	—996	1,560
1,100	1,440	—192	1,450	993	—703	1,220
1,300	1,520	—409	1,570	955	—486	1,070

*The effect of the condition of the distant end of the power circuit is negligible.

3. *Discussion of Results.*

Plots of the results of this test are shown on drawing No. 311, which is attached to this report. The individual observations at each frequency varied over a considerable range. These individual observations were considered in determining the "best representative line." The values given in Table X are not the averages of the individual observations but were taken from the curves on drawing No. 311.

Following is a table giving corresponding values of measured mutual impedance, and of computed total mutual reactance. The computed values are obtained by applying the unit coefficients given in technical report No. 56 to the resultant length of unbalanced exposure, neglecting leakage attenuation and phase-change along the circuits, and assuming that the induction is entirely electrostatic when the circuits are open at the distant end and entirely electromagnetic when closed.

TABLE XII.

Measured and Computed Coefficients of Transverse Induction.

Frequency cycles per second	Distant end open			Distant end closed		
	Measured mutual impedance, ohms	Computed total mutual reactance, ohms	Ratio	Measured mutual impedance, ohms	Computed total mutual reactance, ohms	Ratio
300	0.20	0.018	11	0.10	0.010	10
500	0.09	0.011	8	0.18	0.017	11
700	0.08	0.008	10	0.30	0.023	13
900	0.11	0.006	18	0.60	0.030	20
1,100	0.18	0.005	36	1.16	0.037	31

The values of measured mutual impedance are in all cases several times the computed total mutual-reactance, the discrepancy being far greater than was noted in the case of longitudinal induction. There are several reasons for this greater discrepancy. The effects of attenuation and phase-change are greater and also the error introduced by neglecting electromagnetic induction in the one case, and electrostatic induction in the other is greater, since resonance of the telephone circuit for transverse induction occurs at a lower frequency than for longitudinal induction. There is also the effect of the admittance and series impedance unbalances of the telephone circuit. The effect of these unbalances is, in this case, probably to decrease rather than to increase the measured transverse induction. This is discussed in greater detail in a succeeding section of this report.

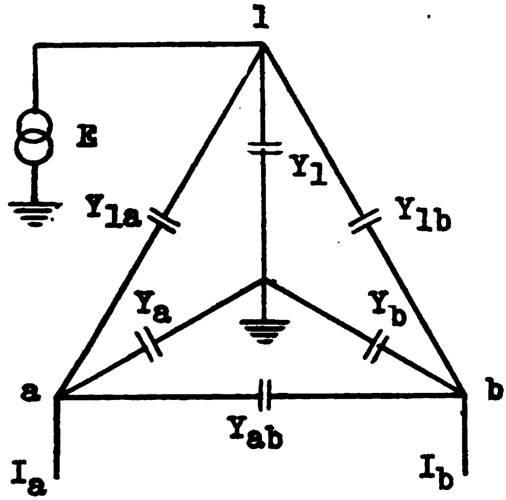
A large part of the discrepancy between the measured mutual impedance and computed total mutual reactance at the lower frequencies, particularly 300 cycles, is probably due to irregularities in the line construction which impair the effectiveness of the transposition system. The effect of irregularities increases relatively as the length of unbalanced exposure is decreased. The unbalanced exposure being, in this case, small, 0.7% of the total length of the line, the effect of irregularities is large.

C—Effect of Admittance Unbalance.

1. *Analytical Treatment.*

The measured transverse induction is in each case the resultant effect of two unbalances: sides of telephone circuit to power conductors, and sides of telephone circuit to ground. Though the two sides of a telephone circuit may be equally "exposed" to a power circuit, if unbalanced to ground, and if the circuit be raised above ground potential, a transverse voltage will be produced. An analytical discussion of these matters is given in the succeeding paragraphs.

The system consisting of the disturbing conductors (power-circuit conductors in parallel), the disturbed conductors (telephone circuit), and ground, may be represented by a network of admittances. In the equivalent network shown in the diagram, the three power conductors are represented by one conductor, designated as 1. The telephone conductors are designated a and b. Y_1, Y_a and Y_b are the direct admittances to ground of the several conductors designated by the subscripts. Y_{1a}, Y_{1b} and Y_{ab} are the direct admittances between the pairs of conductors designated by the subscripts. Should it be desired to consider the case of balanced three-phase components, an equivalent direct admittance for a grounded single-phase circuit, having the same inductive effect as the actual three-phase circuit, could be obtained.



To obtain the ratio of the transverse open-circuit voltage to the longitudinal open-circuit voltage, assume the sending ends of the disturbed conductors isolated, and a voltage E impressed on the disturbing conductors. For this condition I_a and I_b, the currents in the disturbed conductors, are each equal to zero. If E_a and E_b are the potentials of the disturbed conductors, the transverse open-circuit voltage is $E_a - E_b$ and the average longitudinal open-circuit voltage is $\frac{1}{2}(E_a + E_b)$. The ratio of the transverse to the longitudinal open-circuit voltage is given by the following equation:

$$\frac{E_a - E_b}{\frac{1}{2}(E_a + E_b)} = \frac{2\,(Y_{1a}Y_b - Y_{1b}Y_a)}{Y_{1a}Y_b + Y_{1b}Y_a + 2\,(Y_{1a} + Y_{1b})\,Y_{ab} + 2Y_{1b}Y_{1a}} \qquad (7)$$

If, as in the practical case, $Y_b - Y_a$ is small as compared to $Y_b + Y_a$ and $Y_{1a} - Y_{1b}$ small as compared to $Y_{1a} + Y_{1b}$ equation (7) may be reduced to the form,

$$\frac{E_a - E_b}{\frac{1}{2}(E_a + E_b)} = \frac{Y_{1a}Y_b - Y_{1b}Y_a}{2Y_{1a} \cdot Y_{a-b}} \qquad (8)$$

Y_{a-b} is the equivalent transverse admittance of the disturbed circuit, the admittance measured between the two disturbed conductors.

If $Y_{1a} = Y_{1b}$ the ratio of transverse to longitudinal voltage is determined by the admittance unbalance to ground and under this condition,

$$\frac{E_a - E_b}{\frac{1}{2}(E_a + E_b)} = \frac{Y_b - Y_a}{2Y_{a-b}} = \frac{Y_b - Y_a}{Y_b + Y_a} \cdot \frac{Y_a}{Y_{a-b}} \tag{9}$$

If $Y_a = Y_b$ the ratio of transverse to longitudinal voltage is determined by the admittance unbalance to the disturbing conductor, under this condition,

$$\frac{E_a - E_b}{\frac{1}{2}(E_a + E_b)} = \frac{Y_{1a} - Y_{1b}}{Y_{1a} + Y_{1b}} \cdot \frac{Y_a}{Y_{a-b}} \tag{10}$$

Examination of equations (9) and (10) shows that the transverse open-circuit voltage is in each case directly proportional to the longitudinal open-circuit voltage and the percentage admittance unbalance to ground or to the disturbing conductor as the case may be. Hence, the two percentage unbalances are equally important. Furthermore, if the same side of the disturbed circuit has the greater admittance both to the disturbing conductor and to ground, the two effects are approximately in opposition.

The equivalent admittances $Y_b - Y_a$ and Y_{a-b} may be measured at the terminus of the circuit whether or not this point is also the terminus of the exposure. The measured equivalent admittance-unbalance to ground $(Y_b - Y_a)$ includes also the effect of series-impedance unbalance of the disturbed circuit as well as the effect of capacitance and conductance unbalance to ground. If the longitudinal voltage at the terminus of the circuit is known, the open-circuit transverse-voltage due to admittance unbalance to ground may be computed from equation 9.

The circulating current through an instrument of admittance Y, connected at this point is,

$$I = \frac{Y(Y_a - Y_b)}{2Y_{a-b} + 2Y} \cdot \frac{E_a + E_b}{2} \tag{11}$$

and the short-circuit current is,

$$I_{sc} = \frac{Y_a - Y_b}{2} \cdot \frac{E_a + E_b}{2} \tag{12}$$

2. *Application to Experimental Results.*

Measurements of the admittance unbalance to ground of the San Fernando-Somis telephone circuit were made for one condition only of the circuits at the distant end, that of both circuits isolated and open. These measured admittance unbalances are shown on drawing No. 297.

attached. The transverse admittance, Y_{a-b}, is the reciprocal of the transverse self-impedance of the telephone circuit which was measured in connection with the determination of the coefficients of transverse induction (Table XI).

The computed ratio of transverse to longitudinal open-circuit voltage due to admittance unbalance to ground, is shown on drawing No. 320, together with the ratio of the measured transverse to longitudinal open-circuit voltage. The ratio of the measured values is less than the ratio computed from the admittance unbalance to ground, indicating that the transverse induction due to the admittance unbalance to the power conductors is in approximate opposition to that due to the unbalance to ground.

Drawing No. 316 shows the ratio of measured transverse to longitudinal induction for all four conditions of the circuits at the distant end. Sufficient measurements to determine the effect of admittance unbalance to ground in all these cases, were not made.

3. *Effect of Transpositions.*

Transpositions in a telephone circuit tend to balance the direct admittances to ground as well as the direct admittances between the telephone conductors and equivalent power conductor. These transpositions do not reduce the average admittance to ground or to the power conductor, hence do not affect the longitudinal induction.

In the case of a nontransposed telephone circuit subject to induction from balanced components in a three-phase circuit, transpositions in the power circuit reduce the admittances between the equivalent power conductor and the disturbed telephone conductors in the same proportion as their difference, hence, the ratio of transverse to longitudinal induction is not reduced by the power circuit transpositions, the longitudinal and transverse induction being reduced in the same proportion.

After the admittances to ground and to the power conductor are balanced as far as practicable by telephone-circuit transpositions further reduction of the resultant transverse induction is possible by power-circuit transpositions in the case of balanced components, and by reduction of the magnitude of the residuals in the case of residual components.

IV. Resumé.

With the frequencies and length of line used in these tests both electrostatic and electromagnetic effects are present in all cases to such an extent that, although their relative importance varies, it can not be said that the one or the other factor becomes entirely negligible. Furthermore, attenuation and phase-change effects become of practical importance, particularly at the higher frequencies. To compute the induction

under the conditions of these tests requires a solution of the problem for lines of uniformly distributed constants. The coefficients of induction computed for this and other parallels have been for unit sections of short length such that attenuation and phase-change could be neglected and electrostatic and electromagnetic induction separately considered. For both transverse and longitudinal induction and for two conditions, distant ends of the circuits open, and distant ends of the circuits closed, computations were made of the total mutual reactance, obtained by applying the unit coefficients given in technical report No. 56, to the computed unbalanced exposures. A comparison of these results with the measured mutual impedances shows that the approximations involved in such computations are not justifiable with the frequencies and length of line used in the tests discussed in this report.

Terminal conditions at the distant end of either circuit affect the relative magnitude and phase of the electrostatic and electromagnetic components of the induced voltages and currents. Reflection occurs at the distant end except in the special case where both circuits are closed through their characteristic impedances (Standardization Rules—A. I. E. E.—July 1, 1915—paragraph 918) for the frequency considered. In certain cases such an arrangement may prove desirable for test purposes. Unless a single frequency is dealt with it is obvious that reflection will always occur.

None of the four conditions of the circuits at the distant end correspond exactly to the condition which would exist in operating practice. They do, however, represent the extreme range, and measurements under such conditions are especially valuable in determining the effectiveness of a given change in transpositions, as they are definite and easily duplicated, and afford also an index of the severity of the induction. "Terminal conditions" at the ends of a parallel such as exist with the circuits in normal operation vary to some extent and it would be difficult to duplicate these conditions for purposes of tests on the portion of the circuits within the limits of the parallel. On the whole, the four distant-end terminal conditions used in this case are the most satisfactory for testing purposes.

Since the tests were made under but one condition of the telephone circuit as regards transpositions it is not possible to determine by comparisons with other tests the effect of transpositions on the transverse induction. The transposition system installed was designed to give a minimum of unbalanced exposure between the two circuits; the transverse induction coefficients as measured, are therefore indicative of the induction which remains when the unbalanced exposure is reduced to the lowest practicable limits.

The importance of longitudinal induction and admittance unbalance in producing circulating currents in metallic telephone circuits is shown by the equations given in a preceding section. Though the two sides of a telephone circuit may be equally exposed to induction from a power circuit, if the admittance to ground or series impedance of the telephone circuit be unbalanced, there will be a voltage between its sides should the circuit as a whole be raised above ground potential. The circulating current due to this voltage is directly proportional to the product of the longitudinal voltage and the equivalent admittance unbalance, and may be diminished by reducing either or both factors. The best result may be obtained by minimizing both as far as practicable: the unbalance of the telephone circuit, by transpositions throughout its entire length, by the use of well balanced equipment and by careful maintenance; and the longitudinal induction, by power-circuit transpositions within the parallel co-ordinated with the telephone-circuit transpositions, and by limiting the residuals to small magnitudes.

Respectfully submitted.

(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

ATTACHMENTS: P. I. C. Drawings No. 297, 309, 310, 311, 316 and 320.

APPROVED: January 25, 1916.

(Signed) R. W. MASTICK,
Field Engineer.

APPROVED: May 9, 1916.

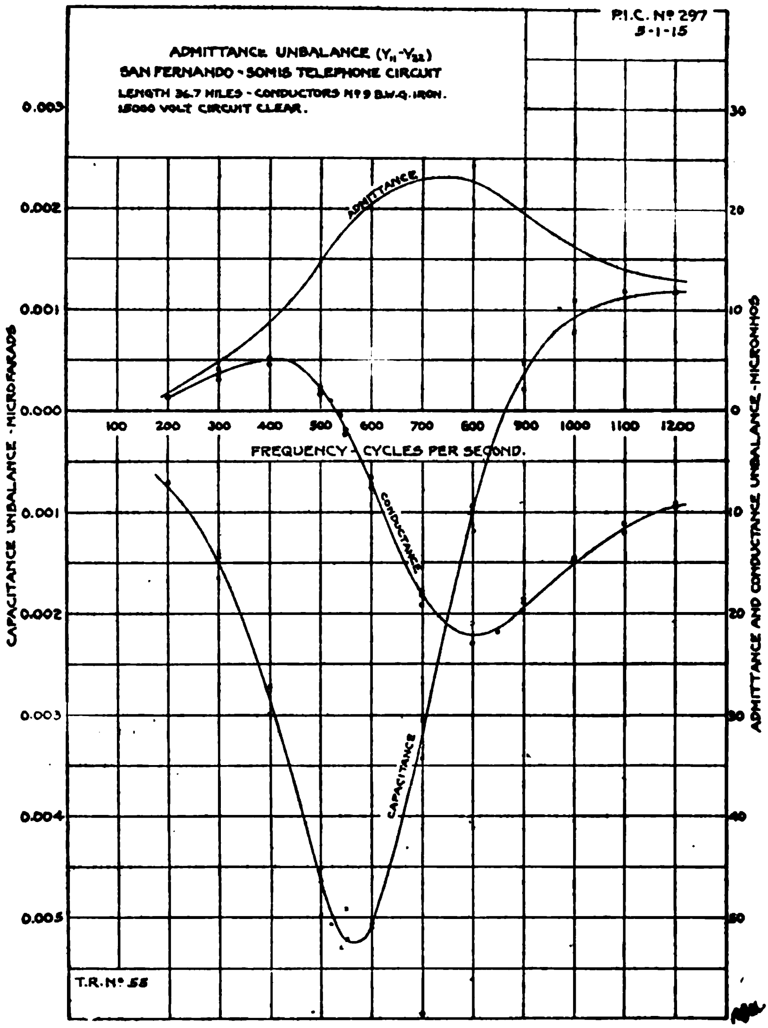
SUBCOMMITTEE ON TESTS,

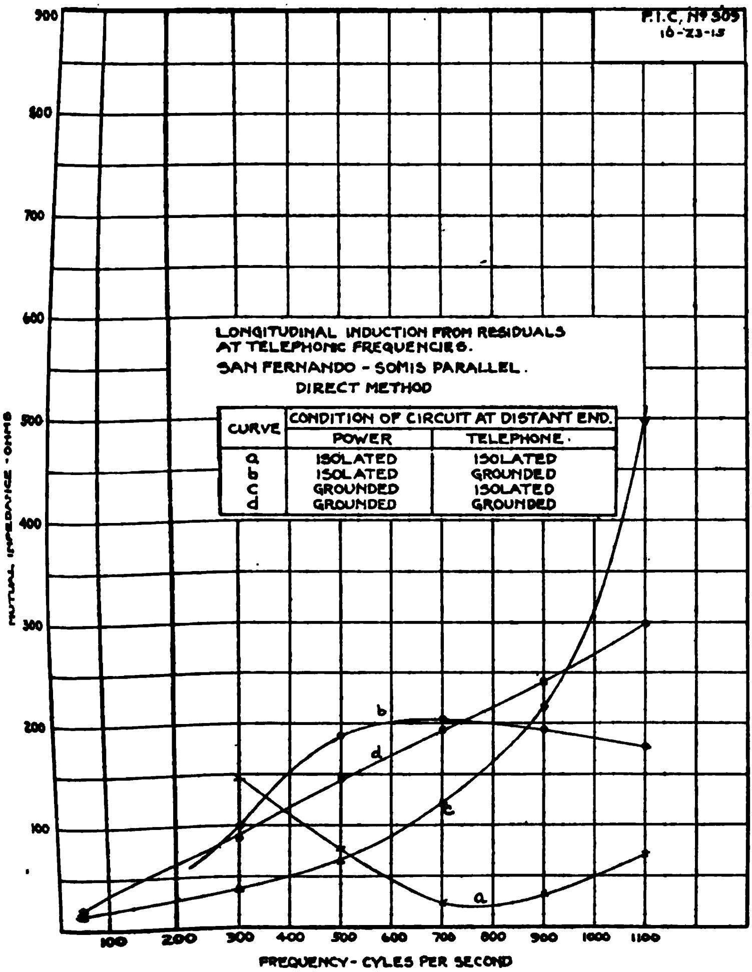
(Signed) J. E. WOODBRIDGE,
Chairman.

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE,

(Signed) ARTHUR F. BRIDGE,
Secretary.

May 19, 1916.

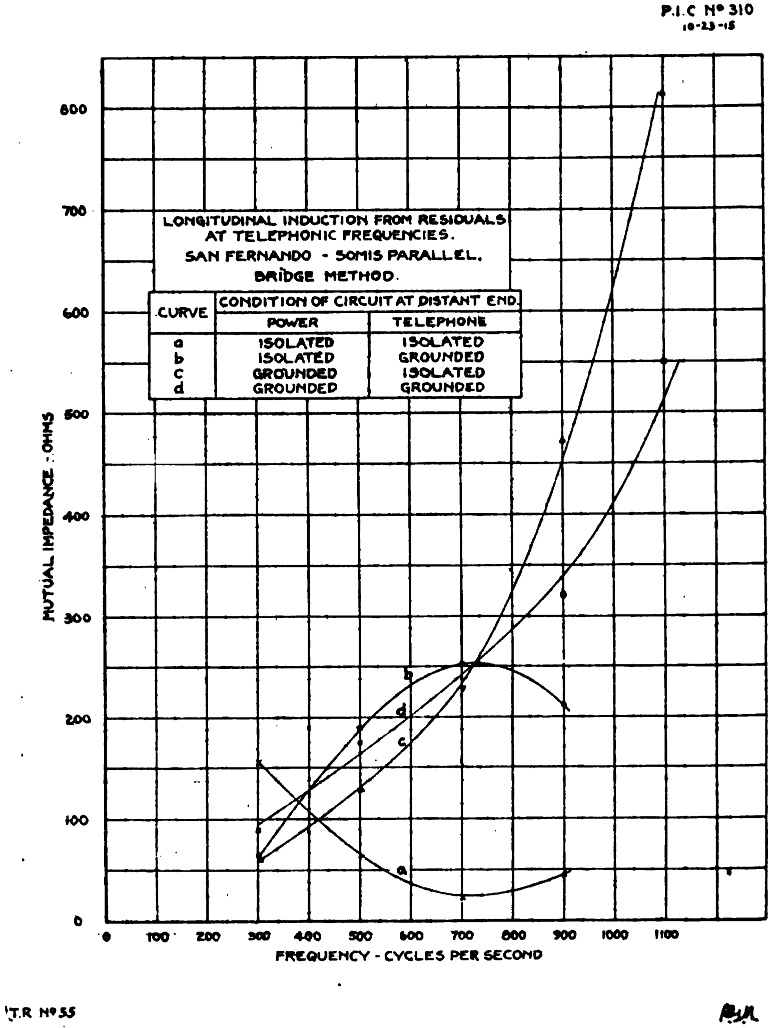

ADMITTANCE UNBALANCE (Y_{11}-Y_{22})
SAN FERNANDO - SOMIS TELEPHONE CIRCUIT
LENGTH 36.7 MILES - CONDUCTORS Nº 9 B.W.G. IRON.
15000 VOLT CIRCUIT CLEAR.
P.I.C. Nº 297
5-1-15
ADMITTANCE
CONDUCTANCE
CAPACITANCE
FREQUENCY - CYCLES PER SECOND.
CAPACITANCE UNBALANCE - MICROFARADS
ADMITTANCE AND CONDUCTANCE UNBALANCE - MICROMHOS
T.R. Nº 58

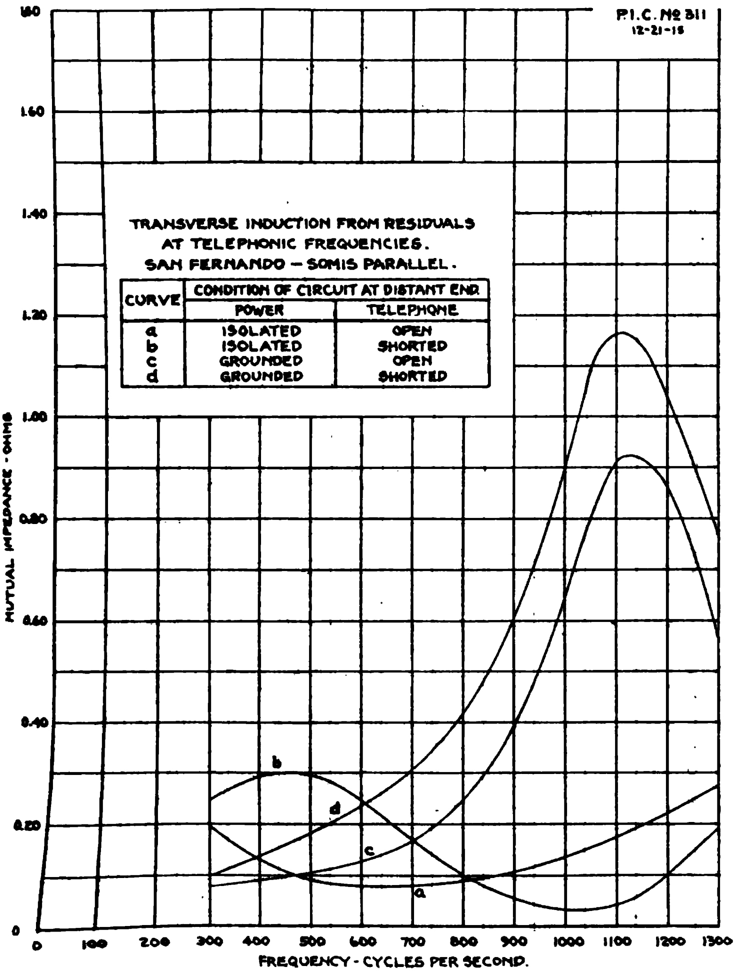


F.I.C. № 305
10-23-13
LONGITUDINAL INDUCTION FROM RESIDUALS
AT TELEPHONIC FREQUENCIES.
SAN FERNANDO - SOMIS PARALLEL.
DIRECT METHOD
CURVE
CONDITION OF CIRCUIT AT DISTANT END.
POWER
TELEPHONE
a ISOLATED ISOLATED
b ISOLATED GROUNDED
c GROUNDED ISOLATED
d GROUNDED GROUNDED
MUTUAL IMPEDANCE - OHMS
FREQUENCY - CYCLES PER SECOND
100 200 300 400 500 600 700 800 900 1000 1100
100 200 300 400 500 600 700 800 900
a b c d

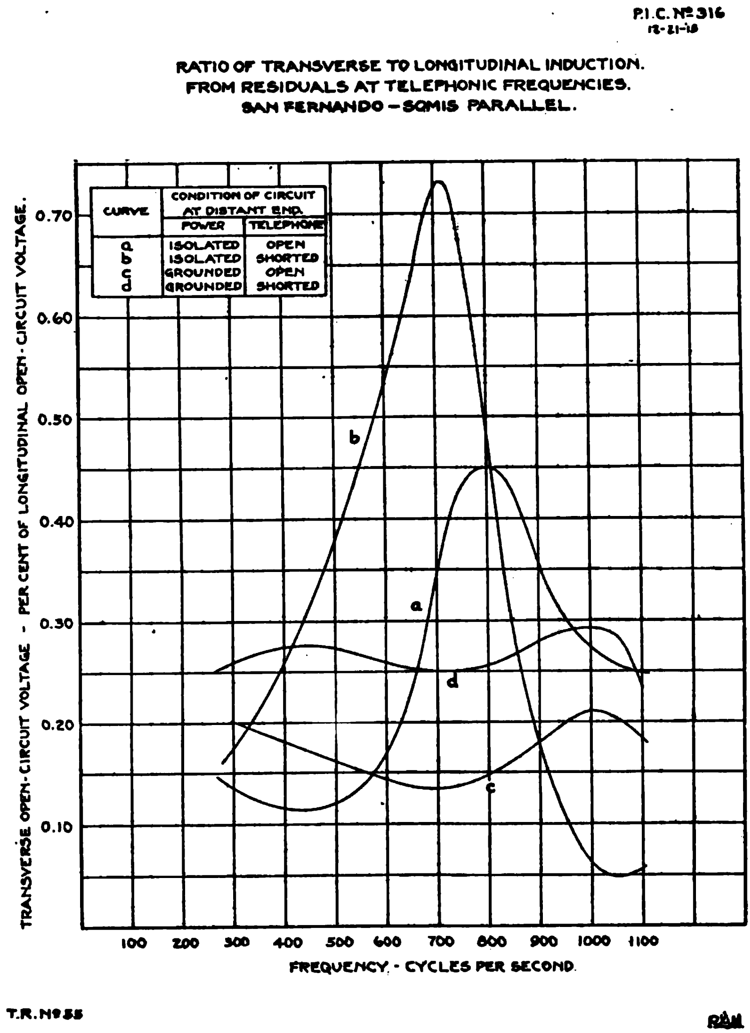
T.R. № 55

RLM

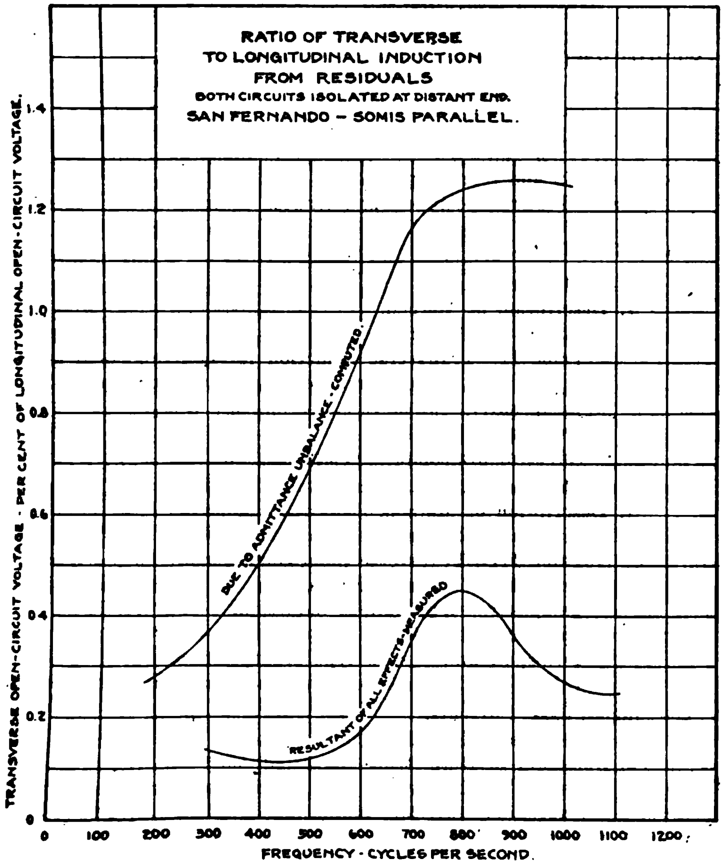

P.I.C Nº 310
10-23-15
LONGITUDINAL INDUCTION FROM RESIDUALS AT TELEPHONIC FREQUENCIES.
SAN FERNANDO - SOMIS PARALLEL.
BRIDGE METHOD.
CURVE
CONDITION OF CIRCUIT AT DISTANT END.
POWER
TELEPHONE
a ISOLATED ISOLATED
b ISOLATED GROUNDED
c GROUNDED ISOLATED
d GROUNDED GROUNDED
MUTUAL IMPEDANCE - OHMS
FREQUENCY - CYCLES PER SECOND
0
100
200
300
400
500
600
700
800
900
1000
1100
a
b
c
d
T.R Nº 55


P.I.C. № 311
12-21-15
TRANSVERSE INDUCTION FROM RESIDUALS
AT TELEPHONIC FREQUENCIES.
SAN FERNANDO — SOMIS PARALLEL.
CURVE
CONDITION OF CIRCUIT AT DISTANT END
POWER
TELEPHONE
a ISOLATED OPEN
b ISOLATED SHORTED
c GROUNDED OPEN
d GROUNDED SHORTED
MUTUAL IMPEDANCE - OHMS
FREQUENCY - CYCLES PER SECOND.
0
0.20
0.40
0.60
0.80
1.00
1.20
1.40
1.60
1.80
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
1300
a
b
c
d

T.R. №55


P.I.C. № 316
12-21-15
RATIO OF TRANSVERSE TO LONGITUDINAL INDUCTION.
FROM RESIDUALS AT TELEPHONIC FREQUENCIES.
SAN FERNANDO – SOMIS PARALLEL.
CURVE
CONDITION OF CIRCUIT AT DISTANT END.
POWER
TELEPHONE
a ISOLATED OPEN
b ISOLATED SHORTED
c GROUNDED OPEN
d GROUNDED SHORTED
TRANSVERSE OPEN-CIRCUIT VOLTAGE - PER CENT OF LONGITUDINAL OPEN-CIRCUIT VOLTAGE.
0.70
0.60
0.50
0.40
0.30
0.20
0.10
a
b
c
d
100 200 300 400 500 600 700 800 900 1000 1100
FREQUENCY - CYCLES PER SECOND.
T.R. № 55
RAM

P.I.C. Nº 320
12-21-15



TR. Nº 55

R.W.M.

Technical Report No. 56.

February 10, 1916.

DETERMINATION OF COEFFICIENTS OF INDUCTION IN A SHORT SECTION OF THE SAN FERNANDO-SOMIS PARALLEL. MEASUREMENTS AND COMPUTATIONS. EFFECT OF TELEPHONE TRANSPOSITIONS.

March 28-April 14, 1915.

OUTLINE.

I. INTRODUCTION.

Purpose and Scope.

II. DESCRIPTION OF PARALLEL.

Configuration and relative position of circuits.
Table I—Transpositions and lengths of unbalanced exposure.

III. MEASUREMENTS.

A—Balanced Voltages.
1. Description of Method.
2. Summary of Results.
Table II—Transverse induction for nontransposed and transposed telephone circuit—relationship of induction and unbalanced exposure.
3. Discussion of Results.
Effect of atmospheric electrification.

B—Balanced Currents.
1. Description of Method.
2. Summary of Results.
Table III—Transverse induction for nontransposed and transposed telephone circuit—relationship of induction and unbalanced exposure—longitudinal induction.
3. Discussion of Results.
Two-span and square type transpositions.
Table IV—Comparison of two-span and square type transpositions.

C—Single-Phase Currents.
1. Description of Method.
2. Summary of Results.
Table V—Transverse and longitudinal induction, nontransposed circuits.
3. Discussion of Results.

D—Residual Voltage.
1. Description of Method.
Grounding crossarms and pins to prevent leakage.
2. Summary of Results.
Table VI—Transverse and longitudinal induction, nontransposed circuits.
Table VII—Effect of shielding.
3. Discussion of Results.
Errors due to induction into leads and atmospheric electrification.

VI. RESUME.

Comparison of measurements and computations.
Effect of capacitance unbalance.
Relative accuracy of computed coefficients and dimensions.
Relationship of induction and unbalanced exposure—effect of irregularities.
Adaptability of computations for study of effect on induction of variations in configuration and relative location of circuits.

I. Introduction.

This report presents the results of measurements and computations of the coefficients of induction for short sections of the San Fernando-Somis parallel. Measurements of the coefficients of longitudinal induction and of the effect of power transpositions for the whole parallel are described in technical report No. 54. It was not practicable, in the tests there reported, to study the effects of telephone transpositions or to measure the coefficients of transverse induction. The length of line used in the tests here reported was such that it was practicable to take out all the telephone transpositions, thus giving an opportunity to obtain the coefficients of transverse induction for the nontransposed condition of both power and telephone circuits, and also to insert transpositions at desired points in the telephone line, in order to determine the effect of different lengths of transverse unbalanced exposure.

Measurements of the coefficients of both longitudinal and transverse induction were made for balanced voltages, balanced currents, single-phase currents, residual voltages and residual currents. All the measurements were made at the fundamental frequency of 50 cycles per second.

The uniformity of the parallel, and the possibility of making measurements on the nontransposed power and telephone circuits, presented an excellent opportunity for comparison between computed and experimentally determined coefficients. Accordingly, the coefficients of induction were computed for all the conditions for which the measurements were made.

II. Description of Parallel.

The parallel involved is a short section, 15156 feet in length (pole No. 84 to pole No. 99), of the San Fernando-Somis 15000-volt, single-circuit, three-phase line of the Pacific Light and Power Corporation. (In a few cases, noted in each instance, a still shorter section was used; 3840 feet in length.) On Drawing No. 230, which is attached to this report, is given a cross-sectional diagram of the line. The sizes and spacings of conductors are uniform throughout this section, the only irregularities being small variations in the sag of conductors, heights of poles and in the separation of the power and telephone circuits.

caused by the use of suspension instead of pin insulators at road crossings and corners. Drawing No. 230 is based on the average height of the conductors above ground, and allows for the sag, on the assumption that the power and telephone conductors sag by the same amount. The possible error due to this assumption is discussed later.

There were no transpositions in the power circuit within the section of line tested. At the beginning of the tests there were sixteen transpositions in the telephone circuit. After making some tests with the telephone line in this condition, the transpositions were all removed. At poles Nos. 67 and 83, located near the half-point and three-fourths point, respectively, of the line, strain insulators and Fahnstock clips were arranged so that the circuit could be readily transposed or untransposed at those points.

Lengths of unbalanced exposure in feet and per cent of the total length of line for each condition of the telephone circuit as regards transpositions are given in Table I below.

TABLE I.

Number of telephone transpositions	Resultant lengths of unbalanced exposure	
	Feet	Per cent of total length
0	15,156	100.0
2	7,592	50.1
1	88	0.6
16	635	4.2

III. Measurements.

A—Balanced Voltages.

1. *Description of Method.*

The distant ends of both power and telephone circuits were isolated, thus favoring electrostatic induction and minimizing electromagnetic induction. The power circuit was energized from the P. L. & P. network through step-down transformers connected delta-delta and step-up transformers connected delta-star with the neutral of the step-up bank grounded. Grounding the neutral practically eliminated the residual voltage due to the capacitance unbalance of the nontransposed power circuit. The third-harmonic residual voltage thus introduced was found, in this case, to be negligible, hence this arrangement was used as giving the nearest practicable approach to balanced voltages.

The voltages induced in the telephone circuit were measured with an electrostatic voltmeter. The impedance of this meter was very large, compared to that of the telephone circuit; hence a direct reading of the

open-circuit voltage was obtained. Transverse measurements were made with the telephone circuit nontransposed and with one and two transpositions in the telephone circuit.

The longitudinal voltages were too large to measure directly with the electrostatic voltmeter (scale 0-80 volts). Measurements of the short-circuit current were made but are not reported, as the line impedance necessary for reducing these observations to open-circuit voltages was not obtained. Satisfactory measurements of the coefficients of longitudinal induction were obtained in the tests reported in technical report No. 54.

Some additional measurements of transverse open-circuit voltage were made on a nontransposed section of line 3840 feet in length, both power and telephone circuits being isolated at pole No. 50. In this section of line the pins and crossarms of the power circuit were all grounded.

2. *Summary of Results.*

TABLE II.

Transverse Induction from Balanced Voltages—Power Circuit Nontransposed.

Number of telephone transpositions	Balanced voltage, kV.	Induced voltage			% induction
		Volts	Volts per kV.	Per cent	% exposure
0*	8.29	63.7	7.69	111	1.11
0	8.18	56.8	6.95	100.0	1.00
2	16.4	58.7	3.58	51.6	1.03
1	16.3	11.0	0.67	9.6	16

*Power and telephone lines isolated at pole No. 50.

3. *Discussion of Results (balanced voltages).*

The measurements of voltage induced in the nontransposed telephone circuit for the two lengths of line, made on different days, differ by about 11 per cent. At the time the measurements on the shorter section were made considerable difficulty was experienced with atmospheric electrification. This effect was not noticed in the measurements on the 15156-foot section. Since an instrument (electrostatic voltmeter) responsive to undirectional potential differences was used, the readings of open-circuit voltage were increased by the atmospheric electrification, hence the measurements in which the readings were smaller (made on the longer line), are probably the more reliable.

The voltages obtained with no transpositions and two transpositions are approximately proportional to the respective lengths of unbalanced exposure. With one transposition the induced voltage is approximately 16 times as great as would be expected from the length of unbalanced

exposure. This may be due in part to the presence of the atmospheric electricity referred to above.

B—Balanced Currents.

1. *Description of Method.*

The distant ends of both power and telephone circuits were short-circuited and grounded, thus favoring electromagnetic induction and minimizing electrostatic induction. Energy was obtained from the P. L. & P. 15000-volt network, using two banks of step-down transformers in parallel, connected delta-star, the neutrals being isolated so as to eliminate residual current. The longitudinal and transverse induced current of the telephone circuit were measured with a thermo-galvanometer, using series resistance. This resistance was sufficiently large in the case of the measurements of longitudinal induction, and of transverse induction on the nontransposed telephone circuit, that nearly open-circuit voltage existed at the telephone-circuit terminals. Thus the values of induced voltage were practically independent of the line impedance, though allowance was made for it in reducing the data.

2. *Summary of Results.*

TABLE III.

Transverse and Longitudinal Induction from Balanced Currents. Power Circuit Nontransposed.

Type of induction and number of telephone transpositions	Balanced current amp.	Induced voltage			Short-circuit current		% induction
		Volts	Volts per amp.	Per cent	mA.	mA. per ampere	% exposure
Transverse induction:							
0	168	2.59	0.0154	100	25.0	0.148	1.00
2	171	1.42	0.0083	54	13.7	0.080	1.08
1	172	0.01—	0.00006—	1—	0.1—	0.001—	1.7—
16	168	0.201	0.0012	8	1.95	0.012	1.9
Longitudinal induction	168.6	21.5	0.127	—	—	—	

3. *Discussion of Results (balanced currents).*

The transverse induction in the telephone circuit is quite closely proportional to the length of unbalanced exposure (see last column of Table III). The results for 1 and 2 transpositions in the telephone circuit are more closely in accord with the percentages of unbalanced exposure than is the case with 16 transpositions. It should be noted in this connection that when there were one and two transpositions they were of the "square" type, *i. e.*, the transposition was completed at the transposition pole, whereas the 16 transpositions originally installed were of the ordinary "two-span" type. With transpositions occurring

at every fourth pole, the plane of the telephone circuit is horizontal for only about one-half the total length of the line. In the other half, the plane of the telephone circuit is rotating from one horizontal position to the other through the vertical position. The coefficient of induction varies with the position; hence the induction is not exactly proportional to the computed unbalanced exposure. To test this matter further some observations of induction from balanced currents on a still shorter section of line (3840 feet) were made, first with four "two-span" type transpositions in the circuit and then with four "square" type transpositions located at the same poles. The computed unbalanced exposure was in each case 312 feet. The results are given in the following table:

TABLE IV.

Comparison of "Two-Span" and "Square" Transpositions. Transverse Induction from Balanced Current—3840-foot Line. Power Circuit Non-transposed.

Type of telephone transpositions	Balanced current, amperes	Induced voltage		% induction
		Volts	Volts per ampere	% exposure
"Two-span"	166	0.12	0.0007	2.2
"Square"	168	0.05	0.0003	1.0

In this instance also the "square" type transpositions give a result more nearly proportional to the length of unbalanced exposure. The chief cause of the disadvantage of the "two-span" transpositions is the inequality of the spans adjacent to the transposition poles. Although it was observed in this test that transpositions of the "square" type were more efficient than those of the "two-span" type it should not be concluded that this will hold in all cases. In a long section of line the irregularities in the transposition spans will tend to be balanced and hence the advantages of the "square" over the "two-span" type will probably be small.

C—Single-phase Currents.

1. *Description of Method.*

Both the power and telephone circuits were short-circuited and grounded at the distant end and the loops formed by pairs of power conductors were energized in turn. The transverse and longitudinal induced currents of the telephone circuit and the longitudinal induced current of the idle power conductors were measured with a thermo-galvanometer. The induced voltage was obtained from the induced current and the meter and line impedances.

2. *Summary of Results.*

TABLE V.

Transverse and Longitudinal Induction from Single-Phase Currents. Power and Telephone Circuits Nontransposed.

Type of induction	Energized power conductors, Nos.	Single-phase current, amperes	Induced voltage	
			Volts	Volts per ampere
Transverse (telephone circuit)	1—2	87.0	1.07	0.0123
	2—3	89.7	0.52	0.0058
	1—3	85.7	1.53	0.0178
Longitudinal (telephone circuit)	1—2	88.8	7.20	0.082
	2—3	90.0	6.15	0.068
	1—3	86.0	12.9	0.150
Longitudinal (idle power conductor)	1—2	88.8	18.5	0.208
	2—3	90.1	18.2	0.202
	1—3	85.6	1.34	0.0157

3. *Discussion of Results (single-phase currents).*

The measurements of transverse induction from single-phase currents were made only for the nontransposed telephone circuit. The results show that the greatest induction in the telephone circuit occurs when the upper and lower power conductors are energized, whereas under the same condition, the longitudinal induction in the idle power conductor, which is in this case equidistant from the two energized conductors, is a minimum.

D—Residual Voltage.

1. *Description of Method.*

Both power and telephone circuits were open and isolated at the distant end. The conductors of the power circuit were energized singly, two in parallel, and three in parallel, the magnitude of the impressed residual voltage being such as to give an open-circuit voltage in the telephone circuit that could be measured directly on the electrostatic voltmeter. Measurements were made of both transverse and longitudinal open-circuit voltage of the telephone circuit with the various combinations of power conductors energized.

The measurements were all made on the section of line to pole No. 50, 3840 feet in length, the pins and crossarms of the power circuit being grounded to prevent direct conduction between the two circuits.

2. *Summary of Results.*

The results of this test are summarized in two tables given below.

TABLE VI.

Transverse and Longitudinal Induction from Residual Voltage. Telephone Circuit Nontransposed. Idle Power Conductors Isolated.

Type of induction	Energized power conductors Nos.	Residual voltage,* volts	Induced voltage	
			Volts	Volts per volt
Transverse	1, 2, 3	24,700	64.6	0.0026
	1	8,190	60.6	0.0074
	2	8,270	27.0	0.0033
	3	8,220	12	0.0015
Longitudinal	1, 2, 3	716	71.0	0.099
	1	240	42.3	0.176
	2	240	34.0	0.141
	3	240	33.6	0.140
	1, 2	477	59.8	0.125
	2, 3	480	52.8	0.110
	1, 3	478	60.8	0.127

TABLE VII.

Transverse and Longitudinal Induction from Residual Voltage. Power and Telephone Circuits Nontransposed. Idle Power Conductors Grounded.

Type of induction	Energized power conductors	Residual voltage,* volts	Induced voltage		Shielded / Unshielded
			Volts	Volts per volt	
Transverse	1	8,180	58.9	0.0072	0.97
Longitudinal	1	240	34.3	0.143	0.81
	2	238	23.8	0.100	0.71
	3	240	20.8	0.087	0.62

*Sum of voltage to ground of *energized conductors only.*

3. *Discussion of Results (Residual Voltage).*

In the longitudinal measurements it was necessary to impress rather low voltage on the disturbing conductor so that the voltage induced on the disturbed conductor would be within the range of the electrostatic voltmeter. The leads from the disturbed conductor to the laboratory were paralleled for a distance of approximately 60 feet by the leads extending from the 15000-volt supply to the substation. Although this parallel was a small proportion of the 3840-foot exposure its voltage to ground was much higher, being approximately 9000 volts, hence the longitudinal measurements are subject to an error from this source.

Both transverse and longitudinal measurements are subject to an error from atmospheric electrification. Every effort was made to elimi-

nate this error by making the measurements in the early morning and in the late evening when this effect was a minimum. At these times, however, the effect of leakage became more appreciable.

The shielding effect on the transverse induction of grounding the two idle conductors was observed to be small when the lowest power conductor was energized. A larger shielding effect was observed in the case of longitudinal induction.

E—Residual Current.

1. *Description of Method.*

The three conductors of the power circuit were grounded at the distant end (pole No. 99) so as to permit of a large residual current with a minimum impressed voltage. At the laboratory the conductors of the power circuit were energized at a single-phase voltage to ground of approximately 540 volts. To determine the effects of individual conductors and combinations of conductors at different separations from the telephone circuit they were connected to the supply singly, in pairs, and all three in parallel. The telephone circuit was short-circuited at the distant end and the induced current measured (by thermo-galvanometer with series resistance) simultaneously with the current supplied to the power circuit, with the telephone circuit nontransposed and with one and two transpositions in the telephone circuit. In the case of the measurements on the nontransposed telephone circuit the combined meter and series resistance was so large that the observed voltage was practically independent of the line impedance.

For the longitudinal measurements the disturbed and disturbing conductors were connected to separate grounds at both ends of the circuits. Special precautions were taken to make the resistances of the ground contacts low, and to have the two grounds at each end of the parallel sufficiently far apart to minimize their mutual effect. Notwithstanding the precautions taken, a small amount of ground resistance, of indeterminate magnitude, remained common to the two circuits.

The longitudinal open-circuit voltage in the telephone circuit was measured with each combination of power conductors energized. In the cases where only one or two power conductors were energized the longitudinal induction in the idle power conductor or conductors was determined. With one power conductor energized the transverse open-circuit voltage of the two idle power conductors was measured.

2. *Summary of Results.*

The results of this test are summarized in four tables given below.

TABLE VIII.

Transverse Induction in Telephone Circuit from Residual Current. Power Circuit Nontransposed. Idle Power Conductors Isolated at Laboratory.

Energized power conductors Nos.	Telephone transpositions	Residual current,* amperes	Induced voltage			% induction / % exposure
			Volts	Volts per ampere	Per cent	
1, 2, 3	0	60.9	1.06	0.0174	100	1.00
	2	60.6	0.56	0.0093	53	1.05
	1	61.0	0.03—	0.0005—	8—	5.—
	16	61.4	0.20	0.0033	19	4.5
1	0	50.0	1.34	0.0267	100	1.00
	2	50.0	0.76	0.0152	57	1.13
	1	50.0	0.02—	0.0004—	2—	4.—
2	0	50.0	0.76	0.0151	100	1.00
	2	50.4	0.42	0.0083	55	1.10
3	0	50.0	0.46	0.0092	100	1.00
	2	50.0	0.26	0.0052	57	1.13
1, 2	0	58.4	1.24	0.0213		
2, 3	0	58.0	0.70	0.0121		
1, 3	0	58.4	1.06	0.0182		

TABLE IX.

Longitudinal Induction in Telephone Circuit from Residual Current. Power Circuit Nontransposed. Idle Power Conductors Isolated at Laboratory.

Energized power conductors Nos.	Residual current,* amperes	Induced voltage	
		Volts	Volts per ampere
1, 2, 3	60.3	79.9	1.32
1	50.0	70.2	1.40
2	49.6	66.2	1.33
3	49.6	62.8	1.26
1, 2	58.0	79.3	1.37
2, 3	58.0	74.3	1.28
1, 3	58.4	77.3	1.32

*Sum of currents in *energized conductors only.*

TABLE X.

Induction in Power Conductors from Residual Current.* Power Circuit Nontransposed. Idle Power and Telephone Conductors Isolated at Laboratory.

Power conductors Nos.		Residual current,* amperes	Induced voltage	
Disturbing	Disturbed		Volts	Volts per ampere
1	2	49.6	83.9	1.69
1	3	49.6	73.0	1.47
2	1	50.2	84.6	1.69
2	3	50.0	84.8	1.69
3	1	50.0	74.3	1.49
3	2	50.0	85.0	1.70
1	2—3	50.0	10.8	0.216
2	1—3	49.8	0.17	0.003
3	1—2	50.2	11.0	0.219

TABLE XI.

Shielding by Grounding Idle Power Conductor. Longitudinal Induction in Power Conductors from Residual Current.* Telephone Conductors Isolated at Laboratory.

Power conductors Nos.			Residual current,* amperes	Induced voltage		
Disturbing	Disturbed	Shield		Volts	Volts per ampere	Shielded / Unshielded
1	2	3	49.6	17.2	0.347	0.21
1	3	2	49.6	3.45	0.070	0.048
2	1	3	50.2	7.64	0.152	0.090
2	3	1	50.0	8.08	0.162	0.096
3	1	2	50.0	4.06	0.081	0.054
3	2	1	50.0	17.7	0.354	0.21

*Sum of currents in *energized conductors only.*

3. *Discussion of Results (Residual Current).*

The transverse induction in the telephone circuit is approximately proportional to the length of unbalanced exposure, the discrepancy noted being probably due to the irregularities. Considering the untransposed condition of the telephone circuit, the greatest coefficient of induction is observed when the lowest power conductor is energized. The coefficient with all three energized is very closely the average of the three coefficients for the power conductors energized singly.

The longitudinal measurements are all subject to an effect due to conduction. The amount of the ground resistance common to disturbed and disturbing circuits is indeterminate since separate ground contacts were used for the two circuits at each end, but as will be shown in a succeeding section of this report, the differences between the computed and observed coefficients can be satisfactorily explained by the assumption of a single value of ground resistance common to the two circuits, which is well within reasonable limits.

Considering induction between power conductors the large degree of shielding obtained (Table XI) is due to the large amount of ground resistance common to the disturbed and shielding circuits, the same ground contacts being used for these circuits at each end. This makes the coupling between the disturbed and shielding circuits very close.

Comparing the coefficients of transverse induction in two power conductors due to "residual current" in the third with the coefficients for the complementary case under "Single-Phase Currents" (page 417) of longitudinal induction in one power conductor due to single-phase current in the loop formed by the other two, it is to be noted that the coefficients recorded in this section are the greater (by 4% to 7%) except in the instance of the middle conductor against the upper and lower conductors, in which case the induction was too small to give good meter readings. They should be the same, since the "disturbing" and "disturbed" circuits were simply interchanged in the two tests.

IV. Computations.

The dimensions given on Drawing No. 230, attached, form the basis for the computations of the coefficients of induction described and discussed in this section of the report. The dimensions are given in inches between centers. The data were obtained from drawings showing standard construction of the Pacific Light and Power Corporation. It was assumed that all the conductors, both power and telephone, have the same sag so that the average distances between conductors and between circuits are those which obtain at the cross-arms. The average height of the conductors was obtained from the average height at the cross-arms and an assumed sag. There is possibility of considerable error in the dimensions, due to a difference in sag and to variations from standard construction. The error in the assumed dimensions is probably about 5%.

As a basis of the formulas used in these computations, it is assumed that the radii of the conductors are small compared to the distances between them and that both are small compared to the length of the conductors. The computations are based upon the further assumption that the mutual resistance between disturbed and disturbing circuits is zero. In all the electromagnetic computations the distortion of the field, due to the iron telephone conductors, was neglected.

For electrostatic induction the positions of the image conductors were reckoned from the earth's surface. For electromagnetic induction the position of the equivalent earth plane was determined by a comparison of the measured self inductance of the three power conductors in parallel, using the full length of line to Somis, and the computed self inductance which is plotted on drawing No. 307 as a function of the

depth of the equivalent earth plane. It was assumed that the position thus determined holds for the short section of line. However, the errors which would have been introduced by neglecting the effect of the image conductors on the coefficients of electromagnetic induction from balanced and single-phase currents, amount to less than 3% for longitudinal induction, and less than 0.004% for transverse induction. For residual currents the coefficients of induction were computed for several positions of the equivalent earth plane and the results are plotted on drawings Nos. 231, 272 and 314; showing the effect of variations in the depth of equivalent earth plane on the induction from residual currents.

The coefficients of longitudinal induction for each telephone conductor and for the two in parallel, and the coefficients of transverse induction, arising from balanced voltages, balanced currents, single-phase currents, residual voltages and residual currents are given in Table XII below. For residual voltage and residual current, besides the normal case of three power conductors energized in parallel, the coefficients for the power conductors energized singly are given. Furthermore, the coefficients of induction from one power conductor into another and the induction between two power conductors with the third energized are given.

The coefficient of transverse induction from balanced voltages is greater than the coefficients of transverse induction from residual voltage. Since residual voltage is defined as the vector sum of the voltages to ground, for balanced and residual voltages of equal magnitude, the relative magnitudes of the voltages from individual conductors to ground are 1 and ⅓, respectively. Also the electric field about the power conductors diminishes more rapidly with increase in separation when balanced voltages are impressed than when residual voltages are impressed, and since the transverse induction is a measure of the change in potential between two points in close proximity, the coefficient of transverse induction may reasonably be greater for balanced voltages than for residual voltage.

TABLE XII.

Computed Coefficients of Induction for Nontransposed Circuits—Length 15,156 Feet.

Disturbing factor	Energized power conductors	Induced voltage—volts per volt or ampere									
		Telephone conductors				Idle power conductors					
		Longitudinal			Transverse	Longitudinal			Transverse		
	Nos.	a	b	a and b	a—b	1	2	3	1—2	1—3	2—3
Balanced voltage	all	0.056	0.049	0.052	0.0067						
Balanced current	all	0.128	0.115	0.122	0.0129						
Single-phase current	1—2	0.082	0.072	0.077	0.0097			0.203			
	2—3	0.066	0.061	0.064	0.0049	0.199					
	3—1	0.148	0.133	0.140	0.0146		0.0042				
Residual voltage	all	0.101	0.098	0.099	0.0035						
	1	0.185	0.176	0.180	0.0086		0.318	0.246			0.071
	2	0.158	0.153	0.155	0.0048	0.313		0.326		0.0042	
	3	0.138	0.135	0.136	0.0080	0.240	0.322		0.082		
Residual current	all	1.07	1.06	1.07	0.0150						
	1	1.15	1.13	1.14	0.0231		1.44	1.24			0.199
	2	1.07	1.06	1.06	0.0134	1.44		1.44		0.0042	
	3	1.00	0.99	1.00	0.0086	1.24	1.44		0.203		

In Table XIII are tabulated values of longitudinal and transverse induction for nontransposed circuits together with their ratios. It will be noted that the ratios for the balanced and single-phase components are greater than for the residual components. Considering the induction from single-phase and residual components, the ratio of transverse to longitudinal induction decreases as the separation between power and telephone conductors is increased, indicating that the rate of change of the transverse induction, with change in separation, is greater than that of the longitudinal induction. The limited number of cases upon which this conclusion is based does not permit of its being stated as general for all conditions.*

*See T. R. No. 65.

TABLE XIII.

Comparison of Transverse and Longitudinal Induction in Telephone Circuit. From Computed Coefficients. For Nontransposed Circuits.

Disturbing factor	Energized power conductors Nos.	Induced voltage, volts per volt or ampere		Transverse / Longitudinal
		Longitudinal	Transverse	
Balanced voltage	all	0.052	0.0067	0.129
Balanced current	all	0.122	0.0129	0.106
Single-phase current	1—2	0.077	0.0097	0.126
	2—3	0.064	0.0049	0.077
	1—3	0.140	0.0146	0.104
Residual voltage	all	0.099	0.0035	0.035
	1	0.180	0.0086	0.048
	2	0.155	0.0048	0.031
	3	0.136	0.0030	0.022
Residual current	all	1.07	0.0150	0.014
	1	1.14	0.0231	0.020
	2	1.06	0.0134	0.013
	3	1.00	0.0086	0.009

For the special cases of residual voltage and residual current with a single conductor of the power circuit energized, the shielding effects of grounding the idle power conductors were computed and the results are given in Tables XIV and XV.

TABLE XIV.

Shielding by Grounding Idle Power Conductors. Induction in Telephone Circuit from Residual Voltage.

Energized power conductor No.	Computed induced voltage					
	Longitudinal			Transverse		
	Volts per volt		Shielded / Unshielded	Volts per volt		Shielded / Unshielded
	Unshielded	Shielded		Unshielded	Shielded	
1	0.180	0.133	0.74	0.00858	0.00777	0.91
2	0.155	0.089	0.57	0.00482	0.00226	0.47
3	0.136	0.075	0.55	0.00296	0.00036	0.12

TABLE XV.

Shielding by Grounding Idle Power Conductors, Longitudinal Induction in Power Conductors from "Residual Current."

Power conductor No.			Computed induced voltage		
			Volts per ampere		Shielded / Unshielded
Disturbing	Disturbed	Shielding	Unshielded	Shielded	
1	2	3	1.44	1.11	0.78
1	3	2	1.24	0.87	0.70
2	3	1	1.44	1.11	0.78
2	1	3	1.44	1.11	0.78
3	1	2	1.24	0.87	0.70
3	2	1	1.44	1.11	0.78

V. Comparison of Measured and Computed Coefficients.

To facilitate comparison of the measured and computed coefficients of induction for the nontransposed condition of the circuits, corresponding values are given in the following tables, together with the ratios of measured to computed values.

TABLE XVI.

Comparison of Measured and Computed Coefficients of Induction. Power Circuit to Telephone Circuit—Nontransposed Circuits—Length 15156 Feet.*

Disturbing factor	Energized power conductors Nos.	Coefficients of induction					
		Longitudinal			Transverse		
		Volts per volt or ampere		Measured / Computed	Volts per volt or ampere		Measured / Computed
		Measured	Computed		Measured	Computed	
Balanced voltages	1, 2, 3	—	0.052	—	0.0070	0.0067	1.04
Balanced currents	1, 2, 3	0.127	0.122	1.04	0.0154	0.0129	1.19
Single-phase currents	1—2	0.082	0.077	1.06	0.0123	0.0097	1.27
	2—3	0.068	0.064	1.06	0.0058	0.0049	1.18
	1—3	0.150	0.140	1.07	0.0178	0.0146	1.22
Residual voltage	1, 2, 3	0.099	0.099	1.00	0.0026	0.0035	0.74
	1	0.176	0.180	0.98	0.0075	0.0086	0.87
	2	0.141	0.155	0.91	0.0033	0.0048	0.69
	3	0.140	0.136	1.03	0.0015	0.0030	0.50
Residual current	1, 2, 3	1.32	1.07	1.24	0.0174	0.0150	1.16
	1	1.40	1.14	1.23	0.0267	0.0231	1.16
	2	1.33	1.06	1.25	0.0151	0.0134	1.13
	3	1.26	1.00	1.26	0.0092	0.0086	1.07

*Except measurement of transverse induction from residual voltage made on 3840-foot section.

TABLE XVII.

Comparison of Measured and Computed Coefficients of Induction from "Residual Current." Between Conductors of Nontransposed Power Circuit—Length 15156 Feet.

Power conductors Nos.		Coefficient of induction					
		Longitudinal			Transverse		
Disturbing	Disturbed	Volts per ampere		Measured / Computed	Volts per ampere		Measured / Computed
		Measured	Computed		Measured	Computed	
1	2	1.69	1.44	1.17			
	2—3				0.216	0.199	1.08
	3	1.47	1.24	1.18			
2	3	1.69	1.44	1.17			
	1—3				0.003	0.004	0.75
	1	1.69	1.44	1.17			
3	1	1.49	1.24	1.20			
	1—2				0.219	0.203	1.08
	2	1.70	1.44	1.18			

It is impossible to make a true comparison of the measured and computed coefficients of longitudinal induction from residual current, since under the conditions of the tests the amount of ground resistance common to both circuits is not known, separate grounds having been used for each circuit at both ends of the parallel. The magnitude of the ground resistance common to both circuits which would be necessary to satisfactorily explain the difference between the measured and computed values of the coefficients of induction has been found to be 0.8 ohm. This figure does not appear unreasonable inasmuch as the disturbing circuit contained about 9 ohms ground resistance and the disturbed circuit about 75 ohms. Accordingly, the values of longitudinal induction from residual current have been recomputed on the assumption of 0.8 ohm ground resistance common to both disturbed and disturbing circuits and are given in Table XVIII. Without such allowance the maximum difference between the measurements and computations amounts to 26%. Had not separate ground connections been used for the two circuits in the residual-current tests the effect of ground resistance would have greatly outweighed the effect due to the mutual inductance between the two circuits, which condition was not desired.

TABLE XVIII.

Comparison of Measured and Computed Coefficients of Longitudinal Induction from Residual Current. 0.8 ohm Common Ground Resistance Assumed.

Disturbing conductors	Disturbed conductors	Induced voltage, volts per ampere: Measured	Induced voltage, volts per ampere: Computed	Measured / Computed
1, 2, 3	Telephone	1.32	1.33	0.99
1	Telephone	1.40	1.38	1.01
2	Telephone	1.33	1.33	1.00
3	Telephone	1.26	1.28	0.98
1	2	1.69	1.65	1.02
	3	1.47	1.47	1.00
2	1	1.69	1.65	1.02
	3	1.69	1.65	1.02
3	1	1.49	1.47	1.01
	2	1.70	1.65	1.03

The measured coefficients of transverse electromagnetic induction are in all cases greater than the computed, the differences ranging from 7 to 27 per cent. The differences are mainly due to two classes of errors which may have any phase with respect to each other, experimental errors which affect the measurements, and errors in the assumed dimensions which affect the computations. In the measurements on the non-transposed telephone circuit the experimental error is less than 5%. With the small separation of circuits which obtains in this parallel, the coefficients of transverse induction change rapidly with changes in separation. Computations show that 1% change in vertical separation causes $2\frac{1}{2}$% change in the coefficients of transverse induction from balanced currents. It was assumed in the computations that the power and telephone conductors sag the same amount. Since the power conductors are copper and the telephone conductors iron, it is reasonable to suppose that the power conductors sag the greater amount. This would cause the actual separation to be less than that assumed, and hence the computed coefficients to be too small.

The error in the computed coefficients of electromagnetic induction due to the assumption of unit permeability for the telephone conductors is small as compared to those discussed above. When a cylindrical iron conductor is placed transversely in an otherwise uniform magnetic field the field is distorted, more lines of induction passing through the iron than through a space of equal extent on either side. The field, however, remains symmetrical with respect to the iron conductor and hence the total flux interlinked with the circuit is the same as if the field were not distorted. Since the magnetic field intensity about the power conductors is not uniform there is a slight increase in the total flux enclosed by the circuit due to the increased permeability of the space occupied

by the iron conductors. On account, however, of the small change in field intensity over the area occupied by the telephone conductors (0.03% for residual current) and between them (4.7%) and the small cross-section area of iron, this effect is not of practical importance, on either longitudinal or transverse induced voltage. This is entirely apart from a consideration of the effect of the iron on the induced current. Its effect in increasing the impedance of the circuit has an important bearing in this regard.

The measured coefficients of transverse induction from residual voltage are in all cases less than those computed, the extreme difference being 50%. As shown below this large difference is due to capacitance unbalance of the telephone circuit, it being assumed in the computations that the telephone circuit was perfectly balanced. Measurements were made of capacitance unbalance and transverse capacitance of the non-transposed telephone circuit to Pole No. 99. The capacitance unbalance is plotted on drawing No. 324, attached. The magnitude of the transverse open-circuit voltage due to the measured capacitance unbalance to ground (assuming same capacitances per unit length applied to section to pole 50) and the computed longitudinal voltage, was determined by using formula 9 of technical report No. 55. Measurements confirm what is to be expected from the position of the power and telephone circuits, with respect to each other and the poles, that the telephone conductor having the greater capacitance to the power circuit also has the greater capacitance to ground. Hence, in accordance with technical report No. 55, the effects of the unbalances to the power circuit and to ground are in opposition. Allowance for this has been made in the computed coefficients of transverse induction given in Table XIX, which are thus brought into much closer agreement with the measurements. The measured coefficients exceed these computed values from 0 to 12% for residual voltages, and 15% for balanced voltages. This relationship of measured and computed coefficients is in accord with the case of electromagnetic induction.

TABLE XIX.

Comparison of Measured and Computed Coefficients of Transverse Electrostatic Induction. Capacitance Unbalance of Telephone Circuit Considered.

Disturbing factor	Energized power conductors Nos.	Induced voltage, volts per volt		Measured / Computed
		Measured	Computed	
Balanced voltages	1, 2, 3	0.0070	0.0061	1.15
Residual voltage	1, 2, 3	0,0026	0.0024	1.08
	1	0.0075	0.0067	1.12
	2	0.0033	0.0031	1.06
	3	0.0015	0.0015	1.00

Table XX compares the shielding, measured and computed, for the case of induction from "residual voltage," one power conductor being energized and the other two grounded. The computed values of shielding for transverse induction take account of the capacitance unbalance of the telephone circuit. For longitudinal induction the observed shielding was smaller than the computations indicated. For the one case of shielding on transverse induction for which measurements were made, the two values check very closely.

TABLE XX.

Comparison of Measured and Computed Shielding. Idle Power Conductors Grounded. Induction in Telephone Circuit from Residual Voltage.

Energized power conductor No.	Induced voltage ratio—shielded to unshielded			
	Longitudinal		Transverse	
	Measured	Computed	Measured	Computed*
1	0.81	0.74	0.97	0.96
2	0.71	0.57	—	0.41
3	0.62	0.55	—	0.32

*Capacitance unbalance of the telephone circuit considered.

The electromagnetic shielding effects observed in the residual-current test are very much greater than the computed shielding effects as a comparison of Table XI and Table XV will show. In the computations it was assumed that the ground resistance was negligible, whereas there was approximately 8 ohms in the disturbing circuit, and 75 ohms common to both the disturbed and shielding circuits. Approximately 0.8 ohm was common to all three circuits. Computing the shielding with these values of ground resistance the computations and observations are in satisfactory agreement. A comparison of Tables XI and XV shows the relative importance of the mutual inductance and mutual resistance. It will be seen that the effect of the mutual resistance is far the more important in this case.

VI. Resumé.

In the preceding sections the features of chief interest have been discussed in connection with the tables presenting the data. In Section V detailed comparisons between measured and computed coefficients are given. The purpose of this resumé is to comment only upon the more important aspects of the report.

The measured coefficients of induction are, with one or two exceptions, greater than the computed values. The coefficients of longitudinal induction are in better agreement than the coefficients of transverse induction. An error in separation of circuits, such as would be caused by a wrong assumption in regard to relative sag of the two circuits, would cause all the computed coefficients to be in error in the same

direction and the transverse coefficients to be in greater error than the longitudinal. The disagreements noted between the computed and measured coefficients are such as to make this explanation of the differences probable. There are no grounds for believing that the iron of the telephone conductors appreciably modifies the mutual inductance.

In the comparison of computed and measured coefficients of transverse electrostatic induction account must be taken of capacitance unbalance to ground of the telephone circuit. It is assumed in the computations based upon the dimensions of this system that the telephone circuit is perfectly balanced. Allowance for the measured capacitance unbalance, made by applying formula 9 of technical report No. 55, brings the measured and computed values of transverse electrostatic induction into reasonably close accord. In this particular case the effect of this unbalance is to decrease the induction as compared to what it would be, were the telephone circuit perfectly balanced to ground. That the effects of the unbalance to ground and unbalance to power circuit are in opposition is in accord with the positions of the two circuits with respect to each other and the poles, causing the same telephone conductor to have the greater capacitance both to the power circuit and to ground. The effect of capacitance unbalance to ground is greater for residual than for balanced voltages, since for the latter the ratio of transverse to longitudinal induction is greater. At the separations used in these experiments the relative effect of capacitance unbalance increases as the separation of circuits increases, since the transverse induction decreases more rapidly than the longitudinal induction with a given increase in separation.

With the relative position of circuits considered in this report, the accuracy of the computed coefficients of longitudinal induction is approximately that of the dimensions upon which the computations are based. The coefficients of transverse induction are less precise than the dimensions, since a change in separation causes a greater change proportionately in the coefficients of transverse induction.

When the length of unbalanced exposure is a considerable part of the total length of line the measured induced voltage is approximately proportional to the computed unbalanced exposure. For an unbalanced exposure, small as compared to the total length of line, the effects of irregularities are more apparent and hence more induction is measured than would be expected from the computed length of unbalanced exposure.

Comparison of the results of the measurements and computations shows very clearly, as is to be expected, that the coefficients of induction for short uniform parallels between nontransposed circuits may be computed with sufficient accuracy for all practical purposes. For the higher harmonics and for long parallels greater refinements in compu-

tations become necessary. In irregular parallels, and in cases of frequently transposed circuits such that the unbalanced exposures are small fractions of the total length of the parallel, accurate computations of the induction are difficult and measurements are to be preferred.

An investigation of the relative merits of different configurations of both power and telephone circuits, based upon the corresponding coefficients of induction and degree of unbalance to ground of circuits for unit nontransposed sections, and an investigation of the variation of induction with separation and relative positions of circuits, can be carried out by means of computations more advantageously than by experimental determinations. Such computations could be made in far less time than would be required for an experimental investigation and with a fraction of the expense, and the information derived would be fully as valuable. The chief sources of disagreement between the computations and measurements recorded in this report are experimental errors, errors in assumed dimensions, capacitance unbalance and ground resistance. In a comparison of different configurations and in a study of the variation of induction with separation, such errors would not affect computations. On the other hand they would be sources of difficulty in an experimental study. The effects of capacitance unbalance and of ground resistance, on the coefficients of induction, could be separately studied if desired. The effect of variations in the depth of the equivalent earth plane, within the range indicated by previous tests, should be considered in the computations. Further experimental data concerning the depth of earth currents under various practical conditions can be obtained independently of any measurements of induction between power and communication circuits, by self-impedance measurements on grounded circuits.

Respectfully submitted.

(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

ATTACHMENTS: P. I. C. Drawings Nos. 230, 231, 272, 307, 314 and 324.

APPROVED: February 18, 1916.

(Signed) R. W. MASTICK,
Field Engineer.

APPROVED: May 2, 1916.

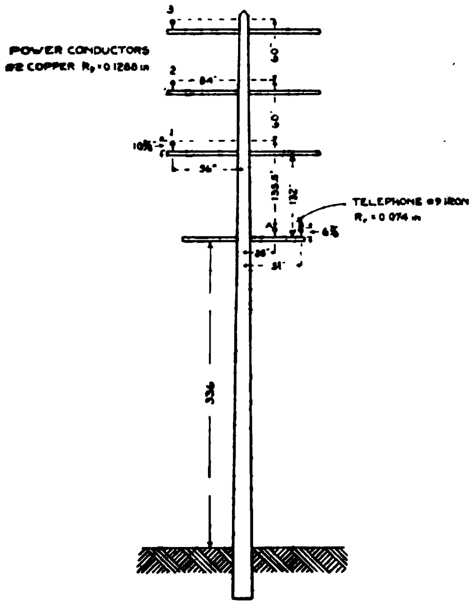
SUBCOMMITTEE ON TESTS,
(Signed) J. E. WOODBRIDGE,
Chairman.

May 19, 1916.

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE,
(Signed) ARTHUR F. BRIDGE,
Secretary.

P.I.C.#250
2/10/15

Typical Pole Diagram
Pacific Light & Power Corporation.
San Fernando - Somis Line
from Pole #34 to Pole #104.
For computations of coefficients of induction.



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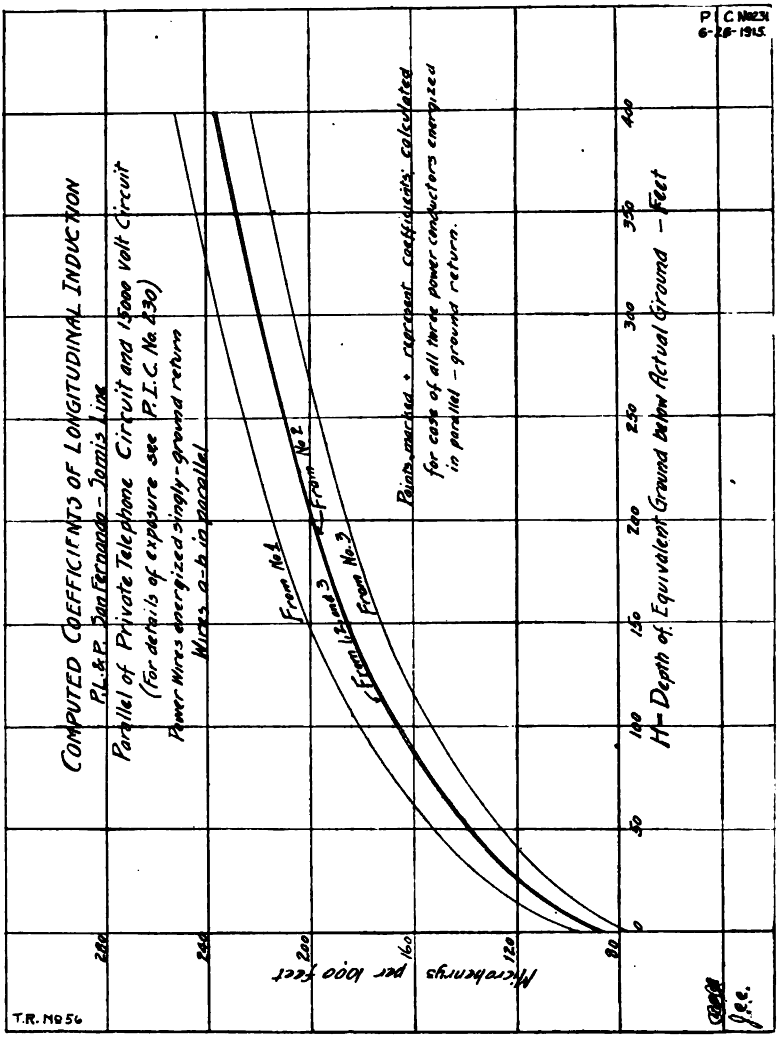
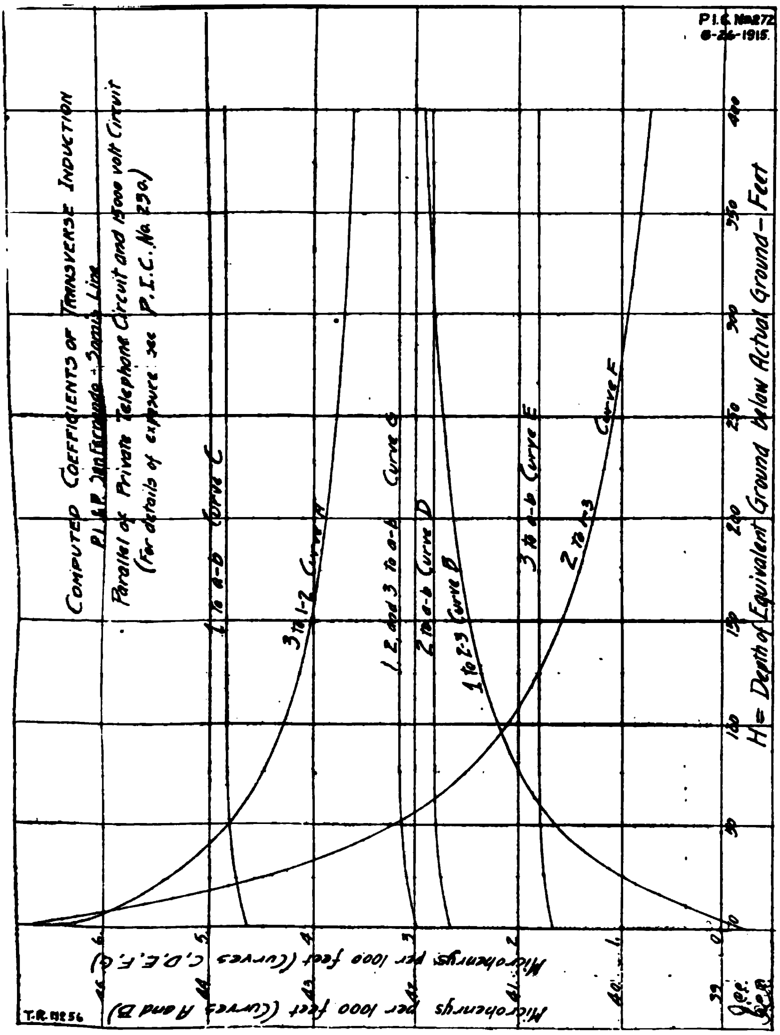
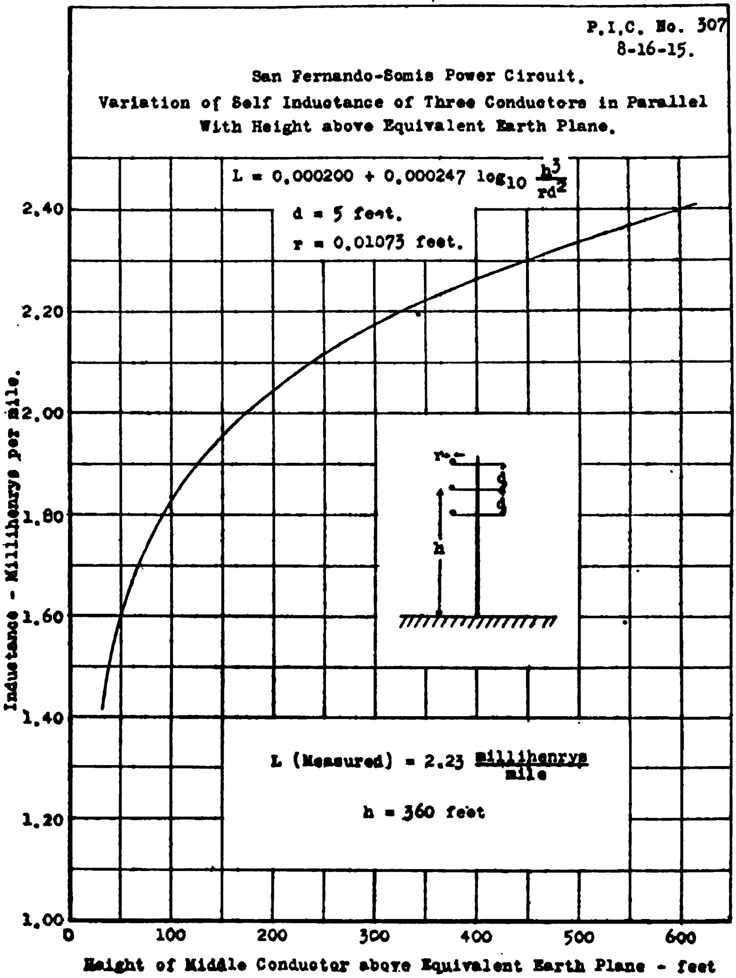
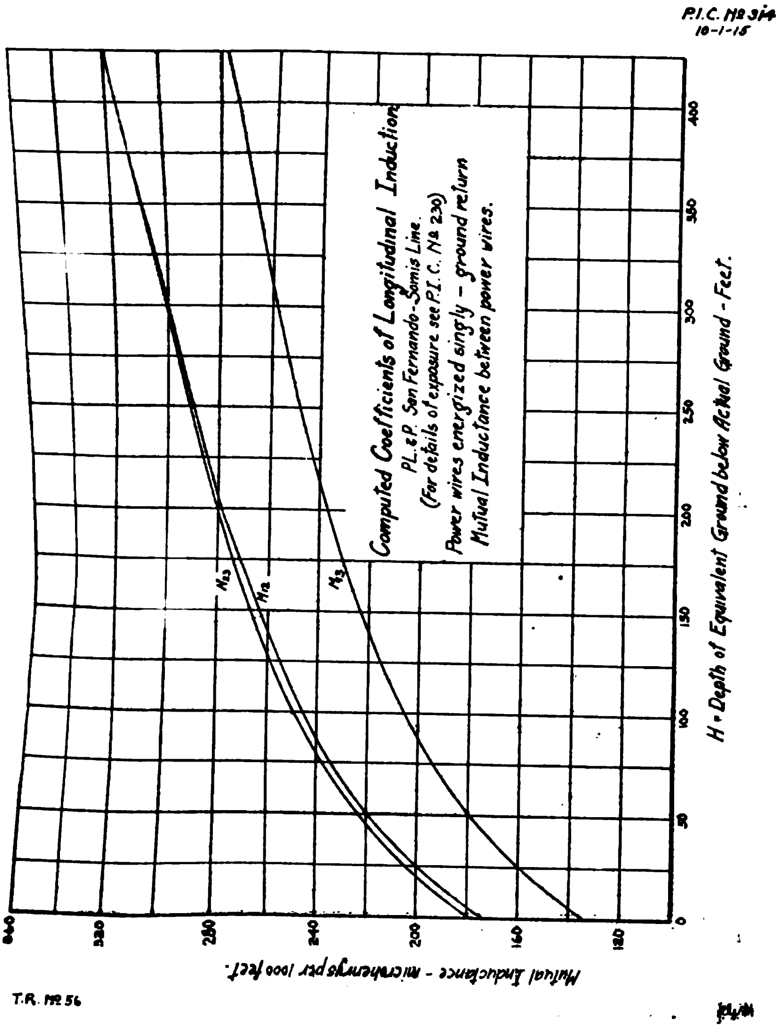
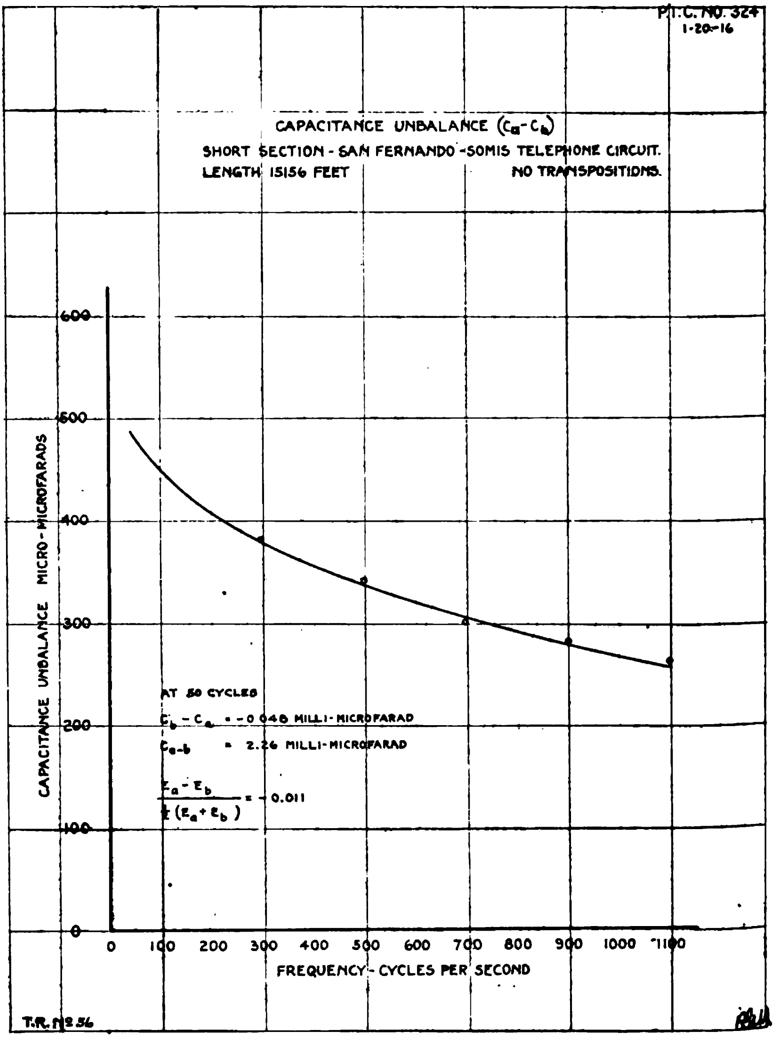

COMPUTED COEFFICIENTS OF LONGITUDINAL INDUCTION
P.L.&P. San Fernando – Somis Line
Parallel of Private Telephone Circuit and 15000 volt Circuit
(For details of exposure see P.I.C. No. 230)
Power Wires energized singly-ground return
Wires a-b in parallel
Points marked + represent coefficients calculated
for case of all three power conductors energized
in parallel – ground return.
From No. 1
From No. 2
From No. 3
From 1, 2 and 3
H – Depth of Equivalent Ground below Actual Ground – Feet
0
50
100
150
200
250
300
350
400
Microhenrys per 1000 feet
80
120
160
200
240
280
P.I.C. No. 231
6-28-1915
T.R. No 56
J.e.c.


P.I.C. No. 272
6-26-1915
COMPUTED COEFFICIENTS OF TRANSVERSE INDUCTION
Parallel of Private Telephone Circuit and 15000 volt Circuit
(For details of exposure see P.I.C. No. 230)
1 to a-b Curve C
3 to 1-2 Curve A
1, 2, and 3 to a-b Curve G
2 to a-b Curve D
1 to 2-3 Curve B
3 to a-b Curve E
2 to 1-3
Curve F
H = Depth of Equivalent Ground below Actual Ground—Feet
Microhenrys per 1000 feet (Curves C, D, E, F, G)
Microhenrys per 1000 feet (Curves A and B)
T.R. No. 56


P.I.C. No. 307
8-16-15.
San Fernando-Somis Power Circuit.
Variation of Self Inductance of Three Conductors in Parallel
With Height above Equivalent Earth Plane.
$L = 0.000200 + 0.000247 \log_{10} \frac{h^3}{rd^2}$
d = 5 feet.
r = 0.01073 feet.
L (Measured) = 2.23 millihenrys / mile
h = 360 feet
r
d
d
h
Inductance - Millihenrys per mile.
2.40
2.20
2.00
1.80
1.60
1.40
1.20
1.00
0
100
200
300
400
500
600
Height of Middle Conductor above Equivalent Earth Plane - feet

T.R. No. 56


P.I.C. No 314
10-1-15
Computed Coefficients of Longitudinal Induction
P.L.&P. San Fernando-Saugus Line.
(For details of exposure see P.I.C. No 230)
Power wires energized singly - ground return
Mutual Inductance between power wires.
M23
M12
M13
H = Depth of Equivalent Ground below Actual Ground - Feet.
0
50
100
150
200
250
300
350
400
Mutual Inductance - microhenrys per 1000 feet.
340
320
280
240
200
160
120
T.R. No 56


P.T.C. NO. 324
1-20-16
CAPACITANCE UNBALANCE (C_a-C_b)
SHORT SECTION - SAN FERNANDO - SOMIS TELEPHONE CIRCUIT.
LENGTH 15156 FEET
NO TRANSPOSITIONS.
CAPACITANCE UNBALANCE MICRO-MICROFARADS
600
500
400
300
200
100
0
AT 50 CYCLES
$C_b - C_a$ = -0.048 MILLI-MICROFARAD
C_{a-b} = 2.26 MILLI-MICROFARAD
$\frac{E_a - E_b}{\frac{1}{2}(E_a + E_b)}$ = -0.011
0 100 200 300 400 500 600 700 800 900 1000 1100
FREQUENCY - CYCLES PER SECOND
T.R. Nº 56

Technical Report No. 58.

April 14, 1916.

INVESTIGATION OF POTENTIAL TRANSFORMERS FOR RESIDUAL VOLTAGE MEASUREMENTS.

March, 1915.

OUTLINE.

I. Introduction.

A—Review of Previous Work.

Measurements of the residual voltage of an electrical system by potential transformers are subject to errors introduced by the transformers; the residual voltage may also be modified by their presence. Two methods of measuring residual voltage* have been used in the Committee's tests. The first, known as the "delta" method, consists of determining the voltage across the open corner of the secondary delta of a bank of transformers, whose primaries are connected in Y with neutral grounded. The second, known as the "Y" method, consists of determining the voltage between the neutral point of the Y-connected primaries of the bank of transformers and ground, their secondaries being

*Of three-phase circuits.

connected in closed delta. Some investigations of these methods have been made and the results set forth in technical reports and memoranda, as follows:

Technical report No. 13 gives results of tests made at Salinas, Cal., in April and July, 1913, on the bank of potential transformers hereinafter described. These tests included measurements of

(a) voltage ratio between pairs of low-tension taps,
(b) internal impedance,
(c) inequality of ratio,
(d) voltage across open corner of high-tension delta when excited from low-tension side with windings connected in delta,
(e) current and voltage across one corner of low-tension delta when excited from high-tension side with windings connected either delta or Y.

The frequency of the supply was 60 cycles.

Technical report No. 38 (7-f) states briefly the principal results of technical report No. 13, with some corrections thereto.

A memorandum to the Chairman of the Subcommittee on Tests, dated January 8, 1914, "summarizes the status of the present (delta) method of measuring residual voltage." The errors are discussed and the requirements of a satisfactory method outlined.

A memorandum dated March 28, 1914, is a "Comparison of Two Methods of Measuring Residual Voltage." Drawing No. 190, attached thereto, has diagrams of the two methods.

A memorandum dated March 17, 1915, discusses conditions of measurement of residual voltage at San Fernando, the advantages there present for tests of the methods in use, and gives an outline of tests.

Copies of these memoranda are attached to the Secretary's file-copy of this report.

B—Purpose.

It was the purpose of the tests here reported, which were carried out in accordance with the memorandum of March 17, 1915, to redetermine the quantities previously measured; and to furnish additional information whereby the two methods of measuring residual voltage might be compared, and their errors made known.

II. Description of Transformers and Line.

A—Transformers.

The tests were made on the three 13200/110-55-volt potential transformers, which have been used at Salinas, Santa Cruz and San Fernando for residual-voltage measurements on the high-tension lines. They are of General Electric Company manufacture; Type P, Form C, 200 watts,

Spec. No. 36499, 25/60 cycles with serial numbers 573938 (herein designated as No. 1), 458889 (No. 2), and 454895 (No. 3).

A 2200-1100/110 volt auxiliary potential transformer was used to measure the voltage from the high-tension neutral of the potential transformers to ground. It is of General Electric Company manufacture; Type H, Form P. A., 10 watt, Spec. No. 12659, 60/125 cycle, Serial No. 150038.

The energy supply for tests described in IV C and IV F was derived from three Westinghouse 37½-kVA. 220/14250-volt step-up transformers, which were connected delta on the low-tension side and either Y or delta on the high-tension side. These were supplied from the Pacific Light & Power Corporation's 15-kV. 50-cycle system through a 15000/220-volt delta-delta bank of similar transformers, stepping down. Both banks were isolated from ground.

B—Transmission Line.

The San Fernando-Somis 15-kV. line, used in tests IV F, is 36.7 miles in length, of vertical construction, with wires of No. 2 and No. 4 B&S gauge copper, for 21% and 79% of the distance, respectively. The conductors are five feet apart and the lowest one is approximately forty feet above the earth's surface. The line is divided by five transpositions into two complete barrels, closely balanced. This line has been used throughout the San Fernando tests, with several transposition systems.

III. Scope of Tests.

In the present investigation, the following tests were made:

A—The ratios of transformation were determined directly, at a low value of impressed voltage.

B—The inequalities of ratio were determined by connecting the transformers in pairs, and measuring the differences in high-tension terminal voltages, with low-tension coils in parallel, high-tension coils in series opposing.

C—The potential transformers were connected delta-delta and energized on the high-tension side, and the voltage across the corner of the low-tension delta (due to ratio inequality and triple harmonics of magnetizing current) was determined.

D—The internal impedance of each transformer was measured at 50 cycles. The internal impedance of No. 1 was measured at higher frequencies. An approximate determination of the exciting admittance was made, at 50 cycles, for a considerable range of impressed voltage.

E—The ratio, internal impedance and exciting impedance of the auxiliary transformer were measured.

F—The potential transformers were connected to the 15-kV. line, which was open and clear at its far end. This line was energized from an isolated transformer bank connected either delta-Y or delta-delta. The potential transformers being connected Y-delta 13200/110 volts, measurements of neutral current or voltage, delta circulating current, and line voltages to ground, were made with various values of resistance in the corner of the delta (from 1.6 to 6.70 ohms) and with the following conditions of the primary neutral:

1. Grounded.

2. Grounded through an air-core inductance coil designed to make the external impedance, to the residual current introduced by the potential transformers, approximately that with a grounded-neutral Y-delta bank of transformers connected to each end of the 15-kV. line.

3. Grounded through a potential transformer, as used for measurement of residual voltage by the "Y" method.

4. Isolated.

IV. Results of Tests.

A—Magnitude of Ratio.

No means were available of measuring directly the ratio of the potential transformers at normal voltage, but the ratios at 0.9% and 1.8% rated voltage were determined by the method shown in drawing No. 257, A. The mean results of all observations are given in Table I. These results are liable to be in error by approximately ½%, but their relative values are somewhat more accurate.

TABLE I.
Transformer Ratios.

Number	Ratio
1	121.1
2	120.4
3	120.8

The impedance drop in the transformers due to the greater exciting current and to the load current will cause the ratio at rated voltage to be a little different from the values here determined.

B—Inequality of Ratio, by Comparison.

By connecting the low-tension sides in multiple, and measuring the differences of the high-tension terminal voltages with an electrostatic voltmeter, the differences in ratio were determined. The connections are shown on drawing No. 257, B. The data are plotted on drawing No. 258, A with variable low-tension impressed voltage. It is evident that at a given impressed voltage, the observed difference of high-tension

terminal voltages of the pair having maximum difference of ratio should be equal to the sum of the differences for the other two pairs. Such is not accurately true by the data here given. The discrepancy has been ascribed to capacitance between the high-voltage leads as indicated by C′ in drawing No. 257, B, which was eliminated as far as possible by shielding the leads to the electrostatic voltmeter. Before the shield was used, erratically varying indications were given by the electrostatic voltmeter, though the unshielded conductor was but a few feet in length.

Let the ratios of the three transformers be

$$\text{No. } 1 = K + x$$
$$\text{No. } 2 = K + y$$
$$\text{No. } 3 = K$$
$$\text{Impressed voltage} = E$$

Potential difference on voltmeter $= V_{mn}$

Induced voltage due to $C' = r$

Then
$$E(K + y) = E(K + x) + r + V_{12} \tag{1}$$
$$EK = E(K + y) + r + V_{23} \tag{2}$$
$$EK = E(K + x) + r + V_{13} \tag{3}$$

Solving for x, y, and r

$$x = \frac{V_{12} + V_{23} - 2V_{13}}{E} \tag{4}$$

$$y = \frac{V_{12} - V_{13}}{E} \tag{5}$$

$$r = V_{13} - (V_{23} + V_{12}) \tag{6}$$

The signs of the V's are not determined by the measurements; but the value of r, which should be negative and small, serves as a criterion. r is negative, because the induced voltage due to capacitance C′ is opposite in phase to the magnetically induced voltage. There are eight possible combinations of signs of the three V's, four being negatives of the other four. Selecting that for which r is small and negative, we have as data:

$$E = 115 \text{ volts} \qquad \text{whence } x = 0.45$$
$$V_{12} = -118 \text{ volts} \qquad y = -0.63$$
$$V_{23} = +79 \text{ volts} \qquad r = -6\tfrac{1}{2}$$
$$V_{13} = -45\tfrac{1}{2} \text{ volts}$$

Assuming that $K = 120.0$, the ratios are:

TABLE II.

Relative Transformer Ratios.

E—volts	115	85	55
Potential Transformer No. 1	120.4	120.3	120.5
Potential Transformer No. 2	119.4	119.4	119.4
Potential Transformer No. 3	120.4	120.0	120.0

By comparison with (A) it is seen that the two determinations agree as to the relative magnitudes of the transformer ratios. As is to be expected and as shown by the fact that the curves of drawing No. 258-A are approximately straight lines, the inequality of ratio is practically unaffected by the variation of impressed voltage, over the range tested.

To determine the error due to inequality of ratios, when connected for service, assume balanced voltages, E, applied between the lines and the neutral of the three high-tension windings, connected in Y. Then the apparent residual voltage E_R across the open corner of the low-tension delta, in terms of the high-tension side, due to the ratio inequalities, is:

$$\frac{E_R}{E} = \frac{K\angle 0^\circ}{K+x} + \frac{K\angle 120^\circ}{K+y} + \frac{K\angle 240^\circ}{K} \tag{7}$$

$$= -\frac{x}{K}\angle 0^\circ - \frac{y}{K}\angle 120^\circ$$

with close approximation.

Substituting

$$x = +0.45 \quad y = -0.63 \quad K = 120.0$$

$$\frac{E_R}{E} = 0.0079$$

or 0.8% of the impressed voltage from line to neutral.

This value is approximately three times as great as that found at Salinas, where the resultant was determined to be 0.25%. The cause of the discrepancy can only be conjectured, as there is no evident reason for doubting either set of measurements. There was an interval of two years between the two sets of tests, and in the meantime the transformers had been much used, at times under severe conditions, which apparently caused the change, although there was no visible evidence of alteration of the transformers.

C—Apparent Residual Voltage by Delta-Delta Connection.

The voltage across the open corner of the low-tension delta was measured, with the high-tension sides of the transformers in delta. With this connection, the vector sum of the high-tension impressed voltages is zero; the voltage across the open corner of the secondary delta is the vector sum of the secondary voltages. The fundamental-frequency component is due to the inequality in ratio, and the triple-frequency component is due to the variation of permeability of the iron.

The results are given in Table III. See drawing No. 257, C for the diagram of connections. The line voltages were nearly sinusoidal.

TABLE III.

Apparent Residual Voltage.

Date. 1915	Osc. No.	$E_R(1)$ volts	$E_R(3)$ volts	$E_{m-n}(1)$ kV.	$\frac{E_R}{E_{m-n}}(1)$	$\frac{E_R}{E_{m-n}}(3)$
3/23	949	140	59	15.5	0.90%	0.38%
3/23	950	140	61	15.6	0.90%	0.39%
4/16	964	116	62	15.9	0.73%	0.39%
5/27	967	71	75	16.2	0.44%	0.46%
5/27	969—1	75	63	16.1	0.47%	0.39%
5/27	969—2	74	66	16.1	0.46%	0.41%
6/15	975	72	66	15.9	0.45%	0.41%

(1) = fundamental. (3) = third harmonic.

The first measurements indicated a delta-corner fundamental voltage of 0.9% of the total impressed voltage on each transformer. The corresponding value as observed at Salinas was 0.25%. Measurements were then made at three later dates, with varying results, as shown in the table.

In Oscillogram 949, the residual voltage and line voltage were photographed simultaneously; in 950, they were taken separately. It was thus determined that no appreciable difference was made in the observed delta-corner voltage, by simultaneously measuring a phase-voltage.

The delta-corner measuring resistance was in each case 50 ohms or more, and for measuring phase-voltages a resistance of 2500 ohms or more was used. Observation of the delta-corner voltage on the oscillograph tracing table indicated no appreciable change of wave-shape, due to change of delta-corner resistance, at least above a value of 40 ohms.

Table III also shows the value of the third-harmonic delta-corner voltage, which represents the voltage required to produce the third-harmonic current in the primary delta.

D—Internal Impedance and Exciting Admittance.

The resistances of the windings were measured by a Wheatstone bridge, and are given in Table IV.

TABLE IV.
Resistances of Potential Transformer Windings—ohms.

P. T. No.	H. T.	L. T.	Total*
1	6550	0.288	0.743
2	6460	0.290	0.739
3	6490	0.290	0.741

*In terms of low-tension side, $R_1 = 0.288 + \frac{6550}{(120)^2}$

Measurements of the internal impedances of the potential transformers were made on the low-tension sides, for currents of 0.35 to 0.70 amperes at 50 cycles, and are given in Table V. The variation of the impedance with the current is small. See drawing No. 257, D for the connection diagram.

TABLE V.
Internal Impedance—in Terms of Low-Tension Side.

P. T. No.	Internal impedance, ohms	Internal resistance,* ohms	Internal reactance,° ohms	Internal inductance, millihenrys
1	0.804	0.743	0.308	0.982
2	0.812	0.739	0.334	1.062
3	0.801	0.741	0.304	0.968
Average	0.806	0.741	0.315	1.004

*From Table IV. °$X = \sqrt{Z^2 - R^2} = 2\pi fL$.

The average values of total internal impedance for the three transformers are, at 50 cycles:

Low-tension side	High-tension side
$R = 0.741$ ohms	$R = 10680$ ohms
$X = 0.315$ ohms	$X = 4540$ ohms
$Z = 0.806$ ohms	$Z = 11600$ ohms

Referred to No. 1, the impedance of No. 2 is 1% high; that of No. 3 is 0.4% low.

The internal impedance of No. 1 was measured at several frequencies from 300 to 900 cycles per second, on both low- and high-tension sides, with the impedance bridge. For the low-tension side the measured value of the internal inductance is 0.93 millihenrys, with variation of less than 0.3% over the range from 300 to 900 cycles per second, with currents of a few milliamperes. In Table VI is given the comparison of internal inductances as measured on the high- and low-tension sides.

TABLE VI.
Internal Inductance.

Frequency cycles per second	L. T. side, millihenrys	H. T. side, henrys	$\left\{\frac{\text{H. T.}}{\text{L. T.}}\right\}^{\frac{1}{2}}$
300	0.927	14.2	124
500	0.930	16.4	133
700	0.930	20.4	148
900	0.928	21.4	152

The internal inductance as given by Table VI, checks the value of Table V within a few per cent. The fourth column gives a rough approximation of the ratio, since the relation between the inductances measured on the two sides is the square of the ratio of turns. The apparent inductance, measured on the high-tension side, increases rapidly with the frequency.

The values here given for internal inductance are about 0.35 of that given in technical report No. 13 for transformer No. 1. It seems most likely that an error was made in that determination, such as, for example, having extra resistance in the circuit.

In some cases it is desirable to know the internal impedance of each winding separately. As an approximation, the leakage inductance may be assumed as equally divided between high- and low-tension windings; the impedances then are, using the average values:

High-tension winding, $Z'_t = 6500 + j\ 2270\ n$ ohms.
Low-tension winding, $Z''_t = 0.29 + j\ 0.158\ n$ ohms.
$n =$ order of harmonic of 50 cycles.

It is theoretically possible to separate the high- and low-tension coil reactances by observations of the low-tension delta and high-tension neutral currents of triple frequency, but the internal reactances are too small and the precision of measurements insufficient to allow a satisfactory solution with the data of these tests.

There is given on page 20, technical report No. 13, the result of an attempt to separate the leakage reactances of the high- and low-tension windings. It is based on the assumptions (1) that the total third harmonic magneto-motive force due to the circulating currents is constant, with the delta-delta connection, for a variation of the external resistance in the low-tension delta corner from 6 to 26 ohms, and (2) that this total magneto-motive force is the same as that which exists when the transformers are connected Y-delta, with the high-tension neutral ungrounded. These assumptions are, at best, only approximate. It is further assumed that the high- and low-tension delta currents are in the same phase, which is true only when the time-constants of high- and

low-tension sides are the same. As the resistances are larger than the reactances, it is evident that small errors in data and assumptions will cause much greater errors in the reactances. Hence the ratio of reactances in technical report No. 13 can not be accepted as reliable.

The exciting admittance at 50 cycles of the transformers measured on the low-tension side, is approximately as follows:

TABLE VII.

Exciting Admittance—Low-Tension Side.

Impressed voltage volts	Exciting admittance mhos
45	0.0059
90	0.0048
135	0.0045

These values were determined from meter measurements made in connection with IV-B.

E—Auxiliary Transformer, Ratios and Impedances.

The transformer has three high-tension terminals, marked 1, 2, 3, and two low-tension terminals. The ratio was determined with applied voltage of 10.4% normal for the windings 1-2 and 2-3, and 5.2% normal for winding 1-3, at 50 cycles. The results are:

High-tension winding	Ratio
1—2	10.1
2—3	10.1
1—3	20.2

Measurements were made of the coil-resistances and of the internal impedance at 50 cycles on the low-tension side, the high-tension winding 1-3 being short-circuited.

$R_{1-2} = 474$ ohms $\qquad R_{LT} = 6.03$ ohms

$R_{2-3} = 548$ ohms $\qquad R_{1-3} = 1022$ ohms

$Z_{LT} = 8.61 = 8.54 + j\,1.10$ ohms or, in terms of the high-tension side

$$Z_{1-3} = 3520 = 3490 + j\,450 \text{ ohms}$$

The leakage reactance is so small compared to the resistance that it has very slight effect on the impedance. Assuming one-fourth of the leakage reactance in each high-tension coil, the short-circuit impedances are

$$Z_{1-2} = 1140 = 1090 + j\,340 \text{ ohms}$$
$$Z_{2-3} = 1210 = 1160 + j\,340 \text{ ohms}$$

The exciting current was measured with 115 volts at 50 cycles applied to each winding.

Reduced to terms of winding 1-3, the exciting impedance is

Impressed voltage volts	Exciting impedance ohms
115	46,000
230	68,800
2320	34,700

F—Tests Under Operating Conditions.

For these tests the potential transformers were connected as in drawing No. 257, E, which corresponds to the conditions of service, where the high-tension side is in Y and the neutral grounded, either

1. directly, for measurement of residual voltage by the delta method; or,
2. through a potential transformer, for measurement of residual voltage by the Y method.

The low-tension side is connected delta, with a measuring circuit inserted in one corner when measuring residual voltage by the delta method. The corner of the delta is closed when measuring residual voltage by the Y method.

By supplying the line and the potential transformers from an isolated source through one or more delta-connected transformer banks, triple-harmonic residuals (from sources other than the potential transformers themselves) were eliminated. The transpositions in the power line served to reduce the residuals caused by unbalanced capacitances to a small value, comparable to that of the errors of measurement.

The triple-harmonic magnetizing currents had one path in the low-tension delta and another through the external impedance from the high-tension neutral to the line terminals of the transformers. In these tests the external impedances in the high- and low-tension circuits for triple harmonics were varied over a wide range. The external impedance of the high-tension sides of the potential transformers was determined by the impedance of the 15-kV. line wires to ground, and by impedance inserted between the ground and the high-tension neutral. The impedance of the three wires of the San Fernando-Somis 15-kV.

line in parallel to ground is approximately $Z_L = 50 - j\,1400$ ohms at 150 cycles with the far end open and clear.

Residual-voltage measurements have been made on this line with a Y-delta bank of power transformers with grounded neutral at each end of the line. Such a case is typical of conditions where the measurement of the third-harmonic residual voltage is important. For this arrangement, the impedance to ground from the line terminals of the three potential transformers is about $25 + j\,85$ ohms at 150 cycles. For the purpose of the potential transformer tests this condition was closely simulated by inserting an air-core-inductance coil in the circuit between high-tension neutral and ground to neutralize the capacitive reactance of the isolated line. On drawing No. 258 is shown the equivalent impedance of the 15-kV. line (open and clear at the far end) in series with this coil. At 150 cycles this impedance is approximately $110 - j\,100$ ohms. Inasmuch as the impedance to neutral current includes the internal impedance of the high-tension windings of the transformers, which is about $2170 + j\,2270$ ohms, at 150 cycles, for the three transformers in parallel, great refinement is not necessary in the external impedance values.

When the external circuit includes an auxiliary potential transformer, the external impedance is dependent on the impedance through which the secondary of this potential transformer is closed and upon its exciting impedance, and may reach high values. For example, if 300 ohms resistance is in the secondary of a 10:1 potential transformer, the corresponding impedance in the high-tension neutral is approximately $(10)^2 \times 300$ or 30,000 ohms, and thus becomes the controlling factor in the impedance to residuals. The exciting impedance of the auxiliary transformer reduces the above value somewhat. Finally, by isolating the high-tension neutral, practically infinite impedance is offered to residuals.

In Tables VIII and IX are given oscillograms taken under the several conditions discussed, at 100% and at 52% of the normal voltage used at San Fernando corresponding to 125% and 64% of the rated voltage of the potential transformers. Drawing No. 305 gives the results in plotted form, from meter readings and oscillograms, for the delta method.

TABLE VIII.

Oscillograms—Potential Transformers Connected to Isolated Line.

Impedance of potential transformers, H. T., neutral to ground, ohms		Grounded 3.8	Grounded 3.6	Grounded 1.3	Coil + 1.3 ohms				Isolated		Coil + 10.2 ohms		
External impedance of potential transformers, L. T. Delta, ohms.		3.8	4.1	103.3	3.7	9.5	103.3	303.1	3.6	103.6	3.0	53.5	103.5
Secondary delta current of potential transformers, I_d-milliamperes.	n 1	212	182	10.4	170	93	11.6	3.5	53	51	110	11.8	6.5
	3	105	96	5.3	66	41	5.8	2.1	254	250	13	1.8	1.0
	5	9	5	0.6	6	6	1.0	—	13	6	3	0.5	—
	7	—	5	—	—	3	0.4	—	9	—	4	0.9	0.8
	9	6	6	0.6	4	3	0.8	—	7	7	—	—	—
	ESW	238	202	11.8	182	102	13.0	4.1	260	255	111	12.0	6.6
Primary neutral current of potential transformers, I_n-milliamperes.	n 1	4.4	4.0	1.9	4.6	3.0	1.8	1.6			2.2	—	—
	3	4.7	5.0	6.7	5.1	5.7	6.8	7.1			1.4	1.5	1.6
	5	—	0.5	—	—	—	—	—			—	—	—
	ESW	6.5	6.4	7.0	6.9	6.4	7.0	7.2			2.6	1.5	1.6
Line voltage, phase 1 to neutral, E_1 kilovolts	n 1	15.9	16.0	16.0	16.3	16.5	16.4			16.1		8.7	8.6
	3	0.8	0.6	0.4	—	0.4	0.4			0.8		0.2	—
	5	0.7	0.8	0.8	0.6	0.6	0.7			0.6		0.2	—
	7	—	—	—	—	—	—			—		0.2	0.2
	ESW	15.9	16.0	16.1	16.3	16.5	16.4	16.4	16.4	16.1	8.6	8.7	8.6
Oscillogram No.		935	934	936	939	940	941	942	943	944	954	955	956

TABLE IX.

Oscillograms—Potential Transformers Connected to Isolated Line.

Impedance of potential transformers, H. T., neutral to ground—ohms		10:1 P. T. and coil	10:1 P. T.	10:1 P. T. and coil
External impedance of potential transformers, L. T. Delta, ohms.		2.8	2.8	1.6
Primary neutral current of potential transformers, I_n-milliamperes.	n 1	2.6	3.8	1.8
	3	1.9	2.4	1.0
	5	—	—	—
	ESW	3.2	4.5	2.1
Residual voltage, "Y" method, $\frac{1}{3}E_R(Y)$ volts	n 1	32	26	11
	3	24	44	11
	5	—	4	—
	7	—	—	1
	ESW	40	51	15
Line voltage, phase 1 to neutral, E_L, kilovolts.	n 1		16.5	8.5
	3		0.5	0.2
	5		1.0	0.2
	7		—	0.2
	ESW	16.3	16.5	8.5
R_n''-ohms		1065	1065	316
Oscillogram No.		946	948	953

R_n''=resistance in secondary of potential transformer by which E_R (Y) was measured.

The effective value of the high-tension neutral current is nearly constant for the whole range of values of resistance in the corner of the delta with and without the coil in the neutral ground connection. But the oscillograms show that this is due to the fact that the third-harmonic neutral current increases, while the fundamental component decreases, as the delta-corner resistance is increased. With the auxiliary potential transformer in the neutral the neutral current is greatly reduced.

Drawing No. 305 shows values of the equivalent sine wave, fundamental and third-harmonic components of the delta-corner voltage and the neutral current as the resistance in the corner is varied. The quantities exhibit large variations for low values of the resistance in the corner of the delta, but become stationary in value for changes in the delta corner resistance when its value is over 100 ohms.

Lowering the voltage to 64% of normal causes a great reduction in the third-harmonic currents in delta and neutral.

The insertion of the air-core inductance coil in the circuit between the high-tension neutral and ground produced no notable change. The discussion which follows applies therefore to both conditions. The data

for the case of the isolated neutral illustrate the extreme case of high impedance in the neutral.

The quantities measured were in some cases very small, *e. g.* neutral current and residual voltage by the Y method, consequently the values, especially from the oscillograms, give only approximate magnitudes.

V. Discussion of Errors in Residual Voltage Measurements.

The connection of potential transformers to a line for the purpose of measuring the residual voltage at a given point thereon, in general alters the actual residual voltage. The type of connection employed, the characteristics and location of the potential transformers, and the character of the line determine the magnitude of the alteration. If the potential transformers used for residual voltage measurements are normally connected to the line under similar conditions, this effect is not an error; but otherwise the actual residual voltage under the conditions of measurement is different than the residual voltage under normal conditions. The error which may be introduced by such alteration of conditions is very small under conditions of power-system practice. But besides this, there are errors due to the characteristics of the potential transformers and measuring apparatus.

The various errors are discussed in some detail below.

A—Residuals of Same Frequencies as the Balanced Voltages, Due to Admittance Unbalances.

1. *Sources of Errors.*

a. Y method.

In the Y method the three transformers act simply as impedances in star. Inequalities in ratios will produce inequalities in the equivalent impedances between the lines and high-tension neutral, and thus introduce errors in residual-voltage measurements, as discussed in the following paragraphs.

If three impedances of identical magnitudes and time constants are connected in star to the line wires, the vector sum of the voltages from the lines to the common neutral is necessarily zero, if the neutral is isolated; and the voltage from this neutral to ground is one-third of the residual voltage of the line, at the point where the impedances are connected. If the three impedances are unequal the vector sum of the three voltages from lines to neutral will in general not be zero. Hence, the voltage to ground from the neutral of the unequal star impedances will not be a true measure of the residual voltage of the line.

The accuracy of the Y method of residual voltage measurement depends on the considerations just stated; the impedances involved would be the exciting impedances of the transformers except for the effect of the closed secondary delta, in which a current of fundamental

frequency may circulate, due to inequalities in ratios. This delta circulating current tends to equalize the equivalent star impedances of the transformers as compared with their open-circuit impedances. The mathematical analysis of the case is complicated and has not been undertaken.

The observed quantity is the secondary terminal voltage of the auxiliary transformer. The secondary external resistance is usually made high so that practically the secondary induced voltage is measured. When measuring residual voltage there is a current from lines to neutral. The drop of potential through the impedances of the three Y-delta connected transformers may be appreciable compared to the impedance-drop of the auxiliary transformer. For the three Y-connected high-tension windings in parallel (low-tension delta being closed) the impedance is $3560 + j\,1510$ ohms at 50 cycles. With 650 volts impressed on its 2200-volt winding and its 110-volt winding closed through 1000 ohms resistance, the auxiliary transformer used has an effective impedance of 76000 ohms. The total impedance from lines to ground is 79000 ohms. Thus the residual voltage exceeds the impressed voltage on the auxiliary transformer by 4%. With the 1100-volt winding, and lower impedance closing the secondary, the error due to neglecting the drop in the Y-delta bank of transformers would be much greater.

b. delta method.

If the three transformers have unequal ratios an apparent residual voltage will exist across the low-tension delta corner, even though the e. m. fs. between lines and neutral of the high-tension Y are balanced. The observations recorded in IV-B indicate a limiting value of 0.8% of the impressed voltage from line to neutral for approximately balanced impressed voltages. This error is practically independent of line voltage and frequency within the range of use. As determined by the delta-delta test (IV-C) in four trials, this error was 0.9, 0.7 and on two occasions 0.45%. A careful consideration of the measurements failed to reveal an explanation of these variations in the delta-corner voltage unless it is assumed that the transformers undergo changes with time. This view is supported by the large discrepancy between the results at Salinas and those herein recorded and by the differences among the latter.

Evidently, the actual residual voltage may have any phase with respect to this apparent residual voltage, and so values of residual voltage less than 1.5% of the magnitude of the voltage of same frequency from line to ground may be in error by 50% or more. Measurements of the residual voltage on an isolated system are especially concerned with this error.

The delta method is affected slightly by internal-impedance inequalities which influence the regulation. The internal impedances of the transformers are nearly equal, and so the effect will be very small. The observed quantity is the voltage across the corner of the secondary delta. The delta-corner resistance is usually so large compared with the secondary resistance and leakage reactance of the transformers that practically the vector sum of the induced secondary voltages is obtained. The errors in transformation from primary to secondary (inequality of ratio and regulation in the high-voltage windings) have been mentioned above. If advisable, the secondary internal impedance of the three transformers in series can be allowed for in the calculation of residual voltage.

2. *Mathematical Discussion.*

It remains to determine the importance of the factors just considered in measurements with the given transformers and line.

a. Y method.

Consider a three-phase line which has a given admittance unbalance to ground (due to capacitance, conductance, or load unbalance, singly or in combination) with corresponding residual voltage. This residual voltage, at any given point, may be called its "open-circuit" residual voltage. When the bank of potential transformers is connected to the line, as for the Y method of measuring residual voltage, there is a corresponding residual voltage due to their impedance unbalance. The line unbalance and the transformer-impedance unbalance combine to determine the true residual voltage at the given point; that is, there is a component of the true residual voltage due to the line and another due to the impedance unbalance of the transformers. Let:

E_L = open-circuit residual voltage due to line unbalance.
E_T = open-circuit residual voltage due to impedance unbalance of the Y-delta bank of potential transformers.
E_R = true residual voltage of the line with potential transformers connected.
E_{RL} = component of E_R caused by line unbalance.
E_{RT} = component of E_R caused by transformer unbalance.
I_n = neutral current of potential-transformer bank.
Z_n = equivalent impedance of auxiliary transformer in neutral of Y-delta bank.
Z_L = impedance of line to ground (three conductors in parallel).
Z'_t* = internal impedance of high-tension winding of one transformer of Y-delta bank.
Z''_t* = internal impedance of low-tension winding of one transformer of Y-delta bank.
Z_t = short-circuit impedance of one transformer of the Y-delta bank.

*It is assumed that the leakage reactance is equally divided between the high- and low-tension windings.

The action of the line and potential transformers may be represented by two generators connected in series through the impedance of the auxiliary transformer, Z_n. E_L is the induced voltage of the generator representing the line; the internal impedance of this generator being the impedance of the line to ground, Z_L. E_T is the induced voltage of the generator representing the Y-delta bank of potential transformers whose internal impedance is $\frac{Z_t}{3}$. The following equation holds.

$$E_T + E_L = 3\,I_n\left(\frac{Z_t}{3} + Z_n + Z_L\right). \tag{9}$$

The true residual voltage E_R is

$$E_R = E_T - 3\,I_n\left(\frac{Z_t}{3} + Z_n\right) = -E_L + 3\,I_n\,Z_L \tag{10}$$

The residual voltage as indicated by the Y method is approximately $3\,I_nZ_n$. If either E_L or E_T is zero the other can be determined from the impedances and the neutral current.

The open-circuit residual voltage E_L on this line due to its capacitance unbalance can be estimated approximately. The length of longitudinal unbalance of the transmission line, transposed for two barrels, is 0.6%. The characteristic residual voltage of the line is about 12% of its Y voltage. From these figures the open-circuit residual voltage should be about 0.12×0.6 or 0.07%. This will be altered considerably by the irregularities of the line. From the measurements of longitudinal induction on the telephone line (technical report No. 54, page 7) it was found that the induction from balanced voltages with two barrels was 1.8% of the induction with no transpositions. Thus the effective unbalance is about three times the computed value, assuming uniformity. It is reasonable to assume that approximately the same factor applies to the residual voltage. Hence E_L is of the order of 0.21% of the line Y voltage. The maximum possible value of the actual residual voltage occurs if I_nZ_L is opposite in phase to the open-circuit residual voltage.

The fundamental neutral current, with balanced Y voltages of 16.2 kV. impressed, is about 3 milliamperes, from Table IX. The impedance Z_L of the isolated line, at fundamental frequency (50 cycles),

is about 4200 ohms, and with the inductance coil in series, about 3800 ohms. Thus, for the present tests,

$$3\,I_nZ_L = 3\ .\ 3\ .\ 4000\ .\ 10^{-3} = 36 \text{ volts}$$

or 0.22 per cent of the line Y-voltage. The true residual voltage is, in per cent:

$$E_R = -0.21 + 0.22 \text{ (vectorially combined)}$$

the limits being 0.01 and —0.43 per cent. Limits can also be set for E_T, for

$$\frac{Z_t}{3} = 3560 + j\,1510 = 3870 \text{ ohms}$$

$$I_nZ_n = 30 \text{ volts (from Table IX)}$$
$$= 0.19 \text{ per cent of Y voltage}$$

$$Z_n = \frac{30 \times 10^3}{3} \text{ ohms} = 10000 \text{ ohms}$$

Z_n consists chiefly of the exciting impedance of the auxiliary transformer, since the secondary external impedance was 1065 ohms and the ratio was 10:1. Assuming a value of 30° for the angle of hysteretic advance, the impedance is 5000 + j 8700 ohms. From (9),

$$E_T = -E_L + 3\,I_n\left(\frac{Z_t}{3} + Z_n + Z_L\right)$$

$$= -E_L + 3\ .\ 3\ (10600)\ .\ 10^{-3} \text{ volts}$$
$$= -0.21 + 0.59 \text{ (vectorially combined)}$$

So $$E_T = 0.38 \text{ to } 0.80 \text{ per cent}$$

The transformer component of the true residual voltage, E_{RT}, is the value of E_R when E_L is zero.

$$\frac{E_{RT}}{E_T} = \frac{Z_L}{\frac{Z_t}{3} + Z_n + Z_L} \tag{11}$$

$$= \frac{4000}{10600}$$

for the isolated line, at 50 cycles.

So $E_{RT} = 0.14$ to 0.30 per cent of line Y voltage, for the fundamental frequency. The ratio $\frac{E_{RT}}{E_T}$ will, in practice, be always less than unity.

In like manner,

$$\frac{E_{RL}}{E_L} = \frac{Z_n + \frac{Z_t}{3}}{Z_n + \frac{Z_t}{3} + Z_L} \tag{12}$$

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In practice this ratio is very close to unity, increasingly so at higher frequencies.

If the line has grounded Y-delta banks at each end the impedance Z_L is of the order of 40 ohms at 50 cycles, hence the numerator of (11) is about 1% of the value for an isolated line. At about 600 cycles, however, the two types of line condition give the same magnitude of impedance. The denominator of (11) is only slightly affected by the line condition and increases almost directly as the frequency. So the effect of the changes in impedance at higher frequencies is to lessen the importance of the transformer component of the true residual voltage.

b. delta method.

With the delta method, there is a neutral current corresponding to the delta current caused by ratio inequality, as well as the neutral current due to the unbalance of the line. There is also a neutral-current component due to the difference in exciting currents of the three transformers. As the total exciting current at 16 kV. is about 5 milliamperes, this unbalanced component is negligible. The third-harmonic magnetizing neutral current is discussed under "B," and is not considered here.

From Table VIII, the fundamental neutral current with 40 ohms or more in the delta corner is $1\frac{1}{2}$ to 2 milliamperes at 16 kV. impressed Y voltages. Equation (10) gives the true residual voltage. Thus

$$E_R = -E_L + 3 \times 2 \times 4000 \times 10^{-3} \text{ volts}$$

or in per cent, $= -0.21 + 0.15$ (vectorially combined), the limits being -0.06 and -0.36 (per cent of Y voltage of the line).

The component of the true residual voltage, due to the ratio inequalities of the transformers, can be determined by considering the voltage unbalance, E_d, of the secondary delta, as due to a generator in its corner, with the secondary external impedance Z_d in series.

Let: $Z = 3\,Z''_t + Z_d$

$=$ leakage impedance plus external impedance of the delta.

$a =$ ratio of turns (120 : 1)

Considering the neutral current, I_n, and delta current, I_d, due to E_d only:

$$a\left(\frac{E_d - ZI_d}{3}\right) = I_n\left(\frac{Z'_t}{3} + Z_L\right) \qquad (13)$$

As long as $Z'_t + 3Z_L$ is small, compared to the exciting impedances of the transformers, it is approximately true that

$$I_d = \frac{a}{3}\, I_n$$

So

$$E_d = I_n \left(\frac{Z'_t + 3Z_L}{a} + \frac{a}{3} Z\right)$$

$$E_{RT} = 3\, I_n\, Z_L$$

Then

$$\frac{E_{RT}}{aE_d} = \frac{3\, Z_L}{\frac{a^2}{3} Z + Z'_t + 3Z_L} \qquad (14)$$

$\frac{E_{RT}}{aE_d}$ is the ratio of the true residual voltage due to the transformer ratio inequalities to the apparent residual voltage due to the same cause.

For the conditions of use of the transformers E_{RT} is entirely negligible, being, for $a = 120$, $Z_d = 100$ ohms, and fundamental frequency, about 2% of the apparent residual voltage due to ratio inequalities.

3. *Comparison of Methods.*

The residual voltages, true or apparent, discussed above, exist for each frequency present in the balanced components of voltage between lines and ground.

From drawing No. 305 for the delta method, and Table IX, for the Y method, the following comparison is made of the fundamental residual voltage of the line as indicated by the two methods.

TABLE X.

Indicated Fundamental Residual Voltage. Comparison of Two Methods.

Line voltage to ground	Delta method		Y method	
kV.	Volts	Per cent*	Volts	Per cent*
8.6	70	0.8	34	0.4
16.3	130	0.8	91	0.6

*of impressed Y voltages.

The values in Table X for the delta method are for a delta-corner resistance of 100 ohms or more, but are sufficiently close for values over 25 ohms. For the Y method the secondary impedance of the auxiliary transformer was high, as shown in Table IX. If the secondary impedance of the auxiliary transformer is made low, the grounded neutral case is approached.

The above discussion indicates:

1. The true residual voltage of fundamental frequency was not greater than 0.4% during these tests.

2. The fundamental residual voltage due to transformer impedance unbalance (Y method) was between 0.1 and 0.3% during these tests,

with the isolated line. With grounded Y-delta banks on the line, this residual voltage would be very much less.

3. The true residual voltage due to ratio inequalities (delta method) was negligible.

4. The indicated residual voltage (either method) was much greater in magnitude than the true residual voltage and the transformer component thereof, and determines the minimum value of residual voltage which can be measured with these transformers.

5. For higher frequencies studies should be made for the given conditions of impedances of the various parts of the system in order to determine the effect of these factors.

For the delta method, therefore, harmonic values of residual voltage below 1.5% of the balanced voltage of the same harmonic from lines to ground are apt to be in error by 50% or more. For the Y method, residual voltages of less than 1% are subject to the same error.

B—Residuals of Triple-Harmonic Frequencies, Due to Variation of Permeability.

The previous discussion has been concerned with errors due to potential-transformer inequalities, which produce apparent and actual residual voltages of the frequencies which exist in the balanced voltages to ground. The present discussion is concerned with apparent and actual residual voltages due to triple-frequency magneto-motive forces, caused by the variable permeability of the iron cores. These triple frequency magneto-motive forces are approximately in phase in the three transformers. In the tests considered in this memorandum the possibility of triple-harmonic balanced-voltage components is negligible. The third-harmonic residual voltage is all introduced by the potential transformers as the conditions preclude its introduction from other sources.

1. *Y method.*

In Table III it was shown that a third harmonic voltage 0.4% of the fundamental voltage on one transformer (16 kV.) was required to circulate third-harmonic magnetizing current in the high-tension delta. In practice with the Y method, a similar voltage drives this current in the low-tension delta. The impedance in the corners of the delta is here relatively larger, so the third harmonic-voltage observed on the high-tension side should be higher. From Table IX the apparent residual voltage (third harmonic) is about 100 volts at 16 kV. impressed or 0.6%; about 30 volts at 9 kV. impressed (isolated or grounded line). The true residual voltage ($3\ I_nZ_L$) introduced on the isolated line ($Z_L = 50 - j\,1400$ ohms) is about 9 and 4 volts in the two cases, respectively. For the case of Y-delta banks at each end of the line, with

impedance $Z_L = 25 + j\,85$ ohms the residual voltage introduced would be about 0.6 volts at 16 kV. impressed. It is to be expected that the apparent triple-harmonic residual voltage will be much the greater, since Z_n greatly exceeds Z_L. The formal mathematical discussion is given in connection with the delta method, below.

The third-harmonic apparent residual voltage increases much more rapidly than the voltage impressed, as is to be expected.

2. *Delta Method.*

The part of the triple-frequency magnetizing current in the high-tension neutral introduces an actual residual voltage on the power line equal to three times the product of neutral current and impedance of the external path from ground to the line side of the potential transformers. When there is a large resistance in the corner of the delta the product of third-harmonic magnetizing current and delta-corner resistance approaches a limit which, with nearly balanced impressed voltages of 16 kV., is 80 volts in primary terms, or 0.5%. With 9 kV. impressed this value is 12 volts, in primary terms.

This delta-corner voltage of the low-tension side is not a measure of the true residual voltage introduced on the high-tension line. The following equations show the relations involved. The discussion is concerned only with triple harmonics, arising in the potential transformers used for the residual voltage measurements.

Let: $E =$ induced voltage in each high-tension winding, due to the triple-harmonic flux in the transformer core.

Then

$$E = I_n\left(\frac{Z'_t}{3} + Z_n + Z_L\right) = \frac{aI_d}{3}\left(3\,Z''_t + Z_d\right) \qquad (15)$$

It is assumed in the above that the triple harmonic fluxes are equal and in the same phase in the three transformers and the ratios equal; as is approximately the case. Only the third harmonic (150 cycles per second on a 50-cycle system) will be considered, as the higher triple harmonics are not appreciable in these transformers under the conditions of use.

$$Z'_t = 6500 + j\,6810 \text{ ohms}$$
$$Z''_t = 0.29 + j\,0.47 \text{ ohms}$$
$$a = 120$$

When measuring residual voltage by the delta method $Z_n = 0$, the neutral being grounded. As $3\,Z''_t$ is usually small compared to $(3\,Z''_t + Z_d)$, $\frac{a}{3}\,I_dZ_d$ is approximately equal to E. Z_L is in most practical cases less than $\frac{Z'_t}{3}$, hence the residual voltage introduced by the potential transformers, $3\,I_nZ_L$, is usually a small fraction of 3 E.

The neutral current for this line is approximately constant for given delta-corner impedance and impressed voltage, being little affected by the changes in the value of Z_L in practical cases. The neutral current is also but slightly affected by the delta-corner impedance over the range appropriate for use in the delta method, being about 7 milliamperes at 16 kV. impressed voltages and less than 2 milliamperes at 9 kV. In the case of the isolated San Fernando-Somis line, $Z_L = 50 - j\,1400$ ohms. At 16 kV. impressed, the third harmonic residual voltage introduced on the line is $3 \times 0.007\ (50 - j\,1400) = 30$ volts or about 40% of the delta-corner voltage which appears on the low-tension side. At 9 kV. impressed, the third-harmonic residual voltage introduced is 8 volts or about 70% of the value measured on the low-tension side.

The measurement of triple-harmonic residuals is obviously not of special importance for isolated systems. For the more practical case of the same line with grounded Y-delta banks on each end, the impedance Z_L is $25 + j\,85$ ohms, and the residual voltage introduced is, at 16 kV. impressed, only $3 \times 0.007 \times (25 + j\,85) = 2$ volts, about 2% of the apparent residual voltage; at 16 kV. impressed. Incidentally, this voltage will appear in the measurements of line Y-voltages.

VI. Conclusions.

With reference to the residual-voltage measuring equipment as used at San Fernando, some of the results are summarized on Drawing No. 306, which shows the variation of the apparent residual voltage for both methods for the given transformers and line as used. One plot relates to the fundamental frequency (typical of frequencies present in balanced components), and the other to the third harmonic. The Y-method shows the least apparent residual voltage of the fundamental frequency; the delta method shows the least apparent residual voltage of the third harmonic. Since the method of supply was designed to preclude the introduction of triple harmonics in the balanced voltages, the observed third harmonic is chargeable entirely to the potential transformers. The true third-harmonic residual voltage (three times product of third-harmonic neutral current and line impedance to ground) is plotted, for the isolated line, being much less than the apparent residual voltage.

The Y method requires that the measured residuals be multiplied by a factor to correct for the impedances of the transformers. For the apparatus here used this factor is 1.04 under the most favorable conditions, with 20:1 ratio of auxiliary transformer and high impedance (500 ohms or more) in the measuring circuit. For high impressed voltage on the auxiliary transformer, or low measuring-circuit impedance, the factor is greater, and must be investigated for the particular conditions.

For frequencies present in the balanced voltages, residual voltages below 1.5% (delta method) or 1% (Y method) of the Y voltages, may be in error by 50% or more.

For the third harmonic, the error varies with the magnetic density. At 16 kV. from lines to neutral, residual voltages below 1% of the Y-voltage may be in error by 50% or more, with either method. The higher triple harmonics are not introduced by the potential transformers at the magnetic densities considered in these tests; hence are not subject to this error.

Each method introduces some residual voltage on the line smaller than the errors in measurement. The delta method introduces least due to unbalances, while the Y method introduces the least from triple harmonics.

The degree of error with given arrangements of line and transformers can be estimated from the data herein given and the line and transformer impedances and voltages.

The data and discussion given in the foregoing pages show that the limit of the accuracy of residual-voltage measurement by either the delta or the Y method is set by fundamental characteristics of the potential transformers. Thus in the delta method the inequalities of ratio of the three potential transformers used, the impedance to third-harmonic magnetizing currents of the potential transformers and the magnitude of these currents determine the lowest measurable residual voltage. In the Y method the inequality among the equivalent impedances of the three transformers and the impedance of the delta circuit to the third-harmonic magnetizing currents are the factors. A choice between the two methods will depend upon the equipment available. The delta method has the advantage of requiring only three potential transformers. These should be capable of withstanding the full voltage between phases, in case they are used on an isolated system. If the transformers are rated at the normal voltage between phases, they will under normal conditions be subjected to only 58% normal voltage, with the attendant advantage of low flux-density and correspondingly small third-harmonic magnetizing current. With a given amount of ratio inequality, the Y method appears to be best, and so, if the ratio inequality is large, this method may be preferable. The auxiliary transformer should be capable of withstanding the Y voltage of the system and its equivalent impedance should be large compared to the internal impedance of the three potential transformers.

The results obtained in the various tests on the equipment used in the present investigation seem to show that variations can occur in the characteristics of the potential transformers. In undertaking residual-voltage measurements with a set of potential transformers the following

tests appear to be the ones of importance to be made: (1) internal impedance, (2) internal impedance and exciting impedance of auxiliary transformer (in Y method), (3) measurement of open-corner voltage of low-tension delta, with high-tension side connected delta, or vice versa. The impedances of lines are of secondary importance and approximate determination by calculation should be sufficient.

The residual voltage under normal operating conditions is a few per cent of the normal voltage of the system. Differences among the transformers that are unimportant in the measurement of delta and Y voltages of the line become of very great importance in the measurement of residual voltage. The factors involved in determining residual voltages include impedances, ratio inequality, frequency, magnetic density, magnitude of residual voltage to be measured and accuracy desired. Consideration must therefore be given to each particular case in order to determine the equipment required or the suitability of the given equipment. It can, however, be stated, that equality of ratios, low magnetic density, low internal impedance, and high exciting impedance are the desiderata for greatest accuracy.

Respectfully submitted.
(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

ATTACHMENTS: P. I. C. Drawings Nos. 257, 258, 305 and 306.

IN FILES OF JOINT COMMITTEE: Oscillograms Nos. 934, 935, 936, 939, 940, 941, 942, 943, 944, 946, 948, 949, 950, 953, 954, 955, 956, 964, 967, 969, 975.

Copies of the following memorada:

The use of Potential Transformer for measuring Residual Voltage, January 8, 1914. (General discussion.)

Comparison of Two Methods of Measuring Residual Voltage, March 28, 1914. (Results of some tests.)

Investigation of Potential Transformers used for Residual Voltage Measurements, March 17, 1915. (Outline of tests reported herein.)

APPROVED: April 21, 1916.

(Signed) R. W. MASTICK,
Field Engineer.

APPROVED: May 2, 1916.

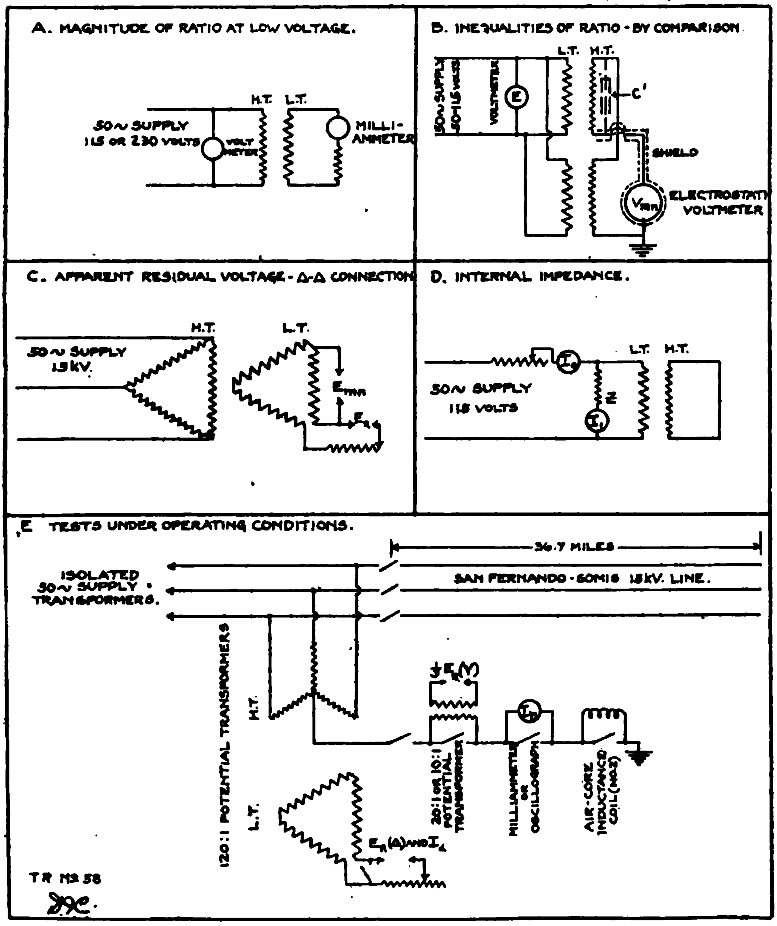
SUBCOMMITTEE ON TESTS,
(Signed) J. E. WOODBRIDGE,
Chairman.

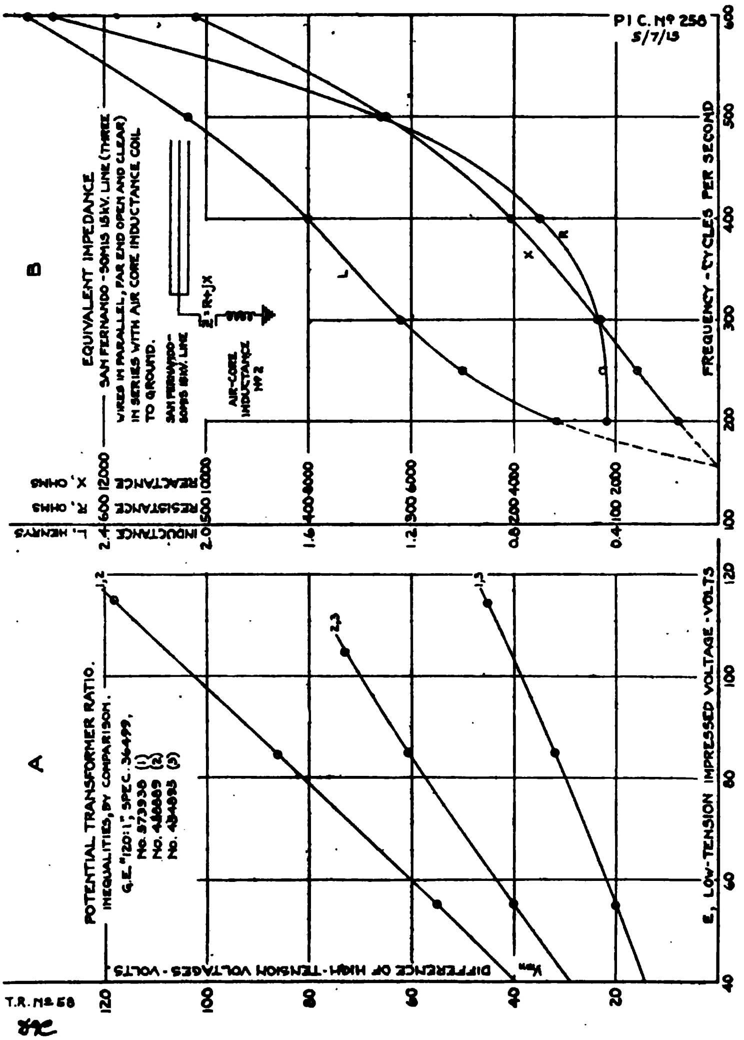
JOINT COMMITTEE ON INDUCTIVE INTERFERENCE,
(Signed) ARTHUR F. BRIDGE,
Secretary.

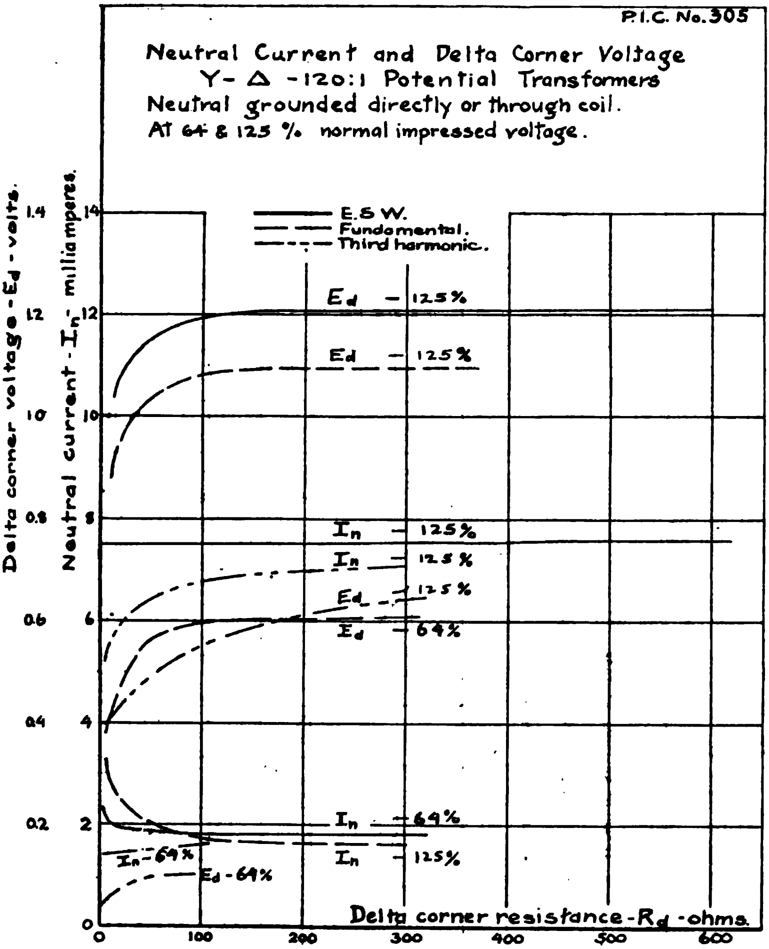
May 19, 1916.

P.I.C. No 257
5/17/15.

POTENTIAL TRANSFORMER TESTS
CONNECTION DIAGRAMS

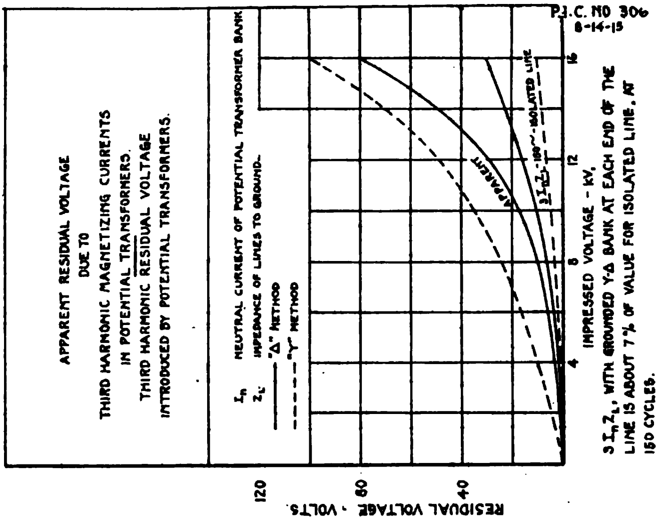


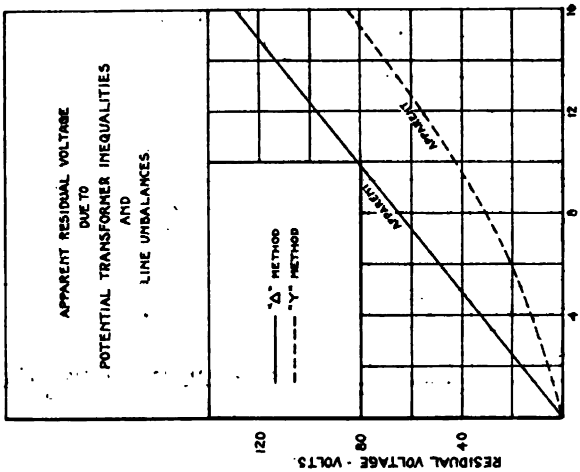

A
POTENTIAL TRANSFORMER RATIO.
INEQUALITIES, BY COMPARISON.
DIFFERENCE OF HIGH-TENSION VOLTAGES - VOLTS
E, LOW-TENSION IMPRESSED VOLTAGE - VOLTS
B
EQUIVALENT IMPEDANCE
SAN FERNANDO-SOMIS 15 KV. LINE (THREE WIRES IN PARALLEL, FAR END OPEN AND CLEAR) IN SERIES WITH AIR CORE INDUCTANCE COIL TO GROUND.
AIR-CORE INDUCTANCE No 2
L, HENRYS
R, OHMS
X, OHMS
INDUCTANCE
RESISTANCE
REACTANCE
FREQUENCY - CYCLES PER SECOND
P.I.C. No 258
5/7/15
T.R. No 58



P.I.C. No. 305
Neutral Current and Delta Corner Voltage
Y-Δ-120:1 Potential Transformers
Neutral grounded directly or through coil.
At 64 & 125 % normal impressed voltage.
E.S.W.
Fundamental.
Third harmonic.
Ed - 125%
Ed - 125%
In - 125%
In - 125%
Ed - 125%
Ed - 64%
In - 64%
In - 125%
In - 64%
Ed - 64%
Delta corner voltage - Ed - volts.
Neutral current - In - milliamperes.
Delta corner resistance - Rd - ohms.
0
100
200
300
400
500
600
1.4
1.2
1.0
0.8
0.6
0.4
0.2
14
12
10
8
6
4
2
0

T.R. No. 58


P.J.C. NO 306
8-14-15
APPARENT RESIDUAL VOLTAGE
DUE TO
THIRD HARMONIC MAGNETIZING CURRENTS
IN POTENTIAL TRANSFORMERS.
THIRD HARMONIC RESIDUAL VOLTAGE
INTRODUCED BY POTENTIAL TRANSFORMERS.
I_n NEUTRAL CURRENT OF POTENTIAL TRANSFORMER BANK
Z_L IMPEDANCE OF LINES TO GROUND.
"Δ" METHOD
"Y" METHOD
APPARENT
$3 I_n Z_L$ - 180~ - ISOLATED LINE
IMPRESSED VOLTAGE - kV.
$3 I_n Z_L$, WITH GROUNDED Y-Δ BANK AT EACH END OF THE LINE IS ABOUT 7% OF VALUE FOR ISOLATED LINE, AT 150 CYCLES.
RESIDUAL VOLTAGE - VOLTS.


APPARENT RESIDUAL VOLTAGE
DUE TO
POTENTIAL TRANSFORMER INEQUALITIES
AND
LINE UNBALANCES.
"Δ" METHOD
"Y" METHOD
APPARENT
APPARENT
IMPRESSED VOLTAGE - kV.
RESIDUAL VOLTAGE - VOLTS.

TR. No 58

Technical Report No. 59.

September 6, 1916.

RELATION OF TRIPLE HARMONIC RESIDUALS IN A TRANSMISSION LINE TO THE MAGNETIC DENSITY IN CONNECTED TRANSFORMER BANKS. EFFECT OF THE LINE CHARACTERISTICS.

OUTLINE.

Introduction.

The variable permeability of iron causes a dissimilarity of the wave-forms of the magneto-motive forces and induced voltages in transformers. Under the conditions of present practice in transformer design the distortion consists principally of a wave of three times the fundamental frequency, though other harmonics appear. When a simple

harmonic voltage is impressed, either the magnetizing current or induced voltage, or both, are distorted. If the harmonics do not appear in the one, they must appear in the other. The magnitudes of the harmonics are dependent on the maximum magnetic density and increase very rapidly as the iron approaches magnetic saturation.

In a three-phase system the triple harmonics due to variation of permeability occur in common time-phase in the three phases of a bank, assuming the fundamental impressed voltages 120° apart in phase, and like transformers. Thus, when the system has a grounded neutral on the line side of the transformer bank, they give rise to residual components of voltage and current in connected transmission lines.

When a circuit exists for triple-harmonic components of magnetizing current in a transformer, the tendency is to cause a certain triple-harmonic current, which is definite for a given magnetic density and magnetic characteristic of the iron core and a given wave-form of induced voltage. That is, the tendency of the "regulation" is toward constant current. The phase and magnitude of the current are also dependent on the impedance of the path. The reactive magnetic flux due to the current combines with the inherent triple-harmonic flux and the induced voltage of triple frequency is determined by the resultant flux. Thus the resultant effect depends on the character of the path for triple-harmonic currents.

In this report there are given, in Section I, results of tests on a three-phase bank of transformers and transmission line to observe the effect upon the residuals of varying the impressed voltage on the transformer bank. In Section II are given results of some theoretical studies paralleling the experimental work.

SECTION I.

EXPERIMENTAL.

I. Scope.

The object of the work here reported was:

A—To determine the characteristics of the transformers.

B—To observe the triple-harmonic residuals of a transmission line energized by a delta-Y or Y-Y bank;

(1) with grounded neutrals at both ends of the line,

(2) line isolated except at energized end, where the neutral was grounded,

(3) line isolated except at receiving end, where the neutral was grounded.

These conditions provided paths of widely diverse characteristics for the triple-harmonic magnetizing currents in the line-side windings of the transformers. The station-side windings were left unchanged during each group of tests, except for the insertion of current transformers at

their midpoints, for the delta-Y test. With the delta-Y connection the delta provided a path for the triple-harmonic currents, in parallel with the circuit through line wires and ground. With the Y-Y connection there was no such parallel path, for the station-side neutral was isolated, consequently the only path for triple-harmonic residual current was through the line-side windings.

II. Description of Apparatus and Procedure.

A—Energy Supply.

Three-phase energy was supplied from the Pacific Light & Power Corporation's 15-kV. network, operated at a frequency of 50 cycles. Three step-down transformers connected delta-delta, delta-Y or Y-delta, constituted an isolated source of energy at the desired voltages for application to the transformers under test, known as the "test-bank." The supply transformers are of the same type and size as those of the test-bank. Drawings Nos. 315 and 328 show the connections used.

B—Test Transformers.

The transformers under test, referred to herein as the "test-bank," are three Westinghouse $37\frac{1}{2}$-kVA., Type SK, Style No. L108764, 50-cycle units, voltages 15000 & 440-110, with extra taps on the high-voltage winding. They have Serial Nos. 352198, 352204, 352203, and are briefly designated herein as (F), (M) and (R), respectively. Further data concerning the transformers are given in Section II of this report.

The transformers were connected as shown on Drawings Nos. 315 and 328. The station-sides had two coils in series and two in parallel, requiring 220 volts per transformer for normal excitation. On the line side, the 14250-volt windings were used, leaving at each end an idle 375-volt coil, except that the induced voltage of one transformer was measured by means of one of these coils. The coil-arrangements were not altered during the tests, except that some of the measurements of induced voltage were made on 7125-volt windings, in the single-phase and Y-Y exciting-current tests.

C—Transmission Line.

The 15 kV. transmission line from San Fernando to Somis was used in these tests; its length is 36.7 miles, and it consists for the most part of three No. 4 B&S solid copper wires, supported in a vertical plane five feet apart, from wooden crossarms, on wooden poles. The average height of the upper conductor above the earth is 50 feet. Technical report No. 54 gives a more complete description. During these tests the transmission line had five transpositions, dividing the line into two complete "barrels," with a resultant unbalanced length of 1200 feet or 0.6% of the total length.

At the far end of the line were three 15-kVA. Westinghouse, Type SK transformers, Style No. L126892 B, 15000 & 440-110 volts, Serial Nos. 352124, 322291, 322293. They were in interconnected-Y on the line side, with two 7500-volt windings in series composing each phase. With this arrangement the voltage rating is reduced by 13%. The low-voltage side was connected delta, 110 volts per phase, with the four coils of each phase in parallel. An air-switch was provided for grounding and isolating the neutral point of the bank.

The object of using the interconnected-Y-delta bank at the receiving end of the line was to reduce to a minimum the triple harmonics introduced by the receiving transformers. This precaution is necessary when the receiving-end neutral is grounded. The measured impedances of the line and receiving transformers to third and ninth-harmonic residual currents are lower with the receiving-end bank in interconnected-Y-delta, than with it in straight-Y-delta, the neutral being grounded. (See drawing No. 270.)

Measurements of line impedance to residuals are given on attached drawings, as follows:

Drawing No. 263—Three line conductors, in parallel, to ground—isolated at far end.

Drawing No. 270—Three line conductors, in parallel, to ground—far end grounded through interconnected-Y-delta and through Y-delta 15-kVA. transformer bank.

D—Procedure.

Measurements were made of the following quantities, using meters and oscillograph:

Quantity	Symbol
Station-side impressed voltage of test-bank, busses A to C	E'_{AC}
Line-side voltages, lines to neutral	E''_1, E''_2, E''_3
Induced voltage—in end winding of one transformer	E_t
Residual voltage of line at San Fernando	E_R
Voltage to ground of line-side neutral of test-bank	E_n
Delta-residual current—vector sum of the currents in the three phases of the station-side delta of the test bank (delta-Y test)	D_R
Line-side neutral current of the test bank	I_n
Line currents	I''_1, I''_2, I''_3
Station-side current, phase A	I'_A

The potential and current transformers* used in the measurement of these quantities were arranged as shown on Drawings Nos. 315 and 328.

Table I shows the supply transformer connections corresponding to the various voltages at which tests were made.

*For results of tests to determine errors due to current transformers, see T. R. No. 53; potential transformers, see T. R. No. 58.

TABLE 1.

Supply transformers — Connection	Supply transformers — Rated voltage	Induced voltage on test bank, per cent normal
Delta-Y connection of test bank:		
Y-delta	15000/110	31
Delta-delta	12980/110	62
Delta-Y	15000/110	96
Delta-delta	15000/220	110
Delta-delta	12980/220	125
Y-Y connection of test bank:		
Delta-delta	15000/220	63
Delta-delta	15000/330	97
Delta-Y	15000/220	109

The single-phase exciting current tests covered the range from 54 to 148 per cent of normal voltage.

The induced voltage of fundamental frequency is directly proportional to the magnetic flux wave of fundamental frequency. In stating the ratio of actual induced voltage to normal voltage, the fundamental component of induced voltage has been used, hence where "per cent normal voltage" appears, it also indicates the per cent of normal magnetic flux density *of the fundamental frequency.* Harmonics in the flux wave generally change the maximum value of the resultant magnetic flux wave as well as its form. In the Y-Y tests this change is of importance.

Assuming a simple harmonic flux wave, the average magnetic density in the net cross-sections of the iron cores of the test transformers is 14100 gausses in the legs, and 10000 gausses in the yokes, which form 65 and 35 per cent of the magnetic circuits, respectively. The cores of the transformers are annealed silicon sheet steel, with laminations 0.014 inches in thickness.

III. Results of Tests.

A—Transformer Exciting-Current Tests.

1. *Exciting Current, Single-phase Connection.*

Measurements of the exciting current and induced voltage were made on one transformer for a range of voltages from 54 to 148 per cent of normal; measurements at 108 per cent were made on the other two.

Tables II and III and drawing No. 329 set forth the results. The transformers were excited on the 220-volt windings and the induced voltage was measured on the 7125-volt windings. The measurements made on March 4, 1915, differed in that the induced voltage was measured on a 375-volt winding, and there was a current transformer con-

nected at the mid-point of the 220-volt winding. The supply was furnished by two transformers similar to those under test, connected in series between phases of the 15-kV. supply.

TABLE II.

Meter Readings of Single-Phase Exciting Current and Induced Voltage.

Transformer	Impressed voltage, volts	Induced voltage kV.	Induced voltage Per cent normal	Exciting current, amperes
R	—	3.86	54	1.4
R	—	5.82	82	2.5
R	—	7.42	104	6.9
R	230	7.68	108	9.8
M	—	7.68	108	11.1
F	—	7.71	108	13.2
R	274	9.07	127	51.7
R	320	10.63	148	Over 80
R	211	6.58*	93	4.1

*Made on 3/4/15; induced voltage measured on a 375-volt coil. The other measurements were made 1/21/15; induced voltage was measured on the 7125-volt windings. In all cases exciting current is that of the 220-volt windings.

TABLE III.

Oscillograms of Single-Phase Exciting Current and Induced Voltage.

Harmonic \ Induced voltage % normal**	54	82	104	108	108	108	108	127	148	93
Induced voltage—kilovolts (7.125-kV. winding).										
1	3.86	5.82	7.40	7.69	7.68	7.70	7.66	9.06	10.5	6.58
3	0.09	0.13	0.10	0.10	0.10	0.10	0.10	0.35	2.1	0.10
5	0.04	0.08	0.05	0.06	0.05	0.09	0.16	*	0.3	*
7	*	*	*	*	*	*	*	*	0.4	*
E. S. W.	3.86	5.82	7.40	7.69	7.68	7.70	7.66	9.07*	10.7	6.58
Exciting current—amperes (220-volt winding).										
1	1.37	2.40	6 0	8.8		10.9	9.4	42.4	100	3.8
3	0.27	0.75	2.7	4.6		6.3	5.2	27.5	50	1.5
5	0.04	0.16	0.8	2.1		3.0	2.3	10.8	7	0.8
7	*	0.04	0.2	0.7		1.2	1.0	1.5	5	0.2
9	*	*	*	0.2		0.4	0.5	*	*	*
E. S. W.	1.40	2.52	6.7	10.2		13.0	11.1	51.7	112	4.1
Impressed voltage—volts.										
1					230				312	
3					6				71	
5					3				6	
7					*				12	
E. S. W.					230				320	211
Transformer	R	R	R	R	R	F	M	R	R	R
Oscillogram No.	735	742	737	736	738	741	739	745	744	863

**Per cent of *fundamental* induced voltage to 7.125 kV., the normal value.

The induced voltage contains small third and fifth harmonics, even at the lower densities. These may be due to the wave-form of the 15-kV. network or to the impedance-drop of the exciting current. When the voltage is 127% of normal, the third harmonic is about 4% of the fundamental and when 148% normal, there is a 20% third harmonic. In the latter case there is 3% of fifth harmonic and 4% of seventh. The ratios of induced voltage to exciting current, for individual frequencies and for the equivalent sine-waves, fall rapidly over the range from 50 to 125% normal voltage, all following similar curves. Two oscillograms of impressed voltage were taken. The noteworthy feature is the large third harmonic, at the highest voltage employed, due to the transformer under test.

For equal induced voltages, there are large differences in the exciting currents of the three transformers, as is shown by both meter readings and oscillograms.

Drawing No. 329 indicates the variations in the components of exciting current of transformer "R." The fundamental and the equivalent sine-wave values increase, at first, approximately in proportion to the voltage. But above about 80% normal voltage the exciting current increases much more rapidly, and from 100 to 130 per cent of normal voltage the fundamental can be represented quite accurately by the formula:

$$I = 4.0\, E^{10} \qquad (1)$$

where

$$E = \frac{\text{Actual fundamental induced voltage}}{\text{Normal fundamental induced voltage}}$$

I = Fundamental exciting current.

The equivalent sine-wave varies in practically the same way.

The rate of variation of the third harmonic with the voltage also changes. At 60% normal it increases as the square of the voltage. At 110% it appears to increase as the fifteenth power of the voltage. The tendency of the third-harmonic current curve to droop at the highest voltage is notable, and corresponds to the observed large third-harmonic induced voltage.

Similar enormous rates of increase are observed for the fifth and seventh harmonics. The fifth apparently rises to a maximum and falls off. The ninth harmonic was observable at 108% normal voltage. At 148% there might have been an ampere or more present without being sufficient to be measured on the oscillogram. The ratios of the higher harmonics of exciting current to the fundamental rise with increasing

rapidity to about 110% normal voltage, reach a maximum at about 125%, and then decrease. The maximum values are

$$\frac{I_3}{I_1} = 65\%, \quad \frac{I_5}{I_1} = 25\%, \quad \frac{I_7}{I_1} = 7\%. \quad \frac{I_{ESW}}{I_1} = 125\%.$$

This falling off in the ratio can be accounted for by the impedance of the exciting-current circuit, which limits the higher harmonic currents.

The values of the harmonics are in all cases approximate, their accuracy being in proportion of their relative values in the complex current waves.

The values of magnetizing current with respect to the load current can be seen on drawing No. 329. Thus at normal voltage the exciting current (E. S. W. value) is 3% of full load current, and at 114% normal voltage the exciting current is 10%.

2. *Exciting Current, Delta-Y Connection.*

One oscillogram was taken of the exciting current and induced voltage of transformer "R," and the delta residual current. The results are given in Table IV.

The supply transformers were in delta-Y, 15000 to 110 volts per phase. The observations were made on the station-side of the test-bank, connected for 220 volts normal excitation.

TABLE IV.

Oscillogram of Exciting Current and Induced Voltage.

Three-Phase—Delta-Y.

(93 per cent of normal induced voltage.)

Harmonic	Delta residual current, amperes	Station bus current, amperes	Induced voltage, kV.
1	0.5	6.4	13.2
3	4.5	0.2	0.3
5	0.2	0.9	*
7	*	0.3	*
9	0.2	*	*
11	*	0.1	*
E. S. W.	4.5	6.5	13.2

Osc. No. 862, 3/4/15.
Line voltages 13.3, 13.5 and 13.2 kV.
Station bus-voltage, 211 volts.

The induced voltage is almost purely fundamental; there is little other than third harmonic in the delta residual current and very little third harmonic in the bus-current.

The average value of third harmonic in the single-phase exciting currents from Table III and drawing No. 329, is 1.46 amperes, which agrees very well with the value $\frac{D_R}{3} = 1.5$ amperes as measured.

3. *Exciting Current, Y-Y Connection.*

Table V presents results of exciting-current measurements with the Y-Y connection. The current in the 220-volt windings and the induced voltage across the 7125-volt windings, were measured.

TABLE V.

Oscillograms of Exciting Current and Induced Voltage. Three-Phase, Y-Y.

Per cent normal** voltage	97			111		
Harmonic	Impressed voltage, volts	Induced voltage, kV.	Exciting current, amperes	Impressed voltage, volts	Induced voltage, kV.	Exciting current, amperes
1	360	6.89	3.04	408	7.9	5.01
3	*	3.27	0.08	*	4.0	*
5	*	0.20	0.44	*	*	1.10
7	*	*	0.11	*	0.19	0.28
9	*	0.17	*	*	0.37	*
11	*	*	*	*	*	*
13	*	*	*	*	0.19	*
15	*	*	*	*	0.29	*
17	*	*	*	*	0.19	*
E. S. W.	360	7.62	3.08	408	8.9	5.20
Osc. No.		747			748	

**Ratio of fundamental induced voltage to 7.125 kV., the normal voltage of the winding.

The measurements, having been made at only two voltages, do not afford data for complete curves. Drawing No. 330 shows the values of Table V. Assuming that the law of variation of the curves is $y = ax^m$ the value of m may be found from the two points. Actually, the exciting current probably varies much more slowly at low density. The rates of variation with the voltage as thus determined from 97 to 111 per cent normal voltage are:

Exciting Current	*m*
Equivalent sine-wave	3.7
Fundamental	3.6
Fifth Harmonic	6.5
Seventh Harmonic	6.6

Induced Voltage	*m*
Equivalent sine-wave	1.12
Fundamental	1.00
Third Harmonic	1.5
Ninth Harmonic	5.5

These figures serve to indicate the large ratio of increase of the exciting-current components to increase of fundamental induced voltage. The third harmonic of induced voltage increases as the 3/2 power of the fundamental. The ninth harmonic apparently increases much more rapidly. The numerous harmonics in the induced voltage, and the prominence of the third and ninth harmonics, are noteworthy.

4. *Energy Loops.*

Energy loops constructed from exciting-current and induced-voltage waves of the single-phase and three-phase Y-Y tests are shown on drawing No. 356. The areas of the loops show all the iron losses in the transformers including both hysteresis and eddy current losses. The difference between these loops and the ordinary "hysteresis" loop is due to this inclusion of other than hysteresis losses. Since the induced voltage was measured on the secondary winding of the transformer the copper loss in the primary winding is not included.

Comparing the loop from the single-phase test at 108% normal voltage with the loop from the three-phase test at 111% normal voltage it will be noted that the single-phase loop reaches the higher flux density and includes the greater area. The loop obtained from the three-phase test is, however, broader at the zero values of magnetomotive-force and flux than is the corresponding single-phase loop. This may be due to the secondary curve in the loop obtained from the three-phase test. This secondary loop corresponds to the "dimple" in the flux wave, obtained with the three-phase test. On drawing No. 357, induced-voltage, magnetomotive-force, and flux waves are plotted for the two values of induced voltage of the three-phase Y-Y tests. It will be noted that due to the suppression of third harmonic in the magnetomotive-force wave the voltage wave is peaked and passes through the zero value three times in each half wave length. Correspondingly, the flux wave is flat-topped and has two maxima and one minimum in each half wave length. The magnetomotive-force wave contains fifth and seventh harmonics and is also double peaked. The flat-topped flux wave noted above accounts for the smallness of the area of the energy loop as compared to that obtained from the single-phase tests at the same value of fundamental induced voltage.

Since the magnetomotive-force required for the high value of flux density of the tests at 127% and 148% normal voltage, is several times that required at 108% normal voltage it is not practicable to show the energy loops for these high values of flux density on drawing No. 356. They are characterized by an extreme lengthening of the beak of the loop, owing to the close approach to saturation of the iron.

The induced voltage is approximately simple harmonic in the single-phase exciting-current tests, and the flux therefore even more closely so.

therefore a drawing of them corresponding to drawing No. 357 is not necessary.

B—Tests With Line Connected.

1. *Delta-Y Connection.*

The measurements summarized in Tables VI and VII and those plotted on drawing No. 333 were taken with the San Fernando-Somis line energized from the test-bank. The conditions of the neutrals of the test-bank and the interconnected-Y-delta bank at Somis are specified. The entire test was conducted on March 3, 1915. The three cases considered were observed in succession, for each value of impressed voltage.

In the tables, values of the impedances of the measuring circuits for delta residual current and neutral current are given. Thus

Z_R = measuring-circuit impedance in secondary of the three 40/1 current transformers (not including leads and impedances of current-transformers).

Z_n = measuring-circuit impedance in secondary of neutral-current transformer (not including leads and impedance of current transformer).

TABLE VI.

Meter Readings of Transformer and Line Voltages and Currents. Three-Phase—Delta-Y.

Per cent normal** voltage	E'_{AC} volts	E''* kV.	E_{13} kV.	D_R amp.	Z_R ohms	I_n amp.	Z_n ohms	I''* amp.
Case 1. Both neutrals grounded.								
31	70	4.71	4.74	0.04	90	0.0010	90	0.80
63	138	—	9.0	1.34	7.6	0.0084	32	1.66
97	210	14.2	13.6	4.8	1.1	0.0078	90	2.32
109	243	16.2	15.7	22	1.1	0.015	90	2.25
125	278	18.4	18.0	88	0.1	0.102	2	1.96
Case 2. San Fernando neutral grounded; Somis neutral isolated.								
31	70	4.70	4.55	0.04	90	0.0005—	90	0.80
62	138	—	8.8	1.18	7.6	0.0084	32	1.65
96	208	14.2	13.7	4.8	1.1	0.0042	90	2.30
110	243	15.8	15.8	25	1.1	0.0060	90	—
125	278	18.4	18.0	101	0.1	0.013	90	1.87
Case 3. San Fernando neutral isolated; Somis neutral grounded.								
32	70	4.69	4.55	0.04	90			0.80
61	138	9.3	8.9	0.97	7.6			1.65
96	210	14.1	13.5	4.8	1.1			2.27
111	245	16.4	15.8	25	1.1			2.28
124	278	18.4	17.8	101	0.07			1.87

*Average of three phases.

**Ratio of fundamental induced voltage to 14.25 kV., the normal value. (From Table VII.)

At the maximum impressed voltage applied the voltage from San Fernando neutral to ground measured only about 20 volts (Case 3), or less than 1/8 per cent of line voltage to neutral.

TABLE VII.

Analyses of Oscillograms. Three-Phase—Delta-Y Connection.

Case 1. San Fernando and Somis Neutrals Grounded.

% normal induced voltage / Harmonic	31	63	97	109	125
			Delta Residual Current, D_R—amperes.		
1	0.76	1.75	2.3	2.9	8.4
3	0.37	1.09	5.3	21.6	88
5	0.11	0.52	0.7	2.5	4
7	0.06	0.10	0.2	*	1—
9	*	*	0.3	2.5	1—
E. S. W.	0.85	2.14	5.8	22	88
Z_R	0.97	0.32	0.38	0.42	0.41
			Neutral Current, I_n—amperes.		
1	0.0105	0.0124	0.040	0.048	0.073
3	0.0014	0.0010	0.005	0.022	0.074
5	0.0011	0.0040	0.007	0.005	0.013
7	0.0004	*	*	*	0.004
9	*	*	*	*	0.006
E. S. W.	0.0107	0.0129	0.041	0.053	0.105
Z_n	2.7	14.9	0.8	3.2	3.2
			Induced Voltage, E_1—kilovolts.		
1	4.7		13.8	15.5	17.8
3	0.11		0.6	0.5	0.5
5	*		0.5	*	*
E. S. W.	4.47	9.0	13.8	15.5	17.8
Osc. No.	858	838	850	845	843

TABLE VII—Continued.

Analyses of Oscillograms. Three-Phase—Delta-Y Connection.

Case 2. San Fernando Neutral Grounded; Somis Neutral Isolated.

Harmonic \ % normal induced voltage	31	62	96	110	125
	Delta Residual Current, D_R—amperes.				
1	0.08	0.24	0.4	2.9	12
3	0.38	1.32	5.5	25	100
5	0.02	0.16	0.3	*	5
7	0.07	0.20	0.2	*	2
9	0.02	0.16	0.3	2.9	*
E. S. W.	0.40	1.38	5.6	25	101
Z_R	0.97	0.32	0.38	0.35	0.42
	Neutral Current, I_n—amperes.				
1	0.0007	0.0006	0.0019	0.0026	0.003
3	0.0004	0.0025	0.0059	0.0138	0.033
5	0.0011	0.0049	0.0049	0.0024	0.005
7	0.0013	0.0029	0.0019	0.0009	0.008
9	0.0005	0.0020	0.0067	0.0101	0.014
E. S. W.	0.0019	0.0057	0.0106	0.0185	0.036
Z_n	2.73	14.9	2.7	3.1	3.2
	Induced Voltage, E_i—kilovolts.				
1	4.48	8.9	13.7	15.7	17.8
3	*	*	0.6	0.4	0.5
5	0.15	0.2	0.4	*	*
E. S. W.	4.48	8.9	13.7	15.7	17.8
Osc. No.	859	839	851	846	842

TABLE VII—Concluded.

Analyses of Oscillograms. Three-Phase—Delta-Y Connection.

Case 3. San Fernando Neutral Isolated; Somis Neutral Grounded.

Harmonic \ % normal induced voltage	32	61	96	96	111	124
	Delta Residual Current, D_R—amperes.					
1	0.11	0.24	0.4		3	4
3	0.40	1.18	5.5		24	100
5	0.06	0.22	0.5		2	7
7	0.02	*	*		1	4
9	*	*	0.3		2	*
E. S. W.	0.42	1.22	5.6		25	101
Z_R	0.97	0.32	0.38	—	0.41	0.42
	Neutral Voltage, E_n—volts.					
1	1.6	3.4	4.6		7.4	9
3	0.4	0.2	1.3		7.8	22
5	0.4	2.8	3.1		2.8	5
7	0.4	0.6	0.7		*	*
9	0.3	*	*		*	*
E. S. W.	1.8	4.2	5.8		11.1	24
Z_n	101	112	151	—	252	512
	Induced Voltage, E_i—kilovolts.					
1	4.56	8.7	13.6	13.7	15.8	17.7
3	0.13	*	0.6	0.5	0.6	0.7
5	*	0.4	0.4	0.3	*	*
7	*	*	*	*	*	*
9	*	*	*	*	*	*
E. S. W.	4.56	8.7	13.6	13.7	15.8	17.7
Osc. No.	860	840	852	853	848	844

The impressed voltages and line currents and voltages to neutral were nearly the same for the three cases. The line current decreases with increase of voltage, owing to the large exciting current taken by the interconnected-Y-delta bank at Somis at the higher voltages. The induced voltage contains from two to four per cent of third harmonic and in some cases a small amount of fifth harmonic. The same is true of the impressed voltage.

The delta residual current, being the vector sum of the currents in the three station-side windings of the test-bank, includes a component of three times the circulating current. The measurements of the delta residual current are somewhat in error due to the apparent third-harmonic residual current caused by the current transformers.* The magnitude of this error is shown in Table VIII.

TABLE VIII.
Error in Third Harmonic of Delta Residual Current Due to Current Transformers.

Per cent normal voltage	Average of $D_R(3)$ amperes	Apparent residual due to C. T. s. amperes	Maximum possible error, per cent
31	0.38	0.10	36
63	1.2	0.24	25
97	5.4	0.40	8.0
110	24	0.40	1.7
125	96	0.29	0.3

For the fundamental frequency the apparent residual current due to the current transformers is 80 per cent of the value given for the third harmonic in Table VIII.
$D_R(3)$ =Third harmonic component of D_R.

Thus the current transformers introduce errors in the measurement of the fundamental and third harmonic of D_R which are of importance at low voltage but of no consequence at the higher voltages.

In the tables, average values of D_R are given. At the higher voltages, the value varied from moment to moment, sometimes over a range of several per cent, due to variations in impressed voltage.

Drawing No. 333 shows the value of the third harmonic of D_R. Above 80% of normal voltage the increase is very rapid. Between 97 and 125 per cent of normal the increase follows, roughly, the 10th power of the voltage. D_R increases in the same general way for the three cases. With both neutrals grounded, the third harmonic is, however, appreciably less than when either neutral is isolated. From Table IV, the delta residual current with the line disconnected is 4.5 amperes at 93% normal induced voltage, or in good agreement with the cases where one of the neutrals is isolated. The fundamental delta residual current varies more with the condition of the neutrals. It is

*T.R. No. 53.

small compared to the third harmonic and represents the equalizing current due to inequalities among the transformers, and to line unbalance.

The fundamental and fifth harmonics of the delta residual current are given on drawing No. 333, to show how small is their increase compared to that of the third harmonic.

The oscillograms of delta residual current, at the lowest density tested, indicate a small second harmonic. The neutral current shows a distinct 29th harmonic. These effects are small and are not appreciable in the wave-shape at higher densities.

With both neutrals grounded, the third-harmonic neutral current bears a nearly constant ratio to the third-harmonic delta residual current.

With the Somis neutral isolated, the neutral current varies less rapidly, and is apparently larger, at lower density, than in the case with both neutrals grounded.

There is intimate relation between the triple-harmonic delta residual current and the neutral current. A discussion of this subject is given in Section III.

In considering the observed values of I_n, account must be taken of the impedance between neutral point and ground, as well as the impedance of the line to residuals. Thus, the meter readings of neutral current are much lower than the oscillogram values, with both neutrals grounded, due to the large fundamental present in the oscillograms, but reduced when the meter (of relatively high impedance) was in circuit. The Somis transformer bank produces a third-harmonic residual voltage which, with both neutrals grounded, affects the third-harmonic neutral current at San Fernando. This is shown by Table IX and discussion thereof, in technical report No. 60. The neutral voltage at San Fernando was between 20 and 30 volts at 113% normal voltage.

The ninth-harmonic neutral current is larger with the Somis neutral isolated than when both neutrals are grounded, owing to the fact that the impedance is much lower. On the other hand, the impedance to the fundamental is much higher and the fundamental neutral current greatly reduced.

The residual voltage was too small to be measured.

The voltage from neutral to ground is chiefly third harmonic, with some fundamental, and at 124% of normal voltage amounts to $\frac{1}{8}$% of the line voltage to ground.

2. *Y-Y Connection.*

The measurements summarized in Tables IX, X, XI, and those plotted on drawing No. 334 were taken on March 12, 1915, with the San Fernando-Somis line energized from the test-bank. The neutrals of the test-bank and of the interconnected-Y-delta bank at Somis were grounded or isolated as specified. The three cases considered were observed successively for each value of impressed voltage.

In the table of neutral currents the impedances of the measuring circuits are given. Thus, Z_n = measuring-circuit impedance in the secondary of the neutral current transformer (not including leads, or impedance of current transformer).

TABLE IX.

Meter Readings of Transformer and Line Voltages and Currents. Three-Phase Y-Y.

Per cent normal voltage	E'_{AC} volts	E''* kV.	E''_{1s} kV.	I'_A (Amperes)	I''_n (Amperes)	I''* (Amperes)
Case 1. Both Neutrals Grounded.						
62	238	—	8.9	102	0.0042	—
98	371	14.4	14.0	146	0.094	2.26
110	418	16.2	15.6	135	0.351	2.28
Case 2. San Fernando Neutral Grounded; Somis Neutral Isolated.						
63	239	—	9.0	102	0.0040	—
98	371	14.4	14.0	144	0.102	2.25
127	416	16.2	18.0	120	1.46	2.52**
Case 3. San Fernando Neutral Isolated; Somis Neutral Grounded.					E''_n, kV.	
66	239	9.4	—	108	4.95	1.54
103	371	14.6	—	147	7.91	2.29
110	402	15.6	—	147	9.39	2.29

*Average of three phases.
**Average of phases 2 and 3. $I''_1 = 2.02$. amp.

TABLE X.

Analyses of Oscillograms. Three-Phase. Y-Y Connection.

Neutrals	Both grounded			At Fernando, grounded At Somis, isolated			At Fernando, isolated At Somis, grounded		
Harmonic \ % normal induced voltage	62	98	110	63	98	128	66	102	110
	Neutral current, I_n—amperes						Neutral voltage, E_n—kV.		
1	0.017	0.021	0.02	0.017	0.018	1.01	0.2	0.1	*
3	0.018	0.095	0.30	0.018	0.110	1.03	5.0	7.7	9.0
5	0.002	0.008	0.01	0.002	0.008	0.19	0.2	*	*
7	0.001	0.002	0.01	0.001	*	0.19	*	0.4	0.3
9	0.001	0.006	0.02	0.001	0.007	*	0.2	2.0	2.7
11	*	*	*	*	0.002	*	*	*	*
13	*	*	*	*	*	*	*	0.1	0.3
15	*	*	*	*	*	*	*	*	0.7
E. S. W.	0.025	0.097	0.31	0.025	0.112	1.46	5.0	8.0	9.4
Z_n	3.2	1.23	0.92	3.2	1.23	1.06			
	Induced voltage, E_1—kilovolts						Line voltage, E''_x—kV.		
1	8.9	14.0	15.7	8.9	14.0	18.2	9.4	14.7	15.6
3	*	0.4	0.1	*	*	1.2	*	*	0.5
5	*	0.3	0.2	*	0.4	0.3	0.5	0.7	*
E. S. W.	8.9	14.0	15.7	8.9	14.0	18.2	9.4	14.7	15.6
	Impressed voltage—E'_{AC} volts.								
1	238	371	425	238	370	414	238	370	402
3	*	*	7	6	10	6	*	*	7
5	*	8	*	*	7	*	*	8	*
E. S. W.	238	371	425	238	370	414	238	370	402
Osc. No.	927	929	922	928	930	924	926	931	925

TABLE XI.

Transformer and Line Currents and Voltages. Three-Phase Y-Y. Oscillogram No. 923.

Both neutrals grounded—110 per cent normal voltage			
Harmonic	Station bus-current, I'_x—amp.	Line voltage, E''_x—kV.	Line current, I''_x—amp.
1	133	16.2	2.21
3	11	0.2	0.15
5	10	0.2	0.18
7	4		*
9	*		*
E. S. W.	135	16.2	2.24

$E_1 = 15.7$ kV. $I_n = 0.37$ amp.

Impressed voltages and line voltages were practically the same for the three cases, and nearly simple harmonic. The measurements of E″ were made on a bank of potential transformers connected in Y on the line side with the neutral isolated and thus do not indicate the voltages of the line wires to ground, but to the neutral as established by the potential-transformer bank. With the Y-Y connection of the test bank it was found that there was a tendency, when the Somis neutral was isolated, for the line voltages and currents to be unbalanced. Further observations of this phenomenon are given in technical reports Nos. 60 and 62. In Tables IX and X, for Case 2 (Somis neutral isolated) it will be noted that at the highest impressed voltage the induced voltage greatly exceeded the line voltage (as measured by the potential transformers) and contained a large third harmonic; the line currents were unequal and the observed station-side current I'_A was notably low. For all the other observations, the line currents and voltages were practically equal and balanced. Oscillograms 918 and 919, taken under similar conditions and given in Table XII, supply the explanation.

TABLE XII.

Oscillograms of Line and Residual Voltages and Neutral Current. Three-Phase Y-Y. Voltages Unbalanced.

Harmonic	Line voltages to ground—kV.			Line voltage, E_1—kV.	Residual voltage, E_R—kV.	Neutral current, I_n—amp.
	E_1''	E_2''	E_3''			
1	13.8	17.9	17.9	13.3	8.4	0.70
3	1.6	0.9	1.1	1.6	3.2	0.83
5	*	0.9	0.7	*	0.2	0.08
7	*	*	*	*	0.3	0.17
E. S. W.	13.9	18.0	18.0	13.4	9.0	1.10
Osc. No.		919			918	

San Fernando Neutral Grounded; Somis neutral isolated.

In this unbalanced condition the fundamental voltages to ground were

$$E''_1 = 13.8 \text{ kV.} \qquad E''_2 = 17.9 \text{ kV.} \qquad E''_3 = 17.9 \text{ kV.}$$

The fundamental residual voltage was 8.5 kV. and the third-harmonic residual voltage, 3.2 kV. Thus one transformer was excited at about 96% normal voltage and the other two at about 126% normal voltage. The impedance of the line is practically pure capacitive reactance, hence the fundamental line current and the required station-side exciting current are, roughly, in the same time phase, since both lead the induced voltage by approximately 90°. The difference of these two constitutes the station-side current.

The exciting current at 126% normal voltage is very large (see drawing No. 329). The voltage unbalance of the transformers is one of four possible solutions satisfying the two conditions for the station-side;

1. the delta voltages are approximately fixed.
2. the sum of the three bus currents is zero.

The three other solutions were also observed during the tests; that is, the inequality of voltages would occur with any one of the three phases at low voltage, the other two being high; besides these, it was possible for the transformer voltages to be practically balanced; but this latter condition rarely occurred with Somis neutral isolated at so high a value of impressed voltage. The third-harmonic neutral current can be roughly estimated from the values of transformer-voltages given by oscillograms 918 and 919.

Phase	Voltage (fundamental) kilovolts	Per cent normal voltage	Third-harmonic exciting current,* amperes
1	13.8	96	0.08
2	17.9	126	0.45
3	17.9	126	0.40
Total			0.88

*From drawing No. 329 and Table III, and ratio of turns 14250/220.

This assumes that the third harmonic is in phase in the three transformers, and exists in the line-side windings only; and that the exciting current of third-harmonic frequency follows the same law of variation with density in all three transformers. The third-harmonic neutral current of oscillogram 918 was 0.83 amperes. In the present tests it was 1.03 amperes. The average of the several observations was 0.90 amperes. With the data available, and considering that small differences in the voltages cause large changes in the third-harmonic currents, the agreement is very satisfactory.

The unbalanced voltages resolve into balanced components of about 16.2 kV. and a residual voltage, opposite in phase to one of the balanced voltages, of about 8.5 kV. For three balanced voltages of 16.2 kV. (114% normal) the required third-harmonic current determined as above. is 0.56 amperes, thus the effect of the distortion of the phase e. m. fs. is to nearly double the third-harmonic neutral current.

Oscillogram 917 was taken when the voltages were balanced, 15.8 kV. per phase, with 0.32 amperes of third-harmonic neutral current. The required third-harmonic exciting current of the three transformers, from drawing 329 and Table III, is 0.38 amperes.

Oscillogram 923 is given in Table XI to present simultaneous measurements of line and station-side currents of one of the transformers, with both neutrals grounded. The third harmonic of the line current, 0.15 ampere, is fairly consistent with the measured neutral current and with the limiting value of third-harmonic current, 0.16 amperes, determined from drawing No. 329. The remarkable feature is the third harmonic observed in the station-side current, which is practically equal to the line-side third-harmonic current multiplied by the ratio of transformation. That is, it acts as if transformed. This is quite contrary to expectation, the station-side neutral being isolated. It does not appear probable that the inequality of the transformers is such as to be responsible for so large a third harmonic on the station-side. The validity of the measurement by this oscillogram is therefore questioned.

In comparing the neutral currents as given in Tables IX and X, account must be taken of the impedance of the measuring circuit to neutral current.

Drawing No. 334 shows the third-harmonic components of the neutral current for the two cases where the neutral at San Fernando was grounded. It will be noted that the condition of the Somis neutral had very little effect. It is not possible to directly compare the values for the highest voltage employed owing to the large unbalance, discussed above, in the case when the Somis neutral was isolated. In this case the third-harmonic neutral current is the result of unequally excited transformers. In the other two observations, where the transformer voltages were approximately balanced, the neutral current, with the Somis neutral isolated, is a little greater than with the Somis neutral grounded, as should be expected from the relative impedances in the two cases to the third harmonic. The ninth-harmonic neutral current is shown for the case with both neutrals grounded. The third-harmonic neutral current begins to increase with enormous rapidity at about 90% of normal voltage, increasing thereafter approximately as the tenth power of the voltage.

The third and ninth harmonics of the neutral voltage are given for the case with the San Fernando neutral isolated. Apparently, the third-harmonic induced voltage increases almost in direct proportion to the fundamental induced voltage. The ninth-harmonic neutral voltage behaves more after the fashion of the third-harmonic neutral current, as observed in the other two cases.

IV. Discussion of Experimental Results.

A more complete discussion of several features of the experimental work is given in Section III, but there are some points which may be appropriately considered here.

The excitation characteristics of the three transformers are quite unlike, as shown by the differences of the exciting currents given in Table III.

The exciting current was 3 to 4 per cent of full load current at normal voltage and increased about as the tenth power of the voltage. At 110% normal voltage the exciting current was 7 to 10 per cent, reaching 10% for the three transformers at voltages from 110 to 114 per cent normal.

The neutral current was predominantly third harmonic and increased about as the tenth power of the voltage at normal and higher voltages.

The residual voltage at the San Fernando end of the line was directly proportional to the neutral current, when the Somis neutral was isolated. With the San Fernando test-bank in delta-Y it varied from 2 to 140 volts for the third harmonic as the voltage varied from 31 to 125 per cent of normal.

With the test bank in Y-Y the third-harmonic residual voltage was much greater, varying from 70 to 1700 volts, as the fundamental voltage varied from 60 to 110 per cent normal, and reaching 3200 volts for the unbalanced condition at a nominal impressed voltage of 114% of normal.

The delta residual current was little affected either by the presence of the transmission line or condition of neutrals at San Fernando and Somis. As an example, consider the case of both neutrals grounded, 109% normal voltage. The third-harmonic ampere-turns are divided between the line and station-side windings of each transformer in the ratio of I_n to D_R, multiplied by the transformer ratio. That is, $\frac{0.022}{21.6} \times \frac{14250}{220} \times 100 = 6.6\%$, the ratio of line-side third-harmonic ampere-turns to those of the station-side. The time-phases are not identical. So the elimination of the line-side path can only have an effect amounting to a few per cent.

From Table X, for 110% normal voltage, Y-Y connection, both neutrals grounded, the third-harmonic neutral current is 0.30 ampere, corresponding to 19.5 amperes of delta residual current.

It was observed in the delta-Y test that isolating the far end neutral of the line increased the ninth-harmonic neutral current, this effect being in accord with the change of impedance.

The values of third-harmonic ampere-turns observed in the transformer bank agree well in all cases with the values obtained by calculation from the single-phase exciting-current curves of drawing No. 329. This is true even under the greatly unbalanced condition encountered with the Y-Y connection. It has not been found possible to closely determine the division of the third-harmonic current between delta and Y windings, when the transformers are in delta-Y. This matter is discussed in Section II.

Section II.

THEORETICAL.

I. Scope.

One of the aims of the present investigation has been to determine the extent to which it is possible to predict the triple-harmonic residuals arising from given transformers and line connections. With this in view the available literature has been examined, and some studies made, particularly from the basis of the design data of the test-bank of transformers, furnished by the manufacturer.

II. Results of Studies.

A—Predetermination of Magnetizing Current.

The relation between magnetomotive force and magnetic induction in magnetic circuits containing iron as expressed by the hysteresis loop, has long been known. With the induction varying harmonically, the current or m. m. f. wave can be separated into two components, (1) in opposite phase with the induced voltage, representing energy expended; (2) in quadrature with the induced voltage, being the magnetizing current, representing energy alternately stored in the magnetic field and returned to the electric circuit. The energy component is simple harmonic. The magnetizing current becomes greatly distorted as the magnetization of the iron approaches the saturation value. Due to the large increase of magnetomotive force required to cause a small increase of magnetization at maximum magnetic density, the component harmonic currents occur in such phase as to produce a sharp peak in the current wave, corresponding to the tip of the hysteresis loop. The present study is concerned with the magnetizing current only. The energy current and the magnetizing current combine to give the exciting current measured, but as stated above, the contribution of the energy current is only to the fundamental-frequency component.

The basic information usually available for the iron of a given bank of transformers is the magnetization curve. Hysteresis loops are rarely available. The magnetization curve is the locus of the tips of the hysteresis loops as the maximum magnetic induction is varied. For the purpose of predetermination of harmonics in magnetizing current, the median line of the hysteresis loop is more accurate (that is, a line determined by the average value of m. m. f. for given magnetization on the "increasing" and "decreasing" sides of the loop). However, at high magnetization the maximum m. m. f. is very large compared to the maximum width of the loop, parallel to the m. m. f. axis, so the mag-

netization curve can be substituted for the median line, with small error.

A magnetization curve for the sheet steel used in the test-transformers was secured and magnetizing-current curves constructed for 50, 75, 100, 110, 125, 135 and 150 per cent of the normal magnetic density used in the transformer design. It was assumed that the induced voltage, and hence the magnetic flux, varied harmonically. Account was taken of the different densities used in the legs and yokes of the core. The joints in the iron magnetic circuit slightly increase the fundamental component of the exciting current, but do not affect the higher harmonics. The effect of the flux in the air-space between the iron core and the surrounding coils is small, and the higher harmonic fluxes therefrom negligible.

Table XIII gives the computed effective values of the various harmonics of the magnetizing current. The harmonics of each wave are in such relation that maxima of all harmonics coincide at the maximum of the fundamental wave. The values are in amperes for the 220-volt winding. They are directly comparable with the measured values shown on drawing No. 329, except that the latter include the energy current required to supply the losses. At normal voltage the iron loss is 300 watts, requiring 1.4 amperes in the 220-volt winding, so the exciting current contains a fundamental component of $\sqrt{5.7^2 + 1.4^2} = 5.9$ amperes. Since the magnetizing current increases very much more rapidly than the energy current the effect is small throughout the range.

TABLE XIII.

Magnetizing Current—Amperes.

Harmonic \ % normal voltage	50	75	100	110	125	135	150
1	1.6	2.4	5.7	10.2	34	68	161
3	0.22	—0.13	—1.7	—4.9	—21	—45	—105
5	0.10	0.25	0.7	2.3	11	22	43
7	0.03	0.07	—	—0.8	—4	—6	—7
9	—	—	0.1	0.4	0.9	—0.6	—3.1
11	—	—	—	—	—	1.8	1.8

On drawing No. 343, are shown for comparison the values of fundamental, third, and fifth harmonics from Table XIII and the corresponding values of exciting current from Table III. The agreement is quite good, particularly above normal voltage. At low voltages the error due to using the magnetization curve is large. For example, the magnetization curve analysis indicates that at 70% normal voltage the third-harmonic current is zero, but this does not occur in practice. The reversal of sign of the computed third harmonic is due to the reversal of curvature which occurs in the magnetization curve, but not in the

median line of the energy loop. The high harmonics are so small compared to the fundamental that their values are not accurately determined by this method, with the data available.

The wave-forms of magnetizing current whose analyses are in Table XIII are also given on drawing No. 343. It will be noted that as the voltage is increased the wave-shape becomes approximately constant. This fact was observed and reported in some recent English papers,* in which it is shown that at high densities the magnetization curve of iron can be represented by the equation

$$H = kB^m \tag{2}$$

For example, above $H = 10$ gilberts per cm., $B = 14000$ gausses, the $B - H$ curve of a sample of "Stalloy" is

$$B = 10900\, H^{0.10}$$

The magnetization curve for the iron of the present tests was analyzed and it was found that it can be fairly well represented by

$$B = 11100\, H^{0.105} \tag{3}$$

or

$$H = \left[\frac{B}{11100}\right]^{9.5}$$

above $H = 6$, $B = 13000$. Such an expression for the relationship of B and H admits of determining mathematically the magnetizing-current wave-shape, with a given induced-voltage wave. For example, let the induced voltage be simple harmonic, then the flux wave is also simple harmonic. Hence

$$B = B_0 \sin x \tag{4}$$

The magnetizing current is proportional to H, so if

$$H = k_1 B^m \tag{5}$$

$$I = k_2 \sin^m x \tag{6}$$

This last equation is readily expanded into a series

$$I = I_1 (\sin x + a_3 \sin 3x + a_5 \sin 5x + \ldots) \tag{7}$$

If, on the other hand, the magnetizing current, and thus H, is simple harmonic

$$B = k_3 \sin^{\frac{1}{m}} x \tag{8}$$

whence

$$B = B_1 (\sin x + a'_3 \sin 3x + a'_5 \sin 5x + \ldots.) \tag{9}$$

and

$$E_1 = E_{1_1} (\sin x + 3a'_3 \sin 3x + 5a'_5 \sin 5x + \ldots.) \tag{10}$$

For $m = 10$, with simple harmonic flux, (7) becomes

$$I = I_1 (\sin x - 0.69 \sin 3x + 0.32 \sin 5x - 0.10 \sin 7x + 0.016 \sin 9x + \ldots. - 3.7 \cdot 10^{-6} \sin 15x)\dagger \tag{11}$$

If $m = 9$, the coefficient for the third harmonic of magnetizing current is reduced to 0.67, and that for the ninth harmonic, to 0.008, being reductions of 3% and 50%, respectively. At $m = 1$, obviously all the higher harmonics vanish.

*Bibliography. Nos. 13 and 14.

†There are, of course, additional smaller terms of frequencies higher than the fifteenth harmonic.

It is of interest to compare the coefficients for magnetizing current given above with the observed values given in Section I.

Harmonic	Observed value (Maximum)	Computed value m=10
3	0.65	0.69
5	0.25	0.32
7	0.07	0.10
E. S. W.	1.25	1.26

In making such comparison, it is to be remembered that in practice the induced voltage is not simple harmonic, and that the value 10 for m is only approximate.

For m = 10, and simple harmonic magnetizing current, (10) becomes

$$E_1 = E_{11} (\sin x + 0.84 \sin 3x + 0.80 \sin 5x + 0.70 \sin 7x + 0.63 \sin 9x + \ldots\ldots + 0.27 \sin 15x)\dagger \quad (12)$$

From drawings 330 and 334, the observed ratio of third harmonic to fundamental induced voltage averages 50% for 90 to 110 per cent normal fundamental voltage. The ninth harmonic is about 10%. The value of third harmonic under these conditions (Y-Y connection with isolated neutrals) is confirmed by other tests.*

These figures are much less than obtained from equation (12). Direct comparison is not possible, however. The magnetizing current, in the Y-Y tests, was not simple harmonic, as is assumed in (12), since only the triple-harmonic components are suppressed by the connection used. There was, of course, a slight triple-harmonic current through the measuring transformer.

It is to be remembered that the exponent m, in order to represent the observed values, must vary from 1 at very low density to about 10 at densities above normal.

The determination of the relationship of magnetizing current and induced voltage from the magnetization curve has not been undertaken in this study for the cases of simple-harmonic magnetizing current and for magnetizing current with triple harmonics suppressed. In the latter case, the fact observed experimentally and shown on drawings 356 and 357 is of interest. That is, the flux and current waves do not follow the simple energy loop as in the single-phase case with simple harmonic flux, but there is a "dimple" in the flux wave, causing a secondary loop within the main energy loop and causing the voltage wave to cross the zero-value three times per half wave.‡ Thus the common "hysteresis"

†There are, of course, additional smaller terms of frequencies higher than the fifteenth harmonic.

*Bibliography Nos. 8 and 9. (Discussion by V. M. Montsinger.)

‡Bibliography, Nos. 8 and 11 show the same phenomenon.

loop, without secondary loops, is not sufficient to determine the flux-current relation at the magnetic densities employed.

In Steinmetz' "Alternating Current Phenomena" (1908 ed., pp. 599 ff.) is given a treatment of one case for simple harmonic magnetizing current, and for the case with triple harmonics suppressed, using a simple hysteresis loop. The results there given indicate the ratio of third and ninth to fundamental induced voltage to be much less for the latter of the two cases.

Another method of attack which has been considerably used in dealing with circuits containing iron involves the linear relation between reluctivity and m. m. f. given by Kennelly.† That is

$$\frac{H}{B} = a + \sigma H \tag{13}$$

where B is the metallic induction and H is the m. m. f. "a" represents the "magnetic hardness" and $\frac{1}{\sigma}$ the magnetic density at absolute saturation of the metal. The application of this formula to the present problem is laborious, and inaccurate at low densities. To summarize the items of principal importance with respect to magnetizing current:

The magnetizing-current wave for simple harmonic induced voltage can be estimated from the magnetization curve, except for low densities.

The magnetization curve can be represented at high densities by a parabolic curve, hence the wave shape approaches a constant value, so that all the harmonics vary in direct proportion to the fundamental or ESW value of magnetizing current.

With Y-Y connection and isolated neutral, the flux-m. m. f. relation does not follow the simple "hysteresis" loop.

B—Division of Magnetizing Current Among the Windings of Transformers.

The division of the third-harmonic magnetizing currents between station and line-side windings of transformers is a matter of great interest, particularly in the case of delta-Y banks, and Y-Y banks with auxiliary delta-Y banks.

The predetermination of such division was undertaken in accordance with the following theoretical basis.

The harmonic magnetizing currents adjust themselves so that the corresponding harmonic fluxes induce e. m. fs. sufficient to drive them through their impedances when there are no external e. m. fs. to interfere. These induced e. m. fs. act alike upon all windings. It is thus possible to predict the division of current among the windings, if the

†Bibliography, No. 2.

impedances are known. Since leakage impedances of the transformer and external impedances are involved, which are little or not at all affected by the amount of current, this division is independent of the actual induced voltage.

The above outlined method is general. It is desired here to investigate the particular case of the third harmonic in the delta-Y test-bank, as used in tests recorded in Section I of this report. It was noted in Section I that the total third-harmonic ampere-turns observed in the Y-delta connection are in agreement with the single-phase exciting-current observations. Thus if the relation between the portions in the two sides of the transformer bank is determined the magnitude of the neutral current can be calculated.

The supply transformers, being delta connected, and isolated from ground, provided a minimum of third harmonics. With the Somis neutral grounded, the transformer bank there was a source of triple-harmonic residuals, but it was in interconnected-Y-delta so as to minimize these. Of course, inequalities among the transformers of a given bank cause some third-harmonic residual, in spite of such precautions.

Assuming no triple harmonics introduced except by the test-bank, with station-side in delta and line-side in grounded-Y the following relations hold, for triple harmonics.

Z'_t. Z''_t = the station-side and line-side internal impedances, respectively, of one transformer.
Z_L = impedance from three line-side taps in parallel through the line to earth.
E_1 = induced third-harmonic voltage in line-side winding of one transformer.
a = ratio of transformation.
D_R = station-side delta residual current.
I_n = line-side neutral current.

Then
$$E_1 = a \frac{D_R}{3} Z'_t = I_n \left(\frac{Z''_t}{3} + Z_L\right) \tag{14}$$
and
$$\frac{D_R}{I_n} = \frac{Z''_t + 3Z_L}{aZ'_t} = \frac{a(3Z_L + Z''_t)}{a^2Z'_t} \tag{15}$$

The above is for the triple harmonics, assuming equal induced voltages and impedances in the three transformers.

The short-circuit impedances of a single transformer and of the three test-transformers connected Y-delta are given on the attached drawings 302 and 304. Assuming that the leakage reactance is equally

divided between line-side and station-side windings, the values are, at 150 cycles

$$Z''_t = a^2 Z'_t = 41 + j\ 165 \text{ ohms}$$
$$a = {}^{14250}/_{220} = 64.8$$

With the far end of the line clear

$$Z_L = 60 - j\ 1400 \text{ ohms}$$

$$\frac{D_R}{I_n} = \frac{(210 - j\ 4050)}{41 + j\ 165}\ 64.8 = 1540$$

With the far end of the line grounded

$$Z_L = 80 + j\ 150 \text{ ohms}$$

$$\frac{D_R}{I_n} = \frac{(280 + j\ 620)}{41 + j\ 165}\ 64.8 = 258$$

From drawing 333, it will be found that the observed ratios of $\frac{D_R}{I_n}$ are as given in Table XIV.

TABLE XIV.

Ratio of Delta Residual to Neutral Current. Three-Phase—Delta-Y.

% normal voltage	Somis neutral	
	Grounded	Isolated
60	1100	530
80	1050	660
100	1060	1040
120	1060	2460

Thus, with the Somis neutral grounded the observed value is quite constant and about four times the computed value. With the Somis neutral isolated, the ratio varies and agrees with the computed value at about 110% normal voltage.

The assumptions made in the computations will now be considered. No allowance was made for the effect of the Somis bank in the case with Somis neutral grounded. In order to judge of the importance of this effect it is of interest to estimate the line-side third-harmonic induced e. m. f. of the test-bank. At 110% normal voltage $D_R = 23$ amperes. If all the leakage reactance were in the station-side delta of the transformers, the voltage to drive D_R in the delta would be

$$23 \times \frac{340}{(64.8)^2} = 1.9 \text{ volts}$$

and the corresponding line-side "residual" voltage would be 120 volts or 40 volts between the neutral point and each line-wire. The third-harmonic voltage due to the Somis bank was between 20 and 30 volts under these conditions, so it is evident that the Somis bank with its neutral grounded has a large effect on the neutral current. If the third-harmonic residual voltage due to the Somis bank opposed that due to the test-bank, the effect observed would take place, that is, the neutral current would be smaller than indicated by the computation.

A very important question concerns the relation between the leakage reactances of the windings. In transformers having their windings interlaced it is to be expected that the leakage flux will divide equally between them. But with the cylindrical windings frequently used in core-type transformers, there is a larger opportunity for difference.

The test transformers are of this latter type, having four cylindrical coils on each leg of the core. The innermost and outmost coils compose the low-voltage winding, while the middle pair compose the high-voltage winding. An attempt was made to estimate the relative leakage fluxes of the high and low-voltage windings, using an extension of the method set forth in "The Magnetic Circuit,"* taking into account the self and mutual fluxes of the several coils in the air space about the iron core. This could be done only approximately owing to the fact that the information concerning the dimensions and the distribution of insulating materials and copper in the winding space is not very exact. A check on the results was obtained by computing the short-circuit reactance, and comparing it with the measured value. The computed value was 71% of the value given by drawing No. 304. It is believed that with accurate dimensional data the method would give much closer agreement.

In terms of the 14250-volt winding, the computed inductances for the coils on one leg of the core, when multiplied by a correction factor of $\frac{100}{71}$ are:

L', Leakage inductance, 220-volt coil = 0.35 henry.
L'', Leakage inductance, 7125-volt coil = 0.44 henry.
M, Mutual inductance in air (exclusive of iron core) = 0.31 henry. The ratio of the effective leakage inductances is approximately $\frac{L' - M}{L'' - M} = 3$. The short-circuit inductance is $L_{sc} = L' + L'' - 2M$, so it is accurate to use $\frac{L_{sc}}{2}$ for the leakage inductance of each winding only in case $L' = L''$.

Considering equation (15) if the ratio 3 is correct the reactance component X'_t, of Z'_t, is decreased while that of Z''_t is increased by one-

*By V. Karapetoff, page 208 ff. 1911 ed.

half. Then since $3Z_L$ is large compared to Z''_t and the value of Z'_t is governed chiefly by its reactance, the value of $\frac{D_R}{I_n}$ is approximately doubled, compared to its value for $L'=L''$. This gives values of $\frac{D_R}{I_n}$ for Somis neutral isolated, 2900 and for Somis neutral grounded, 530.

It is known that the three transformers are not identical with respect to third harmonics as shown by the differences among the measured exciting currents. It is believed that the inequalities among the three transformers are responsible for the marked variation of ratio $\frac{D_R}{I_n}$ observed with Somis neutral isolated. With the Y-delta connection of the test-bank the resultant third-harmonic e. m. fs. are sufficiently small that the inequalities may reasonably be expected to be of importance.

It thus appears that with the methods here used it is practicable to determine only the general order of magnitude of the third-harmonic neutral current and that a more extended analysis would be required to reach an accurate result. Such a study requires the consideration of a complicated network even for the simple case discussed above and the inclusion of the effects of individual transformers. In the practical problem, moreover, the wave-shape and the inequalities of the supply voltages must be taken into account. Small variations in the wave-shape or magnitude of the supply voltages cause large changes in the triple-frequency components, hence these will exhibit large fluctuations under the conditions of normal operation of power systems.

C—Regulation of Magnetizing-Current Harmonics.

In the Trans. A. I. E. E., 1915, p. 2175,* there is given a method of calculation of the regulation of harmonics due to variation of permeability in transformers. This method is helpful in considering the effect of changes of line-impedance on the neutral currents under conditions such as obtained in the tests reported herein and in technical report No. 60, and the observed effects accord with it. A brief discussion is therefore given.

Each transformer is represented by an H-network having the leakage impedance of the two windings in series, with a bridge impedance between them. In series with the bridge impedance is a fictitious generator whose voltage E_n is the harmonic e. m. f. present when the current of that frequency is totally suppressed.

*Bibliography, No. 12, discussion by H. S. Osborne.

The bridge impedance N is the ratio of E_n to the harmonic current I_n, when the transformer is short-circuited, with respect to that frequency. With this network the division of current between the windings is determinable, and is in agreement with the discussion given in "B" above. The application of the theory to delta-Y and Y-Y banks is given in the reference.

It is evident that the impedance N is of primary importance. With transformers of given design and quality of iron, the value of N and of the leakage reactances will vary inversely as the kVA. capacity. Thus with large transformer banks the line impedance more nearly approaches N.

It has been shown above that as the magnetic density is increased above about 13000 gausses, the harmonics of the magnetizing current for simple harmonic flux increase approximately as the tenth power of the fundamental voltage while the voltages of frequencies for which the magnetizing currents are suppressed, increase directly as the fundamental voltage. Hence N decreases very rapidly roughly according to

$$N = CE_1^{1-m} \text{ or } CE_1^{-9} \qquad (16)$$

where m = 10.

This means that increasing voltage from 100 to 120 per cent normal decreases N by 80%.

At lower densities where the exponent m is less than 10, the rate of decrease of N is much smaller, finally approaching O as m approaches unity. That is, N plotted against fundamental voltage begins as a horizontal line and gradually bends downward and reverses curvature, and finally follows the law of equation (16).

Hence, at very high densities N may become small compared to the leakage and external impedances of the transformer to magnetizing currents, in which case the currents would increase directly as the voltage, assuming that these impedances remain constant. This effect accords with the observed falling off of the third-harmonic magnetizing current (see drawing No. 329), but does not admit of the decrease shown for the fifth harmonic. The latter, if correct, must be accounted for by the wave-shape of the supply containing a fifth harmonic.

In the Trans. A. I. E. E., 1914, p. 779 ff.* account is given of the effect of condensers in modifying the third-harmonic voltage across the open corner of the delta of a Y-delta bank of transformers. The result there stated, that increasing the capacitance across the open corner of the delta caused a lowering of the magnetic density at which the sharp peak of delta-corner voltage occurred, is in apparent contradiction to the result to be expected from the theory under consideration. However, the bearing of this theory on the case reported in the refer-

*Bibliography, No. 9, discussion.

ence can not be settled without further study and more detailed information concerning the tests there recorded.

As an example of its application consider the Y-Y test. A value of N for the third-harmonic can be obtained, using the value of E_t from drawing 330 (3450 volts on 7125-volt winding) and D_R from drawing 333 with San Fernando neutral isolated. Then for the 14250-volt line-side windings, at normal voltage and considering the three transformers in parallel,

$E_3 = 6900$ volts.
$I_3 = 7.5/64.8 = 0.116$ amperes.
$N = 60000$ ohms.
N is chiefly inductive reactance.

The station-side neutral being isolated, it may be assumed that the only path for the triple harmonics is through leakage impedance of the line-sides, and the impedance from line-wires to ground, in series with N and the fictitious generator.

That is,

$$I_3 = \frac{E_3}{N + Z''_t + Z_L} \tag{17}$$

Z''_t is smaller than the probable error in N and is unaffected by the line condition. Z_L is 60 — j 1400 ohms or 80 + j 150 ohms, depending on whether Somis neutral is isolated or grounded. It is thus to be expected that the third-harmonic neutral current will be greater with Somis neutral isolated, and less with it grounded, than I_3. This accords with the observations as shown on drawing 334.

It is of interest to consider the application of the theory to a typical problem. Consider the effect on the third-harmonic residual current in a transmission line, of doubling the kVA. capacity of the transformers there, either by doubling the size of bank or using two banks in parallel. Assume delta-Y connection, that there are no other sources of third harmonic and that the impedance of the line is not changed. If $N = 10Z_L$ and the leakage impedance of the delta-winding is $0.1\ Z_L$, doubling the kVA. capacity increases the residual current by approximately 10%. The effect will vary nearly as the ratio of delta leakage impedance to the line impedance to residuals, assuming N large compared to either.

Now consider a Y-Y bank with line-side neutral grounded, the other assumptions being the same as before. For a given design of transformers, N and the leakage impedance vary inversely as the kVA. capacity. The neutral current varies inversely as the sum of N and leakage and line impedances. As long as N is large compared to Z_L,

the neutral current varies in practically direct proportion to the kVA. capacity.

It can be stated for the theory here under consideration that it presents a satisfactory view of the phenomena, for cases where the actual magnetizing current approaches the short-circuit value. This includes the delta-Y and grounded Y-Y cases commonly occurring. For the delta-Y cases there is the practical limitation with respect to inequalities of transformers, hitherto discussed. There is also, no reason to doubt its applicability to the case where the external impedance is large compared to N, that is, where the harmonic current I_n is nearly suppressed.

III. Bibliography.

In connection with the studies on triple-harmonic phenomena in transformers numerous published papers have been consulted. These include, besides the recent papers on transformer connections and attendant wave-shapes, several dealing with the magnetic qualities of iron and some earlier papers of historical as well as technical interest. The discussions published in connection with several of the papers are to be especially noted.

The following are from the Transactions of the American Institute of Electrical Engineers:

1. Ryan—Transformers, vol. 7, 1890, p. 1. Wave-shapes of exciting current and voltage.
2. Kennelly—Magnetic Reluctance, vol. 8, 1891, p. 485.
3. Steinmetz—On the Law of Hysteresis, vol. 9, 1892, p. 3 (Part I) and p. 621 (Part II); vol. 11, 1894, p. 570, (Part III). Magnetic Constants of Iron and other materials. Theory of Ferric Inductances.
4. Emery—Rational and Empirical Formulæ Showing the Relation Between the Magnetomotive Force and the Resulting Magnetization, vol. 9, 1892, p. 192.
5. Huguet—Analysis of Transformer Curves, vol. 13, 1896, p. 207. Study of cause of distortion of exciting current.
6. Blackwell—Y and Delta Connection of Transformers, vol. 22, 1903, p. 385.
7. Bedell & Tuttle—The Effect of Iron in Distorting Alternating Current Wave-Form, vol. 25, 1906, p. 671.
8. Frank—Observation of Harmonics in Current and in Voltage Wave Shapes of Transformers, vol. 29, 1910, p. 809.
9. Blume—Influence of Transformer Connections on Operation, vol. 33, 1914, p. 735.
10. Fortescue—A Study of Some Three-phase Systems, vol. 33, 1914, p. 753.
11. Curtis—The Effect of Delta and Star Connections Upon Transformer Wave-Forms, vol. 33, 1914, p. 1273.
12. Peters—Harmonics in Transformer Magnetizing Currents, vol. 34, 1915, p. 2157, and discussion.

From the Journal of the Institution of Electrical Engineers, vol. 53, 1915.

13. Nicholson—The Magnetization of Iron at High Flux Density With Alternating Currents, p. 248.
14. Tackley—Mathematical Relationship Between Flux and Magnetizing-Current Waves at High Flux Densities, p. 521.
15. McLachlan—The Magnetic Behavior of Iron Under Alternating Magnetization of Sinusoidal Wave-Form, p. 809.

References to other sources are:

16. Rowland—Notes on the Theory of the Transformer, Phil. Mag. Ser. 5, vol. 34, 1892, p. 55.
17. Franklin and Clark—A Normal Course of Magnetization of Iron, Phys. Rev. vol. 8, 1899, p. 304.
18. Ewing—Magnetic Induction, 3d ed., 1900, p. 330.
19. Steinmetz—Alternating Current Phenomena, 1908 ed., chapters 13 and 29.
20. Lloyd—Dependence of Magnetic Hysteresis Upon Wave Form, U. S. Bureau of Standards Bulletin, vol. 5, 1909, p. 381.
21. Thompson—On Hysteresis Loops and Lissajous' Figures, and on the Energy Wasted in a Hysteresis Loop, Phys. Soc. of London, vol. 22, 1910, p. 454.
22. Karapetoff—The Magnetic Circuit, 1911 ed., chap. 12.
23. Ball—Reluctivity of Silicon Steel as a Linear Function of the Magnetizing Force. General Electric Review, September, 1913, p. 750.
24. Williams—Saturation Value of Intensity of Magnetization and Theory of Hysteresis Loop, Phys. Rev., vol. 6, 1915, p. 404.
25. Ball—Investigation of Magnetic Laws for Steel and Other Materials, Jour. of Franklin Inst., April, 1916, p. 459.

SECTION III.

GENERAL CONCLUSIONS.

A brief summary of the studies embodied in this report, which appear to be of most interest, follows:

1. With increasing simple-harmonic impressed voltage, the higher harmonics of the exciting current of a transformer increase at first more rapidly than the fundamental, but for high magnetic densities the wave-shape becomes constant. For modern transformer iron the ultimate relative values of the harmonics are about as follows:

$I_1 = 1.00$	$I_7 = 0.10$
$I_3 = 0.65$	$I_9 = 0.02$
$I_5 = 0.30$	$I_{ESW} = 1.24$

2. The exciting currents required by like transformers for given impressed voltage may vary widely.

3. For magnetic densities above 13000 gausses, it is possible to derive quite accurate magnetizing-current values from ordinary magnetization curves. At the lower densities they give misleading results.
4. The magnetization curve of modern transformer iron is fairly well represented by the expression

$$H = kB^{10}$$

above 13000 gausses. At lower densities the value of the exponent to represent the conditions decreases gradually to unity. For the range of densities normally used, a given reduction of the density results in a much greater reduction of magnetizing current. The extent of such reduction decreases as the density is lessened.
5. Where the triple-harmonic magnetizing currents are suppressed the magnetization does not follow the simple "hysteresis" loop, but secondary loops may be formed.
6. With Y-delta and Y-Y banks the third-harmonic magnetomotive force can be closely predicted from the single-phase tests of magnetizing current, even under conditions of large unbalance.
7. A method involving an H-network for use in the analysis of magnetizing-current problems has recently been published, which is of distinct value in cases where either the harmonic current or flux is nearly suppressed, as in the delta-Y and grounded Y-Y banks commonly occurring.
8. The proportion of triple-harmonic magnetizing current going into the transmission lines is decreased roughly in proportion to decrease of the leakage impedance in the station-side windings of delta-Y banks. Core-type transformers with cylindrical coils are advantageous in this repect, having most of the leakage reactance in the high-voltage coils.
9. It appears that the inequalities among transformers make it very difficult, if not impossible, to accurately calculate the division of triple-harmonic currents between the two sides of a Y-delta bank. The action of other transformer banks in the network can not be neglected.
10. The third-harmonic residual voltages and currents due to both delta-Y and Y-Y banks increase enormously as the magnetic density is increased, and follow approximately the same law as the single-phase magnetizing current for simple harmonic voltage, that is,

$$I_R = KB^m$$

where m increases from 1 at low density to about 10 at values of B used in present practice, and higher.

11. The higher triple harmonics appear to exhibit the same general characteristics as given for the third harmonic, but are generally smaller in magnitude.

Respectfully submitted.

(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

ATTACHMENTS: Drawings Nos. 263, 270, 302, 303, 304, 315, 328, 329, 330, 333, 334, 343, 356, 357.

IN FILES OF SECRETARY: Oscillograms: Nos. 735, 736, 737, 738, 739, 741, 742, 744, 745, 747, 748, 838, 839, 840, 842, 843, 844, 845, 846, 848, 850, 851, 852, 853, 858, 859, 860, 862, 863, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931.

APPROVED: September 15, 1916.

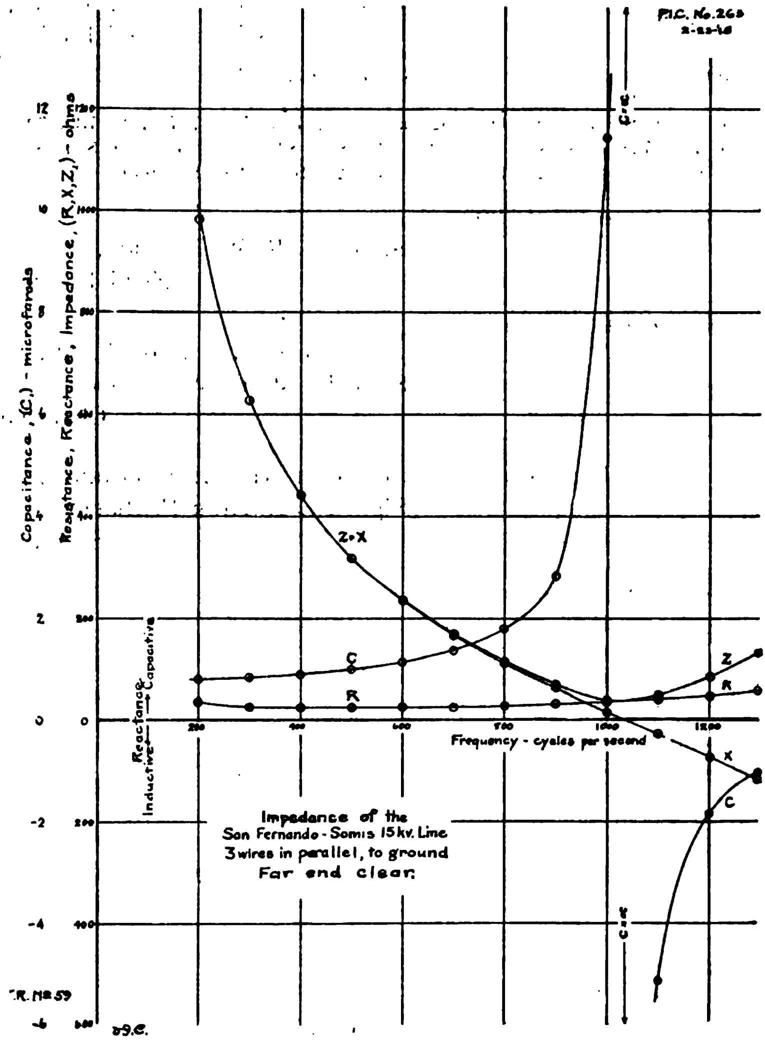
(Signed) R. W. MASTICK,
Field Engineer.

APPROVED: October 11, 1916.

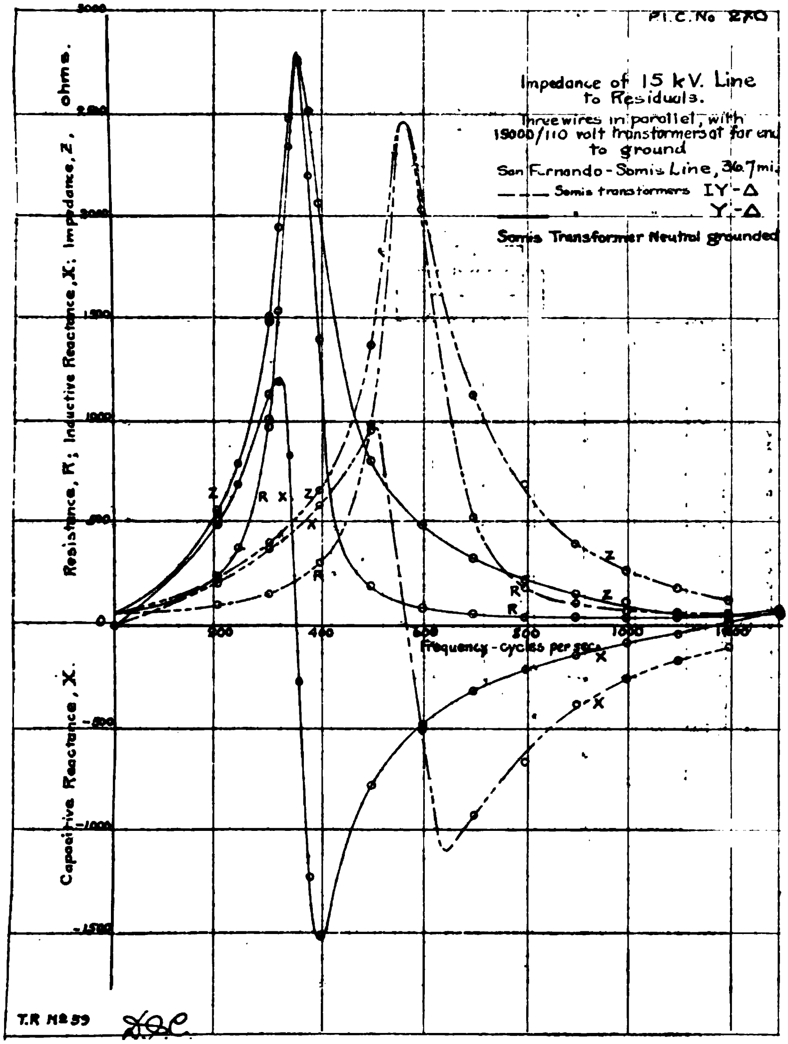
SUBCOMMITTEE ON TESTS,
(Signed) J. E. WOODBRIDGE,
Chairman.

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE,
(Signed) ARTHUR F. BRIDGE,
Secretary.

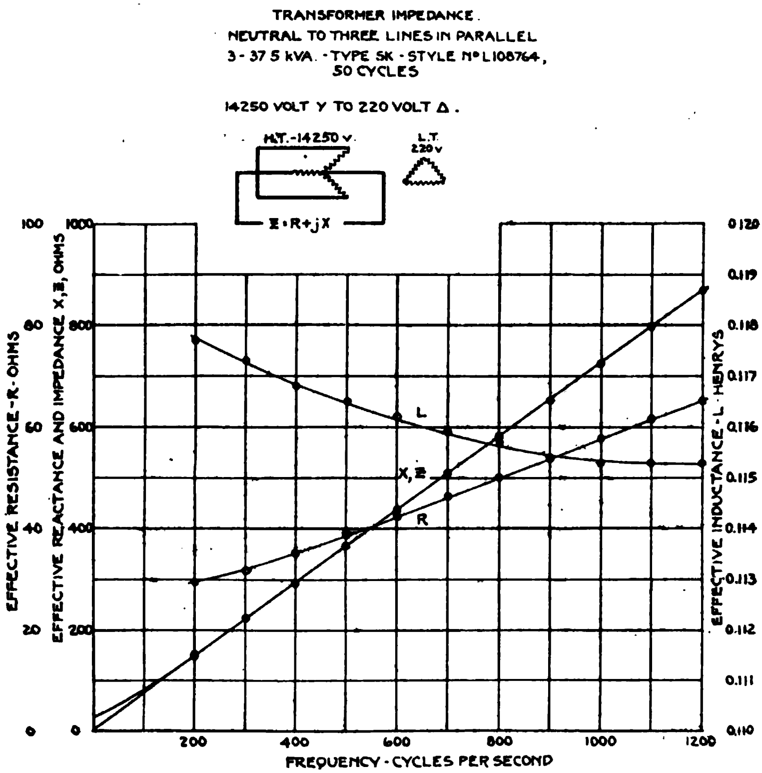
January 8, 1917.



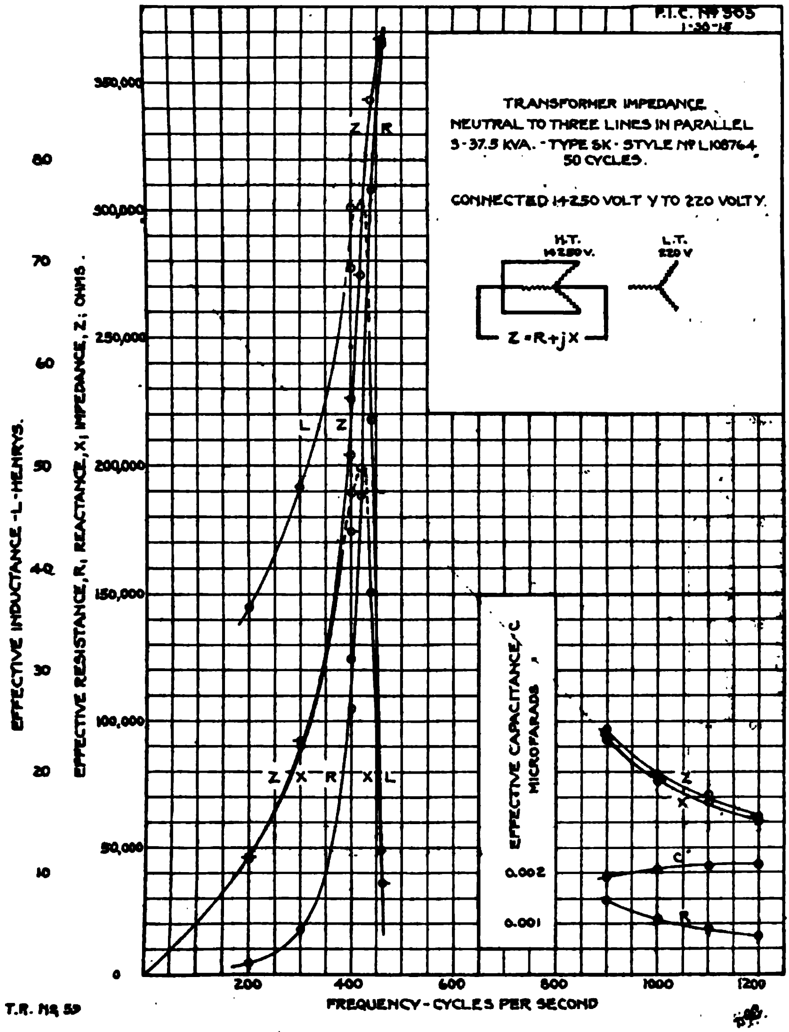
P.I.C. No. 263
2-23-16
Capacitance, (C,) - microfarads
Resistance, Reactance, Impedance, (R, X, Z,) - ohms
Reactance
Inductive ← → Capacitive
Z=X
C
R
Z
X
Frequency - cycles per second
Impedance of the
San Fernando - Somis 15 kv. Line
3 wires in parallel, to ground
Far end clear.
T.R. No 59

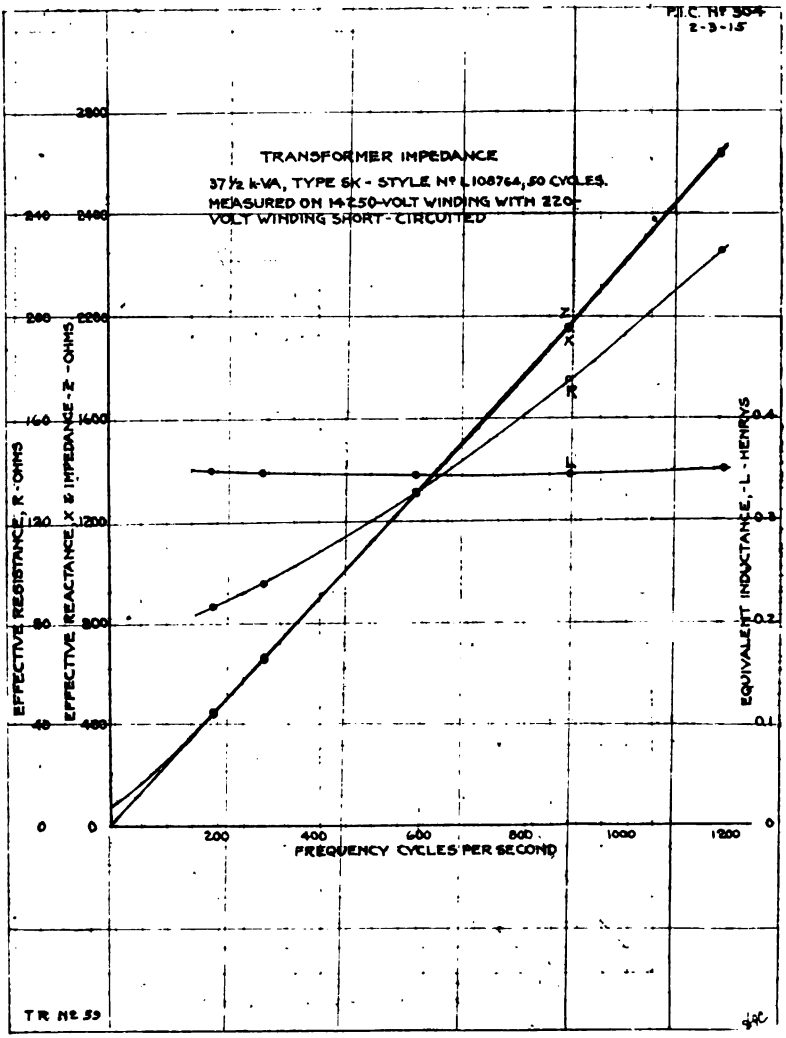

P.I.C. No 270
Impedance of 15 kV. Line to Residuals.
Three wires in parallel, with 15000/110 volt transformers at far end to ground
San Fernando - Somis Line, 36.7 mi.
Somis transformers IY-Δ
Y-Δ
Somis Transformer Neutral grounded
Resistance, R; Inductive Reactance, X; Impedance, Z, ohms.
Capacitive Reactance, X.
Frequency - cycles per sec.
T.R No 59

P I C Nº 302
1-30 15



T.R. Nº 59

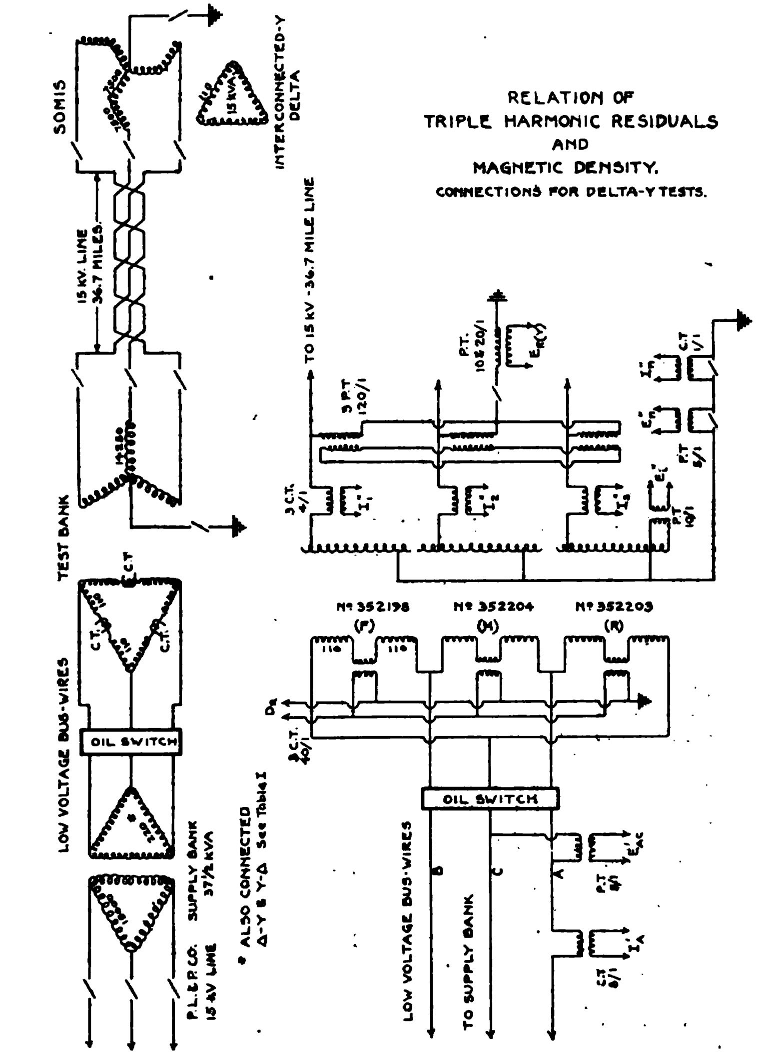

P.I.C. No 505
1-30-15
TRANSFORMER IMPEDANCE
NEUTRAL TO THREE LINES IN PARALLEL
3-37.5 KVA. - TYPE SK - STYLE No L108764
50 CYCLES.
CONNECTED 14250 VOLT Y TO 220 VOLT Y.
H.T.
14250 V.
L.T.
220 V
Z = R + jX
EFFECTIVE INDUCTANCE - L - HENRYS.
EFFECTIVE RESISTANCE, R, REACTANCE, X, IMPEDANCE, Z; OHMS.
EFFECTIVE CAPACITANCE, C
MICROFARADS
FREQUENCY - CYCLES PER SECOND
T.R. No 59



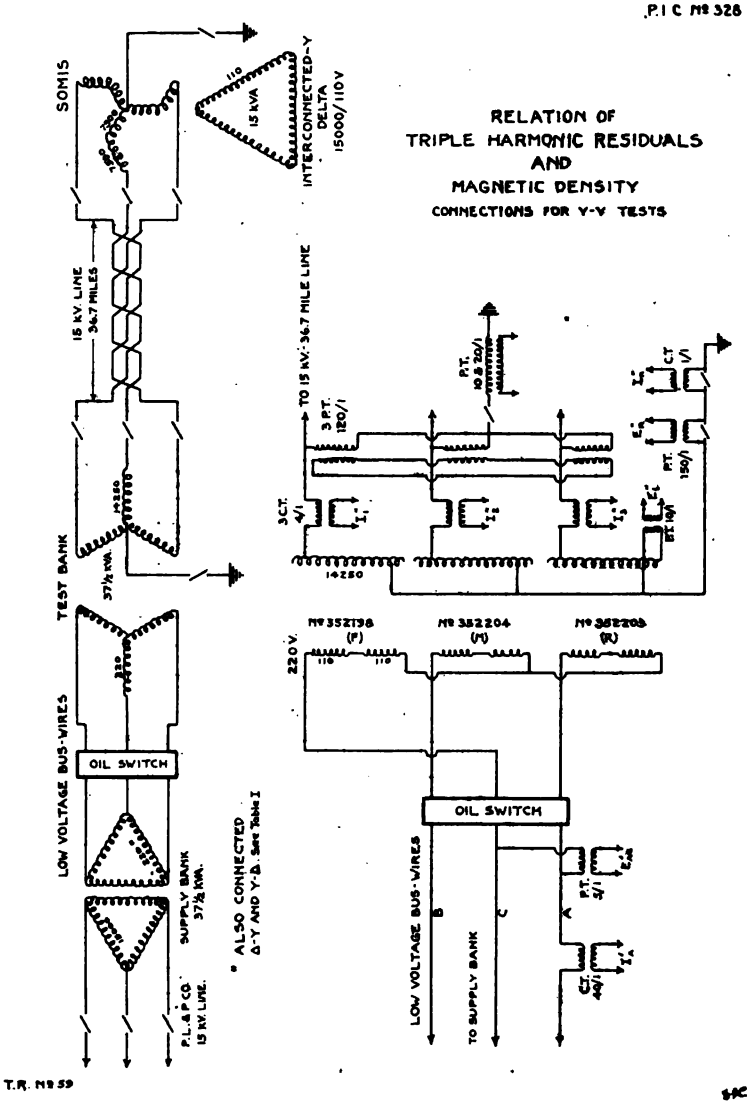
P.I.C. Nº 304
2-3-15
TRANSFORMER IMPEDANCE
37½ k-VA, TYPE SK - STYLE Nº L108764, 50 CYCLES.
MEASURED ON 14250-VOLT WINDING WITH 220-VOLT WINDING SHORT-CIRCUITED
EFFECTIVE RESISTANCE, R-OHMS
EFFECTIVE REACTANCE, X & IMPEDANCE-Z-OHMS
EQUIVALENT INDUCTANCE, -L-HENRYS
FREQUENCY CYCLES PER SECOND
Z
X
R
L
0
40
80
120
160
200
240
400
800
1200
1600
2000
2400
2800
200
600
1000
0.1
0.2
0.3
0.4
TR Nº 59

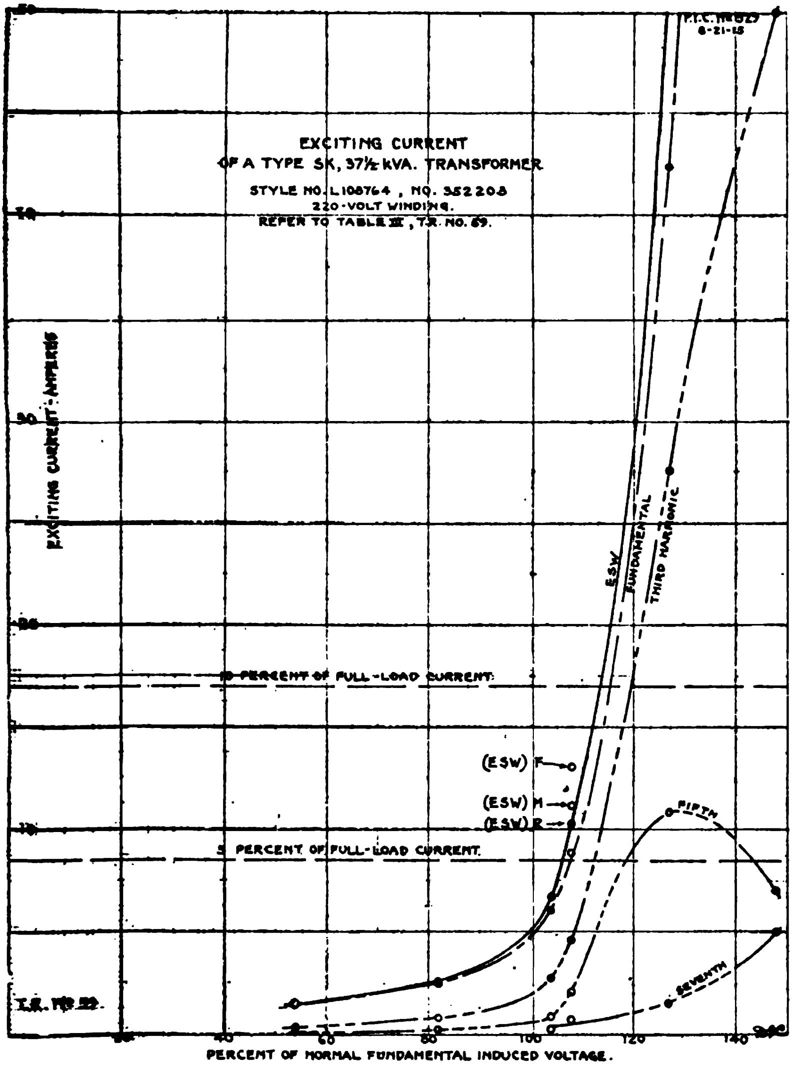
P.I.C. No 315

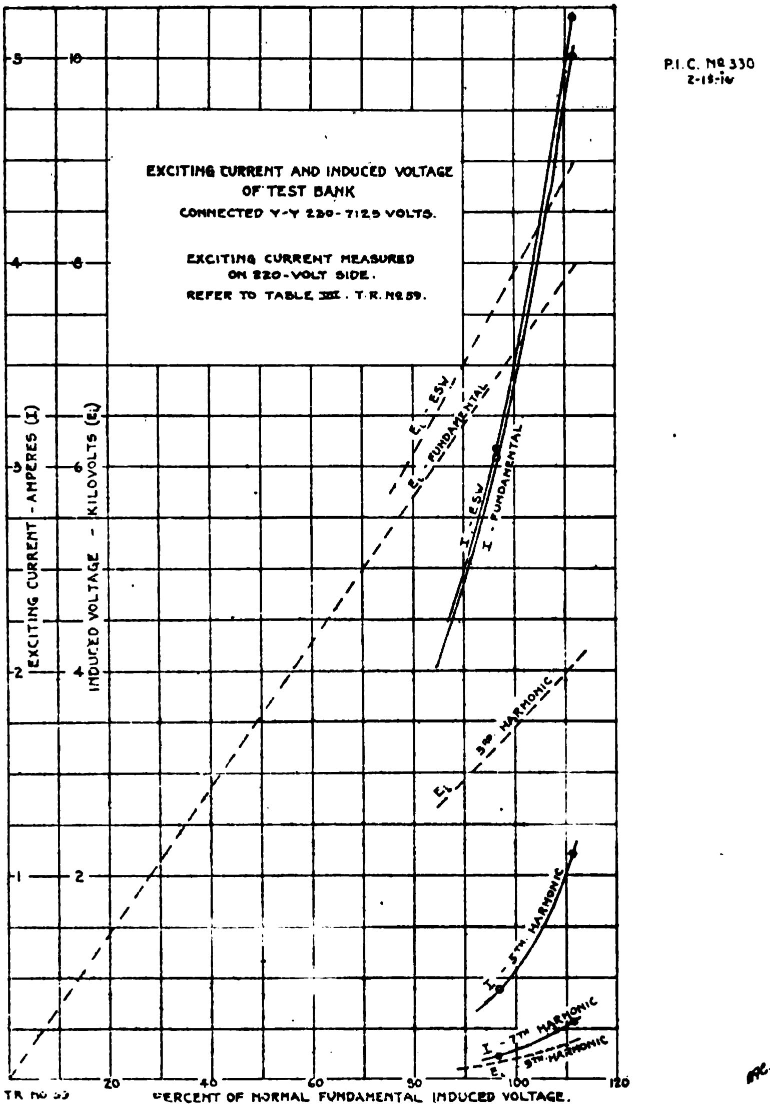
RELATION OF
TRIPLE HARMONIC RESIDUALS
AND
MAGNETIC DENSITY.
CONNECTIONS FOR DELTA-Y TESTS.

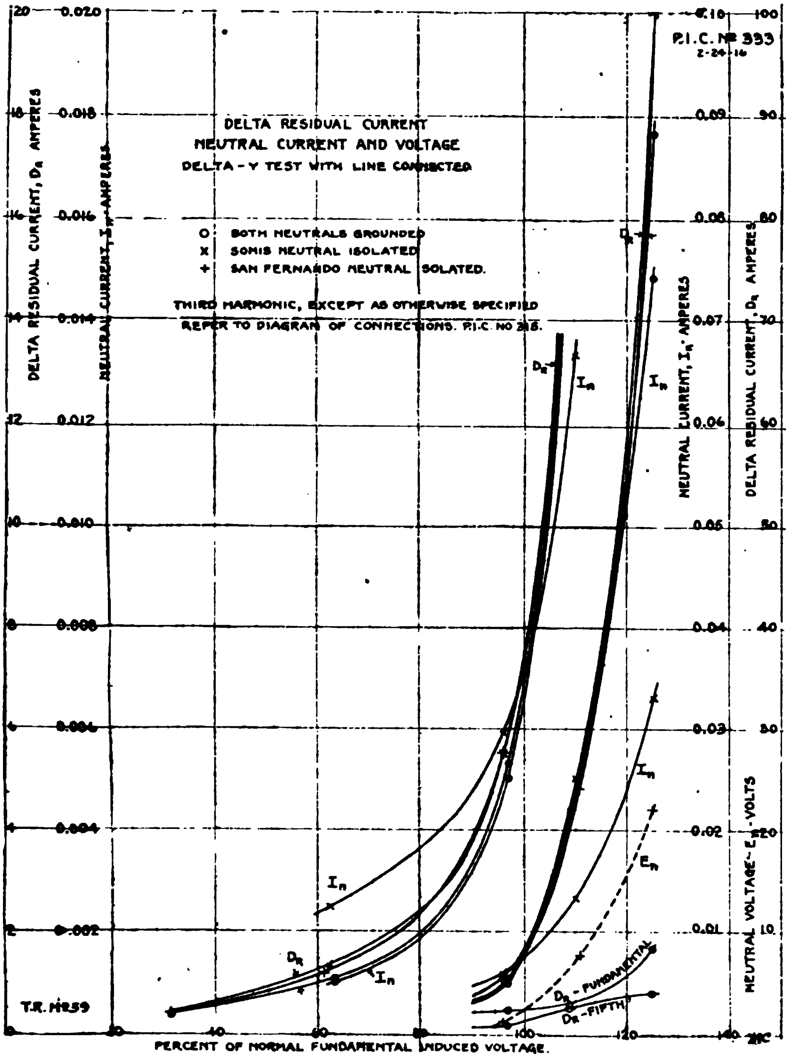


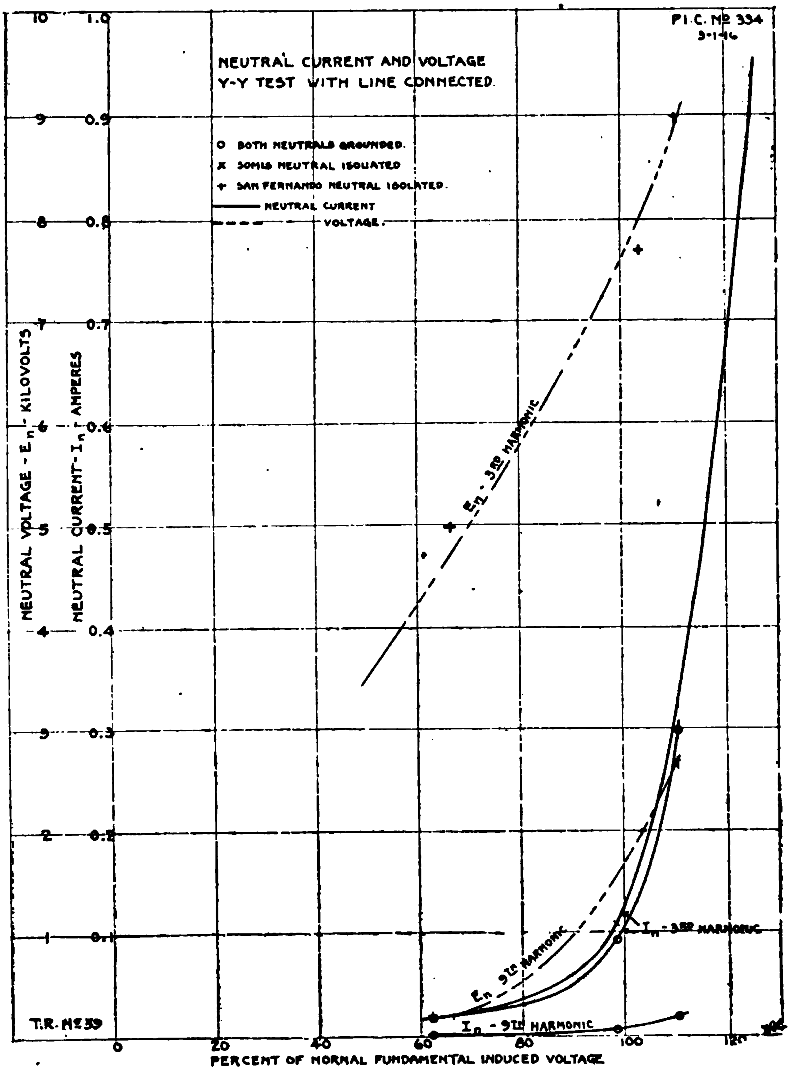
T.R. No 59

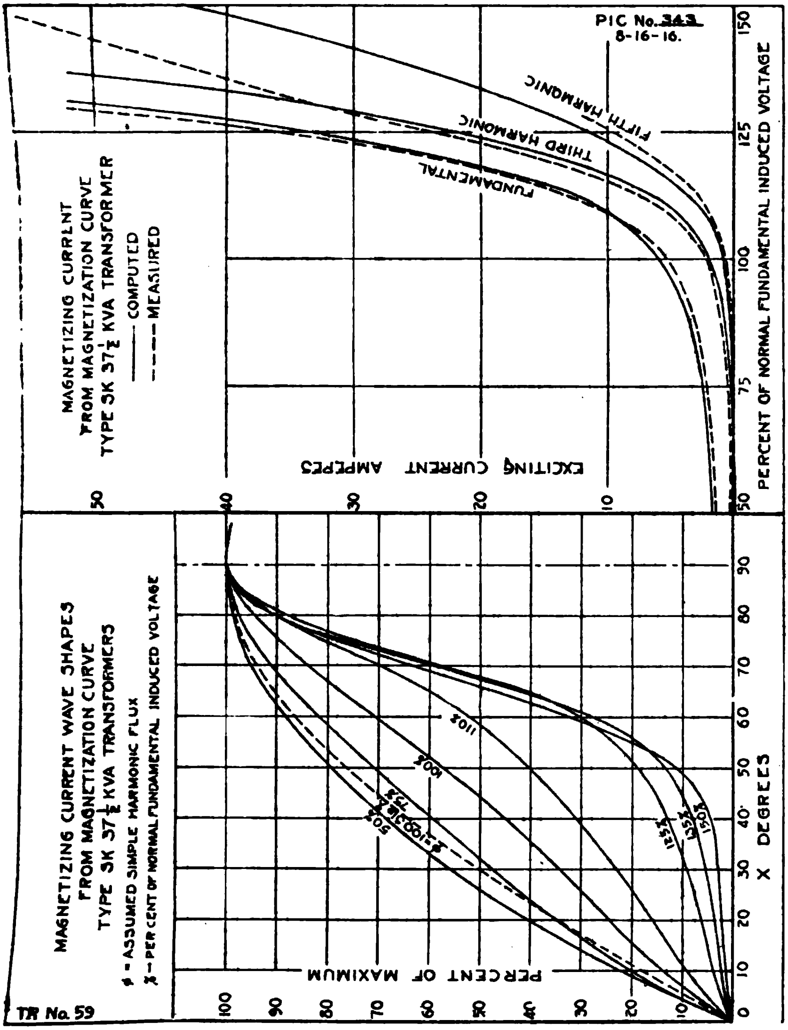

P.I.C. No 328
RELATION OF
TRIPLE HARMONIC RESIDUALS
AND
MAGNETIC DENSITY
CONNECTIONS FOR Y-Y TESTS
SOMIS
INTERCONNECTED-Y
DELTA
15000/110V
15 KVA
15 KV. LINE
36.7 MILES
TO 15 KV. 36.7 MILE LINE
TEST BANK
37½ KVA.
LOW VOLTAGE BUS-WIRES
OIL SWITCH
SUPPLY BANK
37½ KVA.
P.L.&P.CO.
15 KV. LINE.
* ALSO CONNECTED
Δ-Y AND Y-Δ. See Table I
3 P.T.
120/1
P.T.
10.8 20/1
C.T.
1/1
P.T.
150/1
3 C.T.
4/1
14250
No 352198
(F)
No 352204
(M)
No 352208
(R)
220 V.
OIL SWITCH
LOW VOLTAGE BUS-WIRES
TO SUPPLY BANK
P.T.
5/1
C.T.
40/1
T.R. No 59

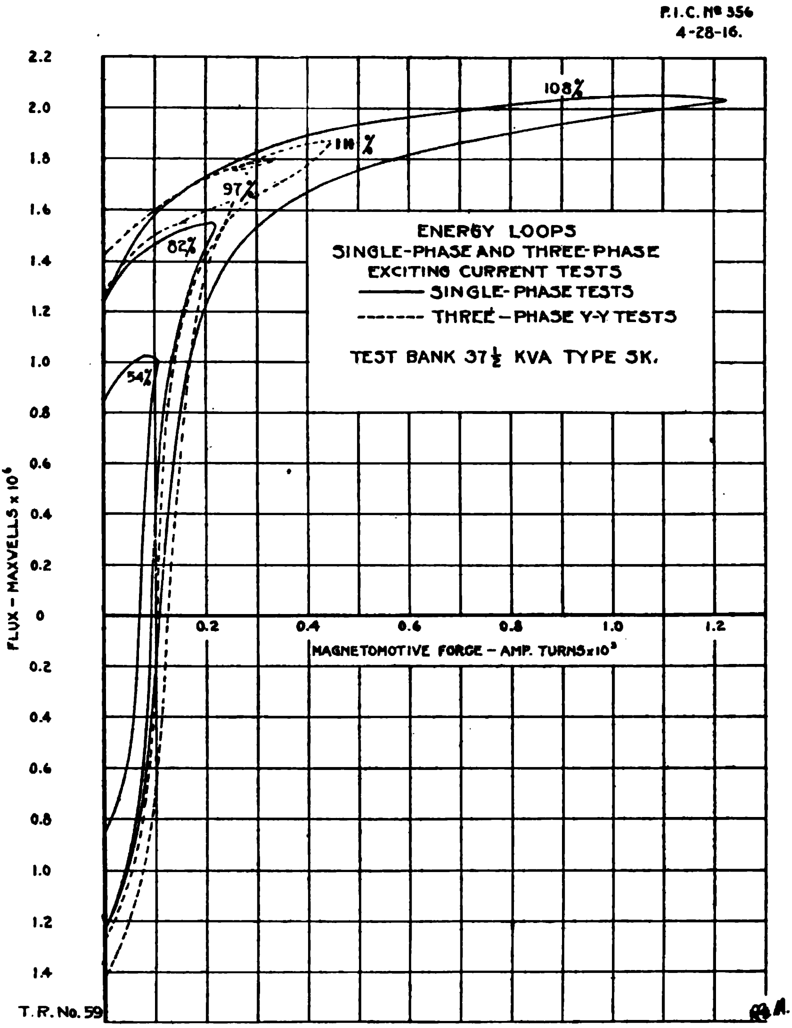

EXCITING CURRENT
OF A TYPE SK, 37½ KVA. TRANSFORMER
STYLE NO. L108764, NO. 352208
220-VOLT WINDING.
REFER TO TABLE III, T.R. NO. 59.
EXCITING CURRENT : AMPERES
PERCENT OF FULL-LOAD CURRENT
5 PERCENT OF FULL-LOAD CURRENT
(ESW) F
(ESW) M
(ESW) R
ESW
FUNDAMENTAL
THIRD HARMONIC
FIFTH
SEVENTH
30
10
40
60
80
100
120
140
PERCENT OF NORMAL FUNDAMENTAL INDUCED VOLTAGE.


EXCITING CURRENT AND INDUCED VOLTAGE
OF TEST BANK
CONNECTED Y-Y 220-7125 VOLTS.
EXCITING CURRENT MEASURED
ON 220-VOLT SIDE.
REFER TO TABLE XIII. T.R. NO 59.
EXCITING CURRENT - AMPERES (I)
INDUCED VOLTAGE - KILOVOLTS (Ei)
PERCENT OF NORMAL FUNDAMENTAL INDUCED VOLTAGE.
Ei - ESV
Ei - FUNDAMENTAL
I - ESV
I - FUNDAMENTAL
Ei - 3RD HARMONIC
I - 5TH HARMONIC
I - 7TH HARMONIC
Ei - 9TH HARMONIC
P.I.C. NO 330

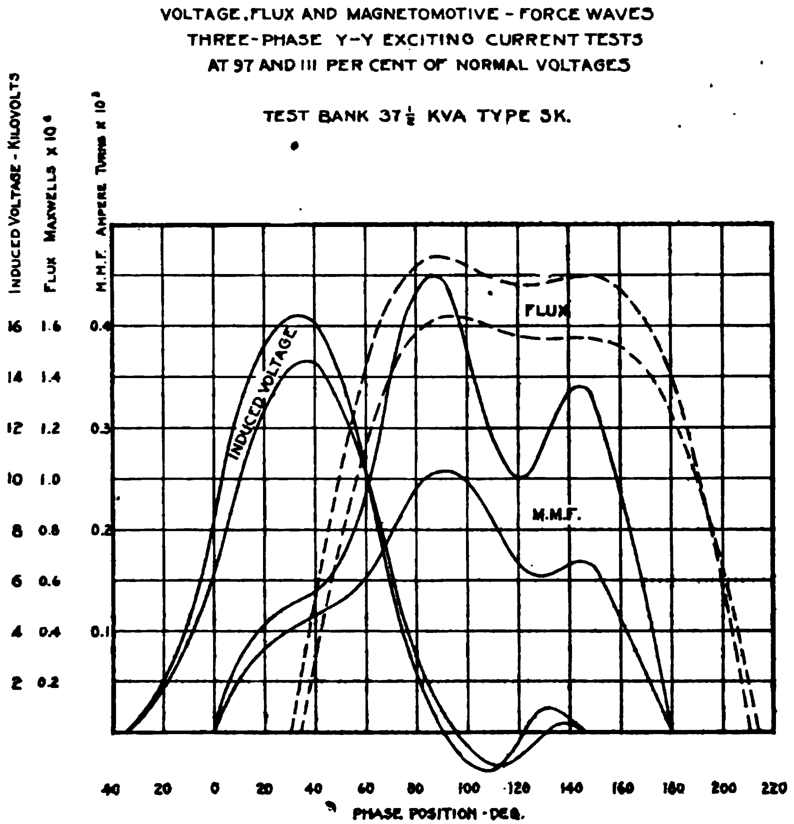

DELTA RESIDUAL CURRENT
NEUTRAL CURRENT AND VOLTAGE
DELTA-Y TEST WITH LINE CONNECTED
O BOTH NEUTRALS GROUNDED
X SOMIS NEUTRAL ISOLATED
+ SAN FERNANDO NEUTRAL ISOLATED.
THIRD HARMONIC, EXCEPT AS OTHERWISE SPECIFIED
REFER TO DIAGRAM OF CONNECTIONS. P.I.C. NO 318.
P.I.C. Nº 333
2-24-16
DELTA RESIDUAL CURRENT, D_R AMPERES
NEUTRAL CURRENT, I_n AMPERES
NEUTRAL VOLTAGE-E_n-VOLTS
PERCENT OF NORMAL FUNDAMENTAL INDUCED VOLTAGE.
D_R - FUNDAMENTAL
D_R - FIFTH
T.R. Nº 59


P.I.C. No 334
NEUTRAL CURRENT AND VOLTAGE
Y-Y TEST WITH LINE CONNECTED.
O BOTH NEUTRALS GROUNDED.
X SOMIS NEUTRAL ISOLATED
+ SAN FERNANDO NEUTRAL ISOLATED.
NEUTRAL CURRENT
VOLTAGE.
NEUTRAL VOLTAGE - E_n - KILOVOLTS
NEUTRAL CURRENT - I_n - AMPERES
E_n - 3RD HARMONIC
E_n - 9TH HARMONIC
I_n - 3RD HARMONIC
I_n - 9TH HARMONIC
T.R. No 59
PERCENT OF NORMAL FUNDAMENTAL INDUCED VOLTAGE


MAGNETIZING CURRENT
FROM MAGNETIZATION CURVE
TYPE SK 37½ KVA TRANSFORMER
—— COMPUTED
- - - MEASURED
PIC No. 343
8-16-16.
FIFTH HARMONIC
THIRD HARMONIC
FUNDAMENTAL
EXCITING CURRENT AMPERES
PERCENT OF NORMAL FUNDAMENTAL INDUCED VOLTAGE
MAGNETIZING CURRENT WAVE SHAPES
FROM MAGNETIZATION CURVE
TYPE SK 37½ KVA TRANSFORMERS
φ = ASSUMED SIMPLE HARMONIC FLUX
X - PER CENT OF NORMAL FUNDAMENTAL INDUCED VOLTAGE
PERCENT OF MAXIMUM
X DEGREES
TR No. 59


P.I.C. No 356
4-28-16.
ENERGY LOOPS
SINGLE-PHASE AND THREE-PHASE
EXCITING CURRENT TESTS
SINGLE-PHASE TESTS
THREE-PHASE Y-Y TESTS
TEST BANK 37½ KVA TYPE SK.
108%
111%
97%
82%
54%
FLUX - MAXVELLS x 10⁶
MAGNETOMOTIVE FORCE - AMP. TURNS x 10³
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0.2
0.4
0.6
0.8
1.0
1.2
T.R. No. 59

P.I.C. No 357
4-28-16.



T.R. No. 59

Technical Report No. 60.

July 26, 1916.

TRIPLE-HARMONIC RESIDUALS AS AFFECTED BY CERTAIN TYPES OF THREE-PHASE STAR CONNECTION OF TRANSFORMERS.

March 5-11, 1915.

OUTLINE.

I. INTRODUCTION.

II. DESCRIPTION OF TRANSFORMERS AND TRANSMISSION LINE.
- A—Transformers
- B—Transmission Line.

III. ARRANGEMENTS FOR TESTS.

IV. RESULTS OF TESTS.
- A—Comparison of Star Transformer Connections.
  1. Description.
  2. Tables.
  3. Discussion.
- B—Effect of an Auxiliary Bank in Reducing Residuals Due to a Star-Star Connected Bank.
  1. Description.
  2. Tables.
  3. Discussion.

V. DISCUSSION AND CONCLUSIONS.

I. Introduction.

The tests recorded in this report were made to study the properties of several different types of star connection of transformers for three-phase service, from the standpoint of the triple-harmonic residual voltages and currents introduced in connected circuits by grounding the neutrals of such banks of transformers. The results of these tests should be considered along with those of the allied tests described in of the experimental investigation contemplated under section 2 of the technical reports Nos. 50, 59 and 61, all dealing with slightly different aspects of the subject of triple-harmonic residuals and constituting part outline of future work in the Joint Committee's report of July 7, 1914.

Rule II-g (2) of General Order No. 39 prohibits the use in a power circuit involved in a parallel, of star-connected transformers or auto-transformers with grounded neutral "unless delta-connected secondary or tertiary windings or other equivalent means are used of suppressing the third-harmonic components of residual voltages and currents introduced by the transformers." In the tests of this report the effective-

ness of various means of suppressing third-harmonic residuals is compared with that of delta-connected secondary windings. These tests are divided into two parts; the first being a comparison with respect to third-harmonic residuals of various connections of a transformer bank, all with line-side in star and neutral grounded; and the second, a study of the efficacy of various connections of a second transformer bank, in parallel on the line side with a star-star connected bank, in reducing the third-harmonic residuals due to the latter.

In order that the impedance offered by the transmission line to residual current should be either capacitive or inductive as desired, a bank of transformers in interconnected star-delta was provided at the distant end of the line with a switch for isolating or grounding the neutral.

The simplicity of the system involved and the ability to control conditions facilitated comparisons of different connections which would have been otherwise more difficult or impossible.

II. Description of Transformers and Transmission Line.

A—Transformers.

There were used in the tests four banks of three transformers each, the designations and ratings of which are given in the following table:

TABLE I.

Designation of bank	Manufacturer	Type	Rating		
			Frequency cycles per second	kVA.	Voltage, volts
Supply	Westinghouse	SK	50	87½	15000/7125—440/110
Test	Westinghouse	SK	50	37½	15000/7125—440/110
Auxiliary	General Electric	H	60	50	22000/19800—2400/480
Somis	Westinghouse	SK	50	15	15000/7125—440/110

Impedances to third-harmonic current of the type H 50-kVA. transformer bank connected in various ways, as indicated, are given in the table below. The measurements were made with the impedance bridge on the 22000-volt side. The full 22000-volt and 2400-volt windings were used in each case.

TABLE II.

Impedance of Auxiliary Test Bank—50-kVA. Units—To Third-Harmonic* Residual

	R, ohms	X, ohms	Z, ohms
Star-Delta	48.3	337	340
Interconnected Star-Delta	980	1770	1960
Interconnected Star-Star	850	1760	1950

*150 cycles per second.

34—30969

Additional data concerning the 50-kVA. transformers are given in technical report No. 50 and concerning the 37½-kVA. transformers, in technical report No. 59.

B—TRANSMISSION LINE.

The transmission line involved in these tests is the San Fernando-Somis, 37 mile single-circuit, three-phase 15-kV. line of the Pacific Light and Power Corporation. It has been described at length in previous technical reports relating to the San Fernando tests, particularly in technical report No. 54. At the time of the tests recorded in this report there were five transpositions, forming two barrels, in the line.

The impedances of the transmission line to residual current are given on drawings Nos. 263 and 270 for each condition of the neutral of the transformer bank at the distant end. These drawings are attached to technical report No. 59.

III. Arrangements for Tests.

The connections for the tests were as shown on drawing No. 219 which is attached to this report. Energy was supplied from the Pacific Light and Power Corporation's 15-kV. system through the "supply bank" which was at all times connected delta-star, 15000-380 volts. The "test bank" (37½-kVA. units) was connected star-star and energized on its low-tension side. The high-tension side of this bank was connected to the transmission line and also to the high-tension side of the "auxiliary test bank." (50-kVA. units.)

Potential and current transformers necessary for measuring the line and neutral currents and voltages were provided at San Fernando as shown on drawing No. 219. A bank of transformers in interconnected star-delta was connected to the line at the distant end, Somis, a switch being provided for grounding or isolating the neutral.

IV. Results of Tests.

A—COMPARISON OF STAR TRANSFORMER CONNECTIONS.

This series of tests constitutes a comparison of various star connections of a transformer bank with respect to the introduction of third-harmonic residual voltages and currents into the connected transmission line.

The observations were made on the auxiliary test bank (50-kVA. units) energized from its high-tension side and with the low-tension side supplying no load. This bank of transformers and the line were supplied through the bank of 37½-kVA. transformers (test bank), the neutral of which was isolated throughout this series of tests. The high-

tension side of the auxiliary test bank was connected in star or interconnected-star with the neutral grounded; the low-tension side was connected in delta and the equivalent of the star connection was secured by opening the delta.

Oscillograms and meter readings of neutral current (residual current of line) and induced voltage, and when the low-tension delta was closed, of delta circulating current, were made under each condition. When the interconnected star was used, the voltage ratio of the 37½-kVA. transformers supplying the line was changed so as to impress the same voltage per coil on the 50-kVA. bank as in the star connection.

The results are summarized in the following tables. The condition number at the top of each column serves as a brief identification of the condition for which the observations apply and is used in discussing and comparing the results obtained under the various conditions. These numbers are the same as those used in the outline of the tests submitted to the Chairman of the Subcommittee on Tests in a memorandum under date of January 19, 1915. The following abbreviations are used in the tables to designate the various transformer connections.

Y-Y—Star-Star.
Y-D—Star-Delta.
IY-Y—Interconnected Star-Star.
IY-D—Interconnected Star-Delta.

TABLE III.

Residual Current—15,000-Volt Line. (Neutral Current of 50-kVA. Bank.)

Condition No.	3		6		10		11	
37½ kVA. Connection	Y-Y		Y-Y		Y-Y		Y-Y	
37½ kVA. Neutral	Isolated		Isolated		Isolated		Isolated	
37½ kVA. Ratio	220-14250		220-14250		220-12340		220-12340	
50 kVA. Connection	Y-D		Y-Y		IY-D		IY-Y	
50 kVA. Neutral	Grounded		Grounded		Grounded		Grounded	
50 kVA. Ratio	22000-2400		22000-2400		19000-2400		19000-2400	
Somis Neutral	Grd.	Isol.	Grd.	Isol.	Grd.	Isol.	Grd.	Isol.
Residual current, amperes — Oscillograph 1	0.032	0.002	0.005	0.005	0.0044	0.0047	0.0024	0.0022
Residual current, amperes — Oscillograph 3	0.011	0.016	0.058	0.059	0.0084	0.0075	0.0045	0.0084
Residual current, amperes — Oscillograph 5	0.002	0.002	0.001	0.002	0.0005	—	—	—
Residual current, amperes — Oscillograph 9	0.005	0.006	0.001	0.001	0.0008	—	0.0017	—
Residual current, amperes — Oscillograph E. S. W.	0.035	0.017	0.058	0.059	0.0057	0.0089	0.0055	0.0088
Residual current, amperes — Meter	0.030	0.019	0.055	0.059	0.0010*	0.0005*	0.0008*	0.0019*
Induced voltage—kV.	16.1	16.1	16.1	16.1	13.5	13.6	13.7	13.7
Osc. No.	870,3	871,2	874	875	908	909	910	911

Grd. = Grounded, Isol. = Isolated.
*Reading unreliable due to high impedance of milliammeter.

TABLE IV.

Delta Circulating Current—50-kVA. Bank.

Condition No.		1		3		10	
37½ kVA.	Connection	Y-Y		Y-Y		Y-Y	
	Neutral	Isolated		Isolated		Isolated	
	Ratio	220-14250		220-14250		220-12340	
50 kVA.	Connection	Y-D		Y-D		IY-D	
	Neutral	Isolated		Grounded		Grounded	
	Ratio	22000-2400		22000-2400		19000-2400	
Somis Neutral		Grd.	Isol.	Grd.	Isol.	Grd.	Isol.
Delta current, amperes — Oscillograph	1	0.01	0.01	0.10	0.01	0.01	0.01
	3	0.17	0.18	0.13	0.16	0.16	0.16
	5	—	0.01	0.01	—	0.01	—
	9	—	—	0.01	0.02	—	—
	E. S. W.	0.17	0.18	0.17	0.16	0.16	0.16
Delta current, amperes	Meter	0.17	0.17	0.18	0.14	0.12	0.14
Induced voltage—kV.		16.1	16.2	16.1	16.1	13.5	13.6
Oscillogram Nos.		884	885	870,3	871,2	908	909

Grd. = Grounded, Isol. = Isolated.

The residual current corresponding to each connection of the 50-kVA. auxiliary test bank is plotted on drawing No. 317, attached. The maximum residual current is obtained with the star-star connection (condition 6), and the third-harmonic residual current is approximately 2.6% of full load current of the bank. The residual current is nearly the same whether or not the neutral is grounded at Somis, it being slightly greater when isolated. The impedance* of the transformer bank is inductive and large compared to the impedance of the line whether or not the neutral is grounded at Somis. The impedance of the transmission line is capacitive when the neutral is isolated at Somis, and inductive when the neutral is grounded, hence the resultant impedance to third-harmonic residual current is less when the neutral at Somis is

*In discussing the reaction of the harmonic currents, in the transformer bank in which they arise, it is sometimes convenient to represent the transformer by an equivalent H network. The series branches of this network are the primary and secondary leakage impedances and the bridge is the mutual impedance. To compute the behavior of any harmonic component, it may be assumed to arise from a fictitious generator in series with the bridge impedance. The voltage E of this fictitious generator is the component of this frequency in the terminal voltage of the transformer when the corresponding component in the magnetizing current is totally suppressed. The mutual impedance in series with this generator is the ratio of the voltage E to current I of the same frequency which would exist in the magnetizing current if the external impedance were negligible.

In the subsequent discussions in this report this method of treatment is followed. (See discussion by H. S. Osborne on "Harmonics in Transformer Magnetizing Currents," Trans. A. I. E. E., vol. 34, 1915, page 2175; and T. R. No. 59.)

isolated than when it is grounded. Connecting the secondary in delta (condition 3) reduces the third-harmonic residual current 74% when the neutral is isolated at Somis and 80% when the neutral is grounded. A still greater reduction is produced by using the interconnected star-delta (condition 10) or the interconnected star-star (condition 11) connections.

When the Somis neutral is isolated the residual voltage is proportional to the residual current and line impedance. With the Somis neutral grounded the third-harmonic voltage due to the Somis bank is of the same order of magnitude as that due to the transformer bank at San Fernando and the residual voltage is not, therefore, proportional to the residual current. The phase relationship between the two voltages was not determined, hence the residual voltage at San Fernando is indeterminate.

The voltage induced in the windings is approximately 12 per cent less than normal voltage (50-cycle rating) for the transformers, hence the triple harmonics are less than if the induced voltage were normal. The ninth-harmonic current is too small at this density to determine the effect thereon of the various types of connection.

B—Effect of Auxiliary Bank in Reducing Residuals Due to Star-Star Bank.

This series of tests was made to determine the relative effectiveness of various connections of an auxiliary transformer bank, for reducing the third-harmonic residuals due to a star-star transformer bank connected in parallel therewith on the line side. The following connections of the auxiliary bank were used: Star-delta, interconnected star-delta, and interconnected star-star. With the auxiliary bank in star-delta the effect of varying the delta impedance by introducing resistance in the external delta circuit was determined. Drawing No. 219 shows the diagram of connections for these tests.

Oscillograms and meter readings were made, under each condition, of the currents in the neutral of each bank and in the common neutral, of induced voltage and of delta circulating current of the auxiliary bank. With the auxiliary bank in star-delta and the neutrals of both banks isolated, the neutral voltages were measured; also the current in and voltage to ground of the common neutral when the two neutrals were interconnected without being grounded. The results of the measurements are given in the following tables:

TABLE V.

Residual Current—15-kV. Line (Current in Common Neutral Connection to Ground).

		Condition No.	2		4		7a		7b		8		9	
37½ kVA.		Connection	Y-Y		Y-Y		Y-Y		Y-Y		Y-Y		Y-Y	
		Neutral	Grounded		Grounded		Grounded		Grounded		Grounded		Grounded	
		Ratio	220-14250		220-14250		220-14250		220-14250		220-14250		220-14250	
50 kVA.		Connection	Y-D		Y-D		Y-D		Y-D		IY-D		IY-Y	
		Neutral	Isolated		Grounded		Grounded		Grounded		Grounded		Grounded	
		R_d	0		0		10.0 ohms		46.0 ohms		0			
		Ratio	22000-2400		22000-2400		22000-2400		22000-2400		19000-2400		19000-2400	
Somis Neutral			Grd.	Isol.	Grd.	Isol.	Grd.	Isol.	Grd.	Isol.	Grd.	Isol.	Grd.	Isol.
Residual current, amperes	Oscillograph	1	0.03	0.79	0.03	—	0.05	—	0.03	0.01	0.03	0.02	0.03	0.02
		3	0.38	0.90	0.26	0.14	0.32	0.17	0.27	0.20	0.34	0.63	0.32	0.53
		5	0.01	0.10	0.02	0.06	0.03	0.06	0.02	0.05	0.02	0.04	0.02	0.02
		9	0.03	—	0.02	0.05	0.03	0.05	0.01	0.04	0.03	0.05	0.03	0.05
		E. S. W.	0.38	1.22	0.27	0.16	0.33	0.19	0.28	0.21	0.35	0.64	0.33	0.54
	Meter		0.34	1.48	0.26	0.17	0.27	0.15	0.28	0.22	0.40	0.64	0.33	0.56
Induced voltage—kV.			15.9	15.9	16.0	16.0	15.9	15.8	15.9	16.0	15.8	15.7	15.8	15.6
Oscillogram No.			914	916,18	876,7	878,9	887,90	888,9	894,5	896,7	900,1	902,3	904,5B	906,7

Isol. = Isolated, Grd. = Grounded, R_d = External resistance in corner of delta.

TABLE VI.

Neutral Current—37½-kVA. Bank.

Condition No.	3		4		5		7a		7b		8		9	
37½ kVA. Connection	Y-Y		Y-Y		Y-Y		Y-Y		Y-Y		Y-Y		Y-Y	
Neutral	Grounded		Grounded		Isolated*		Grounded		Grounded		Grounded		Grounded	
Ratio	220-14250		220-14250		220-14250		220-14250		220-14250		220-14250		220-14250	
50 kVA. Connection	Y-D		Y-D		Y-D		Y-D		Y-D		IY-D		IY-D	
Neutral	Isolated		Grounded		Isolated*		Grounded		Grounded		Grounded		Grounded	
R_d	0		0		0		10.0 ohms		46.0 ohms		0			
Ratio	22000-2400		22000-2400		22000-2400		22000-2400		22000-2400		19000-2400		19000-2400	
Somis Neutral	Grd.	Isol.	Grd.	Isol.	Grd.	Isol.	Grd.	Isol.	Grd.	Isol.	Grd.	Isol.	Grd.	Isol.
Neutral current, amperes: Oscillograph 1	0.03	0.79	0.03	0.04	0.07	0.04	0.04	0.04	0.03	0.03	0.03	0.02	0.04	0.02
3	0.37	0.90	0.42	0.38	0.40	0.39	0.40	0.43	0.30	0.34	0.37	0.42	0.37	0.40
5	0.01	0.10	0.02	0.03	0.03	0.03	0.03	0.02	0.03	0.03	0.02	0.03	0.02	0.02
9	0.03	—	0.03	0.03	0.02	0.03	0.03	0.02	0.02	0.03	0.03	0.04	0.03	0.04
E. S. W.	0.38	1.22	0.43	0.38	0.40	0.39	0.40	0.44	0.30	0.34	0.37	0.42	0.38	0.40
Meter	0.34	1.48	0.39	0.38	0.38	0.37	0.41	0.35	0.33	0.36	0.42	0.42	0.36	0.32
Induced voltage—kV.	15.9	15.9	16.0	16.0	16.1	—	15.9	15.8	15.9	16.0	15.8	15.7	15.8	15.6
Oscillogram No.	914	916,8	876	879	882	883	887	888	894	897	900	903	904	907

Isol. = Isolated, Grd. = Grounded, R_d = External resistance in corner of delta.
*Neutrals of two banks connected together.

TABLE VII.

Neutral Current—50-kVA. Bank.

		Condition No.	4		5		7a		7b		8		9	
37½ kVA.		Connection	Y-Y		Y-Y		Y-Y		Y-Y		Y-Y		Y-Y	
		Neutral	Grounded		Isolated*		Grounded		Grounded		Grounded		Grounded	
		Ratio	220-14250		220-14250		220-14250		220-14250		220-14250		220-14250	
50 kVA.		Connection	Y-D		Y-D		Y-D		Y-D		IY-D		IY-Y	
		Neutral	Grounded		Isolated*		Grounded		Grounded		Grounded		Grounded	
		R_d	0		0		10.0 ohms		46.00 ohms		0			
		Ratio	22000-2400		22000-2400		22000-2400		22000-2400		19000-2400		19000-2400	
Somis Neutral			Grd.	Isol.	Grd.	Isol.	Grd.	Isol.	Grd.	Isol.	Grd.	Isol.	Grd.	Isol.
Neutral current, amperes	Oscillograph	1	0.03	0.05	0.07	0.04	0.04	0.06	0.01	0.02	0.004	0.04	0.002	0.03
		3	0.14	0.56	0.40	0.39	0.15	0.46	0.06	0.39	0.023	0.27	0.024	0.22
		5	0.01	0.09	0.03	0.03	0.01	0.06	0.01	0.04	0.001	0.01	0.002	0.01
		9	0.01	0.02	0.02	0.03	0.02	0.02	0.01	0.02	0.004	0.01	0.004	0.01
		E. S. W.	0.14	0.57	0.40	0.39	0.15	0.47	0.06	0.39	0.024	0.27	0.025	0.22
	Meter		0.13	0.51	0.38	0.37	0.13	0.47	0.07	0.44	†0.008	0.27	†0.007	0.25
Induced voltage—kV.			16.0	16.0	16.1	—	15.9	15.8	15.9	16.0	15.8	15.7	15.8	15.6
Oscillogram No.			876	879	882	883	887	888	894	897	900	903	904	906

Grd. = Grounded, Isol. = Isolated, R_d = External resistance in corner of delta.
*Neutrals of two banks connected together. †Reading unreliable account high impedance of milliammeter

TABLE VIII.

Delta Circulating Current—50-kVA. Bank.

		Condition No.	2		4		5		7a		7b		8	
37½ kVA.		Connection	Y-Y		Y-Y		Y-Y		Y-Y		Y-Y		Y-Y	
		Neutral	Grounded		Grounded		Isolated*		Grounded		Grounded		Grounded	
		Ratio	220-14250		220-14250		220-14250		220-14250		220-14250		220-14250	
50 kVA.		Connection	Y-D		Y-D		Y-D		Y-D		Y-D		IY-D	
		Neutral	Isolated		Grounded		Isolated*		Grounded		Grounded		Grounded	
		R_4	0		0		0		10.0 ohms		46.0 ohms		0	
		Ratio	22000-2400		22000-2400		22000-2400		22000-2400		22000-2400		19000-2400	
Somis Neutral			Grd.	Isol.	Grd.	Isol.	Grd.	Isol.	Grd.	Isol.	Grd.	Isol.	Grd.	Isol.
Delta current, amperes	Oscillograph	1	0.01	0.02	0.16	0.11	0.16	0.15	0.10	0.09	0.05	0.08	0.02	0.02
		3	0.17	0.18	0.65	1.85	1.26	1.31	0.57	1.69	0.33	1.31	0.31	0.31
		5	—	—	0.04	0.23	0.10	0.08	0.04	0.20	0.03	0.14	0.01	0.01
		9	—	0.01	0.05	0.05	0.07	0.07	0.05	0.04	0.04	0.04	0.01	0.01
		E. S. W.	0.17	0.18	0.67	1.87	1.28	1.33	0.58	1.72	0.34	1.32	0.31	0.31
	Meter		0.15	0.16	0.61	1.87	1.4	1.4	0.58	1.7	0.37	1.55	0.32	0.29
Induced voltage—kV.			15.9	15.9	16.0	16.0	16.1	—	15.9	15.8	15.9	16.0	15.8	15.7
Oscillogram No.			914	916,7	877	878	882	883	890	889	895	896	901	902

Grd. = Grounded, Isol. = Isolated, R_4 = External resistance in corner of delta
*Neutrals of two banks connected together.

TABLE IX.

Neutral Voltages.

			37½-kVA. bank		50-kVA. bank		Common neutral	
		No.	1		1		5	
Conditions	37½ kVA.	Connection	Y-Y		Y-Y		Y-Y	
		Neutral	Isolated		Isolated		Isolated*	
		Ratio	220-14250		220-14250		220-14250	
	50 kVA.	Connection	Y-D		Y-D		Y-D	
		Neutral	Isolated		Isolated		Isolated*	
		R_d	0		0		0	
		Ratio	22000-2400		22000-2400		22000-2400	
		Somis Neutral	Grd.	Isol.	Grd.	Isol.	Grd.	Isol.
Neutral voltage, volts	Oscillograph	1	—	300	6.5	11	7	7
		3	9500	9300	3.5	22	129	144
		5	200	500	5.0	—	17	16
		9	1900	2300	10.5	6	24	27
		15	600	1000	7.9	17	4	4
		E. S. W.	9700	9700	17.2	30	133	148
		Meter	9700	9740	18	26	131	148
Induced voltage—kV.			16.1	16.2	16.1	16.2	16.1	—
Oscillogram Nos.			884	885	884	885	882	883

Grd. = Grounded, Isol. = Isolated, R_d = External resistance in corner of delta.
* Neutrals connected together.

During this entire series of tests the 37½-kVA. test bank was operated at approximately 12 per cent above normal voltage and the 50-kVA. auxiliary bank at approximately 12 per cent below normal voltage (50 cycle basis). Hence, the third harmonic introduced by the auxiliary bank is small compared to that introduced by the test bank. This is further borne out by a comparison of the neutral voltages to ground of the neutral of the auxiliary bank connected delta-star and of the common neutral when the neutrals of the two banks are interconnected (see Table IX).

On drawing No. 332 the three neutral currents, the delta circulating current and the impedance of the auxiliary bank for the third harmonic, are plotted for each connection of the auxiliary bank; also, for comparison, the neutral current of the 37½-kVA. bank when the neutral of the auxiliary bank was isolated. On drawing No. 318 the third-harmonic residual currents, and voltages as obtained from the residual current and line impedance, are plotted for each connection of the auxiliary bank and also for the condition when the 37½-kVA. bank is

connected in delta-star without the auxiliary bank. The data for the latter are obtained from technical report No. 59. The effectiveness of the auxiliary bank in reducing residual voltages and currents due to the star-star bank is thus compared to that of changing the low-tension connections to delta.

The third-harmonic residual current observed when the neutral of the main star-star bank is grounded, the neutrals of the auxiliary bank and of the Somis bank being isolated, does not agree with the value to be expected from a consideration of the line and transformer impedances. The impedance of the transmission line, whether the neutral at the distant end is isolated or grounded, is small compared to the impedance of the test bank, and hence the third-harmonic residual current should be little affected by the condition of the Somis neutral. On the contrary, the third-harmonic neutral current measured with the Somis neutral isolated, is nearly three times the value measured with the Somis neutral grounded. This greatly increased third-harmonic residual current is due to an unbalanced condition of the fundamental induced voltages in the three transformers. There are four possible conditions of adjustment of the line voltages and currents with this connection, one in which the voltages and currents of fundamental frequency are balanced, and three others in which any two of the line voltages and currents are high, the third being low. With the unbalanced conditions a large residual voltage of fundamental frequency exists opposite in phase to the lowest line voltage. The residual current also contains a large component of fundamental frequency. By oscillogram No. 919, the voltages to ground of the three transformers were 13.8, 17.9 and 17.9 kV. From the exciting current curves (see drawing No. 329, T. R. No. 59), assuming that the third-harmonic currents of the three transformers of the bank are in phase, the third-harmonic residual current under this condition should be approximately 0.88 amperes. An average value of 0.90 amperes was observed, which is that given on drawing No. 318.

With the neutral isolated at Somis, one oscillogram (No. 917) was obtained in which the voltages of the three transformers are evidently balanced. The third-harmonic neutral current shown by this oscillogram is 0.34 amperes and is quite closely equal to the values obtained when the Somis neutral is grounded.

Comparing the residual current due to the combination of the 37½-kVA. bank connected star-star and the auxiliary bank connected star-delta, with the residual current due to the 37½-kVA. bank alone when connected delta-star at the same voltages per coil as in the former case, the relative magnitudes are 10 and 1 when the Somis neutral is grounded, and 9 and 1 when it is isolated. Since the impedance of the star-delta auxiliary bank to third-harmonic current is much greater

than the impedance of the low-tension delta of the test bank, it is to be expected that the latter should be the more effective in minimizing the third-harmonic residual current. These impedances are $48 + j\ 337$ ohms and $14 + j\ 55$ ohms, respectively (in terms of the line-side windings and assuming that the total leakage impedance of the test bank is equally divided between the two sides).

For all methods of connection the impedance of the auxiliary bank is greater than that of the line with neutral grounded at Somis, and hence the third-harmonic residual current is greater than the current in the neutral of the auxiliary bank. With the auxiliary bank in interconnected star-delta or interconnected star-star, its impedance is greater than that of the transmission line with the Somis neutral isolated; hence, the third-harmonic residual current is greater than the current in the neutral of the auxiliary bank. With this bank in star-delta, however, its impedance is less than that of the line with the Somis neutral isolated, and the third-harmonic residual current is less than the current in the neutral of this bank.

The interconnected star-delta and interconnected star-star arrangements of the auxiliary bank are about equally effective in reducing the residual current introduced by the star-star bank. Neither is as effective as the star-delta connection of the auxiliary bank.

The effect of external impedance in the delta circuit of the auxiliary bank (conditions 7a and 7b) is to increase the impedance of the bank to third-harmonic current introduced by the star-star bank. The delta and neutral currents of the auxiliary bank are decreased and the residual current of the line increased as compared to the values when the delta is closed through a negligible external resistance.

Condition 5 (neutrals of star-star bank and star-delta auxiliary bank interconnected and isolated) shows the interaction of the two banks with respect to third-harmonic residuals without modification by the line reactions. Compared with condition 4, which differs only by the grounding of the common neutral, the effect of such grounding upon the neutral current of the star-star bank is small, decreasing it when the Somis neutral is isolated and increasing it when the Somis neutral is grounded. Grounding the common neutral at San Fernando alters the total impedance to third-harmonic current from the star-star bank but slightly, irrespective of the condition of the Somis neutral; hence the neutral current of this bank is only slightly changed. When the Somis neutral is isolated the neutral current of the auxiliary bank is greater with the common neutral at San Fernando grounded than with it isolated. The line impedance is capacitive and when paralleled with the inductive impedance of the auxiliary bank the resultant impedance is increased. Since the resultant current (neutral current of 37½-kVA. bank) is but slightly altered, the neutral current of the auxiliary bank

is increased. When the Somis neutral is grounded the opposite effect is noted since the line impedance is inductive.

In Table IX are given values by meter and oscillograph of voltages from neutrals to ground of the star-star bank (37½ kVA.) and star-delta auxiliary bank (50 kVA.), and also, the voltage to ground of the common neutral when the neutrals of the two banks are interconnected. From the two readings of third-harmonic voltage from the auxiliary bank neutral to ground, with the Somis neutral isolated and with it grounded, the third-harmonic voltage due to the Somis transformer bank is approximately 20 volts and in opposition to the third-harmonic voltages of the transformer banks at San Fernando.

V. Discussion and Conclusions.

Of the four methods tested for the connection of a bank of transformers with neutral grounded on the line side (see drawing No. 317) the interconnected star-delta arrangement gives rise to the least amount of third-harmonic residual current. The results given in technical report No. 50 were inconsistent in this regard and further work was suggested in order to make the comparisons definite. The results of the tests reported herein are conclusive and they are in accord with theoretical considerations. The delta connection on the station sides provides a path for the third-harmonic current and hence the corresponding harmonic is nearly eliminated from the flux and induced-voltage waves. The third-harmonic voltage which remains is sufficient only to drive the third harmonic current through the impedance of the delta circuit. When the line-side windings are interconnected the small amounts of third-harmonic voltage induced in the halves of the windings between neutral and line are put in opposition and the resultant third-harmonic voltage from neutral to line is very small. With equal impressed voltages 120° apart in phase, if the three transformers be identical with respect to magnetic characteristics and the linkages of flux of the halves of the windings with the core be equal, no third-harmonic voltage can exist between lines and neutral. Since there are small inequalities among the several transformers, a small amount of third-harmonic voltage remains. Although this arrangement of transformers is advantageous from the standpoint of triple-harmonic residuals, it is open to objection from an operating standpoint since it reduces the rating 13% and complicates the station wiring.

As shown on drawing No. 318, none of the connections of the auxiliary bank are as effective in eliminating the third-harmonic residuals due to the star-star bank, as changing the station-side connections to delta. With the most favorable connection of the auxiliary bank (star-delta) the third-harmonic residual current is about ten times as great as when the main test bank is connected delta-star and the auxiliary bank dispensed with. For the star-delta connection the impedance of the auxil-

iary bank is approximately six times as great as that of the delta circuit of the 37½-kVA. test bank (assuming that the total leakage impedance of the latter is equally divided between the two sides)

If the magnetic density of the auxiliary bank be sufficiently low that its third harmonics are small as compared to those due to the star-star bank with which it is in parallel, and there are no other transformer banks with grounded neutrals connected to the line, the division of the third-harmonic current between the line and auxiliary transformer bank is inversely proportional to the line and transformer impedances. If there is appreciable third harmonic due to the auxiliary bank or to other transformer banks with grounded neutrals this distribution of current will not hold.

In order that an auxiliary bank be as effective as a station-side delta on the main bank its impedance should be as low as that of the delta of the main bank and its third harmonics negligible. To fulfill this requirement transformers of low magnetic density and small leakage reactance are necessary. It is not probable that these conditions could be met by transformers of smaller kVA. rating than those of the main bank.

The bank of 50-kVA. transformers used as the auxiliary bank is most effective, in reducing third-harmonic residuals due to the star-star 37½-kVA. bank, when connected star-delta (see drawing No. 318). The interconnected star-star and interconnected star-delta arrangements of the auxiliary bank are much less effective. The construction of these transformers is such that magnetic leakage between the halves of the line-side windings is relatively large and hence the impedance between lines and neutral is larger for the interconnected-star than for the straight star-delta arrangement. In general, this is to be expected unless the transformers are designed so as to minimize this leakage.

Respectfully submitted.

(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

ATTACHMENTS: P. I. C. Drawings Nos. 219, 317, 318 and 332.
IN FILES OF JOINT COMMITTEE: Oscillograms Nos. 870, 871, 874, 875, 876, 877, 878, 879, 882, 884, 885, 887, 888, 889, 890, 894, 895, 896, 897, 900, 901, 902, 903, 904, 905b, 906, 907, 908, 909, 910, 911, 914, 915, 916, 918, 919.

APPROVED: August 29, 1916.

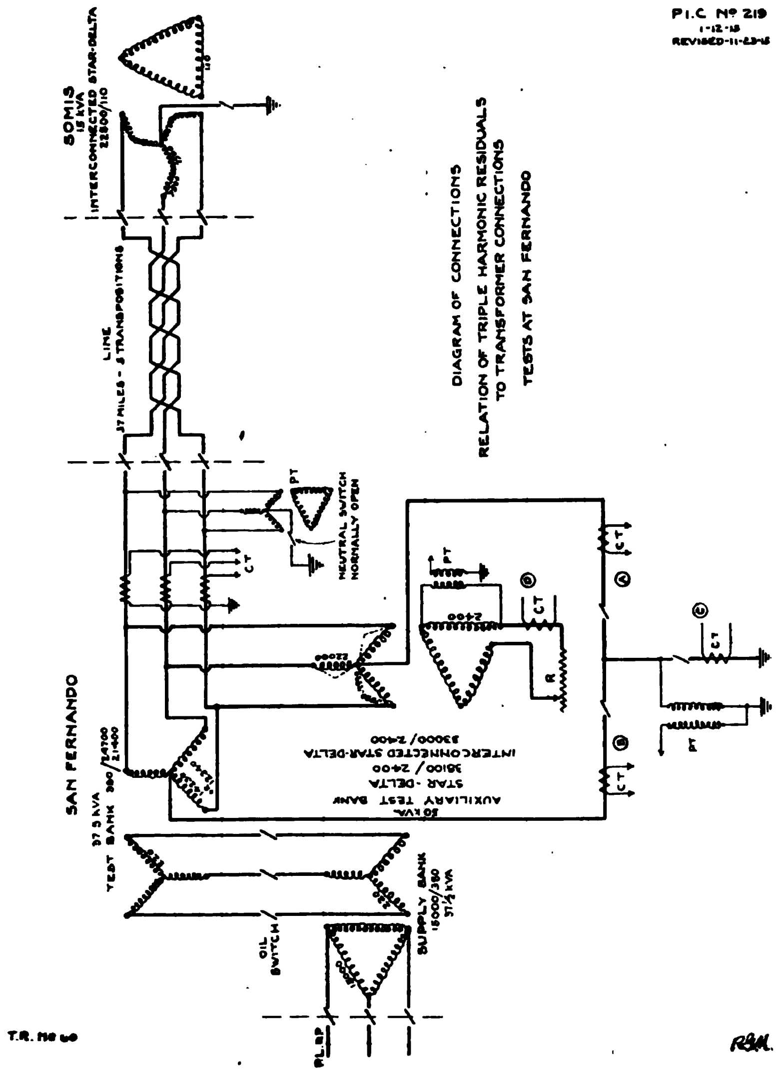
(Signed) R. W. MASTICK,
Field Engineer.

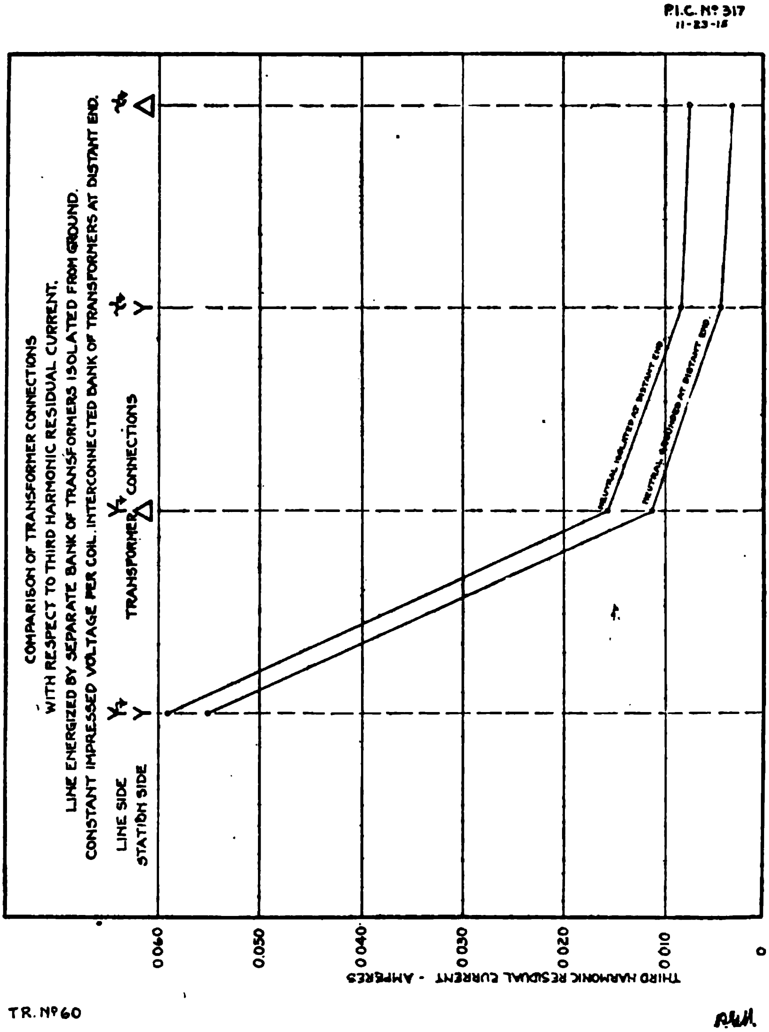
APPROVED: October 10, 1916.

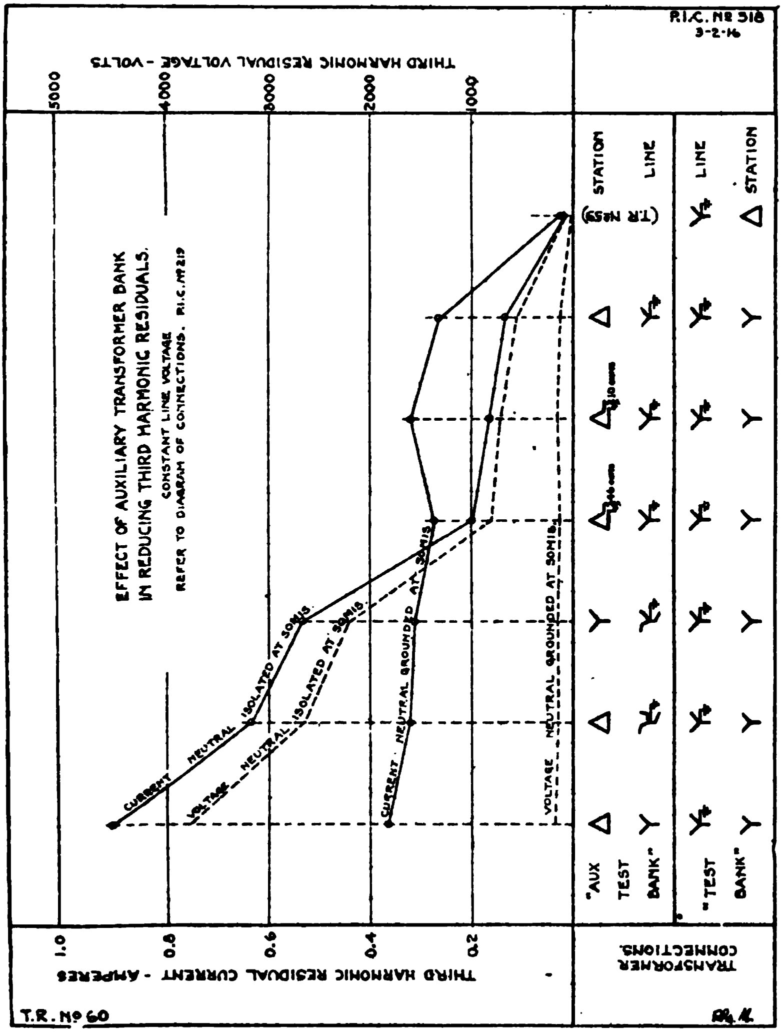
SUBCOMMITTEE ON TESTS,
(Signed) J. E. WOODBRIDGE,
Chairman.

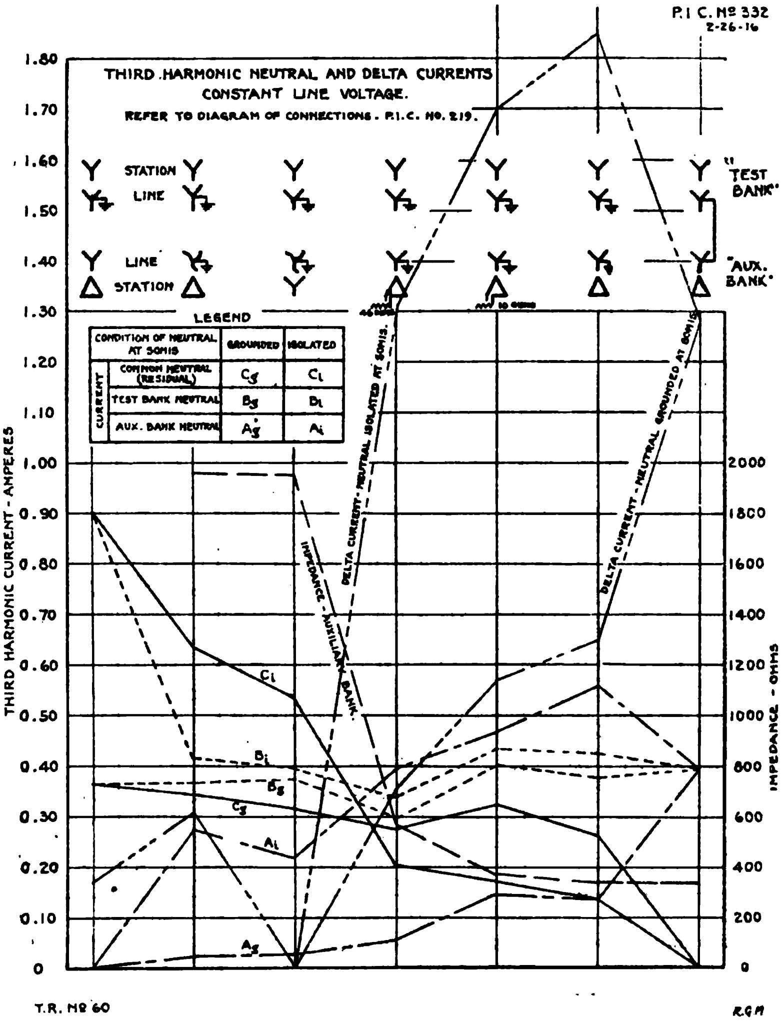
JOINT COMMITTEE ON INDUCTIVE INTERFERENCE,
(Signed) ARTHUR F. BRIDGE,
Secretary.

January 8, 1917.


P.I.C. No 219
1-12-15
REVISED-11-23-15
DIAGRAM OF CONNECTIONS
RELATION OF TRIPLE HARMONIC RESIDUALS
TO TRANSFORMER CONNECTIONS
TESTS AT SAN FERNANDO
SOMIS
15 KVA
INTERCONNECTED STAR-DELTA
22800/110
LINE
37 MILES - 3 TRANSPOSITIONS
SAN FERNANDO
37.5 KVA
TEST BANK 380/24700 21400
50 KVA
AUXILIARY TEST BANK
STAR-DELTA
35100/2400
INTERCONNECTED STAR-DELTA
23000/2400
NEUTRAL SWITCH
NORMALLY OPEN
PT
CT
R
OIL SWITCH
SUPPLY BANK
15000/380
37½ KVA
T.R. No 60
R.G.M.


P.I.C. No 317
11-23-15
COMPARISON OF TRANSFORMER CONNECTIONS
WITH RESPECT TO THIRD HARMONIC RESIDUAL CURRENT.
LINE ENERGIZED BY SEPARATE BANK OF TRANSFORMERS ISOLATED FROM GROUND.
CONSTANT IMPRESSED VOLTAGE PER COIL. INTERCONNECTED BANK OF TRANSFORMERS AT DISTANT END.
LINE SIDE
STATION SIDE
TRANSFORMER CONNECTIONS
NEUTRAL ISOLATED AT DISTANT END
NEUTRAL GROUNDED AT DISTANT END
0.060
0.050
0.040
0.030
0.020
0.010
0
THIRD HARMONIC RESIDUAL CURRENT - AMPERES
TR. No 60


R.I.C. No 318
3-2-16
THIRD HARMONIC RESIDUAL VOLTAGE - VOLTS
5000
4000
3000
2000
1000
EFFECT OF AUXILIARY TRANSFORMER BANK
IN REDUCING THIRD HARMONIC RESIDUALS.
CONSTANT LINE VOLTAGE
REFER TO DIAGRAM OF CONNECTIONS. R.I.C. No 219
CURRENT NEUTRAL ISOLATED AT SOMIS
VOLTAGE NEUTRAL ISOLATED AT SOMIS
CURRENT NEUTRAL GROUNDED AT SOMIS
VOLTAGE NEUTRAL GROUNDED AT SOMIS
STATION
LINE
(T.R. No 59)
"AUX TEST BANK"
"TEST BANK"
1.0
0.8
0.6
0.4
0.2
THIRD HARMONIC RESIDUAL CURRENT - AMPERES
TRANSFORMER CONNECTIONS.
T.R. No 60


P.I.C. Nº 332
2-26-16
THIRD HARMONIC NEUTRAL AND DELTA CURRENTS
CONSTANT LINE VOLTAGE.
REFER TO DIAGRAM OF CONNECTIONS. P.I.C. NO. 219.
STATION
LINE
LINE
STATION
"TEST BANK"
"AUX. BANK"
LEGEND
CONDITION OF NEUTRAL AT SOMIS
GROUNDED
ISOLATED
CURRENT
COMMON NEUTRAL (RESIDUAL)
TEST BANK NEUTRAL
AUX. BANK NEUTRAL
Cg
Ci
Bg
Bi
Ag
Ai
THIRD HARMONIC CURRENT - AMPERES
IMPEDANCE - OHMS
DELTA CURRENT - NEUTRAL ISOLATED AT SOMIS.
DELTA CURRENT - NEUTRAL GROUNDED AT SOMIS
IMPEDANCE - AUXILIARY BANK.
10 OHMS
1.80
1.70
1.60
1.50
1.40
1.30
1.20
1.10
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0
2000
1800
1600
1400
1200
1000
800
600
400
200
0
T.R. Nº 60
R.G.M.

Technical Report No. 61.

July 29, 1916.

MEASUREMENTS OF RESIDUAL VOLTAGES AND CURRENTS OF SEVERAL TRANSMISSION SYSTEMS.

December, 1912, to June, 1915.

OUTLINE.

I. Introduction.

This report presents the results obtained in an investigation of residual voltages and currents of representative power-transmission systems of California, under normal operating conditions. The investigation was outlined under "Future Work (II-A)" of the Joint Committee's report of July 7, 1914. Measurements have been made on both grounded and isolated systems but for none of the systems may the investigation be considered complete.

The transmission systems on which measurements were made are the 22-kV. network of the Coast Counties Gas & Electric Company (isolated from ground) and the following systems having grounded neutral connections: Sierra & San Francisco Power Company (104-, 55-, 33-, 22-kV.

lines), Pacific Gas & Electric Company (60-kV. lines) and the Pacific Light and Power Corporation (15-kV. lines). For two of the systems, Coast Counties Gas & Electric Company and Sierra and San Francisco Power Company, the results of previous tests are again utilized along with those which were made primarily for the purposes of this report. The results for each system are separately described and discussed below.

II. Coast Counties Gas & Electric Company.

A—Description of System.

The Coast Counties Gas & Electric Company's system is a 22-kV. network isolated from ground throughout. The construction varies, being partly flat horizontal and partly triangular. Some of the lines are transposed but the transpositions are neither so located nor sufficient in number to give capacitance balance of the system to ground. This system forms a loop and is connected to the 60-kV. system of the Pacific Gas & Electric Company at Davenport and San Jose through Y-delta transformer banks. The 22-kV. line of the Pacific Gas & Electric Company forms the connecting link between the Coast Counties Gas & Electric system at Morgan Hill and the 60-kV. system at San Jose.

Energy may be supplied from either end of the loop from the Pacific Gas & Electric Company's 60-kV. system, or from the Big Creek generating station, also from the Santa Cruz and Watsonville steam stations of the Coast Counties Gas & Electric Company. The generating apparatus in these steam stations is not used except in cases of emergency and sometimes during the evening peak at Santa Cruz. Under normal operating conditions a sectionalizing switch just north of Watsonville separates the system into two parts, the "north" end (Santa Cruz) of the system being fed from Davenport with the Big Creek station in parallel, and the "south" end (Gilroy) from San Jose.

B—Scope of Measurements.

Measurements were made of voltages to ground and residual voltages at Santa Cruz, Watsonville and Morgan Hill with several different methods of operation. In addition, the residual current was measured at Santa Cruz. Oscillograms were taken at Santa Cruz and Morgan Hill and resonant analyses were made at Watsonville.

C—Data.

Results of the measurements are given in the following tables:

TABLE I.

Measurements of Voltage to Ground and Residual Voltage—Volts. Coast Counties Gas & Electric Company 22000-volt Line—Watsonville and Santa Cruz.

August 28, 1914.

Condition of sectionalizing switch at Watsonville	n	Gilroy line		Santa Cruz line			
		At Watsonville		At Watsonville		At Santa Cruz	
		Voltage to ground	Residual voltage	Voltage to ground	Residual voltage	Voltage to ground	Residual voltage
Open	1	12500	520	12400	500		500
	3	14	13	270	9		18
	5	69	13	210	3		9
	7	75	3	9	3		*
	9	3	17	32	2		1
	11	69	12	81	7		1
	13	170	7	230	11		2
	E. S. W.	12500	520	12400	500		500
Closed	1	11900	150	11900	150		220
	3	52	11	52	11		16
	5	34	4	34	4		6
	7	48	2	48	2		3
	9	5	11	5	11		15
	11	40	13	40	13		4
	13	170	7	170	7		3
	E. S. W.	11900	150	11900	150		220

n = order of harmonic; used also in subsequent tables.

Operating Conditions:

With sectionalizing switch at Watsonville open, the Davenport and Big Creek stations were in parallel supplying north end, Watsonville load being carried on Santa Cruz line.

With sectionalizing switch at Watsonville closed, the Davenport and Big Creek and San Jose stations were in parallel supplying entire system.

TABLE II.

Measurements at Santa Cruz. Coast Counties Gas & Electric Company 22000-Volt System. February 11-12, 1914.

n	Line voltage to ground. volts	Residual voltage, volts	Residual current, amperes
1	13800	360	0.094
3	240	*	*
5	280	*	*
7	80	*	*
9	*	19	0.021
11	*	13	0.017
13	*	8	*
E. S. W.	13800	360	0.094

Operating Condition:

San Jose and Big Creek stations in parallel, Davenport off.

Average values from T. R. No. 45.

D—Discussion.

Residual voltages measured at Santa Cruz and at Watsonville are chiefly of fundamental frequency. The harmonics, while of appreciable magnitude, are small as compared to the fundamental, and in general corresponding harmonics in the line and residual voltages bear approximately the same ratio as the fundamental components. This indicates that the residuals are due to the unbalanced admittances of the several phases.

The third is the largest harmonic of the balanced voltages and arises probably from the comparatively large number of open-delta and Scott-connected transformer banks used on the system. With the sectionalizing switch open at Watsonville the third harmonic is the larger on the line toward Santa Cruz and when the switch is closed the third harmonic voltage is decreased in the Santa Cruz line and increased in the line toward San Jose, *i. e.,* assumes an intermediate value. In partial explanation of this it may be stated that the largest banks of Scott-connected transformers are at Santa Cruz and Big Creek. The other prominent harmonic is the thirteenth. At Watsonville the fundamental components of the residual voltages of the lines toward Santa Cruz and toward Gilroy were approximately the same. When the switch connecting together the two sections of line was closed, the residual voltage was decreased approximately 70 per cent. It is probable that the resultant admittance unbalance to ground is less than the unbalances of the two sections of line taken separately.

The measurements at Morgan Hill were made approximately 18 months previous to the others described in the section, and under somewhat different conditions; hence, it is not possible to compare them. See T. R. No. 20.

III. Sierra & San Francisco Power Company.

A—Description of System.

This system consists of two 104-kV. lines between the Stanislaus generating station and the Bay Shore substation, a distance of 145 miles, with intermediate substations at Manteca and Guadalupe, 55 and 110 miles, respectively, from Stanislaus. There is a bank of auto-transformers at Guadalupe through which a 75-mile 55-kV. line to Salinas is energized. At Salinas there is a bank of auto-transformers energizing a 41-mile 33-kV. line to King City and a 20-mile 22-kV. line to Monterey. Both banks of auto-transformers have grounded neutrals and delta-connected tertiary windings. The transformers at Stanislaus, Manteca and Bay Shore are connected Y-delta with neutrals grounded on the 104-kV. side.

B—Scope of Measurements.

Measurements of the residual currents and voltages of this system were made at the Salinas and Guadalupe substations and at the Stanislaus generating station under several conditions of operation. Resonant shunt and meter measurements were made at Guadalupe and Stanislaus and meter measurements at Salinas simultaneously with those at Guadalupe. In addition, a large number of oscillograph and meter measurements made at Salinas and summarized in technical report No. 38 are again utilized in this report.

C—Data.

The results of the various measurements are summarized in the following tables:

TABLE III.

Residual Current at Stanislaus Generating Station. 104-kV. Lines. September 16, 1914. Amperes.

Measurement on		Line No. 1				Line No. 2			
Line connected to substation	Guadalupe	1	1	2	2	2	2	1	1
	Manteca	1	1	1	2	2	1	1	1
	Bay Shore	1	1 & 2	1 & 2	2	2	1 & 2	1 & 2	1
n	1	0.51	0.49	0.36	0.30	0.32	0.29	0.30	0.39
	3	0.60	0.66	0.77	0.42	0.60	0.24	0.37	0.41
	5	0.04	0.05	0.05	0.05	0.04	0.02	0.05	0.06
	7	*	0.04	0.02	*	*	0.03	0.13	*
	9	0.13	0.11	0.27	0.03	0.12	0.03	0.05	0.02
	11	*	*	*	0.02	0.03	0.03	0.08	0.03
	13	0.02	*	*	*	*	*	*	0.05
	E. S. W.	0.80	0.83	0.94	0.52	0.69	0.38	0.50	0.57

TABLE IV.

Measurements at Guadalupe Substation. September 9, 1916. Amperes.

Quantity		Residual current 55-kV. line						Neutral current	
Line to Stanislaus		No. 1		No. 2		Nos. 1 and 2		Nos. 1 and 2	
Condition of Salinas Neutral		Grd.	Isol.	Grd.	Isol.	Grd.	Isol.	Grd.	Isol.
n	1	0.62	0.67	Not analyzed		0.59	0.70	0.55	0.57
	3	0.12	0.26			0.15	0.36	0.33	0.48
	5	*	*			0.030	0.038	0.017	0.012
	7	0.012	0.007			0.010	0.005	0.013	0.007
	9	0.11	0.008			0.086	0.008	0.098	0.089
	11	*	*			0.014	0.014	0.006	0.006
	13	*	*			0.047	0.047	0.018	0.018
	15	0.016	0.016			0.004	0.004	0.005	0.005
	E. S. W.	0.64	0.73	0.67	0.78	0.63	0.79	0.65	0.75

Line No. 1 between Guadalupe and Bay Shore was out of service during the tests. Manteca substation connected to line No. 1 throughout.

TABLE V.

Average Values of Residual Voltage at Salinas.* Volts.

n	Neutral isolated			Neutral grounded		
	22-kV. line	33-kV. line	55-kV. line	22-kV. line	33-kV. line	55-kV. line
1	90	—	—	50	—	—
3	320	350	—	150	150	—
9	50	50	20	40	80	120
E. S. W.						

*Data obtained from T. R. No. 38.

TABLE VI.

Average Values of Residual and Neutral Currents at Salinas.* Amperes.

n	Neutral isolated			Neutral grounded			
	Residual current			Residual current			Neutral current
	22-kV. line	33-kV. line	55-kV. line	22-kV. line	33-kV. line	55-kV. line	
1	0.01	0.07	0.06	0.01	0.06	0.12	0.13
3	0.05	0.12	0.16	0.02	0.08	0.05	0.09
9	0.02	0.08	0.10	0.02	0.12	0.07	0.22
E. S. W.	0.05	0.16	0.20	0.03	0.16	0.14	0.27

*Data obtained from T. R. No. 38.

TABLE VII.

Simultaneous Meter Measurements at Guadalupe and Salinas on 55-kV. Line. September 9, 1914.

Time, P.M.	Line from Stanislaus supplying Guadalupe	Condition of Salinas Neutral	Guadalupe			Salinas	
			Residual current, amp.	Neutral current, amp.	Line voltage to ground, kV.	Residual current, amp.	Neutral current, amp.
12:13	No. 2	Grd.	0.70	0.93		0.20	—
12:14		Grd.	0.70	1.1	32.6	0.20	—
12:17		Grd.	0.64	0.93	32.0	—	0.093
12:18		Grd.	—	0.80	32.9	—	0.100
12:21		Isol.	0.78	0.87	32.3	0.25	—
12:28	No. 1	Grd.	0.64	0.70	32.6	0.14	—
12:29		Grd.	0.63	0.96	32.3	—	0.076
12:30		Isol.	0.73	0.95	31.7	0.21	—
2:50	Nos. 1 and 2	Grd.	0.62	0.70	32.3	0.15	—
2:51		Grd.	0.64	0.60	32.6	—	0.074
2:52		Isol.	0.79	0.75	32.9	0.26	—
4:08		Grd.	—	—	—	—	0.077

Line No. 2 between Guadalupe and Bay Shore out of service.
Manteca connected to line No. 1.

D—DISCUSSION.

The fundamental and third harmonic are the chief components of the residual currents of the 104-kV. lines at Stanislaus—the two are of approximately the same magnitude. Under certain condition there is also a considerable amount of ninth harmonic. There is no characteristic difference in the residual current due to the type of transmission line used (vertical or triangular), for under the same conditions the residuals of the two lines are practically identical.

When the 104-kV. lines are paralleled at the Stanislaus and Bay Shore terminals, the residual current at Stanislaus is the greater in the line to which the intermediate substations, Manteca and Guadalupe, are connected, but when these are connected to different lines, paralleled as before, the residual current is the greater in the line to which Manteca is connected. This is true of the third-harmonic, ninth-harmonic and equivalent sine-wave values. The difference is especially noticeable in the case of the ninth harmonic, it being several times greater in the line to which Manteca is connected than in the other line.

With the lines under no load, no substations being connected, the ninth harmonic is small. Little change is noted when the line is closed at the far end through the Bay Shore substation. Adding the Manteca substation increases the third harmonic about 80 per cent and the ninth harmonic about 800 per cent. Adding both the Manteca and Guadalupe substations increases the third harmonic 50 per cent and the ninth 300 per cent. Adding the Guadalupe substation only, decreases the third harmonic 40 per cent and the ninth harmonic a small amount.

It appears from the above that the Manteca bank of transformers is responsible for the greater part of the ninth harmonic measured at Stanislaus. However, this can not be stated definitely since banks at any of the four stations may give rise to it, the different amounts measured at Stanislaus with different combinations of lines and transformer banks being due to differences in impedances.

The distance between Stanislaus and Bay Shore is slightly greater than a half-wave length for the ninth harmonic. With the lines separated at the distant ends and paralleled at Stanislaus, but not at any of the substations, the impedances to the ninth harmonic at the various substations may be computed. Citing one of the conditions of tests, all substations being connected to one of the lines, the other carrying charging current only, the impedance of the open-circuited line at Stanislaus is 840 — j 2400 ohms and that of the line closed through the neutrals of the various transformer banks 20 — j 150 ohms. The resultant impedance of the two lines in parallel is capacitive and intensifies somewhat the ninth-harmonic voltage due to the Stanislaus bank of transformers. The ratio of computed impedances of the two lines is approximately

16 to 1, that of the measured ninth-harmonic currents is approximately 5 to 1 (average). Therefore, the effects just noted could be explained if the large ninth harmonic were due to the Stanislaus bank. Assuming the ninth-harmonic voltage to be due to some other transformer bank on the system the computed ratio of ninth-harmonic currents in the two lines at Stanislaus would be approximately 30 to 1.

When the two lines are paralleled at more than one point the network is too complicated to be readily computed. However, since with a number of different combinations, the large ninth harmonic was observed only in the line to which Manteca was connected, it is regarded as probable that the transformer bank there is responsible for the greater part.

The measurements at Guadalupe were made under somewhat different operating conditions than those at Stanislaus. Line No. 1 between Guadalupe and Bay Shore was out of service during the Guadalupe tests because of insulator testing. This changed the impedances of the line to third- and ninth-harmonic currents as compared to the conditions which obtained during the Stanislaus measurements. No analysis of residual current at Guadalupe was made with Manteca and Guadalupe connected to different lines. Since the measurements at Guadalupe were made prior to those at Stanislaus, the desirability of such analysis was not realized.

The fundamental is the chief component of the residual current of the 55-kV. line at Guadalupe, both with Salinas neutral grounded and with it isolated. With the Salinas neutral grounded (the usual operating condition) the third and ninth harmonics are about the same in magnitude when one 104-kV. line is supplying Guadalupe; and when both 104-kV. lines are in parallel supplying Guadalupe the ninth is about 60 per cent of the third. Isolating the neutral at Salinas increases the third harmonic and decreases the ninth harmonic of the residual current of the 55-kV. line at Guadalupe. Meter measurements made simultaneously at Salinas showed that isolating the neutral at Salinas increases the residual current of the 55-kV. line at that point. No analyses of the residual or neutral currents were made at Salinas at this time.

Tables V and VI summarize the results obtained in the earlier tests at Salinas as given in technical report No. 38. The residual current of the 33-kV. and 55-kV. lines, the neutral current, and the residual voltage of the 22-kV. line were measured; the other values were obtained from these measured quantities and the computed line impedances. The results are in agreement with those obtained in the later tests and show that isolat-

ing the Salinas neutral affects the ninth-harmonic residuals at Salinas as follows:

1. increases current and voltage on 22-kV. line.
2. decreases current and voltage on 33-kV. line.
3. increases current and decreases voltage on 55-kV. line.

Computations of impedance to ninth-harmonic residual current of the 55-kV. line at Guadalupe indicate that it is approximately a quarter wave-length for this frequency. The combination of line and transformer impedances at Salinas is rather complicated for exact computation, but an approximate estimate of the resultant impedance between the 55-kV. line and ground at Salinas may be made from the current and voltage measurements given in Tables V and VI, the computed impedances of the 33- and 22-kV. lines and an assumed transformer impedance.

Assuming constant ninth-harmonic voltage, unaffected by the condition of the Salinas neutral, impressed between the 55-kV. line and ground at Guadalupe, with the receiving end at Salinas closed through the values of impedance as estimated above, the ratio of ninth-harmonic currents at the two ends of the line for each condition of the Salinas neutral, and the effect of the condition of the Salinas neutral on the values of the two currents were computed. The results obtained from this computation agree favorably with the results of the measurements as described above.

It is probable, therefore, that the ninth harmonic observed in the 55-kV. line at Salinas and Guadalupe arises from some one or more of the 104-kV. transformer banks and not from the transformer bank at Salinas. Further evidence to support this view is given by the tests made at Salinas and summarized in technical report No. 38 with the voltage lowered 5% by changing the auto-transformer taps at Guadalupe which show little effect on the magnitude of the ninth-harmonic neutral and residual currents at Salinas due to this change. On the other hand, lowering the voltage on the 104-kV. line by approximately the same amount reduces the ninth-harmonic neutral current at Salinas 40%.

Isolating the neutral at Salinas increases the third-harmonic current of the 55-kV. line at Guadalupe and Salinas and increases the third-harmonic residual current and voltage of both the 22-kV. and 33-kV. lines. The effect on the third-harmonic residual voltage of the 55-kV. line at Salinas is uncertain since it was not measured. It is not considered valid to assume the third-harmonic voltage to arise entirely from sources other than the transformer bank at Salinas, and hence the

value of residual voltage at this point can not be computed with any degree of certainty from the measurements on the 22-kV. line and impedances of transformers and lines.

IV. Pacific Gas and Electric Company.

A—Description of System.

The 60-kV. system of the Pacific Gas and Electric Company consists of approximately 1260 miles of electrically connected network. In addition there are normally in electrical connection therewith the 60-kV. lines of the Western States Gas and Electric Company, Northern California Power Company and the Snow Mountain Water and Power Company, thus increasing the total mileage of the 60-kV. electrically connected network to approximately 1500 miles. Also, there are 109.5 miles of 100-kV. line which is connected to the 60-kV. system through Y-connected auto-transformers and a large mileage of 11-kV. lines connected through Y-Y transformers with neutrals grounded on both sides, thus transforming residual currents and voltages from one system to the other. Transformer banks at most of the generating stations are connected Y with grounded neutral on the line side and delta on the station side. At the substations, transformers are connected in Y with grounded neutrals on the 60-kV. side, about half being delta on the low-tension side and the other half Y. In a large number of the smaller stations, switches, normally kept open, are provided in the neutrals of the Y-delta connected banks.

B—Scope of Measurements.

Measurements were made at one generating station (Electra, 14100-kVA.), one substation with Y-delta connected transformers (Davenport, 7800-kVA.) and one substation with Y-Y connected transformers (Banta, 600 kVA.). Tests having been made at such a limited number of points it is evident that a thorough discussion of the residuals of this extensive system can not be made.

C—DATA.

The results obtained from the measurements at these three points are given in the following tables:

TABLE VIII.

Residual Current at Electra Generating Station. September 30-October 1, 1914. Amperes.

n	In parallel with rest of system		60-kV. line open at Stockton** Energized by Generator No. 7	60-kV. line open at Stockton** Energized by Generator No. 1
	9:24-9:42 A.M.	7:25-7:50 P.M.	12:41-1:11 A.M.	2:44-3:02 A.M.
1	0.46	0.85	0.021	0.031
3	0.38	0.52	0.042	0.039
5	*	*	0.002	0.004
7	*	*	0.001	*
9	0.06	0.06	0.008	0.015
11	*	0.02	0.003	0.004
13	0.04	0.04	0.001	0.003
15	*	*	*	*
17	0.04	*	*	*
E. S. W.	0.95	1.00	0.048	0.052

**With line open at Stockton there are no grounded neutrals on the line except at Electra.
Generator No. 7, revolving field type.
Generator No. 1, inductor type.

TABLE IX.

Analysis of Generator Voltage at Electra. September 30-October 1, 1914. Volts.

n	Revolving field type	Inductor type
1	2080	2100
3	6.5	*
5	16	14
7	1.6	0.7
9	0.4	*
11	0.6	*
13	0.5	*
E. S. W.	2080	2100

*Too small to measure.

TABLE X.

Residual Current at Davenport Substation. October 14, 1914. Amperes.

n	Synchronous motor		
	On	Off	On
	10:35 A.M.	12:35 P.M.	1:35 P.M.
1			
3	3.1	3.5	3.6
9	0.06	0.06	0.06
13	0.02	0.02	0.02
E. S. W.	3.3	3.5	3.6

TABLE XI.

Neutral Current of 60-kV. Transformers at Santa Substation. November 18, 1914. Amperes.

n	11:02 A.M.	11:18 A.M.	11:22 A.M.
1			
3	0.66	0.45	0.53
5	0.012		
9	0.022		
15	0.010		
E. S. W.			0.59

D—Discussion.

The measurements at all three points show the third harmonic as the chief component of the residual and neutral currents of this system. The fundamental components are of comparatively small magnitude, indicating that the load unbalance is, in this case, an unimportant factor. The residuals are chiefly those arising from the exciting currents of the transformers. It is evident from the measurements that there is a considerable interchange of third-harmonic exciting current among the stations of this system, the Y-delta banks supplying low impedance paths to ground for third-harmonic current arising from Y-Y banks in some cases at a considerable distance from the former.

The large third-harmonic current at Davenport is due doubtless to the grounded Y-Y banks at substations near Davenport. There are such banks at San Jose (6000 kVA.) and at Mountain View (1500 kVA.) and others of smaller capacity at other points. There are Y-delta banks at San Jose (6000 kVA.) and at Martin (9000 kVA.) substations, a total of 15000 kVA. These banks divide the third-harmonic current arising from the Y-Y banks with the transformers at Davenport. It is probable, therefore, that correspondingly large values of third-harmonic neutral current would be observed in the neutrals of the banks at San Jose and Martin.

The generating station at Electra is at a greater distance from these Y-Y banks, hence it is to be expected that a smaller amount of this third-harmonic current would appear there. The magnitudes of third-harmonic current which were measured are in agreement with this.

The third-harmonic current measured at Banta may be taken as a very rough indication of what may exist at other Y-Y banks on the system. The third-harmonic current of this bank (600 kVA.) varies from 0.4 to 0.7 amperes. Roughly the amount of current which will exist in the neutrals of the other Y-Y banks will be in proportion to their kVA. capacity. For instance, at San Jose the 6000-kVA. bank would have a third-harmonic neutral current of from 4 to 7 amperes and at Mountain View 1 to 2 amperes. It is not surprising, therefore, that the large third-harmonic current measured at Davenport should exist.

The residual current at Davenport was also measured with only one of the two banks of Y-delta transformers in operation. The change in residual current due to disconnecting one bank was so small that it was masked by other changes which occurred from time to time due to voltage fluctuations on the system. This is in agreement with the statement that the third-harmonic current is due largely to sources other than the Davenport transformers.

Residual current at Electra was also measured with the transmission line open at Stockton (approximately 40 miles of line), there being no transformers with grounded neutrals connected to the line beyond Electra. The third-harmonic residual current under this condition is therefore due to the Electra transformers only. The measurements were made with the line energized by two different generators, one of inductor type and the other revolving-field type. There is seen to be little difference in the wave form or the magnitude of the residual current, due to this difference in generators. This is not surprising in view of the fact that the third harmonic is the chief component and that the transformer windings are delta connected on the station side. Analysis of generator-voltage wave forms while energizing the line under the same conditions shows the inductor-type alternator to have a slightly better wave-form, only the fifth and seventh harmonics being of magnitude large enough to measure, the fifth being the larger. The wave form of the revolving-field type generator contains practically the same amount of fifth as that of the inductor type generator and in addition, third, fifth, seventh, ninth, eleventh and thirteen harmonics. The effect of these is, however, on the balanced voltages and currents of the line, rather than on the residuals, as the residual current measurements show.

Some measurements were made at the Davenport substation with a synchronous motor connected and disconnected, to see if its operation

had any effect on the residual current of the 60-kV. line. There was no apparent change in magnitude or wave-form of the residual current that could be attributed to the operation of this motor. This result is to be expected and is in agreement with the results noted at Electra in the tests with the two different types of generators.

V. Pacific Light and Power Corporation.

A—Description of System.

This system consists of approximately 900 miles of electrically connected 15-kV. network. Most of the transformers connected to these lines are isolated from ground. At Vernon there is a 15000-kVA. bank Y-delta connected, with the neutral grounded on the 15-kV. side. At Redondo the neutrals of a turbo-alternator and of a bank of Y-connected auto-transformers, which connect the generator to the line, are both grounded during the time the generator is in service.

B—Scope of Measurements.

Measurements on this system were made at only one point, San Fernando, at which the Committee's laboratory had been set up for the purpose of carrying out other tests. The measurements were made on two different days under somewhat different conditions. At the time of the first series of measurements the section of line from San Fernando to Somis (37 miles) was under the control of the Joint Committee and tests were made of the effect on the residual voltage at San Fernando, of connecting in this section of line and also of connecting in one or two conductors only, thus introducing a capacitance unbalance on the system. At the time of the second series of measurements the line from San Fernando to Somis was in commercial operation for approximately 30 miles, the remainder being disconnected. The residual voltage and current of the line at the laboratory were measured at intervals during the daily cycle of operation, and a record of all important switching operations was obtained from the load dispatcher.

C—DATA.

The measurements obtained in the two series of tests are recorded in the following tables:

TABLE XII.

Residual Voltage and Current at San Fernando. March 26, 1915

Time, P.M.	Line voltage to ground, kV.	Line current, amp.	Residual voltage, volts	Residual current, amp.	Conductors connected to instrument transformers	
					Toward San Fernando	Toward Somis
1:34–2:06	9.46	0	640	0	all	none
2:06–2:20	9.57	1.55	690	0.19	all	all
4:03–4:09		0	3300	0	all	none
4:09–4:18	9.51		3580	0.87	all	all
2:34–2:37	9.49	1.41	540	0.94	all	1 and 2
2:41–2:43	9.52	1.42	1160	0.95	1 and 2	all
2:47–2:49	9.42	1.15	1430	1.12	1	all
2:57½			510		all	1

TABLE XIII.

Analyses of Line and Residual Voltages at San Fernando. March 26, 1915.

Line beyond laboratory		Disconnected	Connected
Time		3:18 P. M.	3:45 P. M.
Residual voltage, volts	n		
	1	100	120
	3	460	2930
	9	40	110
	E. S. W.	470	2930
Voltage phase 1 to neutral, volts	1	9300	9500
	3	*	*
	E. S. W	9300	9500
Oscillogram No.		958	960

TABLE XIV.

Average Values of Line and Residual Voltages and Currents at San Fernando. June 15, 1915.

Time	Line voltage, volts	Line current, amp.	Residual voltage, volts	Residual current, amp.	Redondo turbo-generator
6:24– 7:00 A.M.	9130	1.16		0.127	Off
7:01–10:17 A.M.	9300	1.18	1120	0.246	On
10:43 A.M.– 3:27 P.M.	9320	1.19	530	0.119	Off
3:32– 6:02 P.M.	9420	1.19	1090	0.246	On

TABLE XV.

Analyses of Line and Residual Voltages and Currents. June 15, 1915.

Turbo-generator		Off			On	
Time		6:50½ A.M.	10:37 A.M.	3:25 P.M.	7:05 A.M.	5:37 P.M.
Voltage phase 1 to ground, volts	1	8990			9870	9250
	3	250			*	210
	5	*			*	180
	E. S. W.	9000			9870	9250
Residual current, milliamp.	1	13	17	17	18	11
	3	82	100	100	253	219
	5	10	14	10	18	16
	7	15	21	21	26	25
	9	23	27	38	32	46
	11	8	15	10	20	9
	13	16	31	26	13	*
	15	9	8	15	*	9
	17	*	*	4	*	*
	21	*	*	*	48	46
	27	*	*	*	32	74
	E. S. W.	90	115	115	263	245
Residual voltage, volts	1	110	130	110	120	130
	3	390	580	410	830	1030
	5	20	30	30	30	60
	7	30	40	40	30	30
	9	30	50	30	40	90
	11	*	20	30	10	*
	E. S. W.	410	600	480	840	1040
Oscillogram No.		971	974	976	972	978

D—DISCUSSION.

The results obtained in the tests of March 26 show slight effect on the residual voltage of this system due to adding the San Fernando-Somis line, although of course causing a residual current at San Fernando which otherwise would not exist. There is perhaps a slight increase in the residual voltage at San Fernando, but this is not definitely established as the quantities were varying from time to time on account of other changes of conditions on the system. A capacitance unbalance at San Fernando, caused by connecting one or two conductors of the line to Somis, has small effect on the residual voltage at this point. This is to be expected in view of the large star-delta transformer bank at Vernon. Due to the unbalanced charging current at San Fernando a residual current proportional to the capacitance unbalance is caused. This exists only between the point where the unbalance occurs and the grounded-neutral banks of transformers.

Connecting the turbo-alternator and bank of auto-transformers at Redondo to the 15-kV. network approximately doubles the residual voltage at San Fernando. The large increases, as shown by the oscillo-

gram analyses, are in the third and ninth harmonics. The residual current shows appreciable fifteenth, twenty-first and twenty-seventh harmonics, when the Redondo generator and auto-transformers are connected.

On drawing No. 327 the readings of residual voltage and current, taken at intervals throughout the day,* are plotted. Although the quantities are fluctuating from time to time it is readily seen that the large changes are due to connecting or disconnecting the Redondo turbo-generator and auto-transformers. The residuals of this system are chiefly of triple-harmonic frequencies and are due to the transformer banks and generator with grounded neutrals. Since the voltage was greater with the Redondo generator and auto-transformer connected, it is reasonable to suppose that they give rise to the greater residuals. It is probable that the residual voltage would be still greater were the Vernon neutral ungrounded and that at Redondo grounded. The measurements have not been made at a sufficient number of points to give a very thorough discussion of this subject.

VI. General Discussion.

Detailed discussions of the measurements made on each of the transmission systems investigated, have been given in the preceding sections. It is the purpose of this section to briefly summarize these and to discuss the prominent features which are brought out by the measurements as a whole.

The residual voltage of the Coast Counties Gas and Electric Company's system is chiefly of fundamental frequency. Higher harmonics are present in appreciable magnitudes and bear approximately the same ratio to the corresponding harmonic in line voltage, as do the fundamental components; indicating that the residual voltage is due to the admittance unbalance of the several phases to ground.

Residual voltages and currents of the Sierra & San Francisco Power Company's system are chiefly third and ninth harmonics, the fundamental component being of less importance. Measurements have indicated that the transformer bank at Manteca gives rise to a large part of the ninth-harmonic current and voltage. The measurements have not, however, been sufficiently complete to establish this beyond question. If, as is thought, the Manteca bank is the principal source of ninth harmonic its effect could be eliminated by operating it with neutral isolated. Since there is a delta on the 17-kV. side of this bank and there are other Y-delta connected banks with grounded neutrals on the system, it does not seem that this procedure should be objectionable from an operating standpoint.

*June 15.

The Pacific Gas and Electric Company's system is the most complicated of the four which have been investigated. The residual currents and voltages are chiefly triple harmonics and arise from the Y-connected transformers with grounded neutrals. There is an interchange of third-harmonic exciting current throughout the system, since Y-delta banks furnish low impedance paths to ground for current arising from Y-Y banks sometimes at widely separated points. Residuals of much lower magnitudes would be observed were all the banks delta connected on the low-tension side.

The residual voltage of the Pacific Light and Power Corporation's 15-kV. system is chiefly third harmonic. This is due to the transformers at Vernon which are operated Y-delta with neutral grounded. Connecting the generator and bank of auto-transformers at Redondo to the system practically doubles the third-harmonic residual voltage and current at San Fernando and also increases the higher triple harmonics.

As on all systems the measurements were made at but a few locations the investigation is incomplete. Additional experimental work is particularly desirable on the grounded-neutral networks if specific measures for reducing their residuals are to be proposed.

Respectfully submitted.

(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

ATTACHMENT: P. I. C. Drawing No. 327.

IN FILES OF JOINT COMMITTEE: Oscillograms Nos. 958, 960, 971, 972, 974, 976, 978.

APPROVED: August 29, 1916.

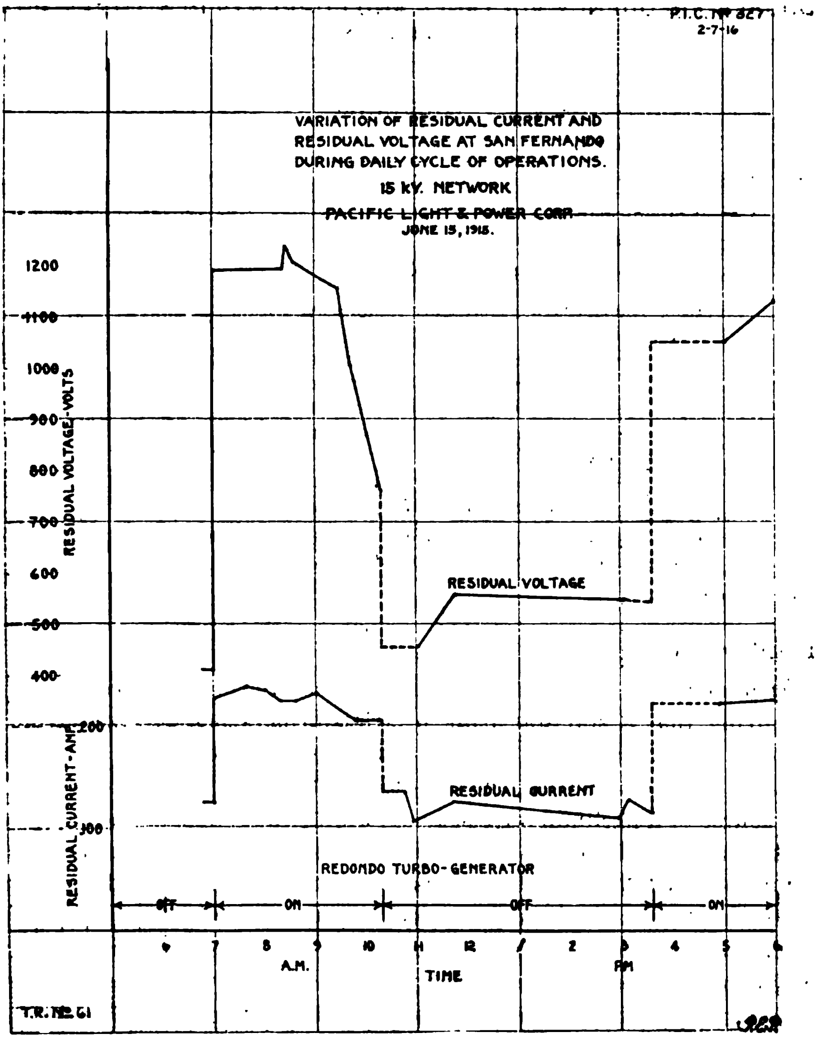
(Signed) R. W. MASTICK,
Field Engineer.

APPROVED: August 31, 1916.

SUBCOMMITTEE ON TESTS,
(Signed) J. E. WOODBRIDGE,
Chairman.

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE,
(Signed) ARTHUR F. BRIDGE,
Secretary.

January 8, 1917.


P.L.C. № 327
2-7-16
VARIATION OF RESIDUAL CURRENT AND
RESIDUAL VOLTAGE AT SAN FERNANDO
DURING DAILY CYCLE OF OPERATIONS.
15 kV. NETWORK
PACIFIC LIGHT & POWER CORP.
JUNE 15, 1915.
RESIDUAL VOLTAGE-VOLTS
RESIDUAL VOLTAGE
RESIDUAL CURRENT-AMP.
RESIDUAL CURRENT
REDONDO TURBO-GENERATOR
OFF
ON
OFF
ON
A.M.
TIME
P.M.
T.R. № 61

Technical Report No. 62.

March 18, 1916.

DOUBLE-FREQUENCY VOLTAGES AND CURRENTS IN A THREE-PHASE TRANSMISSION LINE.

January 6-23, 1915.

OUTLINE.

I. INTRODUCTION.

II. DESCRIPTION OF TRANSFORMERS AND TRANSMISSION LINE.

- A—Transformers.
- B—Transmission Line.

III. DESCRIPTION OF PHENOMENA.

- A—Line Energized by Star-Star Connected Bank.
  1. 37½-kVA. test bank.
     - a—Impressed voltage 58 per cent normal.
     - b—Impressed voltage 50 per cent normal.
     - c—Impressed voltage 43 per cent normal.
     - d—Impressed voltage 29 per cent normal.
  2. 50-kVA. test bank.
     - a—Impressed voltage 55 per cent normal.
     - b—Impressed voltage 73 per cent normal.
- B—Line Energized by Delta-Delta Connected Bank. Grounded Through Neutral of Star-Star Connected Bank.

IV. DISCUSSION.

I. Introduction.

Included in the program of work at San Fernando were some tests requiring that a star-star bank of transformers be energized from its low-voltage side and that the high-voltage side be connected to a 37-mile isolated transmission line, the line-side neutral only being grounded. In the course of a preliminary test to determine the fitness of the equipment certain abnormal phenomena were encountered. Energizing the system, connected as indicated above, was followed by an abnormal noise from the transformers continuing usually for a few seconds.

The occurrence of a disturbance under one of the conditions of the proposed tests necessitated a delay in the regular program pending an investigation, as it was feared the equipment might be damaged, if not already defective. The tests herein described and discussed were made to ascertain the nature of the phenomena and to explain if possible their causes.

It will be observed that some of the tests give no evidence of the presence of double-frequency components in the line voltages and currents although large unbalances, of odd harmonic frequencies were

observed. These unbalances were such as were to be expected from previous experience with the star-star connection with no provision for stabilizing the neutral. There is a marked difference between these effects and the double-frequency phenomena which form the main subject of this report.

II. Description of Transformers and Transmission Line.

A—Transformers.

The tests were made on two different banks of core-type transformers of different sizes and ratings and of different manufacture. The first bank tested is of Westinghouse manufacture, Type SK, 37½ kVA., 50 cycles, 15000/7125–440/110 volts, and the other of General Electric manufacture, Type H, 50 kVA., 60 cycles, 22000/19800–2400/480 volts. These transformers are more fully described in technical reports Nos. 59 and 60. Data with reference to the exciting currents and leakage impedances are there given.

B—Transmission Line.

The transmission line involved in these tests is a 15-kV. circuit of the Pacific Light and Power Corporation, extending approximately 37 miles beyond the Joint Committee's substation at San Fernando. The configuration is vertical. The average height of the lowest conductor above ground is 40 feet, the spacing of conductors 5 feet, and the conductors are of copper, No. 2 B&S gauge for 21% and No. 4 B&S gauge for 79% of the length. At the time of these tests the line was not transposed. The distant end of the line was open and clear.

The direct capacitances between conductors and between each conductor and ground are given below. The values were obtained from measurements with an impedance bridge on a short section of the line.

$$C_{10} = 0.255 \text{ mfd.} \qquad C_{12} = 0.113 \text{ mfd.}$$
$$C_{20} = 0.223 \text{ mfd.} \qquad C_{23} = 0.116 \text{ mfd.}$$
$$C_{30} = 0.245 \text{ mfd.} \qquad C_{13} = 0.068 \text{ mfd.}$$

C_{10}, C_{20} and C_{30} are the direct capacitances between the conductors 1, 2 and 3, respectively, and earth, and C_{12}, C_{13} and C_{23} are the direct capacitances between the conductors indicated by the subscripts. Conductor No. 1 is the lowest, No. 2 the middle and No. 3 the top conductor.

Computations based on these values of direct capacitance indicate that it is necessary to supply a current through the neutral connection, about 5% of the average normal charging current per phase, in order to maintain equal voltages, 120° apart in phase, between the three line conductors and ground.

III. Description of Phenomena.

The tests were made on each bank of transformers at several different impressed voltages. Different effects were observed at the different voltages, and hence the results of the tests for each transformer bank are described separately, according to the voltage at which the test was made. In each case the voltage given is the per cent of normal voltage which would be impressed on the test bank, if rated voltage were impressed on the high-tension side of the supply bank. The supply voltage varied and was in general from 5 to 10 per cent higher than the rated voltage.

Prints of oscillograms taken under various conditions are given and discussed in the appendix to this report.

A—Line Energized by Star-Star Connected Bank.

1. *37½-kVA. Test Bank.*

a. Impressed Voltage 58 Per Cent Normal.

The transformers were connected as in Fig. 1, drawing No. 319, the supply bank being connected for 15000-440 volts and the 37½-kVA. test bank 440-7125 volts, star-star, with neutral grounded on the line side. Energizing the test bank and transmission line by closing the oil-switch on the station side was accompanied by an abnormal noise and vibration of the transformers, which continued for several seconds after the closing of the oil-switch. Measurements of the voltages to ground and of the currents by meter and oscillograph showed them to be greatly distorted and increased during the continuance of the vibration. A summary of the main features brought out by the oscillograms (appendix, pages 568 to 582) and by meter readings and observations on the tracing table of the oscillograph follows:

> The abnormal phenomena commenced from one-tenth to two seconds after the closing of the oil-switch, and lasted for from four to fifteen seconds. Sometimes there was a short recurrence of the condition, but in general it was over by the end of the fifteen-second period.
>
> The voltages from lines to ground were approximately four-times normal, and were composed chiefly of double-frequency components. The double-frequency components did not exist in the voltages between lines, which were of normal magnitude and fundamental frequency with the exception of an increase in the higher harmonics.
>
> The residual voltage was similar in wave-form to the line voltages to ground, and of approximately three times their magnitude (about 43-kV.). Under normal conditions the residual voltage was principally third harmonic and less than 10 volts.
>
> The line currents were also greatly increased over normal values (2.5 amperes as compared to 0.6 amperes) and were made up chiefly of components of double the normal frequency of the system. These double-frequency components were in phase in all three line conductors.

The residual current was approximately equal in magnitude to the numerical sum of the three line currents. The most prominent component was of double normal frequency. Under normal conditions the value of the residual current was less than 0.05 ampere and principally of third-harmonic frequency.

The voltages between supply mains of the station side were normal. The voltages from mains to neutral and the currents were greatly increased and distorted, having components of double frequency, similar to the line voltages and currents.

Analyses of typical oscillograms of line and residual voltages and currents are given in Table I below.

TABLE I.

Line and Residual Voltages and Currents.

Analyses of Oscillograms.

n	Line voltage to ground, E, kV.	Line current, I, amps.	Residual voltage, E_R kV.	Residual current, I_R amps.
1	2.8	0.6	3.7	0.3
2	12.8	1.8	41.0	6.8
3	3.0	0.5	9.0	2.3
4	1.0	0.3	2.4	0.3
5	1.8	0.8	4.2	1.2
6	1.5	0.8	4.4	2.8
7	1.2	0.5	3.1	2.2
8	0.4	0.1	2.4	0.9
9	0.9	0.7	1.4	0.5
10	0.6	0.4	1.4	0.7
11	0.4	0.2	*	1.0
E. S. W.	13.8	2.5	42.9	8.3
Oscillogram No.	712		707	

A test made with the same impressed voltage per coil, changing the low-voltage coils in both the supply and test banks from series to parallel, gave results, as ascertained by inspection of the wave-forms on the tracing table of the oscillograph, identical with those described above.

It was thought that the difficulties which these phenomena presented might be obviated by energizing the test bank and line with the neutral isolated, and then grounding the neutral. Energizing the system in this manner, however, the abnormal phenomena occurred, upon grounding the neutral, as regularly and with the same intensity and characteristics, as when the switching was done with the neutral grounded.

b. Impressed Voltage 50 Per Cent Normal.

For this test the transformers were connected as in Fig. 2, drawing No. 319. Effects similar to those which occurred with 58 per cent normal voltage impressed were observed (by meter readings and by

inspection of wave-form on the tracing table of the oscillograph) when the line-side neutral was grounded and the station-side neutrals not interconnected. If the two station-side neutrals were interconnected stable conditions resulted. During the abnormal condition the voltage between station-side neutrals was approximately 900 volts, while under normal conditions the voltage between neutrals was of negligible magnitude.

c. Impressed Voltage 43 Per Cent Normal.

The connections for this test are shown by Fig. 1, drawing No. 319, the supply bank being connected 330-volts delta on the station side. The abnormal effects observed upon the closing of the oil-switch, the line-side neutral of the test bank being grounded, were very different from those observed at the other values of impressed voltage. The vibration was less violent and of longer duration than was observed with 58 per cent or 50 per cent normal voltage impressed. The voltages to ground of the three phases were greatly unbalanced, the highest voltage being sometimes on one phase and sometimes on another. Occasionally there would be no abnormal noises upon energizing (through the oil-switch as usual) the transformers and line and the voltages from lines to ground would all be normal. Table II below gives a number of voltage readings. The oil-switch was opened and closed between each set of readings. It should be observed that the highest voltage appeared on different phases at different times.

TABLE II.

Line Voltages to Ground—Kilovolts.

No. of switching	E_1	E_2	E_3
1	8.6	8.0	2.9
2	2.9	8.6	8.3
3	3.0	8.6	8.8
4*	3.2	3.5	3.1
5	8.9	3.0	8.0
6	2.9	8.6	8.3

*No vibration, conditions normal.

The line voltages to ground were chiefly of fundamental and third-harmonic frequency. The double frequency components, if present, were too small to be detected by inspection of the oscillogram (No. 733, see appendix, page 584) taken during this test. The residual voltage was composed of fundamental and third harmonic, and was approximately 19 kV.

Observations, on the tracing-table of the oscillograph, of the currents in the low-tension supply mains, showed that the current in the main to which the transformer having the highest voltage was connected, was approximately opposite in phase to the currents in the other two supply

mains. Unbalance effects similar to these were observed in later tests on the same bank of transformers and are recorded and discussed in technical reports Nos. 59 and 60.

d. Impressed Voltage 20 Per Cent Normal.

Connections were made as in Fig. 1, drawing No. 319, the supply bank being connected 220 volts, delta on the station side. Switching under this condition was followed by noise and vibration less frequently and of less violence than 58 and 50 per cent normal voltage impressed. No observations taken under this condition are recorded.

2. *50-kVA. Test Bank.*

a. Impressed Voltage 55 Per Cent Normal.

The 50-kVA. transformers were connected as shown by Fig. 1, drawing No. 319. The supply bank was connected delta-delta 12980-330 volts thus impressing approximately 55 per cent of normal voltage (50-cycle rating) on the test bank. Switching caused vibration of the 50-kVA. bank rather infrequently. During the period of vibration the line voltages and currents were distorted and contained double-frequency components. It was found possible in this case to energize the transformer bank without the appearance of any abnormal phenomena by closing the oil-switch first, and grounding the line-side neutral afterwards. Prints of oscillograms taken under this condition are given and discussed in the appendix (pages 585 to 589).

b. Impressed Voltage 73 Per Cent Normal.

When the supply voltage was increased so as to impress nominally 73 per cent normal voltage on the test bank the vibrations accompanying the closing of the oil-switch, line-side neutral being grounded, were greatly increased in violence. The voltages from lines to ground were increased in magnitude and distorted, containing large double-frequency components. The abnormal condition occurred whether the neutral grounding switch was closed before or after the closing of the oil-switch. No oscillograms were taken under this condition. This description is based on meter readings and inspection of the wave-forms on the tracing table of the oscillograph.

B—Line Energized by Delta-Delta Connected Bank.

Grounded Through Neutral of Star-Star Connected Bank.

The arrangement shown by Fig. 3, drawing No. 319, was suggested as a possible method of avoiding the double-frequency phenomena in operating a star-star bank of transformers with line-side neutral grounded. The arrangement was tested at two different voltages.

Switching at rated voltage of the 50-kVA. bank caused distorted line voltages and currents and abnormal noises from the 50-kVA. bank. It was not possible to distinguish double-frequency components by inspection of the wave shapes on the tracing table of the oscillograph. No oscillograms were taken at this voltage.

The impressed voltage was then raised to 15% above normal voltage of the 50-kVA. bank with the transmission line disconnected. The vibration accompanying switching at this voltage was violent and continued as long as the transformers were energized. Double-frequency components were present in the line currents and voltages as was the case when the star-star bank was used to energize the transmission line. Oscillograms 754 and 755 were taken under this condition and are included and discussed in the appendix (pages 591 to 593).

IV. Discussion.

No adequate explanation of the causes of these phenomena has been advanced. The tests described and discussed above have demonstrated that the phenomena are not due to any defects in the transformers, but rather to the type of connection, star-star connected transformers with the neutral grounded on the line-side only and transmission line otherwise isolated from ground. Tests were conducted on two separate banks of transformers of different ratings and of different makes, and abnormal phenomena were observed in both cases. The insulation of the 37½-kVA. transformers was tested by impressing, between the high-voltage windings and cores, a voltage greater than any encountered in the tests. The individual transformers of the 37½-kVA. bank were likewise tested for loose iron and loose coils by energizing with five-times full-load current. None of the transformers showed any signs of distress under either of these tests.

Transformer connections similar to those employed in the series of tests described in this report were used in the tests described in technical reports Nos. 59 and 60. There, however, the transmission line was transposed, a transformer bank was connected to the line at the distant end (Somis) and the switching was done with the neutral of this latter bank grounded. This bank was in interconnected-star on the line-side and delta on the station-side. After the system was energized the neutral at the distant end was isolated. When the oil-switch at San Fernando was closed, with the line-side neutral grounded and the Somis neutral isolated, abnormal noises indicated the existence of a disturbance. No observations were taken, however, at such times during these later tests. It was possible to reach higher densities before the disturbance occurred than in the tests recorded in this report, due possibly to the improved balance of the transmission line.

From the results of the tests herein recorded and from those made after transposing the line, it is to be expected that the characteristics of the phenomena will vary with different sizes and types of transformers and with the extent and degree of unbalance of the connected transmission system.

So far as is known there is no published record of the occurrence of similar phenomena in other instances. This is somewhat surprising as it seems probable that conditions approximating those of this test must have occurred previously either by accident or design. Although this arrangement of connections is not used in commercial installations it is nevertheless believed that this subject is of considerable interest and perhaps importance to power engineers. Since the occurrence of these phenomena is evidently extremely rare, they do not seem of sufficient importance from the standpoint of inductive interference to justify an extended study, and this record is therefore presented for whatever value it may have to members of the Committee or others who may desire to pursue the subject further from a purely scientific standpoint.

Respectfully submitted.

(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

ATTACHMENT: P. I. C. Drawing No. 310.

APPROVED: April 15, 1916.

(Signed) R. W. MASTICK,
Field Engineer.

APPROVED: May 25, 1916.

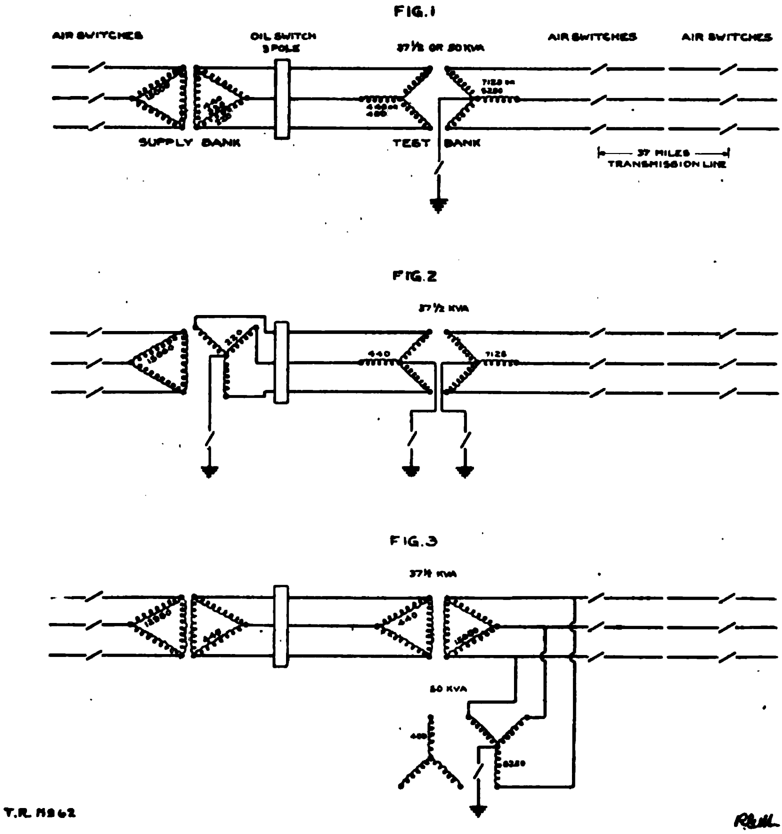
SUBCOMMITTEE ON TESTS,
(Signed) J. E. WOODBRIDGE,
Chairman.

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE,
(Signed) ARTHUR F. BRIDGE,
Secretary.

January 8, 1917.

P.I.C. No 319
12-10-15

TRANSFORMER CONNECTIONS
ABNORMAL DOUBLE FREQUENCY PHENOMENA
SAN FERNANDO
JAN 1915



APPENDIX.

PRINTS OF OSCILLOGRAMS.

Line Energized by Star-Star Connected Bank.

37½-kVA. units.

See Fig. 1—Drawing No. 319.

Impressed Voltage 58 Per Cent Normal.

III—A—1—a—pages 560, 561.

Oscillograms: Nos. 704, 707, 710, 711, 712, 717, 718, 719, 722, 724, 725, 728, 729.

Oscillogram No. 704.
Abnormal Condition.

Three line currents.
Prominent second harmonic in phase in all three (*i. e.* Residual).
Approximately 2.5 amperes per phase.

Oscillogram No. 707.
Abnormal Condition.

Vibrator No. 1.
50-cycle timing wave.

Vibrator No. 2.
Residual voltage.
Principally second harmonic. 43 kV. (See analysis given in report, page 561.)

Compare with Osc. No. 722.

Vibrator No. 3.
Residual current. Principally second and sixth harmonics. 8.3 amperes. (See analysis given in report, page 561.)

Oscillogram No. 710.
Abnormal Condition.

Vibrator No. 1.
50-cycle timing wave.

Vibrator No. 2.
Voltage phase 1 to ground. Principally second harmonic. 15 kV. approx.

Vibrator No. 3.
Current phase 1. Prominent second harmonic. 2.5 amperes approx.

Compare with Osc. No. 711 and No. 712.
Note: End of abnormal condition.

Oscillogram No. 711.
Normal Condition (after abnormal).

Vibrator No. 1.
50-cycle timing wave.

Vibrator No. 2.
Voltage phase 1 to ground. Practically pure fundamental. 4.1 kV. approx.

Vibrator No. 3.
Current phase 1. Principally fundamental. Small eleventh harmonic.

Compare with Osc. No. 710 and No. 712.

Oscillogram No. 712.
Abnormal Condition.

Vibrator No. 1.
50-cycle timing wave.

Vibrator No. 2.
Voltage phase 2 to ground. Principally second harmonic. 14 kV. (See analysis given in report, page 561.)

Compare with Osc. No. 710.

Vibrator No. 3.
Current phase 2. Prominent ninth and second harmonic. 2.5 amperes. (See analysis given in report, page 561.)

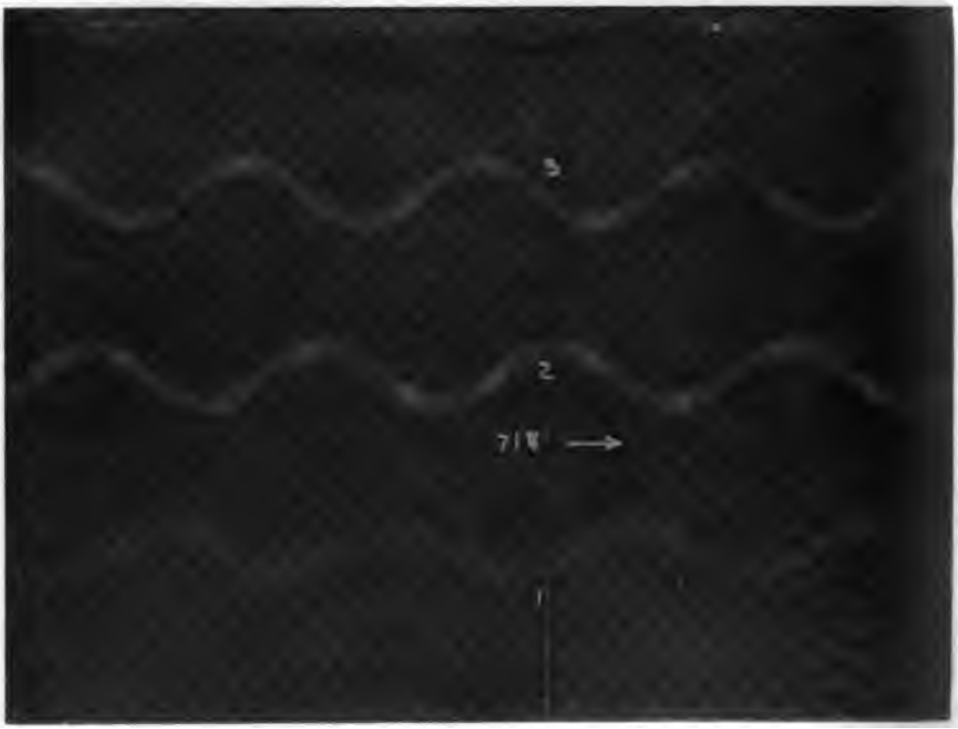
Oscillogram No. 717.
Abnormal Condition.

Vibrator No. 1.
Voltage phase 1 to ground.
15 kV. approx.

Vibrator No. 2.
Voltage phase 2 to ground.
15 kV. approx.

Vibrator No. 3.
Voltage phase 3 to ground.
15 kV. approx.

All phases principally second harmonic. Second harmonic in phase in all phases.
Compare with Osc. No. 721.



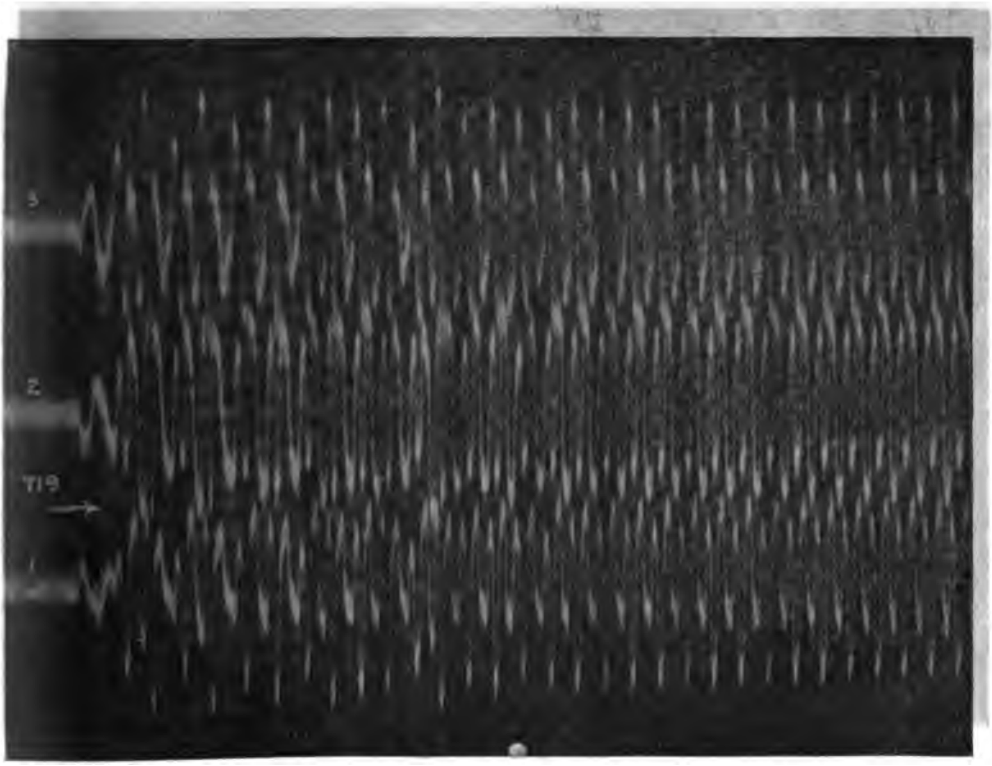
Oscillogram No. 718.
Normal Condition (after abnormal).

Vibrator No. 1.
Voltage phase 1 to ground.
4.1 kV. approx.
Fundamental and low harmonics.

Vibrator No. 2.
Voltage phase 2 to ground.
4.1 kV. approx.
Fundamental and low harmonics.

Vibrator No. 3.
Voltage phase 3 to ground.
4.1 kV. approx.
Fundamental and low harmonics.

Compare with Osc. No. 717.



Oscillogram No. 719.
Beginning of abnormal condition.
Voltages of three phases to ground.

(Phases in order of vibrator numbers.)

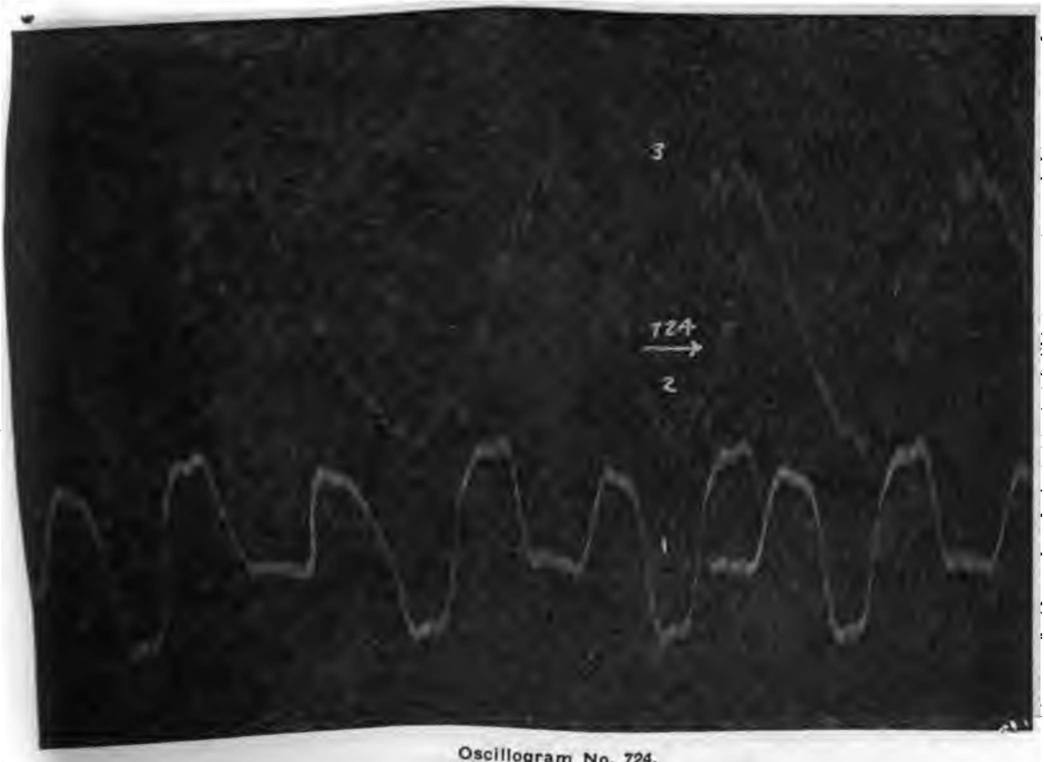
Oscillogram No. 722.
Normal Condition (after abnormal).

Vibrator No. 1.
50-cycle timing wave.

Vibrator No. 2.
Residual voltage. Principally third harmonic. Less than 10 volts.

Vibrator No. 3.
Residual current. Principally third harmonic. Less than 0.05 ampere.

Compare with Osc. No. 707.



Oscillogram No. 724.
Abnormal Condition.

Vibrator No. 1.
Voltage phase 2 to ground. Strong second harmonic.

Vibrator No. 2.
Voltage phase 2 to phase 3. Fundamental and high harmonics. No second harmonic.

Vibrator No. 3.
50-cycle timing wave.

Compare with Osc. No. 725.

Oscillogram No. 725.
Normal Condition (after abnormal).

Vibrator No. 1.
Voltage phase 2 to ground.
No second harmonic.

Vibrator No. 2.
Voltage phase 2 to phase 3. No second harmonic.

Vibrator No. 3.
50-cycle timing wave.

Compare with Osc. No. 724.

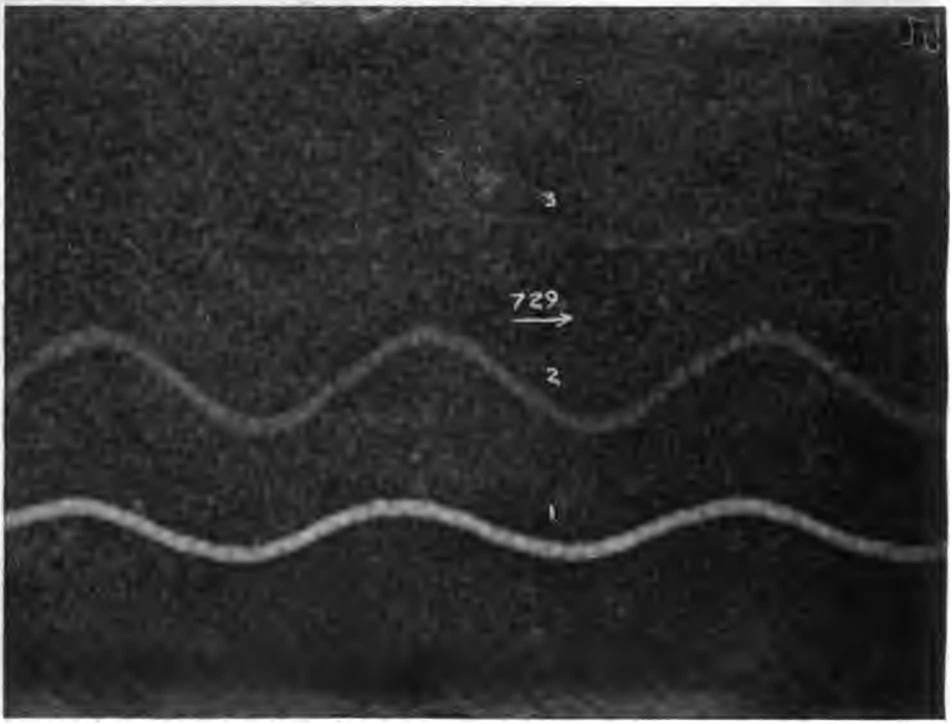
Oscillogram No. 728.
Abnormal Condition.

Vibrator No. 1.
Voltage bus "A" to neutral. Prominent second harmonic. Small fundamental.

Vibrator No. 2.
Voltage bus "A" to bus "C." Almost pure fundamental.

Compare with Osc. No. 729.

Vibrator No. 3.
Current bus "A." Prominent second harmonic.



Oscillogram No. 729.
Normal Condition (after abnormal).

Vibrator No. 1.
Voltage bus "A" to neutral. Fundamental only.

Vibrator No. 2.
Voltage bus "A" to bus "C." Fundamental only.

Vibrator No. 3.
Current bus "A." Fundamental, harmonics small.

Compare with Osc. No. 728.

Line energized by Star-Star Connected Bank.

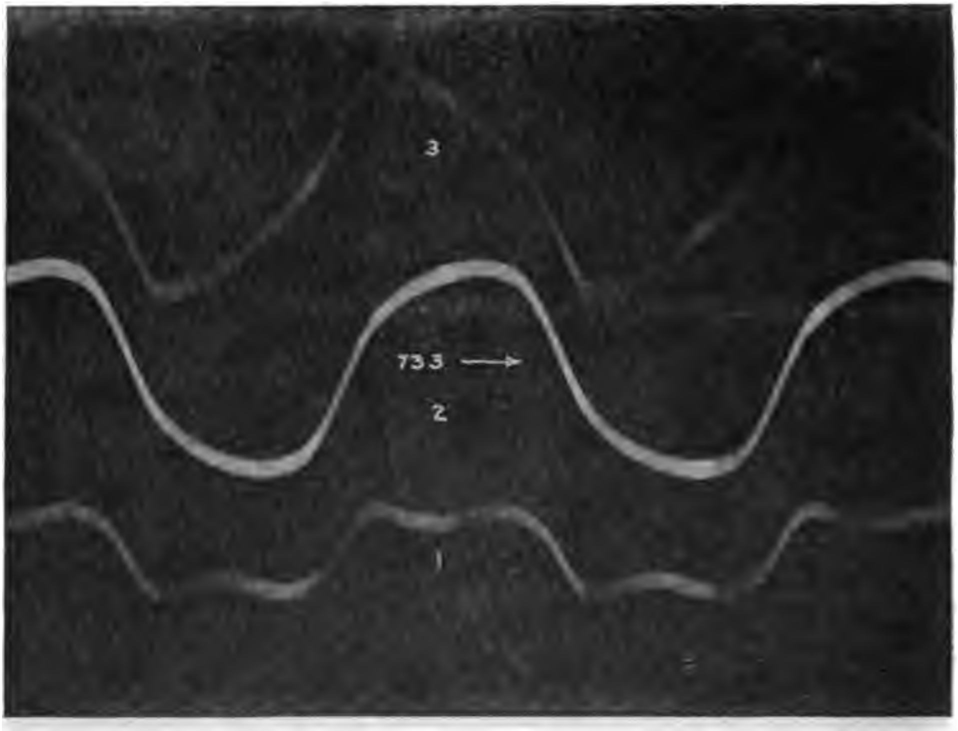
37½-kVA units.

See Fig. 1—Drawing No. 319.

Impressed voltage 43 per cent normal.

III-A-1-c—page 562.

Oscillogram No. 733.



Oscillogram No. 733.
Abnormal Condition.

Vibrator No. 1.	*Vibrator No. 2.*	*Vibrator No. 3.*
Voltage phase 1 to ground.	Voltage phase 2 to ground.	Voltage phase 3 to ground.

Third harmonic prominent.
Same circuit constants in all three vibrators.
Allow ± 5 per cent for differences in sensitivity.
By meter readings phase 1 was 3000, phase 2 was 8640, and phase 3 was 8280 volts during this oscillogram.
Unusual dissimilarity of waves among the voltages from the three line conductors to ground.

Line energized by Star-Star Connected Bank.

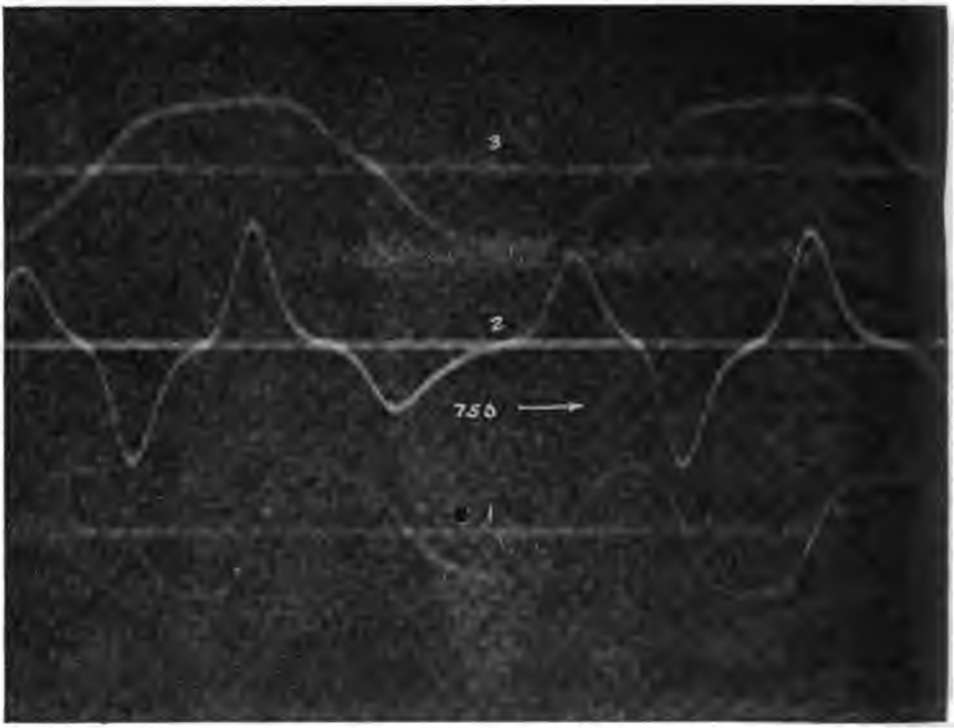
50-kVA units.

See Fig. 1—Drawing No. 319.

Impressed voltage 55 per cent normal.

III-A-2-a—page 563.

Oscillograms Nos. 750, 751, 752, 753.



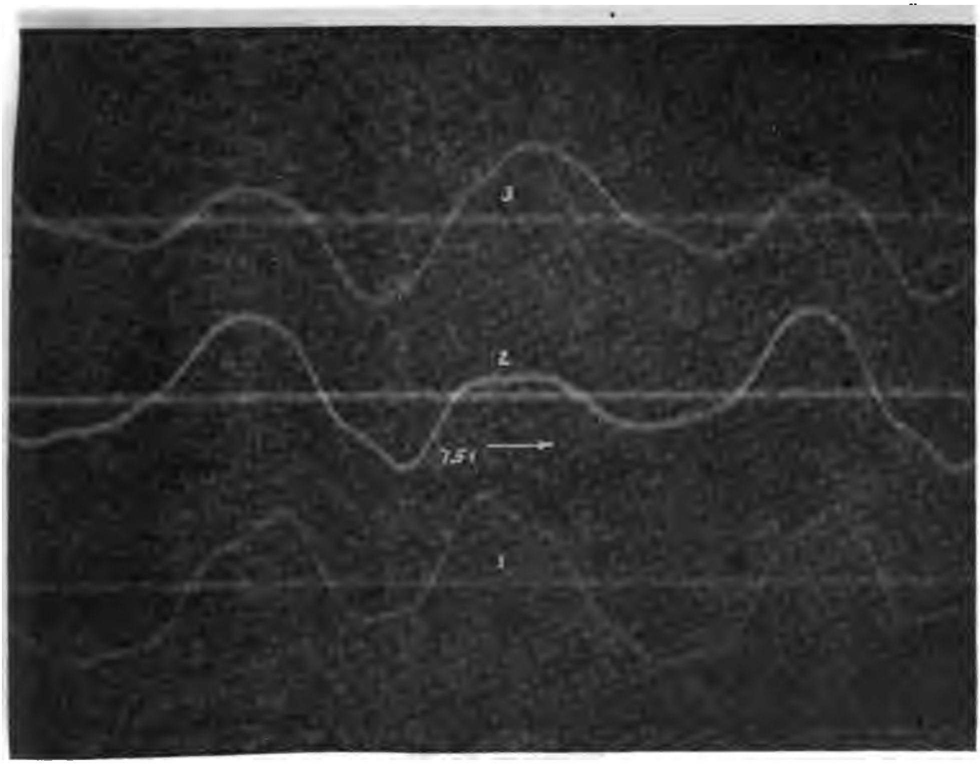
Oscillogram No. 750.
Abnormal Condition.

Vibrator No. 1.
Residual voltage. Prominent second harmonic.

Vibrator No. 2.
Residual current. Prominent second harmonic. Prominent sixth harmonic.

Vibrator No. 3.
50-cycle timing wave. Some second harmonic.

The second harmonic apparently introduced into the Pacific Light and Power system as evidenced by the timing wave.



Oscillogram No. 751.
Abnormal Condition.

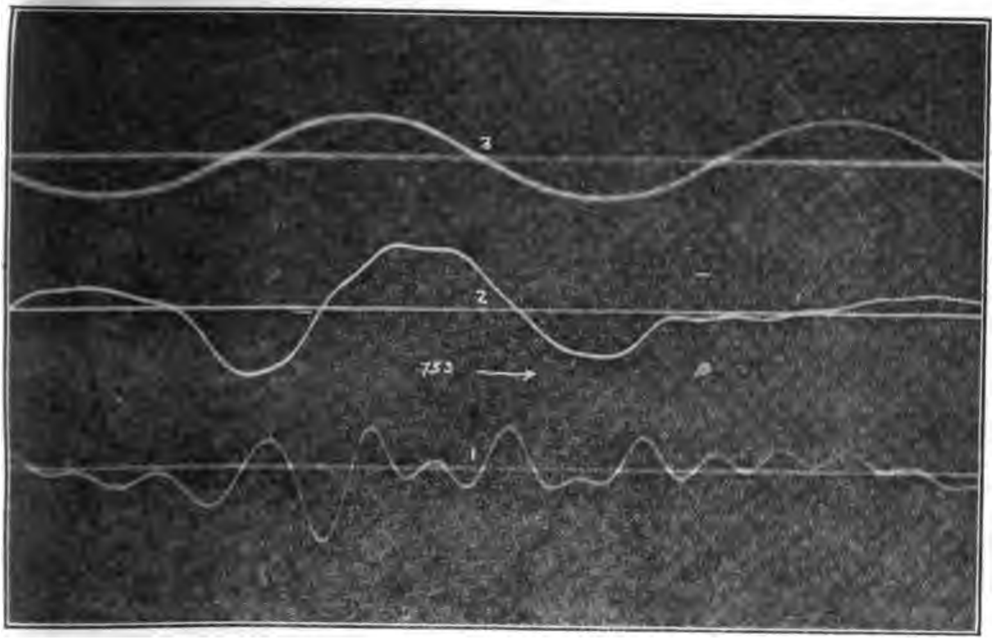
Vibrator No. 1.	*Vibrator No. 2.*	*Vibrator No. 3.*
Voltage phase 1 to ground.	Voltage phase 2 to ground.	Voltage phase 3 to ground.

Prominent second harmonic in phase in all three waves. The comparatively large amount of balanced fundamental voltage causes the dissimilar waves.

Oscillogram No. 752.
Abnormal Condition.

Vibrator No. 1.	*Vibrator No. 2.*	*Vibrator No. 3.*
Current in supply bus "A."	Current in supply bus "B".	Current in supply bus "C."

The whole length of film represents about 4 periods of 50-cycle frequency, *i.e.*, about 0.08 second; the same time as required for Osc. No. 753, so the timing wave on vibrator No. 3 of Osc. No. 753 may be used for studying frequency and harmonics in Osc. No. 752.



Oscillogram No. 753.
End of Abnormal Condition.

Vibrator No. 1.
Current in supply bus "A."

Vibrator No. 2.
Voltage phase 3 to ground.

Vibrator No. 3.
50-cycle timing wave.

Line energized by Delta-Delta Connected Bank.

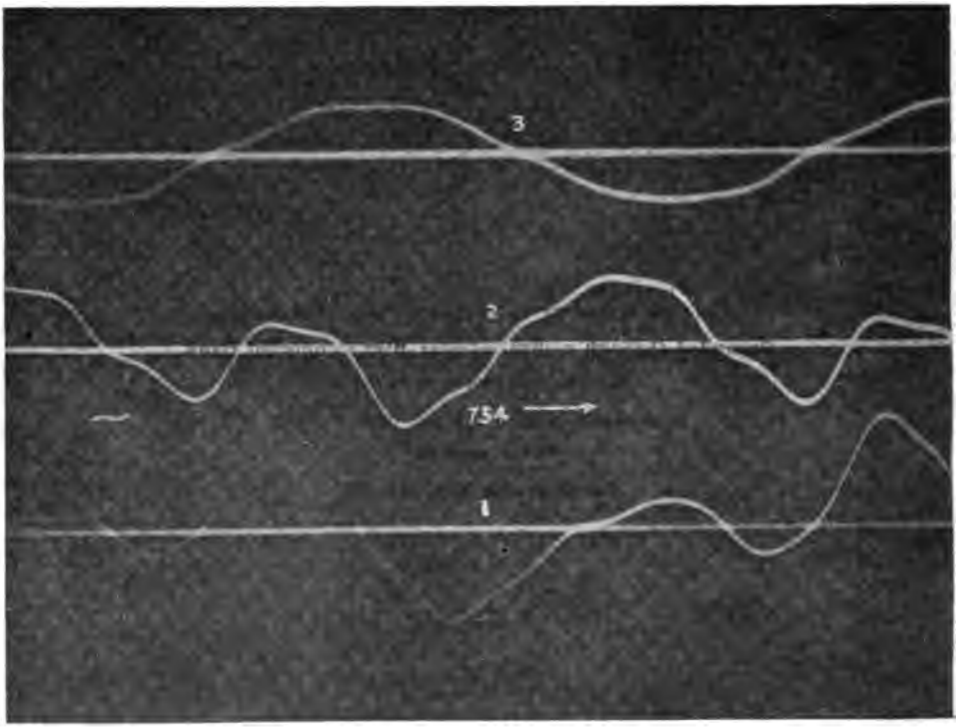
Grounded through neutral of Star-Star Connected Bank.

See Fig. 3—Drawing No. 319.

Impressed voltage 115 per cent normal.

III-B—page 563.

Oscillograms Nos. 754, 755.

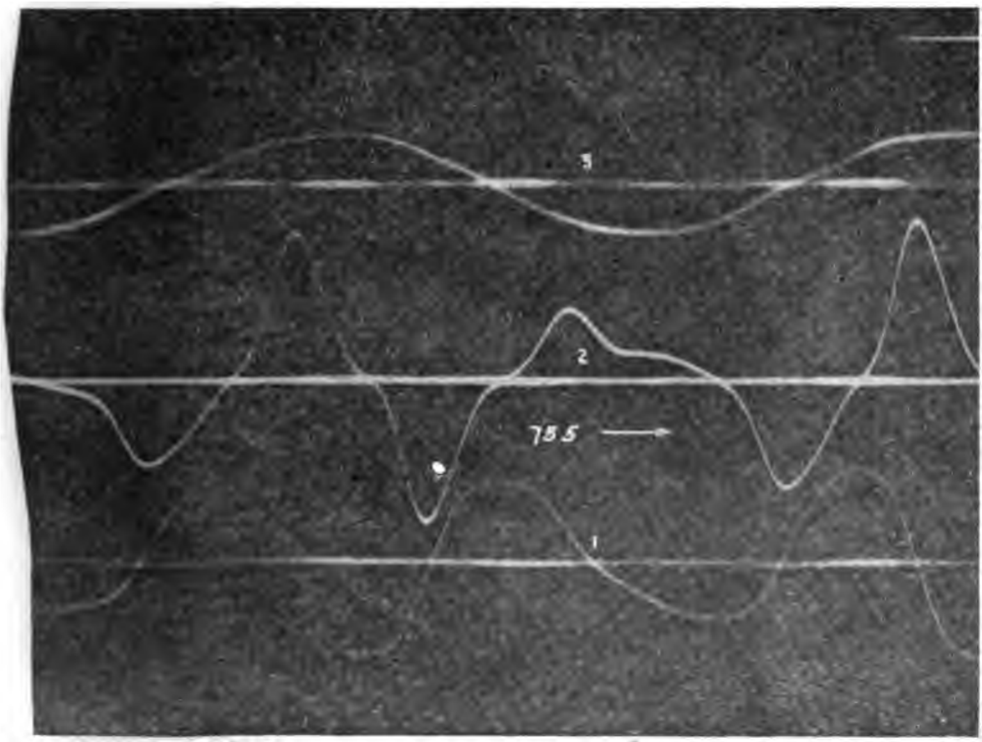


Oscillogram No. 754.
Abnormal Condition.

Vibrator No. 1.
Voltage phase 1 to ground.
Prominent second harmonic.

Vibrator No. 2.
Voltage phase 2 to ground.
Prominent second harmonic.

Vibrator No. 3.
50 - cycle timing wave shows test affected P. L. & P. system.



Oscillogram No. 755.
Abnormal Condition.

Vibrator No. 1.
Residual voltage. Prominent second harmonic.

Vibrator No. 2.
Residual current. Prominent second harmonic. Prominent **sixth harmonic.**

Vibrator No. 3.
50-cycle timing wave.

Technical Report No. 63.

March 8, 1916.

STANDARD FORMS FOR RECORDING DATA AND COMPUTATIONS.

I. Introduction.

At different times during the course of the investigation by this Committee it has been deemed advisable by the field organization to develop standard forms for systematically recording the data and computations involved in frequently recurring types of work. This tendency towards the development and use of standard forms has very naturally increased as the work became better organized, and as greater familiarity with the conditions to be met was gained through experience.

During the course of work at San Fernando the development of standard forms was carried forward so as to provide such means of recording the data and computations incident to practically all commonly occurring tests.

It is the purpose of this report to transmit copies of the various standard forms, to describe them briefly, and to discuss their uses and advantages.

II. Copies and Description of Standard Forms.

Copies of the following standard forms are included herewith.

Title of form.	Form No. (P. I. C.).
Record of Oscillogram	232
Record of (Calibration) Oscillogram	267
Schedule for 6-Ordinate Harmonic Analysis	194
Schedule for 12-Ordinate Harmonic Analysis	239
Schedule for 18-Ordinate Harmonic Analysis.	
Sheet No. 1—original data, results and check. (Printed.)	
Sheet No. 2—computations. (Printed.)	
Schedule for 36-Ordinate Harmonic Analysis.	
Sheet No. 1—original data and results	129
Sheet No. 2—computations	154 and 155
Sheet No. 3—check	197
Record of Circuit Computations	245
Record of Harmonic Analysis by Resonant Shunt	255
Record of Harmonic Comparisons by Resonant Shunt	256
Record of Meter Measurements (small)	235
Record of Meter Measurements (large)	238
Record of Noise Measurements	236
Record of Insulation Measurements	234
Record of Capacitance and Conductance Unbalance Measurements	237
Record of Impedance Measurements	246

Following are brief descriptive notes and comments concerning these forms:

P.I.C. 232

J.C.I.I.

RECORD OF OSCILLOGRAM NO. ______

TEST ______ DATE ______

TIME ______

PLACE ______

GENERAL CONDITIONS ______

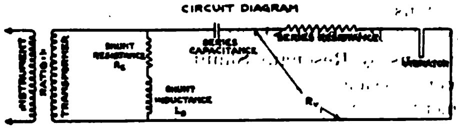
DATA

			1	2	3
VIBRATOR NUMBER			1	2	3
SUBJECT					
CIRCUIT CONSTANTS	INSTRUMENT TRANSFORMER	TYPE			
		IDENTIFICATION			
		RATIO(K)			
	SERIES AND VIB. RESISTANCE R_v	MEASURED	OHMS	OHMS	OHMS
		LEADS	"	"	"
		CORRECTED	"	"	"
	SHUNT RESISTANCE R_s	MEASURED	"	"	"
		LEADS	"	"	"
		CORRECTED	"	"	"
	SERIES CAPACITANCE C_v	CONDENSER			
		SETTING	MICROFARADS	MICROFARADS	MICROFARADS
		CORRECTED	"	"	"
	SHUNT INDUCTANCE L_s	COIL			
		INDUCTANCE	MILLIHENRYS	MILLIHENRYS	MILLIHENRYS
METER READING		IDENTIFICATION			
		TIME			
		READING			
		CORRECTED			
NOTES					
COMMENTS REGARDING ANALYSIS					

GENERAL REMARKS

OSCILLOGRAPH FIELD CURRENT ______ MILLIAMPERES ______ FILM HOLDER NO. ______

CIRCUIT DIAGRAM



CALIBRATION OSCILLOGRAMS:

OPERATOR ______

RECORDER ______

INSTRUCTIONS: MAKE ORIGINAL ENTRIES IN BLACK INK. STATE FULLY ALL CONDITIONS. CORRECTIONS AND COMMENTS MADE SUBSEQUENT TO ORIGINAL RECORD SHOULD BE IN RED INK. AFTER ANALYSIS MOUNT PRINT OF OSCILLOGRAM ON BACK OF THIS FORM AND FILE TOGETHER WITH ANALYSIS AND COMPUTATION SHEETS.

RECORD OF OSCILLOGRAM.

(P. I. C. No. 232.)

This form is used for recording the data of all oscillograms except those taken for purposes of calibration, for which a special form is provided. The number of the oscillogram, date, time, place, name of test, and general conditions are recorded in the upper right-hand corner and across the top. Under the heading of "Data" there are three columns, one for each vibrator, for recording the corresponding subjects (line voltage, residual current, induced current in telephone circuits, etc.), circuit constants and meter readings of the voltages or currents of which the oscillogram is taken, notes, and comments regarding the analysis of the different waves. In order to make the reference to circuit constants clear, a circuit diagram is shown, including all apparatus commonly used in taking oscillograms. If the apparatus referred to is not in circuit the corresponding space in the record is marked with a cross. Provision is made for entering the corrected values of the circuit constants and meter readings, when later determined from calibration data. "General Remarks" applying to the oscillogram as a whole follow the table of data concerning the individual vibrators. The circuit diagram and spaces for reference by number to calibration oscillograms and for initials of ~~operator and recorder occupy the bottom~~ of the form. It is the duty of the operator to check the accuracy and completeness of the the record. The recorder, subsequent to the test, enters the corrected values of circuit constants and meter readings. After completion of the analysis, the measured print of the oscillogram is mounted on the back of the record sheet.

P.I.C. 267
5-27-15

J.C.I.I. RECORD OF OSCILLOGRAM NO. ______

CALIBRATION

DATE ______
TIME ______

TEST ______ PLACE ______

CALIBRATED WITH { DIRECT CURRENT / ALTERNATING CURRENT ______ CYCLES PER SECOND.

OSCILLOGRAPH FIELD CURRENT ______ MILLIAMPERES

VIBRATOR CURRENT (I_v) MILLIAMPERES METER NO.		VIBRATOR* NUMBER	VIBRATOR DEFLECTION (D) MICROMETER UNITS			VIBRATOR CONSTANT (K)
READING	CORRECTED		+	−	AVERAGE	
		1				
		2				
		3				

* ALL VIBRATORS NORMALLY IN SERIES FOR CALIBRATION OSCILLOGRAMS. IF ANY VIBRATOR IS NOT IN USE INDICATE THIS FACT BY CROSSING OUT NUMBER. (+) INDICATES DEFLECTION ABOVE ZERO LINE; (−) DEFLECTION BELOW ZERO LINE.

$K = \frac{1}{\sqrt{2}} \cdot \frac{(I_v)}{D_e}$ (FOR DIRECT CURRENT); $K = \frac{I_v}{D}$ (FOR ALTERNATING CURRENT)

THIS CALIBRATION APPLIES TO OSCILLOGRAMS NUMBERED, ______

REMARKS.

FILM HOLDER NO

OPERATOR ______ RECORDER ______ COMPUTER ______

(MOUNT PRINTS IN THIS SPACE)

INSTRUCTIONS — MAKE ALL ORIGINAL ENTRIES IN BLACK INK. COMPUTED VALUES AND CORRECTIONS MADE SUBSEQUENT TO ORIGINAL RECORD SHOULD BE SHOWN IN RED.

RECORD OF (CALIBRATION) OSCILLOGRAM.

(P. I. C. No. 267.)

Provision is made for the number of the oscillogram, date, time, place, name of test, whether calibration is with direct or alternating current, and if the latter, the frequency in cycles per second; also the value of the oscillograph field current and vibrator current. The deflections are measured, for each vibrator, from a print of the oscillogram and entered in the spaces provided and the vibrator constant computed (from formulas given) and recorded. The numbers of the oscillograms to which the calibration applies are entered as a part of the record. Provision is made for remarks and the initials of the operator, recorder and computer. Space is provided for mounting the print of the oscillogram on the face of the record sheet.

P.I.C. No 194

SCHEDULE FOR SIX ORDINATE HARMONIC ANALYSIS.

OSC. No ______ VIB. No ______

ORIGINAL DATA								RESULTS			
n	y_n	y_n	n	s_n	n	d_n		n	A_n	B_n	C_n
0					0			1			
1			5		1			3			
2			4		2			5			
3					3						

$s_1' = s_1 - s_3$

$d_0' = d_0 - d_2$

		SINE TERMS			COSINE TERMS		
		1	5	3	1	5	3
SINE 30°	0.500	s_1			d_2		
" 60°	0.866		s_2			d_1	
" 90°	1.000	s_0		s_1'	d_0		d_0'
I = SUM 1st COLS.							
II = SUM 2nd COLS.							
I + II							
I − II							
⅓ (I + II)		A_1		A_3	B_1		B_3
⅓ (I − II)			A_5			B_5	

CHECK ON HARMONIC ANALYSIS.

$y_0 = B_1 + B_3 + B_5$

$y_1 = \frac{1}{2}(A_1 + A_5) + A_3 + 0.866(B_1 - B_5)$

$y_2 = 0.866(A_1 - A_5) + \frac{1}{2}(B_1 + B_5) - B_3$

$y_3 = A_1 - A_3 + A_5$

$y_4 = 0.866(A_1 - A_5) - \frac{1}{2}(B_1 + B_5) + B_3$

$y_5 = \frac{1}{2}(A_1 + A_5) + A_3 - 0.866(B_1 - B_5)$

$y_6 = -y_0$

	ORDINATES	
n	COMPUTED	DATA
0		
1		
2		
3		
4		
5		
6		

R.G.H.

SCHEDULE FOR 6-ORDINATE HARMONIC ANALYSIS.

(P. I. C. No. 194.)

This form provides on one sheet for the analysis of waves where it is reasonably certain that the harmonics above the fifth are negligible. The method is similar to that described in technical report No. 41 for 18 ordinates. Space is provided for the original data, consisting of the 6 measured ordinates, the necessary computations, the results and a check of the analysis. Only odd harmonics are considered. As this schedule requires less time than those based on a larger number of ordinates it should be used whenever the absence of harmonics above the fifth permits.

P.I.C. No. 239
3-15-15

J.C.I.I.

RECORD OF ANALYSIS

OSCILLOGRAM No._____ VIBRATOR No._____

ORIGINAL DATA								RESULTS $C_n = \sqrt{A_n^2 + B_n^2}$				
n	y_n	y_n	n		S_n	n	d_n	n	A_n	B_n	C_n	$100C_n/\sqrt{\Sigma C_n^2}$
0						0		1				
1			11			1		3				
2			10			2		5				
3			9			3		7				
4			8			4		9				
5			7			5		11				
6						6						

$S'_1 = S_1 + S_3 - S_5 =$
$S'_2 = S_2 - S_6 =$

$d'_1 = d_1 - d_3 - d_5 =$
$d'_0 = d_0 - d_4 =$

SCHEDULE OF 12 ORDINATE ANALYSIS.

n	SINE $\frac{n\pi}{12}$	SINE TERMS 1	11	3	9	5	7	COSINE TERMS 1	11	3	9	5	7
1	0.2588	S_1				S_5			d_5				d_1
2	0.5000		S_2				S_2	d_4				d_4	
3	0.7071	S_3		S'_1		$-S_3$			d_3		d'_1		$-d_3$
4	0.8660		S_4				$-S_4$	d_2				$-d_2$	
5	0.9659	S_5				S_1			d_1				d_5
6	1.0000		S_6		S'_2		S_6	d_0		d'_0		d_0	
I = Σ 1st.													
II = Σ 2nd.													
I + II													
I − II													
½(I+II)		A_1		A_3		A_5		B_1		B_3		B_5	
½(I−II)			A_{11}		A_9		A_7		B_{11}		B_9		B_7

CHECK ON ANALYSIS

SINE TERMS						COSINE TERMS						n	ORDINATE	
	+	−		+	−		+	−		+	−		COMP	DATA
A_1						B_1								
$-A_3$			A_3						B_3			0		
A_5			$-A_3$						B_5			3		
$-A_7$						B_7						6		
A_9						B_9						9		
$-A_{11}$			A_{11}						B_{11}			12		
$Y_6 =$			$F =$			$H =$			$J =$					

$D = Y_6 + 2F =$
$M = H - J =$

$Y_0 = H + J =$
$Y_{12} = -Y_0 =$

$Y_3 = 0.707(D+M) =$
$Y_9 = 0.707(D-M) =$

MEASURED BY__________
COMPUTED BY__________
DATE FINISHED__________

SCHEDULE FOR 12-ORDINATE HARMONIC ANALYSIS.

(P. I. C. No. 239.)

This form provides on one sheet for the analysis of a wave in which the harmonics above the eleventh are negligible. Only odd harmonics are considered. Provision is made for a check of the analysis. The use of the form is similar to the use of the 18-ordinate forms described in technical report No. 41 but the time required is somewhat less.

CURVE NO.................

VIBRATOR NO.................

18-Ordinate Analysis, Sheet No. 1.

ORIGINAL DATA

RESULTS $F = \sqrt{\Sigma(A^2_n + B^2_n)} =$ ....

$0.707\ F = G =$ ....

n	Y_n	Y_n	n	----SIN X	S_n	n	d_n		n	A_n	B_n	C_n	100 C_n/F
0						0							
1			17			1		$S'_1 = S_1 + S_5 - S_7 =$	1				
2			16			2		$S'_2 = S_2 + S_4 - S_8 =$	3				
3			15			3		$S'_3 = S_3 - S_9 =$	5				
4			14			4		$S''_1 = S'_1 - S'_3 =$	7				
5			13			5		$d'_0 = d_0 - d_6 =$	9				
6			12			6		$d'_1 = d_1 - d_5 - d_7 =$	11				
7			11			7		$d'_2 = d_2 - d_4 - d_8 =$	13				
8			10			8		$d''_0 = d'_0 - d'_3 =$	15				
9						9			17				

CHECK ON HARMONIC ANALYSIS

SINE TERMS											COSINE TERMS											CHECK		
n	A_n	n	+	−	n	+	−	n	+	−	n	B_n	n	+	−	n	+	−	n	+	−	n	ORD. COMP.	ORD. DATA
1		1									1		1									0		
3					3						3								3			3		
5		5						5			5					5						6		
7		−7						−7			7					7						9		
9					−9						9								9			12		
11		−11									11		11									15		
13		13									13		13									18		
15					15						15								15					
17		17						17			17					17								
		A=			B=			F=					H=			J=			K=					

$D = A - 2F =$

$Y_0 = H + J + K =$

$M = H - J =$

$Y_3 = 0.5A + B + 0.866M =$

$Y_{15} = 0.5A + B - 0.866M =$

$Y_{18} = -Y_0 =$

$Y_6 = 0.866D + 0.5Y_0 - 1.5K =$

$Y_{12} = 0.866D - 0.5\ Y_0 + 1.5K =$

$Y_9 = A - B =$

Computed by..............................

Date..............................

SCHEDULE FOR 18-ORDINATE HARMONIC ANALYSIS.

(2 Printed Forms.)

Sheet No. 1—Provision is made for the original data (measured ordinates), a portion of the computations, the final results and a check of the analysis. This form is limited in its use to waves containing only odd harmonics not higher than the seventeenth. The harmonic analysis of waves by the use of this form, in conjunction with Sheet No. 2, is described in detail in technical report No. 41.

CURVE NO.................

18-Ordinate Analysis, Sheet No. 2.

		1	17	3	15	5	13	7	11	9
SIN 10°	0.1737	s_1				$-s_7$		$-s_5$		
" 20°	0.3420		s_2				$-s_4$		$-s_8$	
" 30°	0.5000	s_3		s'_1		s_3		$-s_3$		
" 40°	0.6428		s_4				s_8		s_2	
" 50°	0.7660	s_5				s_1		s_7		
" 60°	0.8660		s_6		s'_2		$-s_6$		s_6	
" 70°	0.9397	s_7				$-s_5$		s_1		
" 80°	0.9848		s_8				s_2		$-s_4$	
" 90°	1.0000	s_9		s'_3		s_9		$-s_9$		s''_1
I = SUM 1st COLS.										
II = " 2d COLS.										
I + II										
I − II										
1/9 (I + II)		A_1		A_3		A_5		A_7		A_9
1/9 (I − II)			A_{17}		A_{15}		A_{13}		A_{11}	

VIBRATOR NO.................

		1	17	3	15	5	13	7	11	9
SIN 10°	0.1737	d_8				$-d_2$		d_4		
" 20°	0.3420		d_7				$-d_5$		d_1	
" 30°	0.5000	d_6		d'_2		d_6		d_6		
" 40°	0.6428		d_5				d_1		$-d_7$	
" 50°	0.7660	d_4				d_8		$-d_8$		
" 60°	0.8660		d_3		d'_1		$-d_3$		$-d_3$	
" 70°	0.9397	d_2				$-d_4$		$-d_2$		
" 80°	0.9848		d_1				d_7		d_5	
" 90°	1.0000	d_0		d'_0		d_0		d_0		d''_0
I = SUM 1st COLS.										
II = " 2d COLS.										
I + II										
I − II										
1/9 (I + II)		B_1		B_3		B_5		B_7		B_9
1/9 (I − II)			B_{17}		B_{15}		B_{13}		B_{11}	

Computed by.................

Date.................

SCHEDULE FOR 18-ORDINATE HARMONIC ANALYSIS.

(2 Printed Forms.)

Sheet No. 2—Upon this sheet are performed most of the computations of the Fourier coefficients as described in detail in technical report No. 41. Used in conjunction with Sheet No. 1.

CURVE NO.------ VIBRATOR NO.------- REMARKS --------------------------------

ORIGINAL DATA								RESULTS $F = \sqrt{\Sigma(A_n^2 + B_n^2)} =$ ------- $G = 0.707F =$ ----------					
n	Yn	Yn	n	----Sin X	Sn	n	Dn	n	An	Bn	Cn	100 Cn/F	
1			35			1		1					
2			34			2		3					
3			33			3		5					
4			32			4		7					
5			31			5		9					
6			30			6		11					
7			29			7		13					
8			28			8		15					
9			27			9		17					$S_1' = S_1 + S_{11} - S_{13} =$
10			26			10		19					$S_2' = S_2 + S_{10} - S_{14} =$
11			25			11		21					$S_3' = S_3 + S_9 - S_{15} =$
12			24			12		23					$S_4' = S_4 + S_8 - S_{16} =$
13			23			13		25					$S_5' = S_5 + S_7 - S_{17} =$
14			22			14		27					$S_6' = S_6 - S_{18} =$
15			21			15		29					$S_1'' = S_1' + S_5' - S_3' =$
16			20			16		31					$S_2'' = S_2' - S_6' =$
17			19			17		33					$d_0' = d_9 - d_{18} =$
18						18		35					$d_1' = d_1 - d_{11} - d_{13} =$
													$d_2' = d_2 - d_{10} - d_{14} =$
0						0							$d_3' = d_3 - d_9 - d_{15} =$
													$d_4' = d_4 - d_8 - d_{16} =$
													$d_5' = d_5 - d_7 - d_{17} =$
													$d_1'' = d_1' - d_5' - d_3' =$
													$d_0'' = d_0' - d_4' =$
													COMPUTED BY------------ DATE--------

CURVE NO VIB. NO

SCHEDULE FOR 36-ORDINATE HARMONIC ANALYSIS.

(3 sheets.)

The time required for the analysis of a wave by the 36-ordinate schedule is, roughly, three times that required for the 18-ordinate schedule. The 36-ordinate schedule should therefore not be used unless the presence of harmonics of a higher order than the seventeenth makes it necessary. This schedule was the first developed for use in the Committee's work. The other schedules, samples of which have been given, were developed in order to shorten the time required for analysis in cases where this longer schedule was not necessary. All of these schedules for harmonic analysis are based on well known principles and are to a large extent adaptations of previously published forms.

Sheet No. 1 (P. I. C. No. 129)—This form provides for a record of measured ordinates, final results and part of the computations.

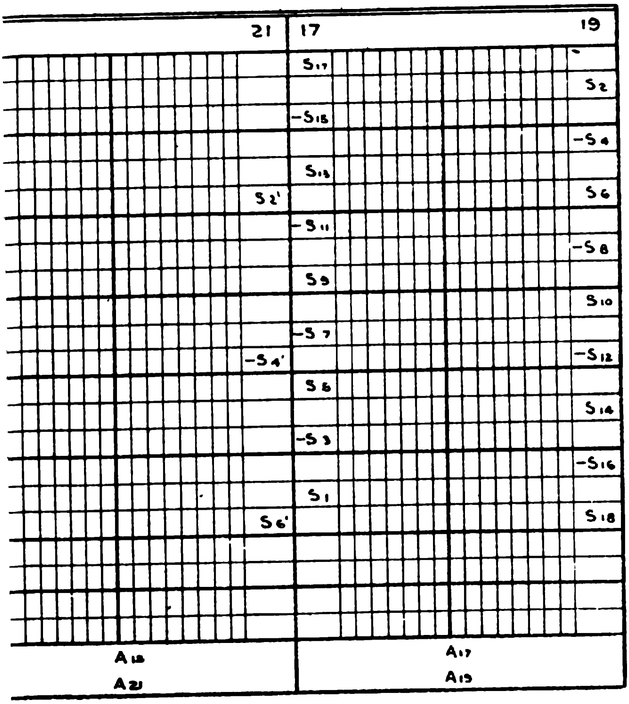
SCHEDULE FOR 36-ORDINATE HARMONIC ANALYSIS.

Sheet No. 2 (P. I. C. Nos. 154 and 155)—Provision is made on these forms for most of the computations involved in determining the 17 sine and 17 cosine coefficients in the Fourier series representing the irregular wave.

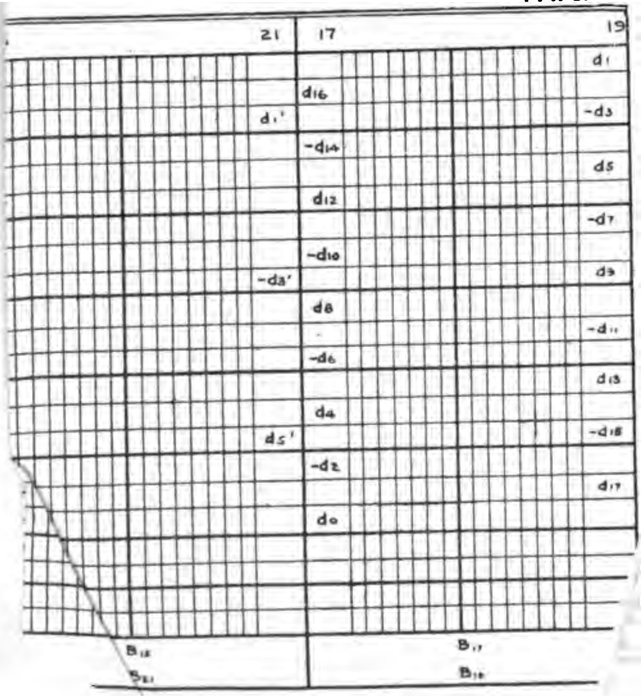
		1	35	3	3.
SIN 5 DEG.	0.0872	S_1			
" 10 "	0.1736		S_2		
" 15 "	0.2588	S_3		S_1'	
" 20 "	0.3420		S_4		
" 25 "	0.4226	S_5			
" 30 "	0.5000		S_6		S
" 35 "	0.5736	S_7			
" 40 "	0.6428		S_8		
" 45 "	0.7071	S_9	.	S_3'	
" 50 "	0.7660		S_{10}		
" 55 "	0.8192	S_{11}			
" 60 "	0.8660		S_{12}		S
" 65 "	0.9063	S_{13}			
" 70 "	0.9397		S_{14}		
" 75 "	0.9659	S_{15}		S_5'	
" 80 "	0.9848		S_{16}		
" 85 "	0.9962	S_{17}			
" 90 "	1.0000		S_{18}		S
I SUM 1ST COLS					
II " 2ND "					
I+II					
I−II					
$\frac{1}{18}$(I+II)		A_1		A_3	
$\frac{1}{18}$(I−II)		A_{35}		A_{33}	

		1	35	3
SIN 5 DEG	0.0872		d_{17}	
" 10 "	0.1736	d_{16}		
" 15 "	0.2588		d_{15}	
" 20 "	0.3420	d_{14}		
" 25 "	0.4226		d_{13}	
" 30 "	0.5000	d_{12}		d_4'
" 35 "	0.5736		d_{11}	
" 40 "	0.6428	d_{10}		
" 45 "	0.7071		d_9	
" 50 "	0.7660	d_8		
" 55 "	0.8192		d_7	
" 60 "	0.8660	d_6		d_2'
" 65 "	0.9063		d_5	
" 70 "	0.9397	d_4		
" 75 "	0.9659		d_3	
" 80 "	0.9848	d_2		
" 85 "	0.9962		d_1	
" 90 "	1.0000	d_0		d_0'
I SUM 1ST COLS				
II 2ND "				
I + II				
I − II				
$\frac{1}{18}$(I + II)		B_1		B_3
$\frac{1}{18}$(I − II)		B_{35}		B_{33}

P. I. C. 154



P. I. C. 155



P.I.C. NO. 197
4-22-14

CHECK ON 36 ORDINATE ANALYSIS

SINE TERMS

OSC. NO.______ VIB. NO.______

	+	−		+	−		+	−
A_1			A_3			A_5		
$-A_{23}$			$-A_{21}$			$-A_{19}$		
A_{25}			A_{27}			A_{29}		
A			B			C		

n	ORDINATES	
	COMP	DATA
0		
6		
9		
12		
18		
24		
27		
30		
36		

	+	−		+	−		+	−
A_7			A_9			A_{11}		
$-A_{17}$			$-A_{15}$			$-A_{13}$		
A_{31}			A_{33}			A_{35}		
D			E			F		

	+	−		+	−		+	−			
A			B			C			C		
-F			-E			-D			-E		
									-F		
H			J			K			L		

COSINE TERMS

	+	−		+	−		+	−
B_1			B_3			B_5		
B_{23}			B_{21}			B_{19}		
B_{25}			B_{27}			B_{29}		
M			N			O		

	+	−		+	−		+	−
B_7			B_9			B_{11}		
B_{17}			B_{15}			B_{13}		
B_{31}			B_{33}			B_{35}		
P			Q			R		

$\alpha = H + K =$

$\beta = \alpha + J - 2L =$

$\gamma = \alpha - 2K =$

$\delta = S - U =$

$\epsilon = Y_0 - 2V =$

$Y_0 = S + T + U$

$Y_6 = 0.5\alpha + J + 0.866\delta$

$Y_9 = 0.707\,(\beta + \epsilon)$

$Y_{12} = 0.866\gamma + 0.5\,Y_0 - 1.5T$

$Y_{18} = \alpha - J$

REVERSE SIGNS OF COSINE TERMS FOR SECOND QUADRANT

	+	−		+	−		+	−		+	−
M			N			O			N		
R			Q			P			O		
									R		
S			T			U			V		

SCHEDULE FOR 36-ORDINATE HARMONIC ANALYSIS.

(3 sheets.)

Sheet 3 (P. I. C. No. 197)—This form is used in checking the analysis

P.I.C. 8248 3/19/15

NOTE: SEE SPECIAL INSTRUCTIONS FOR USE OF FORM.

J.C.I.I. RECORD OF CIRCUIT COMPUTATIONS. OSC. No ____

VIBRATOR NUMBER 1	SUBJECT	CORRECTED CONSTANTS	n	C_n													
			1														
		k ____ (RATIO)	3														
		R_v ____ OHMS	5														
		R_s ____ OHMS	7														
		C_v ____ MICRO-FARADS	9														
		L_s ____ HENRYS	11														
		K_o ____	13														
			15														
		METER VALUE	17														
			E.S.V.														
VIBRATOR NUMBER 2	SUBJECT	CORRECTED CONSTANTS	n	C_n													
			1														
		k ____ (RATIO)	3														
		R_v ____ OHMS	5														
		R_s ____ OHMS	7														
		C_v ____ MICRO-FARADS	9														
		L_s ____ HENRYS	11														
		K_o ____	13														
			15														
		METER VALUE	17														
			E.S.V.														
VIBRATOR NUMBER 3	SUBJECT	CORRECTED CONSTANTS	n	C_n													
			1														
		k ____ (RATIO)	3														
		R_v ____ OHMS	5														
		R_s ____ OHMS	7														
		C_v ____ MICRO-FARADS	9														
		L_s ____ HENRYS	11														
		K_o ____	13														
			15														
		METER VALUE	17														
			E.S.V.														

COMPUTER ____ DATE ____

RECORD OF CIRCUIT COMPUTATIONS.

(P. I. O. No. 245.)

This form is used for determining the values in electrical units of the various harmonics of the voltages and currents from the results of the harmonic analyses of the waves, the vibrator constants as determined by calibration oscillograms, and the corrected circuit constants given on the original record of the oscillogram.

The subject for each vibrator is copied from the record made at the time the oscillogram was taken. The corrected circuit constants, including the vibrator constants (K_e) are entered in the column headed "Corrected Constants." The corrected meter reading of the quantity measured is also entered for comparison with the E. S. W. values as given by the oscillogram. The computations for the individual harmonics are carried out horizontally across the form. From the results of the analyses, the maximum values of the harmonics in terms of the units used in measuring the ordinates of the waves and the calibration oscillograms, are placed in the column headed C_n. The headings of other columns to the right of that headed C_n are left blank, to be appropriately filled in to indicate the steps in the computations for the particular circuit arrangement used with each vibrator.

The form is designed for three waves with harmonics not higher than the seventeenth, which is ordinarily sufficient.

INSTRUCTIONS - MAKE ALL ORIGINAL ENTRIES IN BLACK INK. COMPUTED VALUES AND CORRECTIONS MADE SUBSEQUENT TO ORIGINAL RECORD SHOULD BE SHOWN IN RED.

P.I.C. 8258
4/7/15

J.C.I.I. RECORD OF HARMONIC ANALYSIS BY RESONANT SHUNT.

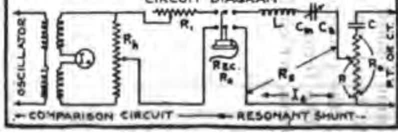
SHEET Nº ____
DATE ____
PLACE ____

TEST ____

GENERAL CONDITIONS ____

	TEST DATA												COMPUTATIONS AND RESULTS							INSTRUMENT TRANSFORMER TYPE ____ RATIO(K) ____
SUBJECT ____ OBSERVER ____ SHUNT Nº ____	TIME	n	f	I_o	C_m	C_s	R_l	R_h	R	R_o	C		I_s	R_s						NOTES
	HOUR MIN. A.M./P.M.		$\sim$	mA READ. CORR.	μF		OHMS	OHMS	OHMS	OHMS	μF NOM. ACT.		μA	OHMS						RECORDER ____ COMPUTER ____
SUBJECT ____ OBSERVER ____ SHUNT Nº ____	TIME	n	f	I_o	C_m	C_s	R_l	R_h	R	R_o	C		I_s	R_s						INSTRUMENT TRANSFORMER TYPE ____ RATIO(K) ____ NOTES
	HOUR MIN. A.M./P.M.		$\sim$	mA READ. CORR.	μF		OHMS	OHMS	OHMS	OHMS	μF NOM. ACT.		μA	OHMS						RECORDER ____ COMPUTER ____

CIRCUIT DIAGRAM



FORMULAS

$I_s = \frac{I_o R_s}{R_s + R_o}$ (APPROX); $E_{oc} = I_s(R_s + R_l)$; $I_{sc} = \frac{E_{oc}}{\sqrt{R_l^2 + X_l^2}}$

$I = K(I_s)\frac{R_s}{R}$ (APPROX); $E = K(I_s)\frac{R_s}{R}\sqrt{R_o^2 + \frac{1}{(2\pi f C)^2}}$ (APPROX)

NOTE - CONDENSER C IS OMITTED WHEN MAKING A CURRENT OR INDUCTION MEASUREMENT. R_o MAY BE OMITTED WHEN MEASURING INDUCTION.

NOTATION

I_s - CURRENT THROUGH RESONANT SHUNT.
R_s - RESISTANCE OF RESONANT SHUNT.
I_o - SUPPLY CURRENT.
E_{oc} - OPEN CIRCUIT VOLTAGE.
I_{sc} - SHORT CIRCUIT CURRENT.
R_l - EFFECTIVE RESISTANCE - LINE
X_l - EFFECTIVE REACTANCE - LINE
E - VOLTAGE, POWER CIRCUIT
I - CURRENT, POWER CIRCUIT

RECORD OF HARMONIC ANALYSIS BY RESONANT SHUNT.

(P. I. C. No. 255.)

This form provides for a complete record of the data, computations and results of two harmonic analyses made by means of the resonant shunt, the principles and use of which are described in technical report No. 41. The circuit diagram with explanation of the notation used and the necessary formulas for reduction of the data are given on the form.

P.I.C. #256
4/13/15

INSTRUCTIONS – MAKE ALL ORIGINAL ENTRIES IN BLACK INK. COMPUTED VALUES AND CORRECTIONS MADE SUBSEQUENT TO ORIGINAL RECORD SHOULD BE SHOWN IN RED.

J.C.I.I. RECORD OF HARMONIC COMPARISONS BY RESONANT SHUNT. SHEET No. ____

DATE ____
TEST ____ PLACE ____
GENERAL CONDITIONS ____

SUBJECT ____ OBSERVER ____ SHUNT No. ____

TEST DATA															COMPUTATIONS AND RESULTS							
TIME			n	f	C_m	C_a	R_1	R_b	R_a'	R_b'	C_1		C_2									
		A.M. P.M.		$\sim$	μF		OHMS	OHMS	OHMS	OHMS	μF NOM.	ACT.	μF NOM.	ACT.								

INSTRUMENT TRANSFORMER
TYPE ____ RATIO(K_1) ____
TYPE ____ RATIO(K_2) ____

NOTES

RECORDER ____
COMPUTER ____

SUBJECT ____ OBSERVER ____ SHUNT No. ____

TIME			n	f	C_m	C_a	R_1	R_b	R_a	R_b'	C_1		C_2									
		A.M. P.M.		$\sim$	μF		OHMS	OHMS	OHMS	OHMS	μF NOM.	ACT.	μF NOM.	ACT.								

INSTRUMENT TRANSFORMER
TYPE ____ RATIO(K_1) ____
TYPE ____ RATIO(K_2) ____

NOTES

RECORDER ____
COMPUTER ____

FORMULAS (EXACT)

$$\frac{E_1}{E_2}=\frac{K_1Y_0Z_1}{K_2Y_0'Z_2} \quad (1)$$

$$\frac{I_1}{I_2}=\frac{K_1Y_0}{K_2Y_0'} \quad (2)$$

$$\frac{E_{oc}}{I_1}=\frac{Y_0'Z}{K_1Y_0} \quad (3)$$

$$\frac{E_{oc}}{E_1}=\frac{Y_0'Z}{K_1Y_0Z_1} \quad (4)$$

$$Y_0=\left(\frac{1}{R_a}+\frac{1}{R_b}\right) \quad (5)$$

$$Y_0'=\left(\frac{1}{R_a'}+\frac{1}{R_b'}\right) \quad (6)$$

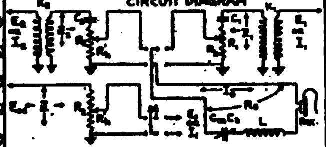
$$Z_0=\left[(R_1+(R_b-R_b')+\frac{R_a'R_b}{R_a'+R_b}\right]+jX_0 \quad (7)$$

$$Z_1=\left[(R_1-R_a)+\frac{R_aR_b}{R_a+R_b}\right]-j\frac{1}{\omega C_1} \quad (8)$$

$$Z_2=\left[(R_2-R_a)+\frac{R_aR_b}{R_a+R_b}\right]-j\frac{1}{\omega C_2} \quad (9)$$

NOTE – E_{oc} IS LONGITUDINAL OPEN CIRCUIT VOLTAGE.

CIRCUIT DIAGRAM



NOTE – OMIT C_1 AND C_2 WHEN COMPARING CURRENT.

FORMULAS (APPROX.)

$$\frac{E_1}{E_2}=\frac{K_1R_0'Z_1}{K_2R_0Z_2} \quad (10)$$

$$\frac{I_1}{I_2}=\frac{K_1R_0'}{K_2R_0} \quad (11)$$

$$\frac{E_{oc}}{I_1}=\frac{R_0Z}{K_1R_0'} \quad (12)$$

$$\frac{E_{oc}}{E_1}=\frac{R_0Z}{K_1R_0'Z_1} \quad (13)$$

$$Z_0=(R_1+R_b)+jX_0 \quad (14)$$

$$Z_1=R_1-j\frac{1}{\omega C_1} \quad (15)$$

$$Z_2=R_2-j\frac{1}{\omega C_2} \quad (16)$$

NOTE – E_{oc} IS LONGITUDINAL OPEN CIRCUIT VOLTAGE.

NOTE

WHEN COMPARING TRANSVERSE INDUCED VOLTAGE WITH POWER VOLTAGE OR CURRENT SHIELDED RECEIVER CIRCUIT MUST BE USED AND THE FORMULAS INVOLVING E_{oc} ARE MODIFIED SOMEWHAT.

RECORD OF HARMONIC COMPARISON BY RESONANT SHUNT.

(P. I. C. No. 256.)

It is sometimes desirable to make a direct comparison of the magnitudes of corresponding harmonics of two related quantities, as for examples, harmonics on primary and secondary sides of current transformers for ratio calibrations, and harmonics of residual current of a power circuit and longitudinal electromagnetic induction in a parallel communication circuit. Where a knowledge of the absolute magnitude of the two quantities is not essential their ratio can be determined more accurately, when the resonant shunt is used, by the method of direct comparison. This form provides space for a record of the data, computations and the results of two such comparisons. The use of the form will be apparent from the notation at the heading of the columns, the circuit diagram, and the formulas.

P.I.C. 235

INSTRUCTIONS: MAKE ALL ORIGINAL ENTRIES IN BLACK INK. COMPUTED VALUES AND CORRECTIONS MADE SUBSEQUENT TO ORIGINAL RECORD SHOULD BE SHOWN IN RED.

J.C.I.I. RECORD OF METER MEASUREMENTS.

DATE ____________

PLACE ____________

TEST ____________

GENERAL CONDITIONS ____________

NUMBER	TIME			A		B		C		D		E		F		G		MULTIPLIER					REFER OSC. NO.	NOTES.
METER TYPE / NO.																								
INST. TRAN. TYPE / RATIO																								
SUBJECT																								
	HOUR	MIN.	A.M. P.M.	READ	CORR.	READ	CORR.	READ	CORR.	READ	CORR.	READ	CORR.	READ	CORR.	READ	CORR.	SUBJECT	SERIES RESISTANCE OHMS	SHUNT RESISTANCE OHMS	SERIES CAPACITANCE MFDS. NOMINAL	ACTUAL		
1																								
2																								
3																								
4																								
5																								
6																								
7																								
8																								
9																								
10																								
11																								
12																								
13																								
14																								
15																								
16																								

CALIBRATION REFERENCES			
APPARATUS	REFERENCE	APPARATUS	REFERENCE

GENERAL REMARKS

OBSERVERS ____________

RECORDER ____________

COMPUTER ____________

PAGE NO.

R.W.H.

NOTE - PLACE (X) AFTER READING FOR WHICH MULTIPLIER IS USED.

RECORD OF METER MEASUREMENTS (small).

(P. I. C. No. 235.)

This form provides space for 16 sets of simultaneous meter readings of seven quantities. The date, place, name of test and general conditions are entered at the top. At the heading of each column of readings, space is provided for the type and number of the meter, and the type and ratio of the instrument transformer, as well as the description of the quantity measured. The readings are entered in the upper left-hand quarters of the unit spaces. The corrected readings are subsequently entered in the lower left-hand quarters and the results in the right-hand half of each column, taking into account the instrument transformer ratio and multiplying factor, in case a multiplier was used. At the right of the seven columns of meter readings data concerning the use of a multiplier are entered. Multiplier data may apply to any one of the quantities measured and provision is made for cross-references indicating those readings with which a multiplier was used. The capital letters designating columns are intended particularly for this purpose. A column is provided for references to oscillograms taken during the test. Notes applying to individual sets of readings are placed in the extreme right-hand column. References to calibration data concerning meters, instrument transformers, and multipliers are entered in the space provided in the lower left-hand corner. A small space is provided for general remarks. The names of the observers and the signatures of the recorder and computer complete the record.

RECORD OF METER MEASUREMENTS (large).

(P. I. C. No. 238.)

This form is similar to form 235, but larger, providing for 29 sets of simultaneous readings of 12 quantities.

P.I.C. 236

J.C.I.I. RECORD OF NOISE MEASUREMENTS

PAGE Nº. ____
DATE ____
PLACE ____

TEST ____
GENERAL CONDITIONS. ____

NUMBER	TIME			CIRCUIT TESTED	NOISE UNITS						CONDITION	
					OBSERVERS					AVERAGE	TESTED CIRCUIT FAR END	OTHER CIRCUITS
	HOUR	MIN.	A.M. P.M.									
1												
2												
3												
4												
5												
6												
7												
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34												
35												
36												
37												
38												
39												
40												

GENERAL REMARKS

FREQUENCY OF NOISE STANDARD ____ CYCLES PER SECOND ____ RECORDER ____

RECORD OF NOISE MEASUREMENTS.

(P. I. C. No. 236.)

This form provides for the date, place, name of test, general conditions and the data and results of 40 sets of noise measurements by 5 different observers. The use of the form will be apparent from inspection.

P.I.C. 234
2-26-15

J.C.I.I. RECORD OF INSULATION MEASUREMENTS.

PAGE NO. ____
DATE ____
PLACE ____

TEST ____

NUMBER	TYPE OF CIRCUIT	TIME			CONDUCTORS TESTED	VOLTMETER READING VOLTS	CONDITION OTHER CONDUCTORS	WEATHER CONDITIONS & TEMP.	INSULATION RESISTANCE	
		HOUR	MIN	A.M. P.M.					TOTAL MEGOHMS	PER MILE MEGOHMS
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										
11										
12										
13										
14										
15										
16										
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18										
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37										
38										
39										
40										
41										

INSTRUCTIONS: MAKE ALL ORIGINAL ENTRIES IN BLACK INK. COMPUTED VALUES AND CORRECTIONS MADE SUBSEQUENT TO ORIGINAL RECORD SHOULD BE SHOWN IN RED.

GENERAL REMARKS.

SUPPLEMENTARY DATA

VOLTMETER NUMBER ____
" ZERO RDG ____ VOLTS
LENGTH OF CONDUCTOR ____ MILES
____ "
____ "
VOLTMETER RESISTANCE ____ OHMS

FORMULAE

$R_T = r\left(\frac{E_B}{V} - 1\right)(10)^{-6}$ MEGOHMS

$R_M = L R_T$ MEGOHMS PER MILE

R_T = TOTAL INSULATION RESISTANCE
R_M = INSULATION RESISTANCE PER MILE
r = VOLTMETER " -OHMS
E_B = BATTERY VOLTAGE -VOLTS
V = VOLTMETER READING - VOLTS
L = LENGTH OF CONDUCTOR -MILES

NOTE

1-G, OR 1,2-G ETC. IN COLUMN "CONDUCTORS TESTED" INDICATES MEASUREMENT OF INSULATION FROM CONDUCTOR 1 OR 1,2 ETC TO GROUND. 1-2, OR 1,2-3,4 ETC INDICATES MEASUREMENT OF INSULATION BETWEEN CONDUCTORS 1&2 ETC.

OBSERVER ____
RECORDER ____

RECORD OF INSULATION MEASUREMENTS.

(P. I. C. No. 234.)

This form provides for the date, place, name of test, data and results of 41 insulation measurements made by means of a battery and high-resistance voltmeter. Space is provided for recording supplementary data concerning the voltmeter and length of line tested and the formulæ used in the computations are given.

P.I.C. 237
8-2-15

INSTRUCTIONS · MAKE ALL ORIGINAL ENTRIES IN BLACK INK. COMPUTED VALUES AND CORRECTIONS MADE SUBSEQUENT TO ORIGINAL RECORD SHOULD BE IN RED INK.

J.C.11 RECORD OF CAPACITANCE & CONDUCTANCE UNBALANCE MEASUREMENTS.

SHEET NO ____

DATE ____

PLACE ____

TEST ____

GENERAL CONDITIONS ____

NO.	TIME: HOUR	TIME: MIN	TIME: A.M. or P.M.	FREQUENCY	TYPE OF CIRCUIT	CONDUCTORS TESTED (a)	CAPACITANCE: NORMAL: SET μμF	CAPACITANCE: NORMAL (a): CONDENSER: NORMAL	CAPACITANCE: NORMAL (a): CONDENSER: ACTUAL	CAPACITANCE: REVERSED (b): SET μμF	CAPACITANCE: MEAN UNBALANCE (c)	CONDUCTANCE: NORMAL: SLIDER SETTING ohms	CONDUCTANCE: NORMAL: UNBALANCE micro-mhos	CONDUCTANCE: SIDE ARMS ohms	CONDUCTANCE: REVERSED: SLIDER SETTING ohms	CONDUCTANCE: REVERSED: UNBALANCE micro-mhos	CONDUCTANCE: MEAN UNBALANCE (c) micro-mhos	CONDITION OTHER CONDUCTORS	WEATHER CONDITIONS TEMP. (d)
1																			
2																			
3																			
4																			
5																			
6																			
7																			
8																			
9																			
10																			
11																			
12																			
13																			
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19																			
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21																			
22																			
23																			
24																			
25																			

GENERAL REMARKS	LENGTH: CKT. NO.	LENGTH: MILES

(a) "NORMAL" CONNECTION OF SET TO "CONDUCTORS TESTED" SHOULD BE SUCH THAT UNBALANCE IS POSITIVE (+, BLACK SCALE) IF THE GREATEST CAPACITANCE, OR CONDUCTANCE AS THE CASE MAY BE, IS ASSOCIATED WITH THE FIRST NUMBERED CONDUCTOR.

(b) CONDENSER SETTING REVERSED IN SIGN BUT SAME IN MAGNITUDE AS FOR NORMAL CONNECTION.

(c) MEAN OF ABSOLUTE VALUES, "NORMAL & REVERSED", TO BE GIVEN SIGN OF NORMAL.

(d) SEE INSULATION MEASUREMENTS MADE THIS DATE.

CAP. SET ____

COND. SET ____

OBSERVER ____

RECORDER ____

COMPUTER ____

RECORD OF CAPACITANCE AND CONDUCTANCE UNBALANCE MEASUREMENTS.

(P. I. C. No. 237.)

On this form space is provided for the date, place, name of test, general conditions, data and results of 25 sets of capacitance and conductance unbalance measurements. The use of the form will be clear from the column headings and explanatory notes on the form, together with the description of the method and diagram of connections

P.I.C. 246
3-17-15

SHEET No. ____

RECORD OF IMPEDANCE MEASUREMENTS

SUBJECT ________________________________ DATE ____________
PLACE ____________

NUMBER	TEST DATA													RESULTS						NOTES CONDITION OF APPARATUS OR OTHER CONDUCTORS.
	TIME			f	I_0 mA	BRIDGE CONNECTION	RATIO ARMS A	RATIO ARMS B	R_n	CAPACITANCE C_1				L	C	•			I_Z	
	[illegible]	[illegible]	A.M. P.M.	CYCLES PER SEC.	READ. CORR.		OHMS	OHMS	OHMS	NOM. TRUE	NOM. TRUE	NOM. TRUE	NOM. TRUE							
1																				
2																				
3																				
4																				
5																				
6																				
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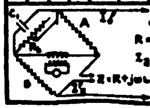
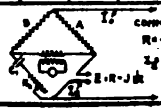
20																				



CONNECTION No. 1

$R = \frac{AB}{R_n}$; $L = C_1 AB$

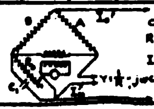
$I_Z = \frac{I_0}{\sqrt{\left(1+\frac{A}{R_n}\right)^2 + (A\omega C_1)^2}}$



CONNECTION No. 2

$R = \frac{A}{B} R_n$; $C = \frac{B}{A} C_1$

$I_Z = \frac{I_0}{\left(1+\frac{A}{B}\right)}$



CONNECTION No. 3

$R = \frac{A}{B} R_n$; $C = \frac{B}{A} C_1$

$I_Z = \frac{I_0}{\left(1+\frac{A}{B}\right)}$

OBSERVER ____________
RECORDER ____________
COMPUTER ____________

RECORD OF IMPEDANCE MEASUREMENTS.

(P. I. C. No. 246.)

Space is provided for the date, place, subject, data and results of 20 impedance measurements. The column headings together with the schematic diagrams of the different bridge connections and formulas given are sufficient explanations of the use of this form. See also technical report No. 29.

In addition to the 18 forms here given and described, three forms for systematized computations of the coefficients of induction for parallel power and telephone circuits have been developed recently. As samples and descriptions will be given in technical report No. 64, dealing with the general subject of computations of induction, they are omitted from this report.

III. Discussion.

The advantages offered by the use of standard blank forms for the recording of data and computations, over the earlier practice of using blank books, are chiefly:

1. Completeness and conciseness of records, as more time and care have been given to the development of these forms than could properly be spent in arranging a form for a single test.
2. Uniformity of records of a given kind, irrespective of the recorder.
3. Neatness.
4. Increased speed and accuracy of recording, due to more systematic provision for necessary data and to greater familiarity with standard forms.
5. Increased facility in reduction of data and checking by the provision of space for corrected observations and computation of results on the same sheet with original data.
6. Saving in time otherwise required to prepare forms (in books or sheets) for records of each test or computation.
7. Flexibility in use and filing, as in any loose-leaf system of records.

Blank forms for recording data of similar tests are conveniently bound together and subsequent to the test the used sheets may be taken out, results computed, and upon completion of a report on a given subject all data and computation sheets upon which that report is based may be assembled and bound for future reference. Under the former system, data upon which a given report is based are scattered among several books.

As the development of some of the forms described was not completed until the latter part of the San Fernando work it was not pos-

sible there to put this method into complete operation. These forms are, however, available for any future work of a similar character and their use should promote efficiency.

Respectfully submitted.

(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

APPROVED: April 2, 1916.

(Signed) R. W. MASTICK,
Field Engineer.

APPROVED: May 2, 1916.

SUBCOMMITTEE ON TESTS,
(Signed) J. E. WOODBRIDGE,
Chairman.

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE,
(Signed) ARTHUR F. BRIDGE,
Secretary.

May 19, 1916.

Technical Report No. 64.

August 4, 1916.

COMPUTATION OF INDUCTION BETWEEN PARALLEL POWER AND COMMUNICATION CIRCUITS.

OUTLINE.

I. Introduction.

In previous technical reports the results of computations of coefficients of induction for parallel power and communication circuits have been given. Although some of the formulas upon which the computations are based have been listed, their derivation and the assumptions and approximations involved have not been given. Technical report No. 12 lists formulas for computing electric and magnetic induction, but several are erroneous, as pointed out later in this report, and some of the fundamental formulas required in calculations of electric induction are omitted. Technical report No. 47, giving the results of computations on the Santa Cruz-Watsonville exposure, also gives the formulas upon which the computations are based. For electric induction, however, the formulas are limited, though not so stated, to a particular power-circuit configuration (isosceles triangle, base parallel to ground).

It is the purpose of this report to present general formulas for the computation of induction, to outline the methods by which these formulas are derived, to give the assumptions and approximations upon which they are based, and to describe systematized methods by which the operations required in carrying out the computations may be facilitated. The formulas of this report are limited to the computation of the voltages induced in grounded or metallic circuits paralleled by transmission lines for distances which are short as compared to a wave length, so that attenuation and phase-change along the circuits may be neglected and electric and magnetic induction separately considered. Within these limitations they may be applied to any configuration or relative position of circuits with the exception that some of the formulas for electric induction are limited to moderate or large separations of disturbed and disturbing circuits.

II. Electric Induction.

"Electric induction" is the term applied to the voltages and currents produced in a circuit due to its presence in the varying electric field set up by the voltages of a neighboring circuit. The term "electrostatic" heretofore used in this connection is deemed inappropriate as in all practical cases the field is varying, generally alternating harmonically at definite frequencies. The fundamental relations all depend upon theorems of electrostatics, hence the origin of the term customarily used.

As a first step in the calculation of the voltage induced in circuits subject to the influence of the electric field surrounding a parallel power-transmission circuit it is necessary to determine the electric charges on the power conductors per unit length of line. These charges depend upon the impressed voltages and upon the sizes of the conductors and their positions with respect to each other and all other neighboring bodies, particularly the earth and conductors of other circuits on the same pole line.

The following equations connecting the potentials and electric charges of an independent electrical system consisting of m conductors and ground are well known:

$$\left.\begin{array}{l} V_1 = P_{11}Q_1 + P_{21}Q_2 + P_{31}Q_3 + \quad . \quad . \quad . \quad + P_{m1}Q_m \\ V_2 = P_{12}Q_1 + P_{22}Q_2 + P_{32}Q_3 + \quad . \quad . \quad . \quad + P_{m2}Q_m \\ V_3 = P_{13}Q_1 + P_{23}Q_2 + P_{33}Q_3 + \quad . \quad . \quad . \quad + P_{m3}Q_m \\ . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ V_m = P_{1m}Q_1 + P_{2m}Q_2 + P_{3m}Q_3 + \quad . \quad . \quad . \quad + P_{mm}Q_m \end{array}\right\} \quad (1)$$

V_1, V_2, V_3 . . . V_m are the potentials referred to ground of the conductors denoted by the subscripts; Q_1, Q_2, Q_3 . . . Q_m are the corresponding charges.

The coefficients P_{11}, P_{12}, P_{13}, etc., termed "potential coefficients," are constants depending upon the dimensions of the system and the nature of the dielectric medium in which the conductors are placed (air in this case), and are independent of how the conductors may be charged or how they may be interconnected (provided the surfaces of the connecting wires are small as compared to the surfaces of the conductors). Each coefficient has two subscripts, the first corresponding to the charge and the second to the potential. A coefficient in which the two subscripts are the same denotes the potential of the conductor designated by the subscripts when its charge is unity, that of all the other conductors being zero. A coefficient in which the two subscripts are different denotes the potential of the conductor designated by the second subscript when the conductor designated by the first subscript receives a

unit charge, the charges of all the other conductors being zero. It may be proved that reciprocal relations exist such that the order in which the subscripts appear is immaterial, *i. e.*, $P_{mn} = P_{nm}$. All the potential coefficients are positive and those with like subscripts are not less than those with unlike subscripts.

The relationship of the charges and potentials may also be expressed in the following form:

$$\left.\begin{array}{l} Q_1 = K_{11}V_1 + K_{12}V_2 + K_{13}V_3 + \quad . \quad . \quad . \quad + K_{1m}V_m \\ Q_2 = K_{21}V_1 + K_{22}V_2 + K_{23}V_3 + \quad . \quad . \quad . \quad + K_{2m}V_m \\ Q_3 = K_{31}V_1 + K_{32}V_2 + K_{33}V_3 + \quad . \quad . \quad . \quad + K_{3m}V_m \\ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ Q_m = K_{m1}V_1 + K_{m2}V_2 + K_{m3}V_3 + \quad . \quad . \quad . \quad + K_{mm}V_m \end{array}\right\} \quad (2)$$

The coefficients K_{11}, K_{12}, K_{13}, etc., will be termed "capacitance coefficients" and correspond to Maxwell's coefficients of capacity and induction; termed by Russell the coefficients of self and mutual induction for electrostatic charges. Like the potential coefficients the capacitance coefficients are constants independent of the manner in which the conductor may be charged or interconnected. Each coefficient has two subscripts, the first conventionally denoting the charge and the second the potential. As with the potential coefficients reciprocal relations make the order of the subscripts immaterial, $K_{mn} = K_{nm}$. A coefficient in which the two subscripts are the same denotes the charge on the corresponding conductor when its potential is unity, and that of all the other conductors zero (grounded). This quantity is variously termed the "capacity," the "total capacity" and the "grounded capacity" of the conductor, also its "coefficient of self-induction for electrostatic charges." A coefficient in which the two subscripts are different denotes the charge induced on the conductor designated by the first subscript when the conductor designated by the second subscript is raised to potential unity, the potential of all the other conductors being zero (grounded). This quantity is variously termed "coefficient of induction" and "coefficient of mutual induction," and is equal in magnitude but opposite in sign to the "direct" or "normal" capacitance between corresponding conductors. The direct capacitance between conductors m and n is defined as the ratio of the charge on m due to the difference of potential between m and n when all conductors of the system except n are at the same potential as m. All the coefficients with like subscripts are positive and all those with unlike subscripts are negative and the algebraic sum of all the coefficients belonging to a single conductor is equal to the direct capacitance to ground of that conductor ($K_{n1} + K_{n2} + \quad . \quad . \quad . \quad K_{nn} = C_{no}$).

In order that both sets of equations (1) and (2), connecting the charges and potentials, may hold simultaneously, the potential and

capacitance coefficients must be so related that any capacitance coefficient K_{mn} in equations (2) is the minor of P_{mn} in the determinant of the potential coefficients in equations (1), divided by the value of that determinant. If the number of conductors exceeds four, the arithmetical computations become very laborious unless symmetrical relations simplify the operations. For a three-conductor line, the following simple solution results:

$$\left.\begin{aligned}
K_{11} &= \frac{P_{22}P_{33} - P^2_{23}}{\sigma} & K_{12} &= \frac{P_{23}P_{13} - P_{12}P_{33}}{\sigma} \\
K_{22} &= \frac{P_{11}P_{33} - P^2_{13}}{\sigma} & K_{13} &= \frac{P_{12}P_{23} - P_{13}P_{22}}{\sigma} \\
K_{33} &= \frac{P_{11}P_{22} - P^2_{12}}{\sigma} & K_{23} &= \frac{P_{13}P_{12} - P_{23}P_{11}}{\sigma}
\end{aligned}\right\} \quad (3)$$

$$\sigma = P_{11}P_{22}P_{33} + 2\,P_{12}P_{13}P_{23} - P_{11}P^2_{23} - P_{22}P^2_{13} - P_{33}P^2_{12}$$

For a six-conductor system it is necessary to determine, in general, six charges; there being 21 potential coefficients and 21 capacitance coefficients involved in the equations. The evaluation of a sixth-order determinant is necessary for a complete solution. When the conductors of the two circuits of a double-circuit line are all of the same size and symmetrically located with respect to an intermediate plane perpendicular to the earth's surface, the number of independent potential and capacitance coefficients is reduced, the total of each being 12. In equations (1), by taking the sums and differences of the potentials of symmetrically placed conductors, two sets of three homogeneous equations result; one set involves the sums of pairs of potentials and of pairs of corresponding charges and the other the differences of pairs of potentials and of pairs of corresponding charges. The solution thus requires the evaluation of two third-order determinants instead of the sixth-order determinant required by the general case. By combining the sums and differences of corresponding pairs of charges obtained from the solutions of two third-order determinants a set of six homogeneous equations is obtained, which expresses the charges on the individual conductors in terms of the six potentials. Equating corresponding coefficients of the potentials in the expressions thus obtained with the coefficients of equations (2), the capacitance coefficients are gotten in terms of potential coefficients.

To evaluate the potential coefficients it is necessary to compute, for each one, the work done by an external agent against the repulsive force of a unit positive charge on one of the conductors in bringing a unit positive charge from an infinite distance, or from any place where the

potential is zero, to the conductor whose potential is desired which may or may not be the charged conductor. To evaluate P_{11} assume unit positive charge on conductor 1, charges on all other conductors being zero, and compute the work done in bringing another unit positive charge from ground to conductor 1; similarly for the other coefficients with like subscripts. To evaluate P_{21} assume unit positive charge on conductor 2, charges on all other conductors being zero, and compute work done in bringing unit positive charge from ground to conductor 1; similarly for the other coefficients with unlike subscripts. The computations involved in this process are simplified by considering the ground replaced by the images of the conductors at a distance below the ground surface equal to the height of the conductors above the surface and having charges of equal magnitude but opposite sign to the charges on the actual conductors. If the radii of the conductors are small as compared to the distances between them and these distances small as compared to the length of the conductors the values of the potential coefficients in terms of dimensions of the system are as follows:

$$\left.\begin{array}{ll} P_{11} = 2 \log \dfrac{S_{11}}{r_{11}} & P_{12} = 2 \log \dfrac{S_{12}}{r_{12}} \\ P_{22} = 2 \log \dfrac{S_{22}}{r_{22}} & P_{13} = 2 \log \dfrac{S_{13}}{r_{13}} \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot & \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ P_{mm} = 2 \log \dfrac{S_{mm}}{r_{mm}} & P_{mn} = 2 \log \dfrac{S_{mn}}{r_{mn}} \end{array}\right\} \qquad (4)$$

S_{11}, S_{22} . . . S_{mm} are the distances from the conductors to their own images. S_{12}, S_{13} . . . S_{mn} are respectively the distances from the axis of the conductor denoted by one subscript to the axis of the image conductor denoted by the other subscript (for example, S_{12} is the distance from conductor 1 to the image of conductor 2 or vice versa). r_{11}, r_{22} . . . r_{mm} are the radii of the conductors. For stranded conductors the perimeters of the cross-sections divided by 2π may be taken as the radii with small error. r_{12}, r_{13} . . . r_{mn} are the distances between the axes of the conductors denoted by the subscripts. The notation used is illustrated on drawing No. 331 attached. If the distances between conductors are greater than ten diameters the above expressions for the potential coefficients are accurate to within less than $\frac{1}{10}\%$.

Having the potential coefficients, the capacitance coefficients may be determined and thence the charges for any impressed voltages from

equations (2). The two cases of greatest importance are balanced and residual voltages, as generally used throughout the Committee's investigation.

In computing the induction from three-phase power circuits having voltages of equal magnitude between pairs of conductors it is convenient to consider the voltages to ground divided into two sets of components. These are, balanced voltages, which are equal in magnitude and 120° apart in phase, their vector sum being zero, and three residual voltages equal in magnitude and in phase, their vector sum being the residual voltage of the system. If, then, the coefficients of induction be obtained for the two sets of components in the power-system voltages, and their phase relationship be known, the resultant voltage induced in the communication circuit may be obtained by a combination of the induced voltages arising from the two sets of components separately considered. (See also technical reports No. 2 and No. 12 and report of Joint Committee to California Railroad Commission, dated July 7, 1914).

For balanced voltages:

$$V_1 = E, V_2 = (-\tfrac{1}{2} + j\tfrac{1}{2}\sqrt{3})E, V_3 = (-\tfrac{1}{2} - j\tfrac{1}{2}\sqrt{3})E \quad (5)$$

the charges on the several conductors of a single-circuit three-phase line are:

$$\left.\begin{aligned} Q_1 &= [K_{11} - \tfrac{1}{2}(K_{12} + K_{13}) + j\tfrac{1}{2}\sqrt{3}(K_{12} - K_{13})]\,E \\ Q_2 &= [K_{12} - \tfrac{1}{2}(K_{22} + K_{23}) + j\tfrac{1}{2}\sqrt{3}(K_{22} - K_{23})]\,E \\ Q_3 &= [K_{13} - \tfrac{1}{2}(K_{23} + K_{33}) + j\tfrac{1}{2}\sqrt{3}(K_{23} - K_{33})]\,E \end{aligned}\right\} \quad (6)$$

For residual voltages:

$$V_1 = V_2 = V_3 = \tfrac{1}{3}E_R \quad (7)$$

where E_R is the vector sum of the voltages to ground, the charges on the several conductors of a single-circuit three-phase line are:

$$\left.\begin{aligned} Q_1 &= \tfrac{1}{3}(K_{11} + K_{12} + K_{13})\,E_R \\ Q_2 &= \tfrac{1}{3}(K_{12} + K_{22} + K_{23})\,E_R \\ Q_3 &= \tfrac{1}{3}(K_{13} + K_{23} + K_{33})\,E_R \end{aligned}\right\} \quad (8)$$

If logarithms to the base e are used in determining the potential coefficients by equations (4) then K_{11}, K_{12}, etc., will be in statfarads per centimeter and if E and E_R are expressed in volts the values of the charges given by equations (6) and (8) must be divided by the factor $9(10)^{11}$ in order to reduce them to coulombs per centimeter length of line. However, this reduction to coulombs is not necessary, as later explained, in determining the voltages induced in parallel circuits.

The expression for the potential of any point "a" in the electric field surrounding the power conductors is,

$$E_a = P_{1a}Q_1 + P_{2a}Q_2 + \quad . \quad . \quad . \quad + P_{ma}Q_m \tag{9}$$

on condition that there are no bodies in the electric field except the power circuit and the earth. The potential coefficients have the following values:

$$\left.\begin{aligned} P_{1a} &= 2 \log \frac{S_{1a}}{r_{1a}} \\ P_{2a} &= 2 \log \frac{S_{2a}}{r_{2a}} \\ &\cdots \\ P_{ma} &= 2 \log \frac{S_{ma}}{r_{ma}} \end{aligned}\right\} \tag{10}$$

In case a conductor of small cross section, paralleled by the power conductors its entire length, passes through "a" it will assume the potential E_a if isolated from ground and from other conductors. This will hold true for a number of such isolated conductors in the field distant apart ten diameters or more, since the area of the field occupied by conductors, which have infinite specific inductive capacity, is relatively inappreciable. To compute the induced potentials of the conductors of a parallel circuit when thus isolated from ground, from each other, from other conductors and from unexposed sections of the circuit, equations (9) and (10) may be applied for the position of each conductor; having previously determined the charges on the power conductors corresponding to the given balanced, residual or other voltages. The potential difference between conductors is of course the vector difference between their potentials referred to ground.

If in equations (4) and (10) logarithms are taken to the same base, either e or 10, and the dimensions are expressed in the same units throughout, the voltages induced in the parallel circuit are obtained in volts per volt impressed upon the power circuit.

While the induced potentials of the conductors of the parallel circuit, when isolated as described above, are useful in indicating the relative severity of the inductive effects under different conditions of parallelism and as a basis for further computations, it becomes necessary to take account of the modifying effects of other circuits on the same pole line, if it is desired to determine the magnitude of the induction under operating conditions or any condition other than complete isolation of

all the disturbed conductors and any others closely associated with them. The following general equations take account of the presence of a number of conductors on disturbed lines, such as telephone lines.

$$
\left.\begin{array}{l}
\qquad\qquad\overbrace{\qquad\qquad\qquad\qquad\qquad\qquad}^{\text{Power}}\qquad\overbrace{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}^{\text{Telephone}}\\
V_a = P_{1a}Q_1 + P_{2a}Q_2 + \ldots + P_{ma}Q_m + P_{aa}Q_a + P_{ba}Q_b + P_{ca}Q_c + \ldots + P_{na}Q_n \\
V_b = P_{1b}Q_1 + P_{2b}Q_2 + \ldots + P_{mb}Q_m + P_{ab}Q_a + P_{bb}Q_b + P_{cb}Q_c + \ldots + P_{nb}Q_n \\
V_c = P_{1c}Q_1 + P_{2c}Q_2 + \ldots + P_{mc}Q_m + P_{ac}Q_a + P_{bc}Q_b + P_{cc}Q_c + \ldots + P_{nc}Q_n \\
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
V_n = P_{1n}Q_1 + P_{2n}Q_2 + \ldots + P_{mn}Q_m + P_{an}Q_a + P_{bn}Q_b + P_{cn}Q_c + \ldots + P_{nn}Q_n
\end{array}\right\} (11)
$$

V_a, V_b, V_c . . . V_n represent the potentials referred to ground of the telephone conductors designated by the subscripts a, b, c, . . . n. The sums of the first three terms of each equation, headed "power," give the resultant potentials of the telephone conductors on condition that all the telephone conductors are completely isolated. The terms in columns headed "telephone" represent the mutual effects of the several telephone conductors in modifying their respective potentials. When all the conductors are isolated Q_a, Q_b, Q_c, . . . Q_n are all equal to zero, hence there are no mutual effects.

On the assumption that the charges on the power conductors are unaffected by the condition of the telephone conductors, the values of E_a, E_b, E_c, . . . E_n determined by equation (9) may be substituted for the first three terms of (11). Rewriting (11) making this substitution:

$$
\left.\begin{array}{l}
V_a - E_a = P_{aa}Q_a + P_{ba}Q_b + P_{ca}Q_c + \ldots + P_{na}Q_n \\
V_b - E_b = P_{ab}Q_a + P_{bb}Q_b + P_{cb}Q_c + \ldots + P_{nb}Q_n \\
V_c - E_c = P_{ac}Q_a + P_{bc}Q_b + P_{cc}Q_c + \ldots + P_{nc}Q_n \\
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
V_n - E_n = P_{an}Q_a + P_{bn}Q_b + P_{cn}Q_c + \ldots + P_{nn}Q_n
\end{array}\right\} (12)
$$

The following relations also hold on the assumption that the potentials of the power conductors are independent of the condition of the telephone conductors:

$$
\left.\begin{array}{l}
Q_a = K_{aa}(V_a - E_a) + K_{ab}(V_b - E_b) + \ldots + K_{an}(V_n - E_n) \\
Q_b = K_{ba}(V_a - E_a) + K_{bb}(V_b - E_b) + \ldots + K_{bn}(V_n - E_n) \\
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
Q_n = K_{na}(V_a - E_a) + K_{nb}(V_b - E_b) + \ldots + K_{nn}(V_n - E_n)
\end{array}\right\} (13)
$$

Any coefficient K_{mn} in equation (13) is the minor of P_{mn} in the determinant of the potential coefficients of the telephone conductors, divided by D, the value of that determinant; assuming that the capacitance coefficients of the telephone conductors are independent of the condition of the power conductors. Though this is not strictly so it is

practically exact for moderate or large separations of power and telephone lines and a similar assumption was made with regard to the capacitance coefficients of the power circuit which were assumed independent of the condition of the telephone conductors.

$$D = \left\{ \begin{matrix} P_{aa} & P_{ba} & P_{ca} & \cdots & P_{na} \\ P_{ab} & P_{bb} & P_{cb} & \cdots & P_{nb} \\ P_{ac} & P_{bc} & P_{cc} & \cdots & P_{nc} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ P_{an} & P_{bn} & P_{cn} & \cdots & P_{nn} \end{matrix} \right\} \tag{14}$$

If the number of the telephone conductors does not exceed four, the corresponding K's may be computed as above indicated without undue labor. For a larger number of conductors the computations become exceedingly laborious and the values of the K's should be determined by measurements or from tables, if available, of the direct capacitances between conductors. The K's with unlike subscripts are equal in magnitude but opposite in sign to the direct capacitances between corresponding conductors, that is, K_{ab}, K_{ac}, etc., equal respectively $-C_{ab}$, $-C_{ac}$, etc. The K's with like subscripts are equal in magnitude and sign to the capacitances to ground of the corresponding conductors when all other telephone conductors are grounded, that is, K_{aa}, K_{bb}, etc., equal respectively, C_{aa}, C_{bb}, etc. Rewriting (13):

$$\left.\begin{aligned} Q_a &= C_{aa}(V_a-E_a)-C_{ab}(V_b-E_b)- \cdots -C_{an}(V_n-E_n) \\ Q_b &= -C_{ba}(V_a-E_a)+C_{bb}(V_b-E_b)- \cdots -C_{bn}(V_n-E_n) \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ Q_n &= -C_{na}(V_a-E_a)-C_{nb}(V_b-E_b)- \cdots +C_{nn}(V_n-E_n) \end{aligned}\right\} \tag{15}$$

The application of equations (12) and (15) to several particular cases are given below. Cases I to IV correspond to conditions considered in technical report No. 12.

Case I. Open-circuit voltage of two conductors in parallel to ground, all other conductors isolated.

$$V_a = V_b, \qquad Q_a = -Q_b$$

$$Q_c = Q_d = \cdots = Q_n = 0$$

From (12)
$$V_a = \frac{E_a(P_{bb}-P_{ab})+E_b(P_{aa}-P_{ab})}{P_{aa}+P_{bb}-2P_{ab}} \tag{16}$$

The denominator of (16) is equal to the reciprocal of the capacitance between a and b with all other conductors isolated. If a and b are the same size and in a horizontal plane, then $P_{aa} = P_{bb}$ and $V_a = \frac{E_a + E_b}{2}$

This latter expression is sufficiently accurate for most practical cases, even though a and b are not in the same horizontal plane.

Case II. Open-circuit voltage of one conductor to ground, all other conductors grounded.

$$Q_a = 0 \text{ and } V_b = V_c = \quad . \quad . \quad . \quad = V_n = 0$$

From (15)

$$V_a = E_a - \frac{E_b C_{ab} + E_c C_{ac} + E_d C_{ad} + \quad . \quad . \quad . \quad + E_n C_{an}}{C_{aa}} \tag{17}$$

If $\quad E_a = E_b = E_c = \quad . \quad . \quad . \quad = E_n$ (Approx.)

then

$$V_a = E_a \frac{C_{ao}}{C_{aa}} \quad \text{(Approx.)} \tag{18}$$

where C_{ao} is the direct capacitance of the conductor a to ground.

Case III. Open-circuit voltage of two conductors in parallel to ground, all other conductors grounded.

$$V_a = V_b, \qquad Q_a = -Q_b,$$
$$V_c = V_d = \quad . \quad . \quad . \quad = V_n = 0$$

From (15)

$$V_a = \frac{E_a (C_{aa} - C_{ab}) + E_b (C_{bb} - C_{ab}) - E_c (C_{ac} + C_{bc}) - \ldots E_n (C_{an} + C_{bn})}{C_{aa} + C_{bb} - 2 C_{ab}} \tag{19}$$

If $\quad E_a = E_b = E_c = \quad . \quad . \quad . \quad = E_n$ (Approx.)

then

$$V_a = E_a \frac{C_{ao} + C_{bo}}{C_{aa} + C_{bb} - 2 C_{ab}} \quad \text{(Approx.)} \tag{20}$$

The denominator of (19) and (20) is the capacitance of conductors a and b in parallel to ground, with all other conductors grounded.

Case IV. Open-circuit voltage between two conductors, all other conductors grounded.

$$Q_a = Q_b = 0$$
$$V_c = V_d = \quad . \quad . \quad . \quad = V_n = 0$$

From equations (15),

$$V_a - V_b = E_a - E_b - \frac{E_c [C_{ab} (C_{bc} - C_{ac}) - C_{aa} C_{bc} + C_{bb} C_{ac}]}{C_{aa} C_{bb} - C_{ab}^2} - \ldots$$

$$\ldots - \frac{E_n [C_{ab} (C_{bn} - C_{an}) - C_{aa} C_{bn} + C_{bb} C_{an}]}{C_{aa} C_{bb} - C_{ab}^2} \tag{21}$$

This is not in agreement with the formula for the corresponding case given in technical report No. 12 (formula 16 of P. I. C. No. 2). The formula there given is in error.

If $\quad C_{aa} = C_{bb}$

then (21) reduces to

$$V_a - V_b = E_a - E_b - \frac{E_c (C_{ac} - C_{bc}) + \ldots + E_n (C_{an} - C_{bn})}{C_{aa} + C_{ab}} \tag{22}$$

Formulas (21) and (22) apply to phantom circuits provided that the capacitances and potentials of single conductors are replaced by the corresponding capacitances and potentials of pairs of conductors forming the sides of the phantom circuits. For horizontal and vertical phantoms the average potentials of pairs of conductors forming the side circuits of phantoms may be used as a satisfactory approximation. Corresponding capacitances for a side circuit and a phantom circuit are tabulated below.

Side Circuit (Conductors a and b)	Phantom Circuit (Pairs of conductors a, b and c, d)
C_{aa}	$C_{aa} + C_{bb} - 2\,C_{ab}$
C_{bb}	$C_{cc} + C_{dd} - 2\,C_{cd}$
C_{ab}	$C_{ac} + C_{bc} + C_{ad} + C_{bd}$
C_{an}	$C_{an} + C_{bn}$
C_{bn}	$C_{cn} + C_{dn}$

Case V. Open-circuit voltage between two pairs of short-circuited conductors, all other conductors isolated.

$$V_a = V_b = V_{ab} \qquad\qquad V_c = V_d = V_{cd}$$
$$Q_a = -Q_b \qquad\qquad Q_c = -Q_d$$
$$Q_e = Q_f = \ . \ . \ . \ = Q_n = 0$$

The general solution for this case is rather involved. Solutions for three arrangements which result in simplifying relations, are given below. These special arrangements cover the cases of most practical interest.

(a) *Horizontal Phantom.* All conductors of same size and in a horizontal plane, equal distances between adjacent conductors arranged in order a, b, c, d.

$$P_{aa} = P_{bb} = P_{cc} = P_{dd}$$
$$P_{ab} = P_{bc} = P_{cd} \text{ and } P_{ac} = P_{bd}$$

then from (12)

$$V_{ab} - V_{cd} = \frac{(E_a - E_d)(P_{aa} + P_{ac} - 2\,P_{ab}) + (E_b - E_c)(P_{aa} + P_{ac} - P_{ab} - P_{ad})}{2\,P_{aa} + 2P_{ac} - 3\,P_{ab} - P_{ad}} \qquad (23)$$

$$= \frac{(E_a + E_b) - (E_d + E_c)}{2} \quad \text{(Approx.)} \qquad (24)$$

For most practical cases the approximate formula gives a value about 5 per cent greater than the exact formula.

(b) *Vertical Phantom.* All conductors of same size, each pair of conductors in a horizontal plane, one pair (a, b) being vertically over the other pair (c, d).

$$P_{aa} = P_{bb} \qquad P_{ad} = P_{bc}$$
$$P_{cc} = P_{dd} \qquad P_{ac} = P_{bd}$$

then from (12)

$$V_{ab} - V_{cd} = \frac{(E_a + E_b) - (E_c + E_d)}{2} \tag{25}$$

Thus the approximate solution for arrangement (a) becomes the exact solution for (b).

(c) *Square or Rectangular Phantom.* All conductors of same size, placed at the corners of a square or rectangle with base horizontal, diagonally opposite conductors forming pairs.

$$P_{aa} = P_{cc} \qquad P_{ab} = P_{cd}$$
$$P_{bb} = P_{dd} \qquad P_{ad} = P_{bc}$$

then from (12)

$$V_{ab} - V_{cd} = \frac{(E_a - E_c)(P_{bb} + P_{ad} - P_{ab} - P_{bd}) + (E_b - E_d)(P_{aa} + P_{ad} - P_{ab} - P_{ac})}{P_{aa} + P_{bb} + 2\,P_{ad} - 2\,P_{ab} - P_{ac} - P_{bd}} \tag{26}$$

$$= \frac{(E_a + E_b) - (E_c + E_d)}{2} \quad \text{(Approx.)} \tag{26a}$$

The approximate formula (26a) gives a result which in general will be in error by less than 0.5%.

The foregoing discussion has all been with reference to nontransposed circuits. If either the power or telephone circuits or both be transposed the induced voltages with all the telephone circuits isolated are equal to the corresponding induced voltages for nontransposed circuits multiplied by the fractional unbalanced exposures between the telephone circuits and the power circuit.

To determine the unbalanced exposures the lengths of parallel between transpositions in uniform sections are added vectorially; transpositions* in a three-phase power circuit change the phase of the induction from balanced voltages by 120° and transpositions in a telephone circuit change the phase of the induction between sides of that circuit by 180° but do not affect the induction between the two sides of the circuit in parallel and ground. Transpositions in the power circuit do not change the phase of the induction from residual voltages. It is convenient to consider separately the unbalanced exposures to balanced and residual

*See T. R. No. 66 and No. 67.

voltages. In addition the unbalanced exposure for the metallic circuit is to be distinguished from that for the two conductors (four for phantom circuits) in parallel, as if forming one side of a grounded circuit. (Refer to technical reports Nos. 39 and 66 for more detailed discussions of unbalanced exposures).

In case the fractional unbalanced exposures thus determined are small the effects of irregularities which occur in practical cases become of importance and the actual induction may differ very largely from that computed on the assumption of a uniform parallel. It is also necessary to consider the balancing effect of the telephone transpositions on the capacitances of the circuits.

To determine the open-circuit voltage of one circuit isolated within the limits of a parallel with all other circuits in normal operation equations (15) may be applied, using, however, the values of admittances corresponding to the particular condition considered, in place of the capacitances for the section of line within the parallel. Thus, in place of the direct capacitances between the conductors the admittances determined by these same capacitances would be used; and in place of the capacitance to ground of a given conductor with all other conductors grounded, its admittance to ground with all other conductors connected not to ground but to their unexposed sections as in normal operation, would be used. This latter can be determined from a knowledge of direct capacitances of the exposed section and the admittances to ground of the unexposed sections. With long unexposed sections of line, the effect on a given isolated circuit of the other telephone circuits under operating conditions, probably approaches more nearly the condition with these other circuits grounded than with them isolated within the limits of the parallel.

To determine the current in a circuit at the junctions of the exposed and unexposed sections of line,* it is convenient to consider the parallel as the equivalent of a generator placed between the sides of the circuit; having an induced voltage equal to the open-circuit voltage of the given circuit, isolated within the limits of the parallel while all other circuits are in normal operation or other assumed condition, and with an internal impedance equal to the open-circuit impedance of the exposed section of the circuit. This generator supplies current to the unexposed sections of the circuit in parallel, the division of current between them being determined by their respective impedances and the internal impedance of the fictitious generator. This method does not take into account the mutual induction between circuits in the unexposed sections of line, which is small between well transposed metallic circuits. The current on short-circuit at one end of the parallel may be obtained from the open-

*Due to electric induction.

circuit induced voltage and the open-circuit impedance of the exposed section of circuit.

Attenuation of the induced currents along the circuits between the ends of the parallel and the terminals of the circuits, and the effect of terminal apparatus, must of course be considered in determining the currents in the telephone receivers.

While for short parallels the voltages induced in parallel circuits by electric induction are independent of the frequency of the disturbing source, the resulting induced currents do depend upon the frequency, since the impedances of the exposed and unexposed sections of the circuit are functions of the frequency. For short parallels the impedance of the exposed section of the circuit (internal impedance of fictitious generator) varies approximately inversely as the frequency, and as the amount of induced current from such a parallel is regulated very largely by this impedance it is obvious that the resulting induced current increases approximately as the frequency of the disturbing source, for a given induced voltage.

The following are references to standard works consulted in preparing this section.

Maxwell's Electricity and Magnetism; vol. 1, sections 73, 87, 88, 89a, 89b, 155, 156, 157 and 161.

Webster's Electricity and Magnetism; sections 136, 138 and 139.

Kelvin's Papers on Electrostatics and Magnetism; Electric Images.

Heaviside's Electrical Papers; vol. 1, p. 42, "On Electrostatic Capacity of Suspended Wires," and p. 116, "On Induction Between Parallel Wires."

Russell's Alternating Current Theory; vol. 1, chapters 4 and 5.

J. J. Thomson's Elements of Electricity and Magnetism; sections 13, 24, 25, 26, 27, 28, 83 and 84.

Pender—American Handbook for Electrical Engineers; p. 182, Capacity and Charging Currents.

III. Magnetic Induction.

The term "magnetic induction" is here applied to the voltages and currents produced in a circuit due to its presence in the varying magnetic field set up by the currents in a neighboring circuit. This term is used in preference to the more usual term, electromagnetic induction, not because of any inconsistency in customary usage but to correspond to the term "electric induction" as used in the preceding section of this report. Induction in both cases signifies the induced voltages and currents, electric or magnetic, referring to the type of field through the mediumship of which the induction is produced.

The electromotive force induced in a circuit is equal to the rate of diminution of the magnetic flux which interlinks with the circuit.

$$e = -\frac{d\phi}{dt} \qquad (27)$$

The first step in computing the voltage or current induced in a circuit due to the current in a neighboring circuit, is to obtain the flux interlinked with the disturbed or secondary circuit due to the current in the disturbing or primary circuit. Since the flux is directly proportional to the current it may be expressed as the product of the current in the disturbing circuit and the mutual inductance M of the two circuits; the mutual inductance being defined as the flux interlinked with one circuit due to unit current in the other.

The magnetic field intensity H at a distance x from the axis of a long-straight conductor carrying a current i is

$$H = \frac{2i}{x} \tag{28}$$

and the flux across a small area at x of unit length parallel to the conductor and of width dx radially, is

$$d\phi = Hdx = \frac{2i}{x}dx \tag{29}$$

The total flux included in the region between points distant r and s is obtained by integration, thus:

$$\phi = \int_r^s \frac{2i}{x}\, dx = 2i \log_e \frac{s}{r} \tag{30}$$

Then

$$\frac{\phi}{i} = 2 \log_e \frac{s}{r} = M \tag{31}$$

the mutual inductance per unit length between this conductor and a parallel circuit whose conductors are distant r and s from the conductor carrying the current i. Under these conditions the voltage induced in the disturbed circuit is

$$e = -M \frac{di}{dt} \tag{32}$$

and if the current is a harmonic function of time

$$E = -j2\pi fMI \tag{33}$$

where E and I are the effective values of voltage and current, respectively, and f is the frequency in cycles per second.

Consider a system of five parallel conductors and ground, as for example a three-phase power circuit and a telephone circuit, the distances between the conductors and their heights above ground being large compared to their diameters and their lengths great as compared to the other dimensions. The earth, as a common conductor of the circuits and as a path for eddy currents, may be replaced by the images of the conductors.

In locating the image conductors, experience has shown that it is not permissible, as in the case of electric induction, to consider them at a

distance below the earth's surface equal to the height of the conductors above it. Since the earth is not a perfect conductor the locus of earth currents is a considerable distance below the earth's surface. This distance varies with the frequency of the current and the character of the country.

The results of tests made by the Joint Committee at three different localities show that the image conductors for computations of magnetic induction should be taken with respect to an "equivalent earth plane" from 300 to 500 feet below the actual surface. The figures given are for 60 cycles. For higher frequencies the "equivalent earth plane" is nearer the surface.

Designating the power conductors by the numbers 1, 2 and 3, carrying currents I_1, I_2 and I_3, respectively, and the telephone conductors by the letters "a" and "b," the expression for the voltage E_a induced along telephone conductor "a," is

$$E_a = -j2\pi f\,(M_{1a}I_1 + M_{2a}I_2 + M_{3a}I_3) \tag{34}$$

where
$$M_{1a} = 2\log\frac{S_{1a}}{r_{1a}},$$

$$M_{2a} = 2\log\frac{S_{2a}}{r_{2a}},$$

and
$$M_{3a} = 2\log\frac{S_{3a}}{r_{3a}}.$$

S_{1a}, S_{2a} and S_{3a} are respectively the distances from the axis of the conductor denoted by one subscript to the axis of the image conductor denoted by the other subscript (for example, S_{1a} is the distance from power conductor 1 to the image of telephone conductor a or vice versa). r_{1a}, r_{2a} and r_{3a} are the distances between the axes of the conductors denoted by the subscripts. The three terms in parentheses must be combined vectorially at angles determined by the phase angles of the currents I_1, I_2 and I_3.

In computing the inductive effect of currents in a three-phase circuit it is convenient to consider the currents divided into three sets of components,* namely:

(1) balanced three-phase currents in each of the three conductors, whose vector sum is zero, and which are therefore displaced one-third cycle in time phase with respect to one another;
(2) a single-phase current in a loop composed of two of the conductors;
(3) a residual current divided equally among the three conductors and returning through the earth.

*See page 23 of report of Joint Committee to California Railroad Commission, dated July 7, 1914; also technical report No. 12.

If the phase relationships are known the resultant induced voltage may be obtained. The following formulas apply for the separate components as above defined:

For Balanced Currents.

$$E_a = -j2\pi fKI\,(M_{1a}\underline{/0^\circ} + M_{2a}\underline{/120^\circ} + M_{3a}\underline{/240^\circ}) \qquad (35)$$

or

$$E_a = -j2\pi fKI\,[M_{1a} - \tfrac{1}{2}(M_{2a} + M_{3a}) + j\tfrac{1}{2}\sqrt{3}\,(M_{2a} - M_{3a})] \qquad (36)$$

where I = magnitude of balanced currents.

In magnitude

$$E_a = 2\pi fKI\sqrt{[M_{1a} - \tfrac{1}{2}(M_{2a} + M_{3a})]^2 + [\tfrac{1}{2}\sqrt{3}(M_{2a} - M_{3a})]^2} \qquad (37)$$

The phase angle of E_a with respect to I_1 is,

$$\tan^{-1}\frac{\tfrac{1}{2}\sqrt{3}\,(M_{2a} - M_{3a})}{M_{1a} - \tfrac{1}{2}(M_{2a} + M_{3a})} - 90^\circ \qquad (38)$$

Substituting the values of M_{1a}, M_{2a} and M_{3a} in terms of the distances between conductors, including the image conductors,

$$E_a = 2\pi fKI\sqrt{\log^2\frac{S^2_{1a}r_{2a}r_{3a}}{r^2_{1a}S_{2a}S_{3a}} + 3\log\frac{S_{2a}r_{3a}}{r_{2a}S_{3a}}} \qquad (39)$$

For Single-Phase Current.

Assume single-phase component I_s in conductors 1 and 3.

$$E_a = -j2\pi fKI_s\,(M_{1a} - M_{3a}) \qquad (40)$$

Substituting the values of M_{1a} and M_{3a},

$$E_a = -j4\pi fKI_s\log\frac{S_{1a}r_{3a}}{r_{1a}S_{3a}} \qquad (41)$$

For Residual Current.

$$E_a = -j2\pi f\frac{K}{3}I_R\,(M_{1a} + M_{2a} + M_{3a}) \qquad (42)$$

where I_R = magnitude of residual current.

Substituting the values of M_{1a}, M_{2a} and M_{3a},

$$E_a = -j4\pi f\frac{K}{3}I_R\log\frac{S_{1a}S_{2a}S_{3a}}{r_{1a}r_{2a}r_{3a}} \qquad (43)$$

If the induced voltage due only to the residual current is desired, the individual power-circuit conductors may usually with small error be considered as replaced by a single conductor at the center of gravity o of the cross-section of the power circuit. The accuracy of this approxima-

tion increases as the separation of the circuits is increased. Based on this assumption formula (43) becomes

$$E_a = -j4\pi fKI_R \log \frac{S_{oa}}{r_{oa}} \qquad (44)$$

If in formulas (35) to (44), inclusive, the current is in amperes and logarithms are taken to the base 10, $K = 70.183\ (10)^{-6}$ to give E_a in volts per 1000 feet of parallelism.

The voltage induced in metallic circuits is the difference of the voltages induced along the two sides of the circuit, thus

$$E_{a-b} = E_a - E_b = -j2\pi f[I_1(M_{1a} - M_{1b}) + I_2(M_{2a} - M_{2b}) + I_3(M_{3a} - M_{3b})] \qquad (45)$$

Formulas (35), (36), (37), (38), (40) and (42) apply to a metallic circuit a, b if $(M_{1a} - M_{1b})$, $(M_{2a} - M_{2b})$ and $(M_{3a} - M_{3b})$ are substituted for M_{1a}, M_{2a} and M_{3a}, respectively. These formulas may also be used to obtain the average of the voltages induced along two conductors by substituting $\frac{1}{2}(M_{1a} + M_{1b})$, $\frac{1}{2}(M_{2a} + M_{2b})$ and $\frac{1}{2}(M_{3a} + M_{3b})$ for M_{1a}, M_{2a} and M_{3a}, respectively. If the separation of the two conductors a and b is small compared to their average separation from the power circuit then the average of their induced voltages is sensibly equal to that along a single conductor midway between them and to the induced voltage along a and b in parallel. Application of this fact is made in obtaining the voltages induced in phantom telephone circuits; the induced voltage along one side of a phantom circuit being taken as the average of induced voltages for the two conductors composing it, and the induced voltage for the phantom being, as for a two-conductor metallic circuit, the difference in the voltages induced along its two sides.

In the case of two or more disturbing circuits, as for example a twin-circuit power line, the resultant induced voltage is the vector sum of the induced voltages due to the individual circuits separately considered. The mutual effects between the several power circuits do not enter to complicate the computations as in the case of electric induction.

Voltages determined by the above formulas are for unit lengths of exposure; for short parallels with no transpositions the voltage induced along the circuits is proportional to the length. With transposed circuits the resultant induced voltage is the product of the voltage per unit length and the length of unbalanced exposure.* If the unbalanced exposure is a small fraction of the total length of the parallel the effect of irregularities becomes of much importance and voltages computed on the assumption of a uniform parallel may be considerably in error.

*See page 650 of this report and technical reports 39 and 66.

The influence of the condition of other circuits on the same pole line in modifying the magnetically induced voltages of a disturbed telephone circuit is less important than in the case of electric induction. In order for the effects in the two cases to be comparable, conductors of practically negligible resistance would be required. In practice the resistance of the conductors is an important factor, greatly diminishing the magnetic shielding effects.

The current resulting from the induced voltage will be determined by the impedances of the telephone circuit beyond the ends of the parallel, and also within the parallel. For short parallels the circuit may be considered as a generator whose open-circuit voltage is the induced e. m. f. and whose internal impedance is the series impedance of the telephone circuit within the parallel, closed through the equivalent impedances of the sections of line beyond either end. The current at the terminals is further determined by the amount and character of the intervening line and apparatus.

For short parallels the induced voltage is directly proportional to the frequency and to the length of the parallel. The induced current is dependent upon the induced voltage and impedance of the circuit. The resistance and reactance of the circuit are proportional to the length and the reactance is proportional to the frequency, hence the short-circuit current will be independent of the length of the parallel and will increase much less rapidly than the voltage with increased frequency. If the resistance of the conductor were negligible compared to the reactance the short-circuit current would also be independent of the frequency. For long parallels, to which these statements are not intended to apply, the attenuation and phase change along the power and telephone circuits must be considered.

The fundamental laws upon which the formulas above given are based, are to be found in most text books dealing with the principles of electricity and magnetism, hence a list of references for this section would be superfluous.

IV. Systematized Methods of Computation.

Several computation forms have been developed by the Field Department to facilitate computations of the coefficients of induction from single and twin-circuit three-phase power lines. They are designed for use with a calculating machine. Much more extended schedules would have been required were the work performed by means of logarithms. Since all the operations required are indicated on the forms with spaces provided for each result the numerical solution of the equations for particular cases becomes a routine operation, so that most of the work can be done by persons without technical training. Copies of

42—39969

the forms are attached, sample computations being shown on the three referring to single-circuit lines for a parallel of dimensions as indicated on drawing No. 323x. A brief description of the different forms follows.

A—Single-Circuit Three-Phase Power Lines.

Form No. 322, headed *Electric Charges and Characteristic Residual Voltage,* is used to obtain the potential and capacitance coefficients of the power-circuit conductors and the electric charges, when the conductors are energized with balanced or with residual voltages. With slight additional labor the characteristic residual voltage* of the system is obtained. On Form No. 323, headed *Electric Induction—Balanced and Residual Voltages,* the potential coefficients connecting the potentials of the telephone conductors and the charges on the power conductors are obtained. These are combined with the charges on the power conductors, giving the induced voltages between the telephone conductors and between one of them (a) and ground; per volt of the balanced voltages between the power conductors and ground, and per volt residual.

On Form No. 321, headed *Magnetic Induction—Balanced and Residual Currents,* the inductance coefficients per unit length of parallelism of the power and telephone circuits are computed. These are combined to give the mutual inductance per 1000 feet of parallel of the telephone circuit with a single-conductor power circuit equivalent to the three-phase circuit when energized with balanced or residual currents. Provision is made on this sheet for either considering or neglecting the image conductors in computing the coefficients of induction from balanced currents. In some cases neglect of the image conductors is permissible without introducing large errors. In the case of the sample computations the error due to neglecting the images is 50% of the voltages along one of the conductors (longitudinal), and less than 0.2% for the difference in voltages along the two conductors (transverse).

Another form, No. 336, which has been developed for computing the magnetic induction, has been found more advantageous than No. 321. It is designed to give directly the induced voltages expressed as millivolts per 1000 feet of parallelism per ampere at 60 cycles, instead of the mutual inductances. The arrangement differs somewhat from No. 321 and no provision is made for neglecting the image conductors.

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*See technical report No. 51.

B—Twin-Circuit Three-Phase Power Lines.

For twin-circuit lines the formulas are more complicated and a total of seven sheets is required for a complete solution of any given case. The forms are arranged to give the characteristic residual voltages of the two power circuits and the induction in a parallel two-conductor circuit for six different methods of parallel operation of the twin power circuits. The six forms of this series are given identifying letters for purposes of cross reference.

On Form No. 337 (Sheet A) the potential coefficients connecting the potentials and charges of the six power conductors are computed. Because of the symmetrical arrangement of the conductors of the two circuits the number of independent coefficients is reduced to twelve. The solutions of the two sets of simultaneous equations are performed on Form No. 338 (Sheet B) giving the capacitance coefficients. In addition, the characteristic residual voltages of the twin circuits are computed and expressed in per cent of the balanced three-phase voltages between pairs of conductors, for each of six different arrangements for parallel operation. The characteristic residual voltages of the twin circuits are equal.

On Form No. 339 (Sheet C), the capacitance coefficients are combined to give the components of the charges on the six conductors when balanced three-phase voltages of unit magnitude are impressed between the conductors of each circuit and ground; for six methods of parallel operation.

The potential coefficients connecting the charges on the power conductors and the potentials of two isolated telephone conductors are computed on Form No. 340 (Sheet D). On Form No. 341 (Sheet E) these are combined with the charges from Sheet C, giving the induced voltages between the telephone conductors and the average of the voltages between the two conductors and ground; in volts per volt impressed between pairs of conductors of the power circuits.

Form No. 341 (Sheet E) is also used for computing the inductance coefficients connecting the currents in the power conductors and the induced voltages per unit length along the telephone conductors. These are combined in proper phase relation on Form No. 342 (Sheet F), giving the average of the induced voltages along the two telephone conductors and their difference, expressed as millivolts per 1000 feet

of parallelism per ampere of balanced current at 60 cycles; for the two circuits individually and combined in six different ways in accordance with six methods of parallel operation.

Respectfully submitted.

(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

ATTACHMENTS: P. I. C. Drawings Nos. 323X, 331; copies of Forms Nos. 321, 322, 323, with sample computations; Nos. 336, 337, 338, 339, 340, 341, 342, blank.

APPROVED: August 29, 1916.

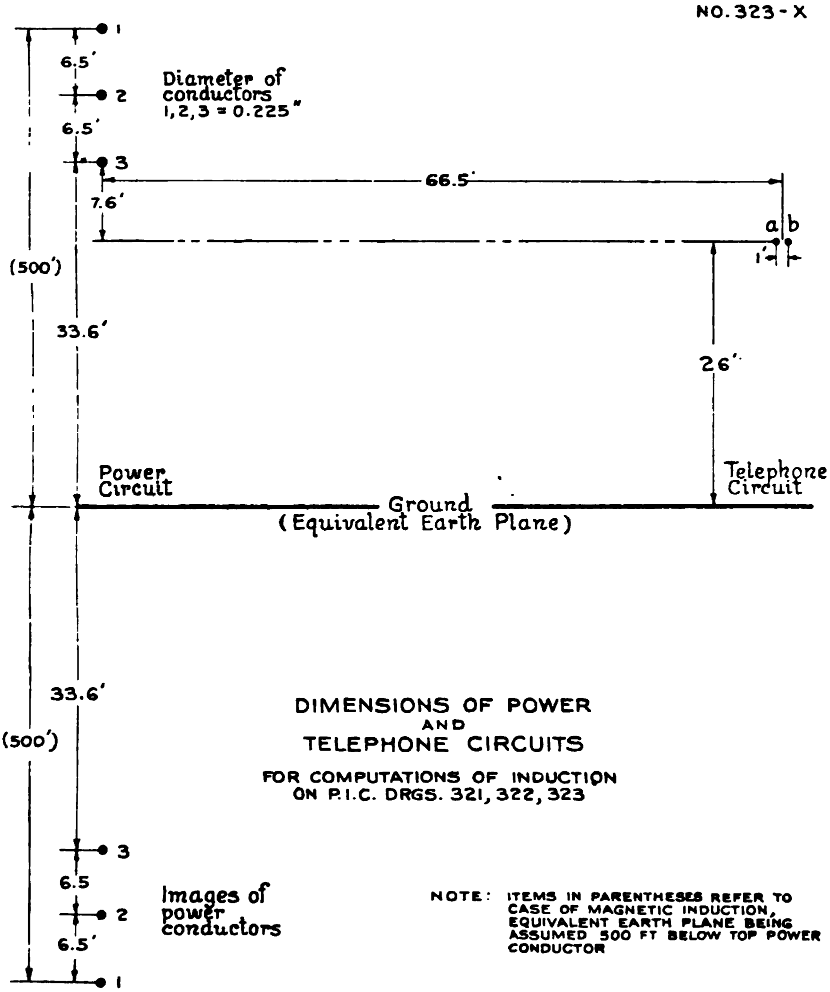
(Signed) R. W. MASTICK,
Field Engineer.

APPROVED: September 5, 1916.

SUBCOMMITTEE ON TESTS,
(Signed) J. E. WOODBRIDGE,
Chairman.

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE,
(Signed) ARTHUR F. BRIDGE,
Secretary.

January 8, 1917.



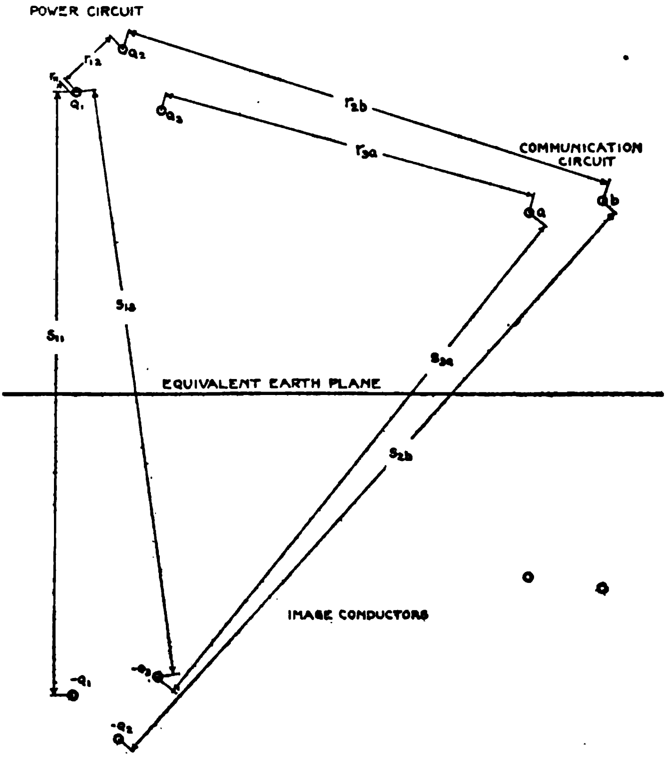
DIMENSIONS OF POWER AND TELEPHONE CIRCUITS

FOR COMPUTATIONS OF INDUCTION ON P.I.C. DRGS. 321, 322, 323

NOTE: ITEMS IN PARENTHESES REFER TO CASE OF MAGNETIC INDUCTION, EQUIVALENT EARTH PLANE BEING ASSUMED 500 FT BELOW TOP POWER CONDUCTOR

P.I.C. No 331
2-21-16

DIAGRAM SHOWING NOTATION USED IN FORMULAS FOR COMPUTING COEFFICIENTS OF INDUCTION.



T.R. No 64

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE.
(For use with Calculating Machine.)

P.I.C. M9381
1-19-16

MAGNETIC INDUCTION—BALANCED AND RESIDUAL CURRENTS.

Power-circuit configuration. Vertical No. Sample
Height of telephone conductors 26.0 ft. Spacing 1.0 ft.
Horizontal separation of circuits 66.5 ft. Reference Drawing No. 323 X.

INDUCTANCE COEFFICIENTS

Images Considered

*	Horizontal	Vertical	$(\text{Resultant})^2$	$\left(\frac{s}{r}\right)^2$	$2\log_{10}\frac{s}{r}$	
s_{1a}	66	979.4	963580.36	201.5707	2.304427	M_{1a}
r_{1a}	66	20.6	4780.36			
s_{2a}	66	972.9	950890.41	208.7662	2.319660	M_{2a}
r_{2a}	66	14.1	4554.81			
s_{3a}	66	966.4	938284.96	212.5818	2.327526	M_{3a}
r_{3a}	66	7.6	4413.76			
s_{1b}	67	979.4	963713.36	196.1414	2.292569	M_{1b}
r_{1b}	67	20.6	4913.36			
s_{2b}	67	972.9	951023.41	202.8716	2.307221	M_{2b}
r_{2b}	67	14.1	4687.81			
s_{3b}	67	966.4	938417.96	206.3927	2.314698	M_{3b}
r_{3b}	67	7.6	4546.76			

Images Neglected

	$(\text{Resultant})^2$	$\left(\frac{r_{mb}}{r_{ma}}\right)^2$	$2\log_{10}\frac{r_{mb}}{r_{ma}}$		$2\log_{10} r_{ma}$	
r_{1b}	4913.36	1.027822	0.011918	δ_1	3.679460	γ_1
r_{1a}	4780.36					
r_{2b}	4687.81	1.029200	0.012500	δ_2	3.658470	γ_2
r_{2a}	4554.81					
r_{3b}	4546.76	1.030133	0.012893	δ_3	3.644809	γ_3
r_{3a}	4413.76					

INDUCTION FROM BALANCED CURRENTS — Microhenrys per 1000 feet

Images Considered

Longitudinal				Transverse			
$M_{1a}-\frac{1}{2}(M_{2a}+M_{3a})$	-0.019166	A		$M_{1a}-M_{1b}-\frac{1}{2}(M_{2a}+M_{3a}-M_{2b}-M_{3b})$	-0.000777	α	
$\sqrt{\frac{3}{4}}(M_{2a}-M_{3a})$	-0.006812	B		$\sqrt{\frac{3}{4}}(M_{2a}+M_{3b}-M_{3a}-M_{2b})$	-0.000339	β	
$K\sqrt{A^2+B^2}$	1.428			$K\sqrt{\alpha^2+\beta^2}$	0.0595		
$\frac{B}{A}$ 0.3554		$\tan^{-1}\frac{B}{A}$	$-160^\circ.4$	$\frac{\beta}{\alpha}$ 0.436		$\tan^{-1}\frac{\beta}{\alpha}$	$-156^\circ.4$

Images Neglected

Longitudinal				Transverse			
$-\gamma_1+\frac{1}{2}(\gamma_2+\gamma_3)$	-0.027820	A		$\delta_1-\frac{1}{2}(\delta_2+\delta_3)$	-0.000778	α	
$\sqrt{\frac{3}{4}}(\gamma_3-\gamma_2)$	-0.011830	B		$\sqrt{\frac{3}{4}}(\delta_2-\delta_3)$	-0.000340	β	
$K\sqrt{A^2+B^2}$	2.122			$K\sqrt{\alpha^2+\beta^2}$	0.0595		
$\frac{B}{A}$ 0.4252		$\tan^{-1}\frac{B}{A}$	$-157^\circ.0$	$\frac{\beta}{\alpha}$ 0.437		$\tan^{-1}\frac{\beta}{\alpha}$	$-156^\circ.4$

INDUCTION FROM RESIDUAL CURRENTS — Microhenrys per 1000 feet

Longitudinal		Transverse	
$\frac{K}{3}(M_{1a}+M_{2a}+M_{3a})$	162.6	$\frac{K}{3}(M_{1a}+M_{2a}+M_{3a}-M_{1b}-M_{2b}-M_{3b})$	0.8686

• $K = 70.183$ $\sqrt{\frac{3}{4}} = 0.8660254$ $\frac{K}{3} = 23.394$

Date 2-15-16 Computed by R.G.M. Checked by

R.I.C. No 322
1-19-16

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE.
(For use with Calculating Machine.)

ELECTRIC CHARGES AND CHARACTERISTIC RESIDUAL VOLTAGE

SYSTEM OF THREE CONDUCTORS AND GROUND.

Power-circuit configuration: Vertical — No. Sample
Spacings of conductors: 6.5 ft., 13 ft. — Size 0.45 in. dia
Height of lowest conductor: 33.6 ft. — Reference Drawing No. 323 X.

POTENTIAL COEFFICIENTS

	Horizontal	Vertical	(Resultant)^2	$(\frac{s}{r})^2$	$2\log_{10}\frac{s}{r}$	
s_{11}		93.2 x 12	1250818.56	24707527.	7.392829	p_{11}
r_{11}		0.225	0.050625			
s_{22}		80.2 x 12	926213.76	18295581.	7.262346	p_{22}
r_{22}		0.225	0.050625			
s_{33}		67.2 x 12	650280.96	12845056.	7.108736	p_{33}
r_{33}		0.225	0.050625			
s_{12}		86.7	7516.89	177.91455	2.250211	p_{12}
r_{12}		6.5	42.25			
s_{13}		80.2	6432.04	38.059408	1.580462	p_{13}
r_{13}		13	169.			
s_{23}		73.7	5431.69	128.56071	2.109109	p_{23}
r_{23}		6.5	42.25			

CAPACITANCE COEFFICIENTS

$p_{11}\,p_{22}\,p_{33}$	381.66293	$\frac{1}{\sigma}(p_{22}\,p_{33}-p_{23}^2)$	0.152361	k_{11}
$2\,p_{12}\,p_{13}\,p_{23}$	15.00156	$\frac{1}{\sigma}(p_{11}\,p_{33}-p_{13}^2)$	0.161656	k_{22}
$-p_{11}\,p_{23}^2$	-32.88582	$\frac{1}{\sigma}(p_{11}\,p_{22}-p_{12}^2)$	0.157038	k_{33}
$-p_{22}\,p_{13}^2$	-18.14032	$\frac{1}{\sigma}(p_{23}\,p_{13}-p_{12}\,p_{33})$	-0.040894	k_{12}
$-p_{33}\,p_{12}^2$	-35.99473	$\frac{1}{\sigma}(p_{12}\,p_{23}-p_{13}\,p_{22})$	-0.021741	k_{13}
$\Sigma=\sigma$	309.64362	$\frac{1}{\sigma}(p_{13}\,p_{12}-p_{23}\,p_{11})$	-0.038870	k_{23}

CHARACTERISTIC RESIDUAL VOLTAGE

PERCENT OF BALANCED THREE-PHASE VOLTAGE BETWEEN CONDUCTORS.

$k_{11}+k_{12}+k_{13}$	0.089726	C_{10}	$3(C_{10}-C_{20})$	0.023502	α
$k_{12}+k_{22}+k_{23}$	0.081892	C_{20}	$\sqrt{3}(C_{10}+C_{20}-2C_{30})$	-0.036782	β
$k_{13}+k_{23}+k_{33}$	0.096427	C_{30}	$\sqrt{\alpha^2+\beta^2}$	0.043649	λ
Σ	0.268045	C_{123-0}	$100\frac{\lambda}{2C_{123-0}}$	8.142	E_{RC}

CHARGES ON CONDUCTORS

Balanced Voltages: $V_1 = E$, $V_2 = (-\frac{1}{2} + j\frac{\sqrt{3}}{2})E$, $V_3 = (-\frac{1}{2} - j\frac{\sqrt{3}}{2})E$

$k_{11}-\frac{1}{2}(k_{12}+k_{13})$	0.183678	X_1	$\frac{\sqrt{3}}{2}(k_{12}-k_{13})$	-0.016587	Y_1
$k_{12}-\frac{1}{2}(k_{22}+k_{23})$	-0.102287	X_2	$\frac{\sqrt{3}}{2}(k_{22}-k_{23})$	+0.173661	Y_2
$k_{13}-\frac{1}{2}(k_{23}+k_{33})$	-0.080825	X_3	$\frac{\sqrt{3}}{2}(k_{23}-k_{33})$	-0.169661	Y_3

$\sqrt{3} = 1.7320508$
$\frac{\sqrt{3}}{2} = 0.8660254$

Residual Voltage: $V_1 = \frac{1}{3}E_R$, $V_2 = \frac{1}{3}E_R$, $V_3 = \frac{1}{3}E_R$

$\frac{1}{3}C_{10}$	0.029908	Q_1	$\frac{1}{3}C_{20}$	0.027297	Q_2	$\frac{1}{3}C_{30}$	0.032142	Q_3

Date 2-15-16 Computed by R.G.M. Checked by F.E.F.

P. I. C. No 323
1-19-16

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE

(For use with Calculating Machine.)

ELECTRIC INDUCTION – BALANCED AND RESIDUAL VOLTAGES.

Three-phase Power Circuit – Two Isolated Telephone Conductors.

Power-circuit configuration Vertical No Sample

Telephone conductors: Relative position 26.0 ft. Spacing 1.0 ft

Separation of circuits: Horizontal 66.6 ft. Vertical Drawing No. 323 X

POTENTIAL COEFFICIENTS.

	Horizontal	Vertical	(Resultant)^2	$(\frac{s}{r})^2$	$2\log_e\frac{s}{r}$	
s_{1a}	66	72.6	9626.76	2.013815	0.304019	P_{1a}
r_{1a}	66	20.6	4780.36			
s_{2a}	66	66.1	8725.21	1.915603	0.282306	P_{2a}
r_{2a}	66	14.1	4554.81			
s_{3a}	66	59.6	7908.16	1.791706	0.253266	P_{3a}
r_{3a}	66	7.6	4413.76			
s_{1b}	67	72.6	9759.76	1.986371	0.298061	P_{1b}
r_{1b}	67	20.6	4913.36			
s_{2b}	67	66.1	8858.81	1.889626	0.276376	P_{2b}
r_{2b}	67	14.1	4687.81			
s_{3b}	67	59.6	8041.16	1.768547	0.247617	P_{3b}
r_{3b}	67	7.6	4546.76			

INDUCTION FROM BALANCED VOLTAGES - VOLTS PER VOLT.

Conductor "a" to ground.				
$P_{1a}X_1$	+ 0.0558416	$P_{1a}Y_1$		- 0.0050428
$P_{2a}X_2$	- 0.0288762	$P_{2a}Y_2$		+ 0.0490255
$P_{3a}X_3$	- 0.0204702	$P_{3a}Y_3$		- 0.0429694
Σ_x	+ 0.0064952	Σ_y		+ 0.0010133
$\sqrt{\Sigma_x^2+\Sigma_y^2}$	0.0065738	$\frac{\Sigma_y}{\Sigma_x}$ 0.15601	$\tan^{-1}\frac{\Sigma_y}{\Sigma_x}$	+ 8.°9
Conductor "a" to Conductor "b"				
$(P_{1a}-P_{1b})X_1$	+ 0.00109435	$(P_{1a}-P_{1b})Y_1$		- 0.00009883
$(P_{2a}-P_{2b})X_2$	- 0.00060656	$(P_{2a}-P_{2b})Y_2$		+ 0.00102981
$(P_{3a}-P_{3b})X_3$	- 0.00045658	$(P_{3a}-P_{3b})Y_3$		- 0.00095841
Σ_x	+ 0.00003121	Σ_y		- 0.00002743
$\sqrt{\Sigma_x^2+\Sigma_y^2}$	0.0000416	$\frac{\Sigma_y}{\Sigma_x}$ - 0.8789	$\tan^{-1}\frac{\Sigma_y}{\Sigma_x}$	- 41.3

INDUCTION FROM RESIDUAL VOLTAGE - VOLTS PER VOLT.

Conductor "a" to Ground		Conductor "a" to Conductor "b"	
$P_{1a}Q_1$	0.00909260	$(P_{1a}-P_{1b})Q_1$	0.000178192
$P_{2a}Q_2$	0.00770611	$(P_{2a}-P_{2b})Q_2$	0.000161871
$P_{3a}Q_3$	0.00814048	$(P_{3a}-P_{3b})Q_3$	0.000181570
Σ	0.02494	Σ	0.0005216

Date 2-15-16 Computed by R.G.M. Checked by

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE
(For use with Calculating Machine.)

MAGNETIC INDUCTION—BALANCED AND RESIDUAL CURRENTS.

Three-phase Power Circuit — Two Telephone Conductors.

Sheet No. ____________

Power-circuit configuration ________________________________

Spacing of power conductors ________________________________

Height of lowest power conductor above equivalent earth plane ____________

Telephone conductors: Arrangement ____________________ Spacing ________

Separation of circuits: Horizontal ________________ Vertical ____________

INDUCTANCE COEFFICIENTS

	Horizontal	Vertical	(Resultant)^2	$\left(\frac{S}{r}\right)^2$	$2\log_{10}\frac{S}{r}$	
S_{1a}						M_{1a}
r_{1a}						
S_{2a}						M_{2a}
r_{2a}						
S_{3a}						M_{3a}
r_{3a}						
S_{1b}						M_{1b}
r_{1b}						
S_{2b}						M_{2b}
r_{2b}						
S_{3b}						M_{3b}
r_{3b}						

INDUCTION FROM BALANCED CURRENTS. Millivolts per ampere per 1000 feet at 60 cycles

$M_{1a}-\frac{1}{2}(M_{2a}+M_{3a})$		A_x	$\sqrt{\frac{3}{4}}(M_{2a}-M_{3a})$			A_y
$M_{1b}-\frac{1}{2}(M_{2b}+M_{3b})$		B_x	$\sqrt{\frac{3}{4}}(M_{2b}-M_{3b})$			B_y
A_x-B_x		X	A_y-B_y			Y
$K\sqrt{A_x^2+A_y^2}$		E_a	$\frac{A_y}{A_x}$		$\tan^{-1}\frac{A_y}{A_x}$	
$K\sqrt{X^2+Y^2}$		E_a-E_b	$\frac{Y}{X}$		$\tan^{-1}\frac{Y}{X}$	
$K\sqrt{B_x^2+B_y^2}$		E_b	$\frac{B_y}{B_x}$		$\tan^{-1}\frac{B_y}{B_x}$	
A_x+B_x		C	A_y+B_y			D
$\frac{K}{2}\sqrt{C^2+D^2}$		E_{ab}	$\frac{D}{C}$		$\tan^{-1}\frac{D}{C}$	

INDUCTION FROM RESIDUAL CURRENT. Millivolts per ampere per 1000 feet at 60 cycles.

$\frac{K}{3}(M_{1a}+M_{2a}+M_{3a})$		E_a	E_a-E_b	
$\frac{K}{3}(M_{1b}+M_{2b}+M_{3b})$		E_b	$\frac{1}{2}(E_a+E_b)$	

$K = 26.458$ $\frac{K}{2} = 13.229$ $\frac{K}{3} = 8.819$ $\sqrt{\frac{3}{4}} = 0.8660254$

Date ____________ Computed by ____________ Checked by ____________

Form No 337

POTENTIAL COEFFICIENTS
TWIN THREE-PHASE POWER CIRCUITS

Sheet No. A ____________

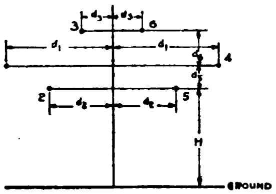
Power-circuit configuration ____________

Spacings d_1 ______ d_2 ______ d_3 ______ d_5 ______ d_6 ______ Size of conductors ____________

Height of lowest power conductor above ground (H) ____________

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE
(For use with Calculating Machine)

	Horizontal	Vertical	$(\text{Resultant})^2$	$(\frac{S}{r})^2$	$2 \log_{10} \frac{S}{r}$	
s_{11}						P_{11}
r_{11}						
s_{14}						P_{14}
r_{14}						
s_{22}						P_{22}
r_{22}						
s_{25}						P_{25}
r_{25}						
s_{33}						P_{33}
r_{33}						
s_{36}						P_{36}
r_{36}						
s_{12}						P_{12}
r_{12}						
s_{15}						P_{15}
r_{15}						
s_{13}						P_{13}
r_{13}						
s_{16}						P_{16}
r_{16}						
s_{23}						P_{23}
r_{23}						
s_{26}						P_{26}
r_{26}						



$P_{11}+P_{14}$		A_{11}	$P_{11}-P_{14}$		B_{11}
$P_{22}+P_{25}$		A_{22}	$P_{22}-P_{25}$		B_{22}
$P_{33}+P_{36}$		A_{33}	$P_{33}-P_{36}$		B_{33}
$P_{12}+P_{15}$		A_{12}	$P_{12}-P_{15}$		B_{12}
$P_{13}+P_{16}$		A_{13}	$P_{13}-P_{16}$		B_{13}
$P_{23}+P_{26}$		A_{23}	$P_{23}-P_{26}$		B_{23}

Date ____________ Computed by ____________ Checked by ____________

Form No. 338

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE

(For use with Calculating Machine)

CAPACITANCE COEFFICIENTS AND CHARACTERISTIC RESIDUAL VOLTAGES

TWIN THREE-PHASE POWER CIRCUITS

Sheet No. B. ________

For Dimensions And Data. See Sheet No. A ________

CAPACITANCE COEFFICIENTS

$A_{11}\ A_{22}\ A_{33}$			$\frac{1}{\alpha}(A_{22}A_{33}-A_{23}^2)$		C_{11}
$2A_{12}\ A_{13}\ A_{23}$			$\frac{1}{\alpha}(A_{11}\ A_{33}-A_{13}^2)$		C_{22}
$-A_{11}A_{23}^2$			$\frac{1}{\alpha}(A_{11}\ A_{22}-A_{12}^2)$		C_{33}
$-A_{22}A_{13}^2$			$\frac{1}{\alpha}(A_{23}A_{13}-A_{12}A_{33})$		C_{12}
$-A_{33}A_{12}^2$			$\frac{1}{\alpha}(A_{12}A_{23}-A_{13}A_{22})$		C_{13}
$\Sigma=\alpha$			$\frac{1}{\alpha}(A_{13}A_{12}-A_{23}A_{11})$		C_{23}
$B_{11}\ B_{22}\ B_{33}$			$\frac{1}{\eta}(B_{22}B_{33}-B_{23}^2)$		D_{11}
$2B_{12}\ B_{13}\ B_{23}$			$\frac{1}{\eta}(B_{11}\ B_{33}-B_{13}^2)$		D_{22}
$-B_{11}B_{23}^2$			$\frac{1}{\eta}(B_{11}\ B_{22}-B_{12}^2)$		D_{33}
$-B_{22}B_{13}^2$			$\frac{1}{\eta}(B_{23}B_{13}-B_{12}B_{33})$		D_{12}
$-B_{33}B_{12}^2$			$\frac{1}{\eta}(B_{12}B_{23}-B_{13}B_{22})$		D_{13}
$\Sigma=\eta$			$\frac{1}{\eta}(B_{13}B_{12}-B_{23}B_{11})$		D_{23}
$\frac{1}{2}(C_{11}+D_{11})$		K_{11}	$\frac{1}{2}(C_{11}-D_{11})$		K_{14}
$\frac{1}{2}(C_{22}+D_{22})$		K_{22}	$\frac{1}{2}(C_{22}-D_{22})$		K_{25}
$\frac{1}{2}(C_{33}+D_{33})$		K_{33}	$\frac{1}{2}(C_{33}-D_{33})$		K_{36}
$\frac{1}{2}(C_{12}+D_{12})$		K_{12}	$\frac{1}{2}(C_{12}-D_{12})$		K_{15}
$\frac{1}{2}(C_{13}+D_{13})$		K_{13}	$\frac{1}{2}(C_{13}-D_{13})$		K_{16}
$\frac{1}{2}(C_{23}+D_{23})$		K_{23}	$\frac{1}{2}(C_{23}-D_{23})$		K_{26}

CHARACTERISTIC RESIDUAL VOLTAGES FOR SIX METHODS OF PARALLEL OPERATION
PERCENT OF BALANCED THREE-PHASE VOLTAGE BETWEEN CONDUCTORS

Direct Capacitances			No.1; $V_1=V_4,\ V_2=V_5,\ V_3=V_6$		
$C_{11}+C_{12}+C_{13}$		C_{10}	$3(C_{10}-C_{20})$		α
$C_{12}+C_{22}+C_{23}$		C_{20}	$\sqrt{3}(C_{10}+C_{20}-2C_{30})$		β
$C_{13}+C_{23}+C_{33}$		C_{30}	$\sqrt{\alpha^2+\beta^2}$		λ
2Σ		C_0	$100\frac{\lambda}{C_0}$		E_{RC}
No.2; $V_1=V_4,\ V_2=V_6,\ V_3=V_5$			No.3 $V_1=V_6,\ V_2=V_5,\ V_3=V_4$		
$3(2C_{10}-C_{20}-C_{30})$		α	$3(C_{10}+C_{30}-2C_{20})$		α
$\sqrt{3}(2C_{10}-C_{20}-C_{30})$		β	$\sqrt{3}(2C_{20}-C_{10}-C_{30})$		β
$\sqrt{\alpha^2+\beta^2}$		λ	$\sqrt{\alpha^2+\beta^2}$		λ
$100\frac{\lambda}{2C_0}$		E_{RC}	$100\frac{\lambda}{2C_0}$		E_{RC}
No.4; $V_1=V_5,\ V_2=V_4,\ V_3=V_6$			No.5; $V_1=V_5,\ V_2=V_6,\ V_3=V_4$		
0	0	α	$3(C_{10}-C_{30})$		α
$\sqrt{3}(C_{10}+C_{20}-2C_{30})$		β	$\sqrt{3}(2C_{20}-C_{10}-C_{30})$		β
β	(ditto)	λ	$\sqrt{\alpha^2+\beta^2}$		λ
$100\frac{\lambda}{C_0}$		E_{RC}	$100\frac{\lambda}{2C_0}$		E_{RC}
No.6;	$V_1=V_6,\ V_2=V_4,\ V_3=V_5$		E_{RC} Same as for No.5		

$\sqrt{3}=17320508$

Date ________ Computed by ________ Checked by ________

Form No. 339
Revised

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE
(For use with Calculating Machine)

COMPONENTS OF ELECTRIC CHARGES

TWIN THREE-PHASE POWER CIRCUITS

Sheet No C. ________

For Dimensions And Data See Sheet No. B ________

$K_{11}-\frac{1}{2}(K_{21}+K_{31})$		R_1	$K_{12}-\frac{1}{2}(K_{22}+K_{32})$		S_1	$K_{13}-\frac{1}{2}(K_{23}+K_{33})$		T_1
$K_{21}-\frac{1}{2}(K_{31}+K_{11})$		R_2	$K_{22}-\frac{1}{2}(K_{32}+K_{12})$		S_2	$K_{23}-\frac{1}{2}(K_{33}+K_{13})$		T_2
$K_{31}-\frac{1}{2}(K_{11}+K_{21})$		R_3	$K_{32}-\frac{1}{2}(K_{12}+K_{22})$		S_3	$K_{33}-\frac{1}{2}(K_{13}+K_{23})$		T_3
$K_{14}-\frac{1}{2}(K_{24}+K_{34})$		R_4	$K_{15}-\frac{1}{2}(K_{25}+K_{35})$		S_4	$K_{16}-\frac{1}{2}(K_{26}+K_{36})$		T_4
$K_{24}-\frac{1}{2}(K_{34}+K_{14})$		R_5	$K_{25}-\frac{1}{2}(K_{35}+K_{15})$		S_5	$K_{26}-\frac{1}{2}(K_{36}+K_{16})$		T_5
$K_{34}-\frac{1}{2}(K_{14}+K_{24})$		R_6	$K_{35}-\frac{1}{2}(K_{15}+K_{25})$		S_6	$K_{36}-\frac{1}{2}(K_{16}+K_{26})$		T_6
$\sqrt{\frac{3}{4}}(K_{21}-K_{31})$		U_1	$\sqrt{\frac{3}{4}}(K_{22}-K_{32})$		W_1	$\sqrt{\frac{3}{4}}(K_{23}-K_{33})$		O_1
$\sqrt{\frac{3}{4}}(K_{31}-K_{11})$		U_2	$\sqrt{\frac{3}{4}}(K_{32}-K_{12})$		W_2	$\sqrt{\frac{3}{4}}(K_{33}-K_{13})$		O_2
$\sqrt{\frac{3}{4}}(K_{11}-K_{21})$		U_3	$\sqrt{\frac{3}{4}}(K_{12}-K_{22})$		W_3	$\sqrt{\frac{3}{4}}(K_{13}-K_{23})$		O_3
$\sqrt{\frac{3}{4}}(K_{24}-K_{34})$		U_4	$\sqrt{\frac{3}{4}}(K_{25}-K_{35})$		W_4	$\sqrt{\frac{3}{4}}(K_{26}-K_{36})$		O_4
$\sqrt{\frac{3}{4}}(K_{34}-K_{14})$		U_5	$\sqrt{\frac{3}{4}}(K_{35}-K_{15})$		W_5	$\sqrt{\frac{3}{4}}(K_{36}-K_{16})$		O_5
$\sqrt{\frac{3}{4}}(K_{14}-K_{24})$		U_6	$\sqrt{\frac{3}{4}}(K_{15}-K_{25})$		W_6	$\sqrt{\frac{3}{4}}(K_{16}-K_{26})$		O_6

COMPONENTS OF CHARGES ON CONDUCTORS FOR SIX METHODS OF PARALLEL OPERATION

Nos. 1 & 2	R_1+R_4		X_1	S_1+S_4		X_2	T_1+T_4		X_3
	R_1+R_4	Same as X_1	X_4	S_1+S_4	Same as X_2	X_5	T_1+T_4	Same as X_3	X_6
Nos. 4 & 5	R_1+R_5		X_1	S_1+S_5		X_2	T_1+T_5		X_3
	R_2+R_4		X_4	S_2+S_4		X_5	T_2+T_4		X_6
Nos. 3 & 6	R_1+R_6		X_1	S_1+S_6		X_2	T_1+T_6		X_3
	R_3+R_4		X_4	S_3+S_4		X_6	T_3+T_4		X_6
No. 1	U_1+U_4		Y_1	W_1+W_4		Y_2	O_1+O_4		Y_3
	U_1+U_4	Same as Y_1	Y_4	W_1+W_4	Same as Y_2	Y_5	O_1+O_4	Same as Y_3	Y_6
No. 2	U_1-U_4		Y_1	W_1-W_4		Y_2	O_1-O_4		Y_3
	$-Y_1$		Y_4	$-Y_2$		Y_5	$-Y_3$		Y_6
No. 4	U_1-U_5		Y_1	W_1-W_5		Y_2	O_1-O_5		Y_3
	U_4-U_2		Y_4	W_4-W_2		Y_5	O_4-O_2		Y_6
No. 5	U_1+U_5		Y_1	W_1+W_5		Y_2	O_1+O_5		Y_3
	U_4+U_2		Y_4	W_4+W_2		Y_5	O_4+O_2		Y_6
No. 3	U_1-U_6		Y_1	W_1-W_6		Y_2	O_1-O_6		Y_3
	U_4-U_3		Y_4	W_4-W_3		Y_5	O_4-O_3		Y_6
No. 6	U_1+U_6		Y_1	W_1+W_6		Y_2	O_1+O_6		Y_3
	U_4+U_3		Y_4	W_4+W_3		Y_5	O_4+O_3		Y_6

$V_1 = 1$, $V_2 = -\frac{1}{2} + j\sqrt{\frac{3}{4}}$, $V_3 = -\frac{1}{2} - j\sqrt{\frac{3}{4}}$

METHOD OF PARALLEL OPERATION	No. 1	$V_1=V_4, V_2=V_5, V_3=V_6$.	No. 4	$V_1=V_5, V_2=V_4, V_3=V_6$	No. 3	$V_1=V_6, V_2=V_5, V_3=V_4$.
	No. 2	$V_1=V_4, V_2=V_6, V_3=V_5$.	No. 5	$V_1=V_5, V_2=V_6, V_3=V_4$	No. 6	$V_1=V_6, V_2=V_4, V_3=V_5$.

$\sqrt{\frac{3}{4}} = 0.8660254$

Date ____________ Computed by ____________ Checked by ____________

Form No 340

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE
(For use with Calculating Machine.)

POTENTIAL OR INDUCTANCE COEFFICIENTS OF POWER AND TELEPHONE CONDUCTORS.

TWIN THREE-PHASE POWER CIRCUITS – TWO TELEPHONE CONDUCTORS.

Sheet No D________

Power Circuit Configuration ____________________

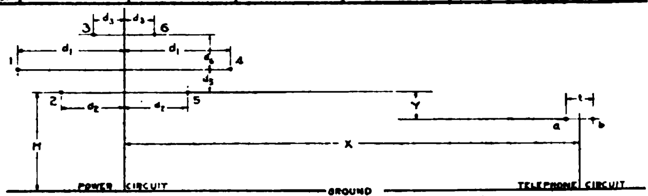
Power Conductor Spacings d_1_______ d_2_______ d_3_______ d_5_______ d_6_______

Height of Lowest Power Conductor above Equivalent Earth Plane (H)__________

Telephone Conductors Arrangement __________ Spacing (t) __________

Separation of Circuits: Horizontal (X)__________ Vertical (Y) __________

	Horizontal	Vertical	(Resultant)^2	$\left(\frac{S}{r}\right)^2$	$2\log_{10}\frac{S}{r}$		
S_{1a}						P_{1a}	M_{1a}
r_{1a}							
S_{2a}						P_{2a}	M_{2a}
r_{2a}							
S_{3a}						P_{3a}	M_{3a}
r_{3a}							
S_{1b}						P_{1b}	M_{1b}
r_{1b}							
S_{2b}						P_{2b}	M_{2b}
r_{2b}							
S_{3b}						P_{3b}	M_{3b}
r_{3b}							
S_{4a}						P_{4a}	M_{4a}
r_{4a}							
S_{5a}						P_{5a}	M_{5a}
r_{5a}							
S_{6a}						P_{6a}	M_{6a}
r_{6a}							
S_{4b}						P_{4b}	M_{4b}
r_{4b}							
S_{5b}						P_{5b}	M_{5b}
r_{5b}							
S_{6b}						P_{6b}	M_{6b}
r_{6b}							



Date __________ Computed by __________ Checked by __________

Form No. 341

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE
(For use with calculating Machine)

Sheet No. E ________

ELECTRIC INDUCTION – BALANCED VOLTAGES

TWIN THREE-PHASE POWER CIRCUITS – TWO TELEPHONE CONDUCTORS

For Dimensions And Data See Sheet No. C ________ And No. D ________

Volts Per Kilovolt Between Conductors of Power Circuit

Method of Parallel Operation	No. 1	No. 2	No. 4	No. 5	No. 3	No. 6	
	$V_1=V_4, V_2=V_5, V_3=V_6$	$V_1=V_4, V_2=V_6, V_3=V_5$	$V_1=V_5, V_2=V_4, V_3=V_6$	$V_1=V_5, V_2=V_6, V_3=V_4$	$V_1=V_6, V_2=V_5, V_3=V_4$	$V_1=V_6, V_2=V_4, V_3=V_5$	
$P_{1a}X_1+P_{2a}X_2+P_{3a}X_3+P_{4a}X_4+P_{5a}X_5+P_{6a}X_6$		Same as No. 1		Same as No. 4		Same as No. 3	A_x
$P_{1b}X_1+P_{2b}X_2+P_{3b}X_3+P_{4b}X_4+P_{5b}X_5+P_{6b}X_6$		" " "		" " "		" " "	B_x
A_x+B_x		" " "		" " "		" " "	C
A_x-B_x		" " "		" " "		" " "	X
$P_{1a}Y_1+P_{2a}Y_2+P_{3a}Y_3+P_{4a}Y_4+P_{5a}Y_5+P_{6a}Y_6$							A_y
$P_{1b}Y_1+P_{2b}Y_2+P_{3b}Y_3+P_{4b}Y_4+P_{5b}Y_5+P_{6b}Y_6$							B_y
A_y+B_y							D
A_y-B_y							Y
$288.7\sqrt{C^2+D^2}$							E_{ab}
$\frac{D}{C}$							
$\tan^{-1}\frac{D}{C}$							
$577.4\sqrt{X^2+Y^2}$							E_a-E_b
$\frac{Y}{X}$							
$\tan^{-1}\frac{Y}{X}$							
$577.4\sqrt{A_x^2+A_y^2}$							E_a
$\frac{A_y}{A_x}$							
$\tan^{-1}\frac{A_y}{A_x}$							
$577.4\sqrt{B_x^2+B_y^2}$							E_b
$\frac{B_y}{B_x}$							
$\tan^{-1}\frac{B_y}{B_x}$							

Date ________ Computed by ________ Checked by ________

Form No. 342

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE

(For use with calculating Machine)

MAGNETIC INDUCTION—BALANCED CURRENTS

TWIN THREE-PHASE POWER CIRCUITS—TWO TELEPHONE CONDUCTORS

Sheet No. F ____________

For Dimensions And Data See Sheet No. D ____________

Millivolts per 1000 feet per ampere at 60 Cycles

Single Circuit Farthest from Telephone Conductors			$M_{1a}-\frac{1}{2}(M_{2a}+M_{3a})$		A_x	$\sqrt{\frac{3}{4}}(M_{2a}-M_{3a})$		A_y
			$M_{1b}-\frac{1}{2}(M_{2b}+M_{3b})$		B_x	$\sqrt{\frac{3}{4}}(M_{2b}-M_{3b})$		B_y
			A_x+B_x		V_x	A_y+B_y		V_y
			A_x-B_x		W_x	A_y-B_y		W_y
			$\frac{K}{2}\sqrt{V_x^2+V_y^2}$		E_{ab}	$K\sqrt{W_x^2+W_y^2}$		E_a-E_b
			$\frac{V_y}{V_x}$	$\tan^{-1}\frac{V_y}{V_x}$		$\frac{W_y}{W_x}$	$\tan^{-1}\frac{W_y}{W_x}$	
Single Circuit Nearest Telephone Conductors			$M_{4a}-\frac{1}{2}(M_{5a}+M_{6a})$		C_x	$\sqrt{\frac{3}{4}}(M_{5a}-M_{6a})$		C_y
			$M_{4b}-\frac{1}{2}(M_{5b}+M_{6b})$		D_x	$\sqrt{\frac{3}{4}}(M_{5b}-M_{6b})$		D_y
			C_x+D_x		T_x	C_y+D_y		T_y
			C_x-D_x		U_x	C_y-D_y		U_y
			$\frac{K}{2}\sqrt{T_x^2+T_y^2}$		E_{ab}	$K\sqrt{U_x^2+U_y^2}$		E_a-E_b
			$\frac{T_y}{T_x}$	$\tan^{-1}\frac{T_y}{T_x}$		$\frac{U_y}{U_x}$	$\tan^{-1}\frac{U_y}{U_x}$	
Method of Parallel Operation	No. 1	$I_1=I_4$	V_x+T_x		X_1	V_y+T_y		Y_1
		$I_2=I_5$	W_x+U_x		x_1	W_y+U_y		y_1
		$I_3=I_6$	$\frac{K}{2}\sqrt{X_1^2+Y_1^2}$		E_{ab}	$K\sqrt{x_1^2+y_1^2}$		E_a-E_b
	No. 2	$I_1=I_4$				V_y-T_y		Y_2
		$I_2=I_6$				W_y-U_y		y_2
		$I_3=I_5$	$\frac{K}{2}\sqrt{X_1^2+Y_2^2}$		E_{ab}	$K\sqrt{x_1^2+y_2^2}$		E_a-E_b
	No. 3	$I_1=I_6$	$M_{6a}-\frac{1}{2}(M_{5a}+M_{4a})$		F_x	$\sqrt{\frac{3}{4}}(M_{5a}-M_{4a})$		F_y
			$M_{6b}-\frac{1}{2}(M_{5b}+M_{4b})$		G_x	$\sqrt{\frac{3}{4}}(M_{5b}-M_{4b})$		G_y
		$I_2=I_5$	$V_x+F_x+G_x$		X_3	$V_y+F_y+G_y$		Y_3
		$I_3=I_4$	$W_x+F_x-G_x$		x_3	$W_y+F_y-G_y$		y_3
			$\frac{K}{2}\sqrt{X_3^2+Y_3^2}$		E_{ab}	$K\sqrt{x_3^2+y_3^2}$		E_a-E_b
	No. 6	$I_1=I_6$				$V_y-F_y-G_y$		Y_6
		$I_2=I_4$				$W_y-F_y+G_y$		y_6
		$I_3=I_5$	$\frac{K}{2}\sqrt{X_3^2+Y_6^2}$		E_{ab}	$K\sqrt{x_3^2+y_6^2}$		E_a-E_b
	No. 4	$I_1=I_5$	$M_{5a}-\frac{1}{2}(M_{4a}+M_{6a})$		H_x	$\sqrt{\frac{3}{4}}(M_{4a}-M_{6a})$		H_y
			$M_{5b}-\frac{1}{2}(M_{4b}+M_{6b})$		J_x	$\sqrt{\frac{3}{4}}(M_{4b}-M_{6b})$		J_y
		$I_2=I_4$	$V_x+H_x+J_x$		X_4	$V_y+H_y+J_y$		Y_4
		$I_3=I_6$	$W_x+H_x-J_x$		x_4	$W_y+H_y-J_y$		y_4
			$\frac{K}{2}\sqrt{X_4^2+Y_4^2}$		E_{ab}	$K\sqrt{x_4^2+y_4^2}$		E_a-E_b
	No. 5	$I_1=I_5$				$V_y-H_y-J_y$		Y_5
		$I_2=I_6$				$W_y-H_y+J_y$		y_5
		$I_3=I_4$	$\frac{K}{2}\sqrt{X_4^2+Y_5^2}$		E_{ab}	$K\sqrt{x_4^2+y_5^2}$		E_a-E_b

$I_1=1$ $I_2=-\frac{1}{2}+j\sqrt{\frac{3}{4}}$ $I_3=-\frac{1}{2}-j\sqrt{\frac{3}{4}}$ $K=26.458$ $\frac{K}{2}=13.229$ $\sqrt{\frac{3}{4}}=0.8660254$

Date ____________ Computed by ____________ Checked by ____________

Technical Report No. 65.

January 6, 1917.

COEFFICIENTS OF INDUCTION FOR COMMUNICATION CIRCUITS PARALLELED BY THREE-PHASE POWER CIRCUITS. VARIATION WITH RELATIVE POSITION AND CONFIGURATION.

OUTLINE.

I. Introduction.

The magnitudes of the induction in communication circuits caused by parallel power circuits is dependent upon a large number of factors including the cross-sectional dimensions of the parallel. This report presents the results of a study of the variation of the induction in communication circuits paralleled by three-phase power circuits, with the relative position and configuration of the two classes of circuits. These results are expressed in terms of the coefficients of induction which are defined as the ratios of the induced voltages in the communication circuit to the inducing factors (components of voltage or current) of the parallel power circuit. The effects of the balanced and residual components of the power circuit voltages and currents are considered separately both for metallic circuits and for a single conductor or pair of conductors with reference to ground. In all cases the coefficients are given for a uniform short section of parallel in which both classes of circuits are assumed nontransposed and the communication circuit nonshielded.

The coefficients were obtained by computations based upon the physical dimensions ordinarily occurring with the types of circuits involved. The investigation at San Fernando* demonstrated that the coefficients of induction for nontransposed circuits obtained by computations are in close agreement with those obtained by experiment, the differences being due to experimental errors, and errors in the dimensions assumed for the computations. For the purpose of the study described in this report these errors are of no effect on computations,

*Technical Reports 54 and 56.

but would be sources of difficulty in an experimental study. Other important reasons for choosing the computation method rather than the experimental method are, however, economy and flexibility. The expense of such an extensive study as is here reported would be practically prohibitive if carried out experimentally on actual lines.

The calculation of the currents and voltages induced in a communication circuit by a power circuit under operating conditions of both circuits presents many difficulties. The length of the parallel may be great for the important frequencies; the shielding effects of neighboring circuits and other objects, as well as the effects of unexposed portions of the communication circuit, require consideration; the magnitudes and wave-shapes of the power-circuit voltages and currents must be known and the effects of the different components properly combined; and allowance must be made for irregularities of exposure. Thus at best only a roughly approximate solution is obtainable.

For given wave-forms of voltage and current in a power circuit the severity of the possible induction in a parallel communication circuit may be taken as proportional to the product of the voltage or current in the power circuit, the coefficient of induction and the length of parallel. With the voltage and current of the power circuit and the length of parallel fixed, the coefficients of induction may be used as criteria for comparing various configurations of power and communication circuits and for judging the relative severity of the inductive effects of parallels. Based upon a maximum value of induced voltage permissible in the communication circuits, and given the voltages and currents in the power circuit, the coefficients of induction furnish means of determining the minimum lengths of parallel construction which will produce this induced voltage in the communication circuit, for various configurations and relative positions of the two classes of circuits. Thus the coefficients of induction may be used in formulating a definition of parallelism, *i. e.,* in specifying the limiting conditions of association of power and communication circuits which are liable to create inductive interference in the latter class of circuits.

II. Scope.

A—General Statement.

Parallels of single-circuit three-phase power lines and one or two-wire communication circuits receive chief consideration herein. Parallels involving twin-circuit three-phase power lines are also considered, but less extensively, a few representative cases being investigated. In general, the conductors of two-wire communication circuits were assumed to lie in a horizontal plane. However, some computations were made for other arrangements of the communication-circuit conductors.

The voltage to ground and between the disturbed conductors induced by the power-circuit voltages, and the mean voltage and difference of voltages along the disturbed conductors induced by the power-circuit currents, were computed in each case. The coefficients of induction due to balanced and residual components of the voltages and currents were considered separately.

To take advantage of the fact that the coefficients of induction are functions of the relative magnitudes of the cross-sectional dimensions and independent of the absolute values, the dimensions have been expressed in terms of the spacing of the power conductors; the base of the triangle, or if all are in the same plane, the greatest spacing of conductors. This method has the effect of reducing the number of variables by one and increases the flexibility of the results.

The coefficients of induction were computed for communication circuits having a wide range of positions relative to the power circuits so that nearly all practical cases of parallelism with power circuits of the types studied are included. The scope of the investigation is described in detail in the sections immediately following.

B—Single-Circuit Power Lines.

The configurations and range of dimensions of single-circuit power lines are given in Table I.

The values given in the table are the limiting dimensions which were used in the computations. In most cases four heights, and for induction from the power circuit voltages four sizes of power conductors, were considered.

The heights of conductors given in the table are referred to an equivalent ground plane. For induction from the power-circuit voltages this plane is considered coincident with the earth's surface. For induction from the power-circuit currents it is in general much below the earth's surface. In addition to the range of heights indicated in the table, the computations of induction from balanced currents were made on the assumption that the presence of the earth could be neglected, expressed as infinite height of conductors.

TABLE I.

Configurations and Dimensions of Single-Circuit Three-Phase Lines.

Configurations	Spacings of conductors		Diameter of conductors	Height of lowest conductor above equivalent ground plane	
	Base	Altitude		Voltages	Currents
Equilateral triangle, base horizontal, vertex upward	d	$\frac{1}{2}\sqrt{3}d$	0.002d to 0.016d	3d to 20d	20d to 200d
Equilateral triangle, base vertical (both sides of line)	d	$\frac{1}{2}\sqrt{3}d$	0.002d to 0.016d	3d to 20d	20d to 200d
Isosceles triangle, base horizontal, vertex upward	d	0.4d	0.002d to 0.016d	3d to 20d	20d to 200d
		1.25d	0.006d	5d to 10d	
Symmetrical, horizontal	d, d/2, d/2		0.001d to 0.008d	1.5d to 10d	10d to 100d
Unsymmetrical, horizontal (both sides of line)	d, d/3, 2d/3		0.001d to 0.008d	2.5d to 8d	10d to 100d
	d, d/4, 3d/4		0.001d to 0.008d	1.5d to 8d	10d to 100d
Vertical	d, d/2, d/2		0.001d to 0.008d	1.5d to 8d	10d to 100d

The coefficients of induction were computed for positions of the disturbed conductors in three or four different horizontal planes varying from the plane of the lowest power conductor to a plane at approximately half this height, and in general for five horizontal separations varying from five times to one hundred times the unit spacing of power conductors. Additional computations for other positions were made where necessary to properly determine curves.

For induction from the power-circuit voltages the complete schedule of relative positions of circuit was carried out for one size of power conductors for each configuration; having a diameter of 0.006d for the triangular configuration and of 0.003d for the horizontal and vertical configurations. For other sizes of conductor fewer coefficients were computed, sufficient to determine the effect of the size of conductor for a number of representative cases.

In computing the voltage induced in metallic circuits, a constant ratio of the spacing of the disturbed conductors to that of the power conductors was used for each power-circuit configuration. In addition, however, for a few selected positions with respect to power circuits of each configuration, the effect of varying the spacing of the disturbed conductors was determined.

In addition to the program as indicated in Table I, coefficients for an isosceles triangular power circuit (base horizontal, vertex upward) with various altitudes ranging from zero to 1.6 times its base, were computed for two horizontal separations at one vertical separation. Similarly, computations were made for an isosceles triangular power circuit with base vertical, for various altitudes ranging from twice the base with vertex away to the same value with vertex toward the disturbed conductors. A like procedure was followed for a horizontal power circuit, varying the position of the inside conductor so that the spacing of power conductors nearest the disturbed conductors varied from 0.2 to 0.8 the spacing of the outside power conductors.

C—Twin-Circuit Power Lines.

The twin-circuit lines studied include only certain cases in which the conductors of the two circuits are symmetrically located with respect to an intermediate plane perpendicular to the ground plane. The computations involved in considering arrangements not fulfilling this condition are too complicated to be included here. For each of the twin-circuit configurations, the coefficients were computed for six different methods of interconnection of the circuits for parallel operation. These include all possible combinations, subject to the limitation that the individual circuits be kept on opposite sides of the supporting structure, which is the usual procedure.

The schedule of configurations and dimensions is given in Table II. The horizontal separation of the disturbed conductors from the power lines varied from four to one hundred times the unit spacing of power conductors, except in the case of vertical lines of dimensions, marked with an asterisk (*) for which only one horizontal separation was considered. One representative vertical separation was considered in each case.

For induction from balanced voltages, a diameter of power conductors of 0.006d was used for the equilateral triangular and horizontal configurations and 0.003d for the isosceles triangular and vertical configurations. The residual components of the voltages and currents were not considered.

TABLE II.

Configurations and Dimensions of Twin-Circuit Three-Phase Lines.

Configuration	Spacings of conductors of one circuit		Minimum distance between circuits	Height of lowest conductor above equivalent ground plane	
	Base	Altitude		Voltages	Currents
Equilateral triangles, bases horizontal, vertices downward	d	$\frac{1}{2}\sqrt{3}d$	d	10d	40d ∞ d
Equilateral triangles, bases horizontal, vertices upward	d	$\frac{1}{2}\sqrt{3}d$	d	10d	40d ∞ d
Isosceles triangles, bases vertical, vertices outward	d	0.125d	0.9d	2d	20d ∞ d
Vertical	d, d/2, d/2		0.9d	2d* 2.5d 5d*	20d ∞ d
			0.75d*	2.5d	20d ∞ d
Horizontal	d, d/2, d/2		d	10d	100d

D—Telephone-Conductor Arrangements.

As noted above, two disturbed conductors in the same horizontal plane were assumed in most of the computations. To compare various configurations of telephone circuits some additional computations were undertaken. For two positions with respect to three single-circuit power lines (triangular, symmetrical horizontal and vertical) the coefficients of induction were computed for "horizontal," "vertical pole-pair" and "square" arrangements of phantom telephone circuits having the same average separation from the power circuit. "Horizontal" and "vertical pole-pair" phantom circuits are in common use by telephone companies throughout the United States. "Square" phantoms are in use by the British Post Office.

In addition, for the same separations and power-circuit configurations used in the above mentioned study, the variation of the induction in two-wire metallic circuits with the angular position of the plane of their conductors was determined. This was done largely as a matter of scientific interest. However, the results have a practical application to cases of bracketed circuits.

In both of these studies the coefficients of induction due to the balanced components of the power-circuit voltages and currents were computed for all three power-circuit configurations. For the residual

components, only the triangular power circuit was considered, since the results in this case depend but little upon the configuration of the power circuit.

III. Method of Procedure.

A—Computations.

Formulas for computing the coefficients of induction are given in technical report No. 64. Forms for systematizing the solution of these formulas, fully described therein, were used throughout the computations for this study. These forms, which are designed especially for use with calculating machines, reduce the solution of the formulas to routine arithmetical operations. The combined use of computation forms and calculating machines greatly increased the accuracy and speed of the work, which was done by a corps of from five to seven computers.

Great care was observed throughout to secure accurate results. Various means of checking were used both during the course of the work and as a test of the completed results. A number of simple rules for detecting errors were standardized and each computer required to apply them to each sheet. In general the computations were performed by one computer, after the fundamental dimensional data had been checked by another. The "electric charge" sheets, upon which the coefficients of induction for several positions of the disturbed conductors depend, were checked by independent computations. The effort was made to have the coefficients determining any one curve worked out by different computers to reduce the chance of personal errors affecting all points similarly. Whenever the plotting showed a value to be questionable, it was carefully looked up and recomputed if necessary.

All operations were carried out so as to insure four significant figures in the result, with the exception of a few cases which would require the use of more than ten place logarithmic tables. The formulas involve logarithms of certain quantities, hence their use was necessary to a limited extent though all operations of multiplication and division were performed with calculating machines.

A total of 2630 computation forms (letter size) was required for the work, exclusive of wastage. The total number of coefficients computed on these forms is approximately 7750.

B—Graphical Representation.

Practically all the results are represented graphically upon the 214 curve sheets attached. In general each curve sheet contains families of curves showing the variation of coefficients of induction with some one factor for constant values of all others.

The classification of the curve sheets is in substantial accord with the following:

(1) Power circuit configurations.

(2) Induction due to voltages or currents.

(3) Balanced or residual components.

(4) Mean or difference of induced voltages.

(5) Variable dimensions.

The detailed scheme of classification is clearly shown by the index of the curve sheets.

Curves for variable horizontal separation and variable height of conductors, and in a few cases for variable horizontal separation and variable vertical separation are grouped on the same sheet. In all other cases curves for only one variable dimension are shown on a sheet.

Several sizes and types of cross-section paper were used to fit the varying requirements. Logarithmic cross-section paper was required to adequately show the variation of the coefficients with horizontal separation and height of conductors, on account of the extreme range in magnitude of the coefficients, 1,000,000 to 1, and on account of the nature of the functions. No logarithmic paper exactly suitable could be obtained so it was necessary to join several squares of paper in order to cover the range. For a few representative cases the variation of the coefficients with horizontal separation was plotted on uniformly divided cross-section paper in order to give an undistorted picture of this relation and as a contrast to the curves on logarithmic paper.

In nearly all cases the effect of vertical separation of the two classes of circuits is shown on uniformly divided cross-section paper, groups of curves for several heights of conductors being given on one sheet, each group consisting of curves showing the variation of the coefficients with vertical separation for several values of horizontal separation.

The effect of the diameter of the power-circuit conductors is shown on uniformly divided cross-section paper as a correction factor, all coefficients being expressed as percentages of the coefficients for the standard diameters assumed for the major portion of the computations.

The coefficients of induction due to twin-circuit power lines are plotted on logarithmic paper. The curves are for variable horizontal separation only, and show the effect of the individual circuits and the resultant effect of the twin circuits for six methods of parallel operation. For comparing the induction from single and twin power circuits special cross-section paper having a logarithmic scale of abscissas and a uniformly divided scale of ordinates was developed and used for showing the coefficients of induction for the six methods

of parallel operation of the twin circuits as percentages of the coefficients for the single power circuits.

Charts of "isoinduction lines" were constructed for picturing the character of the electric and magnetic fields surrounding different types of power circuits. These "isoinduction lines" are analogous to the contour lines of a topographical map.

A number of other plots, most of which are of a special nature, such as ratios of the effects of different configurations and other factors, are shown on uniformly divided cross-section paper.

Each curve sheet has been made a complete unit in itself, independent of other curve sheets, tables or the text of this report. For this purpose each sheet contains a title including a brief description of the dimensions and a cross-sectional diagram of the parallel, showing the configuration and relative positions of power circuit and disturbed conductors.

The completed curves have been checked against the original computation sheets to insure accuracy.

C—TABULATION.

The coefficients are given for purposes of record and convenient reference in 92 tables. These tables, together with two index sheets, are attached. As in the case of the curve sheets, each table is a unit complete in itself, and each contains references to the curve sheets upon which the data given therein are plotted.

A few points computed for the purpose of determining critical positions of the curves are not tabulated in the main tables due to waste space such tabulation would require, but are given according to configurations in a miscellaneous table.

IV. Discussion of Results.

A—SINGLE-CIRCUIT POWER LINES.

1. *Effect of Dimensional Variations.*

The different power-line configurations are, with the exception of the vertical, quite similar as regards manner of variation of the coefficients of induction with variation of the cross-sectional dimensions of the parallel. Therefore, with the exception of the vertical, which receives special mention, the following discussion applies, in general, to all the configurations studied.

The variable dimensions are: spacing, height and diameter of power conductors, horizontal and vertical separation of disturbed conductors from power conductors and spacing of disturbed conductors. In the succeeding paragraphs discussing the effect of varying any one dimension all others are assumed as fixed, unless otherwise stated.

Horizontal Separation. The decrease of the coefficients of induction with increase of horizontal separation is expressed approximately by the formula, $E = K \left[\frac{x}{d}\right]^{-n}$, since the curves are approximately straight lines on logarithmic paper, with diminishing values of coefficients with increasing separation. On some of the curves the slope increases as the horizontal separation increases, thus indicating that the coefficients decrease more rapidly than is indicated by the above approximate formula.

Table III presents average values of the exponent n for the several configurations and classes of coefficients. The largest values, about 3, are observed for induction between the conductors of metallic circuits, from power-circuit voltages. For other cases the values of n are less, falling to 1 for induction from residual current into grounded circuits. With this chief exception the induction into grounded circuits decreases roughly as the square and induction into metallic circuits as the cube of the horizontal separation.

Height of Conductors. Increase in the height of the conductors, of *both* power and disturbed circuits, is, in general, accompanied by an increase in the coefficients of induction, the rate of variation of the coefficients with height becoming greater as the horizontal separation of circuits is increased. This effect is somewhat greater on the induction from the residual than on induction from the balanced components of both voltages and currents.

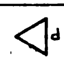
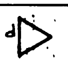
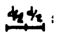
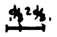
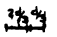
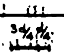
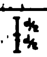
Owing to the imperfect conductivity of the earth, the neutral plane of the magnetic field set up by the currents of the power circuit is not coincident with the earth's surface but several hundred feet below it. The practical range of variation in height of circuits is relatively small when referred to this neutral or equivalent ground plane. Hence the effects of variations in height are much less pronounced on the coefficients of induction due to the currents than on the coefficients of induction due to the voltages; the neutral plane in the electric field set up by the voltages being practically coincident with the earth's surface. For the triangular and horizontal configurations and at small separations of lines the effect of the earth on the induction from balanced currents may be neglected with small error, particularly in computations of induction in metallic circuits. For illustration see Curve Sheets Nos. 14 and 18.

Increase in the height of power conductors, height of disturbed conductors remaining constant, may cause either an increase or decrease in the coefficients of induction depending upon particular conditions. No curves are presented to show this relation. Variation in height of

TABLE III

EXPONENTS (n) FOR FORMULA EXPRESSING VARIATION OF COEFFICIENTS WITH HORIZONTAL SEPARATION

$$E = K\left(\frac{x}{d}\right)^{-n}$$

Case No.	Configuration	Induction from Power-Circuit Voltages: Balanced Voltages: Voltage to ground of a single isolated conductor	Induction from Power-Circuit Voltages: Balanced Voltages: Voltage between two isolated conductors	Induction from Power-Circuit Voltages: Residual Voltage: Voltage to ground of a single isolated conductor	Induction from Power-Circuit Voltages: Residual Voltage: Voltage between two isolated conductors	Induction from Power-Circuit Currents: Balanced Currents: Mean voltage along a pair of conductors	Induction from Power-Circuit Currents: Balanced Currents: Difference of voltages along pair of conductors	Induction from Power-Circuit Currents: Residual Current: Mean voltage along a pair of conductors	Induction from Power-Circuit Currents: Residual Current: Difference of voltages along pair of conductors
1		1.8	2.6	2.0	2.9	1.1	2.0		
2		1.8	2.6			1.1	2.0		
3		1.9	2.6						
4		1.9	2.8			1.1	1.5		
5		1.5	2.3						
6		2.4	3.5	2.0	2.9	1.4	2.4	1.1	1.5
7		2.4	3.3			1.4	2.2		
8		2.0	3.0						
9		2.3	3.1						
10		1.9	3.0						
11		1.9	2.9	2.0	2.9	2.0*	3.0*		

DISTURBED CONDUCTORS AT RIGHT OF POWER CIRCUIT   * WHEN $H/d > 100$

disturbed conductors, height of power conductors remaining constant, is discussed below under "Vertical Separation."

Vertical Separation. Positions of disturbed conductors above the plane of the lowest power conductor were not considered. With disturbed conductors below the power conductors, the coefficients of induction decrease with increase in vertical separation, measured from the plane of the lowest power conductors. In other words, as the height of the disturbed conductors approaches that of the power conductors the induction increases. This relationship is approximately linear except at the smaller horizontal separations. When the vertical separation is equal to the height of power conductors above the equivalent ground plane (that is, when the disturbed conductors are in this neutral plane) the coefficients are equal to zero. The practical variations of vertical separation have much less effect on induction from the currents than on induction from the voltages since for the former the equivalent ground plane is much below the surface. Circuits on and even somewhat below the earth's surface are subject to the inductive effects of the currents of parallel power circuits though unaffected by the voltages.

Diameter of power conductors. The electric charges on the power conductors, for given voltages, increase slowly with conductor diameter, thus causing an increase in the coefficients of induction from the power-circuit voltages. For a variation of power-conductor diameter of from 30 to 270 per cent of the diameters assumed in the major portion of the computations, the greatest corresponding variation in the magnitudes of coefficients is from about 70 to 140 per cent for balanced voltages and 90 to 110 per cent for residual voltage. Induction from the power-circuit currents is not affected by the size of the power conductors.

Spacing of power conductors. In general, for all types of power circuit an increase in the spacing of the conductors causes an approximately proportionate increase in the magnitudes of the coefficients of induction from the balanced voltages and currents, as shown by Curve Sheets Nos. 137 to 141, inclusive. The effect of the spacing of the power conductors on the coefficients of induction from residual current is very small, as shown by Curve Sheet No. 214. For large separations and also if the center of gravity of the conductor arrangement remains the same, variations of spacing have even less effect than shown by these curves. The coefficients of induction from residual voltage increase slowly with the spacing of the conductors due to increased charge per unit of voltage. This increase, as shown by Curve Sheet No. 213, may amount to 50 per cent, for an increase in spacing from 2 to 20 feet.

Displacement of intermediate conductor of horizontal power circuit. Displacements from the median position up to 30 per cent of the spacing of the outside conductors were considered. This extreme displacement corresponds to a ratio of the spacings from the intermediate conductor to the outside conductors of 4:1. The coefficients of induction from balanced currents are approximately minimum when the intermediate conductor is in the median position, and for equal displacements in the two directions, toward or away from the disturbed conductors, the coefficients are approximately equal. The total change in magnitude of the coefficients, due to the extreme displacements, amounts to only a few per cent. The coefficients of induction from balanced voltages vary between limits of 60 and 190 per cent of those for the median position, the lowest values occurring when the intermediate conductor is displaced toward the disturbed circuit. The effect on the coefficients from residual voltage and current is small, the extreme variation being less than 4%. The effects discussed in this paragraph are shown on Curve Sheets Nos. 134, 135 and 136.

Altitude of power-circuit triangle. Large variations occur in the coefficients of induction from the balanced voltages and currents of triangular power circuits, as the altitude is varied. The coefficients for the residual components are affected but little. These effects are shown on Curve Sheets Nos. 128, 129 and 130 for horizontal-base triangles and Nos. 131, 132, 133 for vertical base. In the latter case, base vertical, the coefficients for the balanced components of both voltage and current are nearly minimum when the altitude is zero (all conductors in a vertical plane) and for the same altitude the coefficients are nearly the same whether the vertex of the triangle is toward or away from the disturbed conductors. The magnitude of the effect varies greatly with the height of the power conductors and the relative position of the circuits. With the base horizontal the coefficients of induction from balanced currents are smallest when the altitude is zero (all conductors in horizontal plane) and increase with increase of altitude. The coefficients for balanced voltages may first decrease to a minimum with increase of altitude and then increase. The magnitude of the effect varies greatly with the height of power conductors and relative position of circuits.

Spacing of disturbed conductors. Induction in two-wire metallic circuits is directly proportional, within one-half per cent, to the spacing between the two conductors; up to 0.5 the average spacing of the conductors of the power circuits. No curves are presented to show this relation. The data upon which this statement is based are recorded in the attached tables.

Exceptional effects with vertical configuration. The foregoing statements disregard some marked peculiarities of the electric and magnetic fields about vertical power-circuits, due to the balanced voltages and currents. For example, as the horizontal separation increases the coefficients of induction from balanced currents decrease very rapidly at first, pass a sharp minimum, increase rapidly, reach a maximum and thereafter decrease at a rate approximating that of curves for other configurations. Curves of a somewhat similar character occur for other variables. For balanced currents the peculiarities are much more pronounced than for balanced voltages. The variation of the coefficients of induction from balanced currents with horizontal separation is such that the minimum points of the curves giving the difference of the voltages induced along two closely adjacent conductors are practically coincident with the maximum points of the curves for the mean induced voltage. An example of this relation is afforded by comparing Curve Sheets Nos. 114 and 120.

2. *Isoinduction Lines.*

Curve Sheets Nos. 148, 149, and 150 show the electric and magnetic fields existing in the neighborhood of power circuits of vertical, horizontal and equilateral triangular configurations, due to the balanced voltages and currents. "Isoinduction lines" connect points having the same magnitude of induced voltage, differences of phase being disregarded. It will be noted that the isoinduction lines are closely crowded near the power circuit and that their spacing increases with increase of horizontal separation, corresponding to the rapid decrease in magnitude of the coefficients with increasing horizontal separation. The peculiar characteristics of the fields about the vertical power circuit are especially noteworthy.

3. *Comparisons.*

Induction from voltages and currents. The general forms of corresponding curves of coefficients of induction from voltages and currents are similar. The rate of decrease of the coefficients with horizontal separation is considerably greater for induction from voltages than for induction from currents, as may be seen from an inspection of Table III. The effect of practical variations in the height of conductors and vertical separation of circuits is much greater for induction from voltages than for induction from currents. These effects are due chiefly to the great difference in the position of the equivalent ground plane for the two cases.

Induction in grounded and metallic circuits. Curve Sheets Nos. 195, 197 and 199 show the variation with horizontal separation of the ratio

of the mean to the difference of induced voltage to ground or along a pair of conductors for three configurations. The ratios are approximately proportional to the horizontal separation and may range from 10 to 1000 in practical cases.

Effect of balanced and residual components. For equal magnitudes of balanced and residual voltages and currents the coefficients of induction from residual components are, in general, the greater. For the voltages this ratio ranges from 2 to 9 and for the currents from 5 to 100, and even greater at critical separations from vertical power circuits. Excepting the vertical configuration, the ratios increase with increase of horizontal separation, tending toward constant values at large separations. The variations of the ratios with horizontal separation are shown on Curve Sheets Nos. 196, 198 and 200.

By definition the residual voltage or current of a three-phase circuit is three times the residual component of any one of the conductors. The above ratios and those given by the curves do not, therefore, give a true comparison of the cumulative and differential effects, respectively, of residual and balanced components of voltages and currents. For this purpose the ratios should be multiplied by three, making the residual and balanced components per conductor the same in magnitude.

Power-circuit configurations. To obtain a comparison of different configurations of single-circuit power lines with respect to induction in a parallel communication circuit, representative dimensions were chosen for 100, 60, 30 and 11-kV. lines and the induced voltages in a parallel communication circuit determined from the curves. The configurations considered were equilateral triangular, with base horizontal and base vertical, symmetrical horizontal, vertical, and for the 11 and 30-kV. lines, the unsymmetrical horizontal.

A communication circuit of two conductors one foot apart was assumed at a height of 25 feet above ground. The horizontal separation from the power circuit was varied from a minimum of 30 feet to a maximum of from 200 to 1000 feet, depending on the power-circuit voltage.

The induced voltages to ground and between the disturbed conductors, caused by the balanced voltages to ground of the power circuit, and the mean voltage and difference of voltages induced along the conductors by the balanced currents were obtained in each case. These, expressed in terms of the coefficients for the corresponding triangular power circuit (base horizontal) are plotted on Curve Sheets Nos. 201 to 204, inclusive, showing the variations with the horizontal separation of circuits. Full data concerning the dimensions and relative positions of the two classes of circuits are given on these curve sheets.

It is not possible to draw a generally applicable conclusion regarding the most advantageous type of power circuit for minimizing the inductive disturbances in neighboring communication circuits, as it varies with the spacing of the power conductors (determined by the voltages), the type of induction and the horizontal separation of circuits.

The figures in Table IV show the range of horizontal separations within which the specified configuration gives the minimum coefficients of induction.

TABLE IV

LIMITS OF HORIZONTAL SEPARATION—FEET,
FOR CONFIGURATION GIVING MINIMUM INDUCTION.

		Grounded circuit				Metallic circuit			
Configuration		△ ◁ ▷	—•—	—•—	|	△ ◁ ▷	—•—	—•—	|
Balanced voltages	100 kV	95—170	170—1000		30—95	140—280	280—1000		30—140
	60 kV	70—170	170—600		30—70	110—250	250—600		30—110
	30 kV	65—125		125—350	30—65	95—185		195—350	30—95
	11 kV	40—120		120—200	30—40	70—180		180—200	30—70
Balanced currents	100 kV	560—1000			30—560	30—50			50—1000
	60 kV	460—600			30—460	30—45			45—600
	30 kV				30—350	30—55			55—350
	11 kV				30—200	30—45			45—200

Considering the range of separations from 50 to 75 feet, which will include most cases of parallel lines separated by the width of a highway, the vertical configuration has the smallest coefficients of induction from balanced voltages for power circuits over 30-kV. For power circuits of less than 30-kV. the vertical and equilateral triangular configurations have nearly equal coefficients, which are smaller than those of the other

configurations studied. The horizontal configuration has the largest coefficients for highway separations.

For practically the entire range of horizontal separations the coefficients of induction from balanced currents are smallest for the vertical configuration.

For large separations of power and communication circuits the coefficients of induction from balanced voltages are smallest for the horizontal configuration (symmetrical and unsymmetrical) and largest for the vertical configuration.

The induction from the equilateral triangular power circuit is practically the same whether the base is horizontal or vertical, and if the latter, whether the vertex is toward or away from the communication circuit.

With the horizontal configuration, the coefficients of induction from balanced currents are less when the intermediate conductor is in the median position than if displaced toward or away from the communication circuit. The magnitudes are approximately the same for corresponding positions of the intermediate conductor toward or away from the communication circuit. The coefficients of induction from the balanced voltages are greater if the intermediate conductor is displaced away from the communication circuit than if symmetrically placed or displaced toward the communication circuit. Except for small separations (less than 90 feet for induction in metallic circuits and 40 feet for induction in grounded circuits) from 11-kV. lines, the induction from balanced voltages is less when the intermediate conductor is displaced toward the communication circuit than when in the median position.

For average highway separations, considering the induction due to balanced voltages and balanced currents, the configurations in order of preference are:

	Lines over 30-kV.	Lines under 30-kV.
1.	Vertical	Equilateral Triangular
2.	Equilateral Triangular	Vertical
3.	Horizontal	Horizontal

For low-voltage lines a symmetrical horizontal configuration is better than an unsymmetrical. If an unsymmetrical horizontal configuration is used, it is advantageous to have the intermediate conductor on the side towards the communication circuit.

With high-voltage lines, when the separation of the circuits is 200 feet or more a symmetrical horizontal configuration is best. Unsymmetrical horizontal configurations were not considered for high-voltage lines.

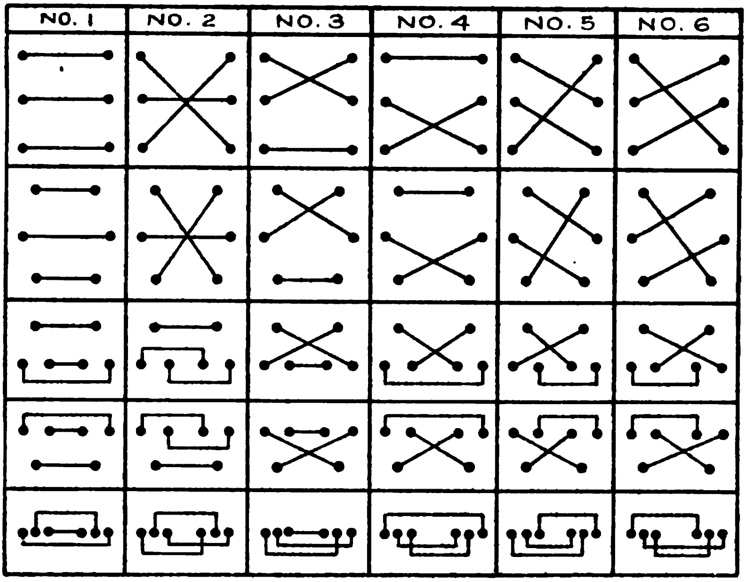
It is to be noted that the vertical configuration is not usually employed for a single-circuit line except with the ultimate construction of twin circuits in view.

B—Twin-Circuit Power Lines.

The six methods of interconnection of the twin circuits for parallel operation which were considered are shown and indicated by number in Table V, for the five different configurations of line studied. In the following discussion and on the curves these numbers are used in briefly identifying the different methods.

Table V

Methods of Interconnection for Parallel Operation



1. *Vertical.*

(Curve Sheets Nos. 155 to 160 and 179 to 182.)

The dimensions used for this configuration correspond to those generally employed for 60-kV. circuits of this type.

The curves of variation with horizontal separation are similar in form to those for the single vertical circuits and contain similar maximum and minimum points. Throughout the range of horizontal separations the largest values of induced voltage to ground from the balanced voltages (about double those of one of the single circuits) occur with Methods Nos. 1 and 4 and the smallest (about half those of one of the

single circuits) with Method No. 2. For the voltage induced between conductors by the balanced voltages the smallest values occur with Method No. 2 except for small separations. The largest values occur with Methods Nos. 1 and 4.

When the effect of the earth is neglected the curves of the coefficients of induction from balanced currents are nearly straight lines and parallel, on logarithmic paper. Throughout the range of separation Method No. 1 gives the largest coefficients, about double those of the single circuits and No. 2 gives the smallest, less than half of those of the single circuits. This is true both for the mean voltage and for the difference of the voltages induced along the conductors. With the equivalent ground plane 20d below the plane of the lowest power conductors there are maximum and minimum points in the curves. Except for a small portion of the range Method No. 2 gives the smallest coefficients and No. 1 the largest. In general, Method No. 2 is the best, No. 5 and No. 6 give very nearly the same result as the single circuits while Nos. 1 and 4 are the worst.

2. *Isosceles Triangles—Bases Vertical—Vertices Outward.*

(Curve Sheets Nos. 161 to 166 and 183 to 186.)

This configuration differs from the vertical in that the middle conductors are each displaced outward $\frac{1}{8}$ the distance between the top and lowest conductors and the height of the lowest power conductors is 2d instead of 2.5d. The dimensions correspond to those ordinarily used for 100-kV. circuits of this type.

The curves for balanced voltages are similar to the corresponding curves for the vertical configuration. Throughout the range of horizontal separations the induced voltages are the smallest when the two circuits are paralleled according to Method No. 2. For nearly the entire range of separations the values for this method of operation are less than half of those for the single circuits.

Considering the induction from balanced currents Method No. 2 gives rise to the smallest values of the mean voltage induced along the conductors throughout the range considered, when the effect of the earth is neglected, and for all except a small portion of the range when the equivalent ground plane is 20d below the plane of the lowest conductors. With minor exceptions, Method No. 2 gives the smallest values of the difference of the voltages induced along the conductors for both positions of the equivalent ground plane.

3. *Equilateral Triangles — Bases Horizontal — Vertices Downward.*

(Curve Sheets Nos. 167 to 172 and 187 to 190.)

The dimensions used for this configuration correspond to those of a 22-kV. line.

No method of parallel operation gives values of induced voltage to ground or between conductors from balanced voltages less than a single circuit except for large horizontal separations. For large separations Method No. 3 gives the smallest values, less than those of a single circuit, No. 6 about the same as a single circuit and the other methods greater, No. 1 giving the maximum values.

The method of operation (No. 3) which gives the smallest values of induced voltages from balanced voltages with largest separations, gives values of induced voltages from balanced currents about equal to the average of the single circuits. Method No. 1 gives the smallest values of induced voltages from balanced currents, less than a single circuit for the greater part of the range of separations, its advantage increasing as the height of the conductors above equivalent ground is increased. (Compare curves for $H/d = 40$ and infinity, Curve Sheets Nos. 169 and 170.) This method which gives the minimum value of induction from balanced currents, gives the maximum value of induction from balanced voltages.

4. *Equilateral Triangles — Bases Horizontal — Vertices Upward.*

(Curve Sheets Nos. 173 to 178 and 191 to 194.)

The dimensions assumed for this configuration correspond to those assumed for triangular circuits with vertices downward and apply to 22-kV. lines.

Except at large horizontal separations the induced voltages to ground and between conductors due to the balanced voltages of the twin circuits, for all methods of parallel operation, are greater than the corresponding values due to either of the single circuits. At large separations Method No. 4 gives rise to smaller values than either of the single circuits, and No. 5 to values about equal to those of the single circuits.

Method No. 4 which is advantageous with respect to the induction from balanced voltages gives about the same coefficients of induction from balanced currents as the average of the single circuits. Method No. 1, which gives the maximum effect from balanced voltages, gives the minimum effect from balanced currents, less than for either of the single circuits throughout the greater part of the range of separations The advantage of Method No. 1 for induction from balanced

currents increases as the height of the conductors above equivalent ground plane increases. (Compare curves for H/d = 40 and infinity).

5. *Horizontal.*

(Curve Sheets Nos. 205 to 212.)

The dimensions assumed correspond to those of 11-kV. circuits of this type.

The coefficients of induction from balanced voltages of the twin circuits are in all cases greater than those for one of the single circuits. Method No. 3 gives the smallest coefficients and No. 5 the largest at small separations, No. 6 the smallest, and Nos. 1 and 4 the largest at large separations.

Method No. 1 gives the smallest coefficients of induction from balanced currents throughout the range of horizontal separations and No. 2 the largest. The coefficients corresponding to Method No. 1 range from 50 to 5 per cent of those for a single circuit, the advantage increasing with horizontal separation.

6. *Summary.*

With twin circuits of the vertical type or having the middle conductors displaced slightly outward from the plane of the top and lowest conductors, the two circuits may be so operated that the induction from them in a parallel communication circuit is less than from a single circuit of the same type. For separations in excess of 100 feet the most advantageous method of parallel operation in this respect is No. 2 which consists of connecting diagonally opposite conductors of the two circuits. At separations smaller than 100 feet the best method varies with separation and differs for induction from the balanced voltages or currents and for induction in grounded or metallic circuits. However, in the absence of a special study in a particular case Method No. 2 is to be recommended. With respect to induction from balanced currents the comparisons herein made are based on the assumption that the currents in the two circuits are equal. It is thus possible with a power line of this type, while transmitting double the load of a single circuit, to decrease the induced voltages in parallel communication circuits due to the balanced components of the power-circuit voltages and currents.

No general recommendation can be made respecting the best methods of interconnection of circuits of the horizontal or equilateral triangular type. The method of operation giving the smallest coefficients depends on the horizontal separation and whether induction from balanced voltages or currents is considered. Method No. 1 gives the smallest values of induction from balanced currents, which are usually less than those of one of the single circuits. This is a common method of interconnect-

ing circuits of this type, the conductors symmetrically placed with respect to the supporting structure being at common potential. No method of interconnection of the horizontal power circuits gives induction from the balanced voltages less than that due to a single-circuit of the same type. At large separations from the triangular circuits Method No. 4 gives induction less than a single circuit if the vertices are upward and No. 3 if the vertices are downward.

No advantage is gained from particular methods of parallel operation with respect to coefficients of induction from the residual components of voltage and current. With a given magnitude of residual voltage of each of the circuits, the induction in a parallel communication circuit, as compared to the value due to a single circuit at the same separation, is approximately equal to the ratio of the direct capacitance to ground of the six conductors of the twin circuits in parallel, to that of the three conductors of one single circuit, the other being absent.* For all the circuits assumed herein, this ratio is approximately 1.4. The induced voltage in a parallel communication circuit, due to a given total residual current in the twin circuits, is approximately equal to the average effect of the same residual current in each of the single circuits. For both residual voltage and residual current, these approximations become more nearly true, as the separation of the power and communication circuits is increased.

A discussion of the effects of different methods of interconnection of twin circuits on the magnitude of the residual voltages due to unbalances among the capacitances to ground of their conductors is given in technical report No. 51.

C—Telephone-Conductor Arrangements.

1. *Phantom Circuits.*

The comparisons of the telephone-conductor arrangements are given in three tables, those for induction due to balanced voltages in Table VI, balanced currents in Table VII, and residual voltage and current in Table VIII.

Actual dimensions rather than the relative values are used in expressing the results, since the spacing of telephone conductors is usually standard and since the comparative values given in the tables are not independent of this spacing. The coefficients for the "square" and "vertical" phantoms and their side circuits are expressed as percentages of the corresponding coefficients for the "horizontal" phantom and its side circuits.

The square phantom has much the smallest coefficients. If the voltage induced between the telephone conductors and ground by the power-

*See T. R. No. 51, p. 282.

circuit voltages, and the voltages induced along the conductors by the power-circuit currents, were linear functions of both the horizontal and vertical separations of the two classes of circuits, the coefficients for the square phantom would be zero. Since the spacing of telephone conductors is small, compared to the separation of circuits, the above condition is approximately met and the coefficients for the square phantom are less than 4 per cent and in one case as low as 0.02 per cent of the corresponding coefficients for the horizontal phantom. The induction into the square phantom increases about as the square of the spacing, while the induction into the horizontal phantom is nearly directly proportional to the spacing. Thus their percentage relation varies directly as the spacing.

Although the coefficients for the square phantom are much less than those for the horizontal phantom, in most cases the coefficients for the side circuits of the former are greater than the coefficients for the side circuits of the latter. In all cases the coefficient for one side circuit of the square phantom is greater than that for the side circuits of the horizontal phantom, while the coefficient for the other is, in a few cases, considerably smaller. For induction from the balanced components of voltage and current of vertical power circuits the coefficients for both side circuits of the square phantom are several times the values for the side circuits of the horizontal phantom.

The relative magnitudes of the coefficients for the horizontal and vertical phantoms depend largely upon the configuration of the power circuit and relative position of the two classes of circuits, power and telephone, and whether the induction is due to the voltage or current of the power circuit. For the balanced components of voltage and current of the vertical power circuit, the coefficients for the vertical phantom are several times the corresponding values for the horizontal phantom. No general conclusions can be drawn in the case of the other power-circuit configurations.

The coefficient for the side circuits of the vertical phantom is ⅓ greater throughout than for the side circuits of the horizontal phantom. This is due to the increased conductor spacing of the former, 16 inches as compared to 12 inches. Since for purposes of comparison the two circuits are assumed both in the same horizontal plane and at the same separation from the power circuit, their coefficients are closely proportional to the spacing of the telephone conductors.

It should be noted that the method of arranging the telephone conductors in forming the phantom group has no appreciable effect on the mean voltage induced between the conductors and ground by the power-circuit voltages or upon the mean voltage induced along the conductors by the power-circuit currents.

COMPARISON OF CONDUCTOR ARRANGEMENTS FOR METALLIC TELEPHONE CIRCUITS

WITH RESPECT TO

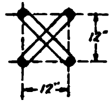
COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES

POWER CIRCUIT { *Greatest Spacing of Conductors* = *d*; *Diameter of Conductors* = *D*; *Height of Lowest Conductor* = *H* }

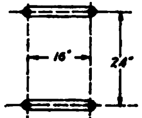
TELEPHONE CONDUCTORS

HORIZONTAL PHANTOM

SQUARE PHANTOM



VERTICAL PHANTOM



SEPARATION { MEAN HORIZONTAL *(from plane midway between outside power conductors)* = *X*; MEAN VERTICAL *(from plane of lowest power conductor)* = *Y* }

POWER-CIRCUIT CONFIGURATION	*d*	*H*	*D*	*Y*	*X*	PHANTOM CIRCUIT			SIDE CIRCUIT OF PHANTOM			
	feet	*feet*	*inches*	*feet*	*feet*	HORIZONTAL	SQUARE	VERTICAL	AVERAGE HORIZONTAL	SQUARE	SQUARE	AVERAGE VERTICAL
VERTICAL	13	32.5	0.468	7.8	65	100	3.32	802	100	.819	711	133
					520	100	1.06	1127	100	1168	969	133
HORIZONTAL SYMMETRICAL	6	30	0.216	6.6	30	100	1.01	67.0	100	163	37.3	133
					240	100	1.07	449	100	524	328	133
EQUILATERAL TRIANGLE-BASE HORIZONTAL	3	30	0.216	6.6	15	100	2.80	84.9	100	147	105	133
					120	100	0.99	268	100	350	161	133

TABLE VI

J.C.I.I.-T.R. No. 65
R.M.

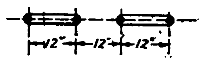
NOTE: Coefficients for square and vertical phantoms and their side circuits are expressed as percentages of the coefficients for the horizontal phantom and its side circuits respectively

COMPARISON OF CONDUCTOR ARRANGEMENTS FOR METALLIC TELEPHONE CIRCUITS
WITH RESPECT TO
COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE CURRENTS

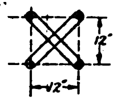
POWER CIRCUIT { *Greatest Spacing of Conductors* = *d*
Height of Lowest Conductor = *H*

TELEPHONE CONDUCTORS

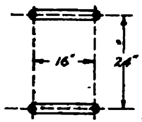
HORIZONTAL PHANTOM



SQUARE PHANTOM



VERTICAL PHANTOM



SEPARATION { MEAN HORIZONTAL (*from plane midway between outside power conductors*) = *X*
MEAN VERTICAL (*from plane of lowest power conductor*) = *Y*

POWER-CIRCUIT CONFIGURATION	*d* *feet*	*H* *feet*	*Y* *feet*	*X* *feet*	PHANTOM CIRCUIT HORIZONTAL	PHANTOM CIRCUIT SQUARE	PHANTOM CIRCUIT VERTICAL	SIDE CIRCUIT OF PHANTOM AVERAGE HORIZONTAL	SIDE CIRCUIT OF PHANTOM SQUARE	SIDE CIRCUIT OF PHANTOM SQUARE	SIDE CIRCUIT OF PHANTOM AVERAGE VERTICAL
VERTICAL	13	∞	7.8	65	100	1.43	217.3	100	119	316	133
				520	100	1.72	1803	100	1703	1904	133
HORIZONTAL SYMMETRICAL	6	∞	6.6	30	100	1.11	46.8	100	147	53.2	133
				240	100	0.017	5.61	100	106	94.7	133
EQUILATERAL TRIANGLE-BASE HORIZONTAL	3	∞	6.6	15	100	2.84	85.9	100	140	122	133
				120	100	0.412	99.2	100	139	143	133

J.C.I.I.– T.R. No. 65

NOTE: Coefficients for square and vertical phantoms and their side circuits are expressed as percentages of the coefficients for the horizontal phantom and its side circuits respectively.

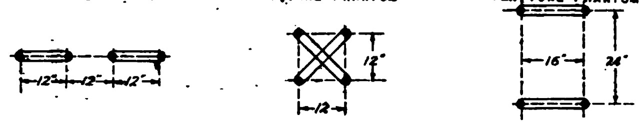
TABLE VII

COMPARISON OF CONDUCTOR ARRANGEMENTS FOR METALLIC TELEPHONE CIRCUITS
WITH RESPECT TO
COEFFICIENTS OF INDUCTION FROM RESIDUAL VOLTAGE AND CURRENT OF THREE-PHASE POWER CIRCUITS

POWER CIRCUIT { *Equilateral Triangle, Base Horizontal, Vertex Upward*; *Spacing of Conductors = 9 feet*; *Diameter of Conductors = D*; *Height of Lower Conductors = H* }

TELEPHONE CONDUCTORS

HORIZONTAL PHANTOM — SQUARE PHANTOM — VERTICAL PHANTOM



SEPARATION { Mean Horizontal *(from plane of vertex of power triangle)* = X; Mean Vertical *(from plane of lower power conductors)* = 6.6 *feet* }

	H	D	X	PHANTOM CIRCUIT			SIDE CIRCUIT OF PHANTOM			
	feet	*inches*	*feet*	HORIZONTAL	SQUARE	VERTICAL	AVERAGE HORIZONTAL	SQUARE	SQUARE	AVERAGE VERTICAL
RESIDUAL VOLTAGE	30	0.216	15 120	100 100	1.62 0.99	94.4 278	100 100	190 363	10.4 164	133 133
RESIDUAL CURRENT	300	any	15 120	100 100	1.33 0.030	52.9 26.7	100 100	153 127	47.0 73.2	133 133

TABLE VIII

J.C.I.I. - T.R. No. 65
NOTE: *Coefficients for square and vertical phantoms and their side circuits are expressed as percentages of the coefficients for the horizontal phantom and its side circuits respectively.*

2. *Effect of Rotating Plane of Telephone Conductors.*

On Curve Sheets Nos. 151 to 154, inclusive, the coefficients of induction in metallic telephone circuits are shown as functions of the angular position of the plane of the telephone conductors. The coefficients are expressed in each case in terms of the coefficient for the horizontal position.

It will be noted that curves for residual voltage and residual current are sinusoidal. This is explained by the close approach to uniformity over the space between the telephone conductors of the fields induced by the residual voltage and current. The coefficient for any angular position of the plane of the telephone conductors is nearly proportional to the sine of the angle made with a plane parallel to the lines of the field. The angular position, for which the coefficient is zero, varies with the relative position of the circuits.

For balanced voltages and currents the field about the power conductors in general changes in phase as well as in magnitude, with changes in separation of the two classes of circuits. The field may be resolved into two single-phase components in time quadrature and their effects separately considered. The coefficient of induction for the telephone circuit may also be resolved into two components corresponding to the two components of the field. Over the small distance between the telephone conductors, each component of the field is approximately uniform. Thus the component of the induction in the telephone circuit is proportional to the sine of the angle made by the plane of the conductors with a plane parallel to the direction of the corresponding component of the field. If the direction of the two components is the same, which in general is not the case, the resultant induction in the telephone circuit has a zero value for one of the angular positions and the curve of variation is approximately a simple sine curve. This condition is approximately fulfilled for induction from balanced currents of the horizontal power circuit. For induction from balanced currents of the triangular power circuit, at the larger separation, the two components of the field have nearly equal magnitudes and their directions are nearly at right angles Hence, the resultant induction is nearly constant for all angular positions of the telephone conductors.

V. Use of Curve Sheets.

A—General Directions.

To use the curves of this report, in obtaining the coefficients of induction for a given parallel it is first necessary to express the cross-sectional dimensions in terms of the spacing of the power conductors, d. In the general case none of the given dimensions will coincide with those for which the curves are plotted and it will be necessary to interpolate.

Having expressed the cross-sectional dimensions in terms of the spacing of power conductors, d, the order of procedure in obtaining the coefficients is, in general, as follows:

By reference to the curves (see index of curve sheets) of variation of the coefficient with the vertical separation, Y/d, four values corresponding to the two heights of power conductors, H/d, and horizontal separation, X/d, nearest to the actual case, are obtained. Plotting these four values among the H/d curves, two auxiliary curves may be drawn, showing the variation of the coefficient over a limited range of heights, including the actual height; for two values of horizontal separation, including between them the actual horizontal separation. From these auxiliary H/d curves two values may be obtained at the actual H/d which, plotted among the X/d curves on the same curve sheet, determine a final auxiliary curve from which may be obtained the value of the coefficient for the actual horizontal separation. In some cases where the variation of the coefficients with vertical separation is very slight, the first step is to be omitted, no Y/d curves being given.

In finding coefficients of induction from the balanced components of the vertical power circuit, it is generally necessary to obtain three or more points for each of the auxiliary curves in order to obtain the result to a satisfactory degree of accuracy.

If the coefficient for the induced voltage along the disturbed conductors due to the power-circuit currents is desired, the above outlined procedure is sufficient.

For induction due to the power circuit voltages it is necessary to correct the coefficient obtained as above indicated, for the deviation of the power-conductor diameter from the standard for which the Y/d, H/d and X/d curves apply. A correction factor for this purpose is obtained from the curve sheets showing the variation of the coefficients with conductor-diameter, choosing the curve or curves for conditions nearest those of the actual case.

If the difference of the induced voltages along the disturbed conductors, due to the power-circuit currents, or the voltage induced between them by the power-circuit voltages, is desired, it is further necessary to multiply the coefficients as above obtained by the ratio of the actual spacing of the disturbed conductors to that given on the curve sheets.

The curves of induction from the residual components of voltage and current have been plotted for the equilateral triangular (base horizontal), symmetrical horizontal and vertical configurations. The curves for the equilateral triangular configuration (base horizontal) may be applied to other equilateral triangular configurations with an error of less than 10 per cent, and those for the horizontal symmetrical to the

unsymmetrical cases, with an error of less than 4%. For triangles with altitude small as compared to the base, the curves for the horizontal configuration should be applied if the base is horizontal, and those for the vertical configuration if the base is vertical.

B—Illustrative Example.

As an illustration of the method of using the curves, the steps required in obtaining the coefficients for the San Jose-Gilroy parallel between the lines of the Sierra and San Francisco Power Company and The Pacific Telephone and Telegraph Company are given in considerable detail. The following table gives the cross-sectional dimensions of this parallel in feet and in terms of d as a unit.

TABLE IX.

San Jose-Gilroy Parallel—Vertical Power Circuit—Cross-sectional Dimensions.

Dimension		Feet	"d units"
Greatest spacing of conductors	d	13	1
Height of lowest conductor:			
Above actual ground	H	33.6	2.58
Above equivalent ground plane	H	400	30.8
Diameter of conductors	D	0.0375	0.00288
Vertical separation of circuits	Y	7.6	0.585
Horizontal separation of circuits	X	66.5	5.11
Spacing of telephone conductors	t	1.0	0.0769

1. *Induction from Balanced Voltages.*

The various steps necessary in obtaining the coefficients for balanced voltages are given in Table X. Referring to the curves of variation of the coefficients with Y/d, Curve Sheet No. 103, for the induced voltage to ground and No. 107 for the induced voltage between conductors, the nine coefficients of induced voltage to ground and nine of induced voltage between conductors corresponding to three values of H/d and X/d nearest the values for this parallel, are read. These are tabulated in Table X in the spaces corresponding to $H/d = 1.5$, 2.5 and 5, and $X/d = 2.5$, 5 and 10. These coefficients will determine three auxiliary curves of variation of the induced voltage to ground and three of induced voltage between conductors at $X/d = 2.5$, 5 and 10. With the aid of the H/d curves given on Curve Sheets No. 102 for induced voltage to ground and Nos. 105 and 106 for induced voltage between conductors, these auxiliary curves may be sketched in and the coefficients at $H/d = 2.58$ determined for three values of X/d. These are given in the corresponding spaces of Table X. From the values thus determined, with the aid of the X/d curves (on the same sheet with the H/d curves) the coefficients for $X/d = 5.11$ are obtained. These are given in Table X for $X/d = 5.11$ and $H/d = 2.58$.

These coefficients are for values of D/d = 0.003 and t/d = 0.1. The induced voltage to ground is independent of t/d, but the induced voltage between conductors is directly proportional to t. The factors by which the coefficients are corrected for D/d are given by the curves on sheets No. 104 for voltage to ground and No. 108 for voltage between conductors. The correction factors and the corrected coefficients are shown in Table X. These coefficients may be compared with those given in the illustrative example of technical report No. 64, dividing the coefficients there given by $\sqrt{3}$ so as to express them in terms of the voltage between power conductors. The two sets of values differ by less than 1 per cent.

TABLE X.

Coefficients of Induction from Balanced Three-Phase Voltages.

Y/d = 0.585

	Voltage to ground				Voltage between conductors			
H/d \ X/d	1.5	2.5	2.58	5	1.5	2.5	2.58	5
2.5	2.6	2.62	2.77	7.55	289	598	606	739
5	2.92	3.82	3.82	8.8	59.5	31	31.2	107
5.11			3.81				31.8	
10	1.1	2.0	2.07	2.94	19	28.2	28.2	25.8

Applying correction factors for D/d = 0.00288.

Induced Voltage to ground = 3.81 × 0.993 = 3.78 V/kV.

Induced Voltage between conductors = 31.8 × 0.985 = 31.3 mV/kV.

Correction for t/d = 0.0769.

$$\text{Induced Voltage between conductors} = 31.3 \times \frac{0.0769}{0.1} = 24.1 \text{ mV/kV.}$$

2. *Induction from Balanced Currents.*

The intermediate results of the process of finding the coefficients of induction from balanced currents are given in Table XI. The curves in the region determined by the values of Y/d, H/d and X/d for this parallel are fairly straight; hence two points are considered sufficient to determine the auxiliary curves.

The Y/d curves for the mean voltage induced along the conductors are given on Curve Sheet No. 119 and for the difference of induced voltages on No. 125. From these curves the values corresponding to Y/d = 0.585, H/d = 20 and 50 and X/d = 5 and 10 are obtained. The auxiliary H/d curves are constructed from these values on Curve Sheet No. 118 for the mean, and on No. 124 for the difference of the induced voltages. The values for H/d = 30.8 and X/d = 5 and 10 are obtained from these auxiliary curves. These values in turn determine the auxiliary X/d curves from which the values at X/d = 5.11 are taken. This completes the procedure for the mean induced voltage along the conductors. The coefficient for the difference of the voltages is corrected for the spacing of the telephone conductors by multiplying by the ratio of 0.0769 to 0.1.

TABLE XI.

Coefficients of Induction from Balanced Three-Phase Currents.

$Y/d = 0.585$

H/d → X/d ↓	Mean voltage			Difference of voltages		
	20	30.8	50	20	30.8	50
5	0.347	0.505	0.68	30.8	30.8	30.8
5.11		0.487			29.1	
10	0.256	0.112	0.03	3.68	3.95	4.16

Mean Voltage induced along conductors
at 60 cycles = 0.487 mV/A per 1000 feet.
Correcting for $t/d = 0.0769$.
Difference of Voltages induced along conductors
at 60 cycles $= 29.1 \times \frac{0.0769}{0.1} = 22.4\ \mu V/A$ per 1000 feet.

3. *Induction from Residual Voltage.*

The results for residual voltage are given in Table XII. The method of tabulation is the same as in Tables X and XI. There are no maximum or minimum points in the curves for residual voltage and two points are sufficient to determine a given auxiliary curve.

The Y/d curves are plotted on Curve Sheet No. 110 for voltage to ground and No. 113 for voltage between conductors. Both the H/d and X/d curves are on sheet No. 109 (voltage to ground) and No. 112 (voltage between conductors). The coefficients corresponding to the two values of X/d and H/d are first read from the Y/d curves and listed in the table. The auxiliary H/d curves for $Y/d = 0.585$ and $X/d = 5$ and 10 are plotted and the values corresponding to $H/d = 2.58$ read from them. From these values the auxiliary X/d curves are constructed and the coefficients for $X/d = 5.11$ obtained.

The curves for correcting the coefficients for the diameter of the power conductors are given on Curve Sheet No. 111. The same curves apply both to the induced voltages to ground and the induced voltages between conductors. The corrections for diameter of power conductors and spacing of telephone conductors are shown.

TABLE XII.

Coefficients of Induction from Residual Voltage.

$Y/d = 0.585$

H/d → X/d ↓	Voltage to ground			Voltage between conductors		
	2.5	2.58	5	2.5	2.58	5
5	24.2	25	52.2	686	705	1020
5.11		24.7			678	
10	8	8.4	22.8	142	150	330

Applying correction factors for $D/d = 0.00288$.
Induced Voltage to ground $= 24.7 \times 0.996 = 24.6$ V/kV.
Induced Voltage between conductors $= 678 \times 0.996 = 675$ mV/kV.
Correcting for $t/d = 0.0769$.

Induced Voltage between conductors $= 675 \times \frac{0.0769}{0.1} = 519$ mV/kV.

4. *Induction from residual current.*

The variation of the coefficient for residual current with the vertical separation is so small that the curves have not been plotted and the coefficients for the given Y/d at the two values of X/d are read directly from the H/d curves. The auxiliary curves of variation with X/d are drawn as in the previous cases and the coefficients for X/d = 5.11 read from them. The H/d and X/d curves for the mean voltage induced along the conductors are given on Curve Sheet No. 126 and for the difference of voltages on Curve Sheet No. 127. The various steps in obtaining the coefficients are shown in Table XIII.

TABLE XIII.

Coefficients of Induction from Residual Current.

Y/d = 0.585. H/d = 30.8

	Mean voltage			Difference of voltages		
X/d	5	5.11	10	5	5.11	10
	56.8	56.0	41.5	435	420	220

Mean Voltage induced along conductors at 60 cycles = 56.0 mV/A per 1000 feet.
Correcting difference of voltages induced along conductors at 60 cycles, for spacing of telephone conductors:

$$420 \times \frac{0.0769}{0.1} = 323\ \mu V/A \text{ per } 1000 \text{ feet.}$$

VI. Conclusion.

As far as is known no comprehensive study of this character has been carried out elsewhere. Such data have long been needed in the work of the Joint Committee and if made available should be of value to any one interested in the study of inductive interference. Aside from the importance of this information as one of the essential steps in defining the conditions of parallelism which will create interference, it affords a graphical picture of some of the complex relationships involved, enabling an estimate to be made of the relative importance of different factors. By taking advantage, in the design and operation of lines, of some of the facts disclosed by this study, the induction can in many cases be greatly reduced at no additional cost. An important use of the information presented should be found in the study of practical cases of parallelism. The example given should amply illustrate the use of the curves in determining the coefficients of induction in such cases.

Within the time and means available the endeavor was made to cover as far as possible the practical range. Doubtless cases will occur which are outside the range of the data presented in this report and it will be desirable in the future to extend these results.

In spite of the care taken in the work it is too much to expect that the results are entirely free from errors, but it is hoped that all of any consequence have been eliminated.

Respectfully submitted.

(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

ATTACHMENTS: Index of Curve Sheets; Curve Sheets, Nos. 1 to 214, inclusive. Index of Tables of Results; Tables, Nos. 1 to 92, inclusive.

APPROVED: January 8, 1917.

(Signed) R. W. MASTICK,
Field Engineer.

APPROVED: January 8, 1917.

SUBCOMMITTEE ON TESTS,
(Signed) J. E. WOODBRIDGE,
Chairman.

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE,
(Signed) ARTHUR F. BRIDGE,
Secretary.

January 8, 1917.


CASE NO

CURVE SHEET No. 3

INDUCTION FROM BALANCED THREE-PHASE VOLTAGES

EFFECT OF VARYING SIZE OF POWER CONDUCTORS.

POWER CIRCUIT:
- Equilateral Triangle, Base Horizontal, Vertex Upward
- Spacings of Conductors = d
- Diameter of Conductors = D/d
- Height of Lower Conductors = H/d

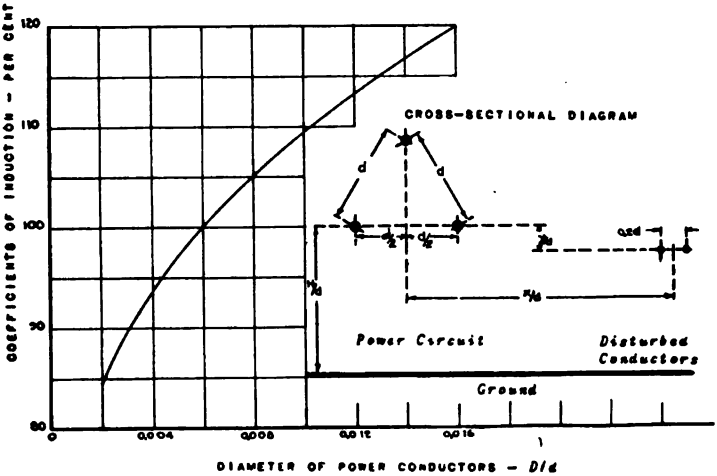
DISTURBED CONDUCTORS: In Horizontal Plane, Spaced 0.2d

SEPARATION: HORIZONTAL = X/d   VERTICAL = Y/d

NOTE: *Curves represent average change of coefficients with change of conductor diameter, for induction between conductors and between conductors and ground, for relative positions of circuits as follows:*

H/d	Y/d	
2	0.5	X/d = 5 and 40
5	1	
10	2	
20	4	

The deviation of any case from the average is less than 0.1 per cent.



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J.C.I.I.
T. R. No. 65.

Curve Sheet No. 4

INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
Effect of Varying Size of Power Conductors.

POWER CIRCUIT
- Equilateral Triangle
  - (1) Base Horizontal, Vertex Upward
  - (2) Base Vertical
    - (a) Vertex Toward Disturbed Conductors
    - (b) Vertex Away from Disturbed Conductors
- Spacings of Conductors = d
- Diameter of Conductors = D/d
- Height of Lowest Conductor = H/d

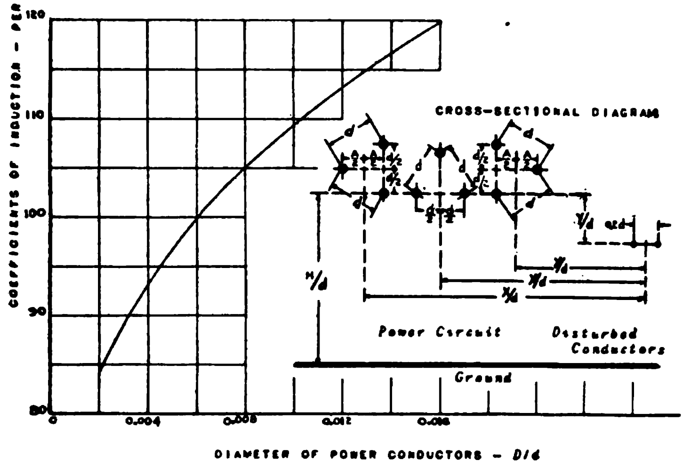
DISTURBED CONDUCTORS: In Horizontal Plane, Spaced 0.2d

SEPARATION: Horizontal = X/d   Vertical = Y/d

NOTE: Curves represent average change of coefficients with change of conductor diameter, for induction between conductors and between conductors and ground, for relative positions of circuits as follows:

H/d	Y/d	
3	0.5	X/d = 5 and 40
5	1	
10	2	
20	4	

The deviation of any case from the average is less than [illegible] per cent.



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J.C.I.I.
T. R. No. 65

CURVE SHEET NO. 10

INDUCTION FROM RESIDUAL VOLTAGE OF THREE-PHASE CIRCUITS

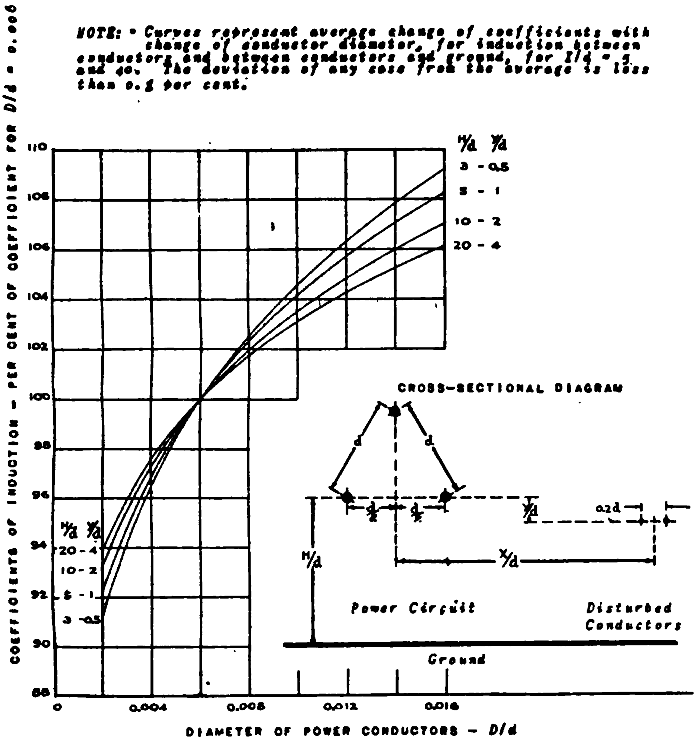
EFFECT OF VARYING SIZE OF POWER CONDUCTORS.

POWER CIRCUIT { Equilateral Triangle, Base Horizontal, Vertex Upward
Spacings of Conductors = d
Diameter of Conductors = D/d
Height of Lower Conductors = H/d

DISTURBED CONDUCTORS: In Horizontal Plane, *Spaced 0.2d*

SEPARATION: HORIZONTAL = *X/d*   VERTICAL = *Y/d*

NOTE: = Curves represent average change of coefficients with change of conductor diameter, for induction between conductors and between conductors and ground, for H/d = 5 and 40. The deviation of any case from the average is less than 0.5 per cent.



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J.C.I.I.

T. R. No. 65.

CURVE SHEET No. 23

INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
EFFECT OF VARYING SIZE OF POWER CONDUCTORS.

POWER CIRCUIT { Equilateral Triangle, Base Vertical
Vertex Away from Disturbed Conductors.
Spacings of Conductors = d
Diameter of Conductors = D/d
Height of Lowest Conductor = H/d

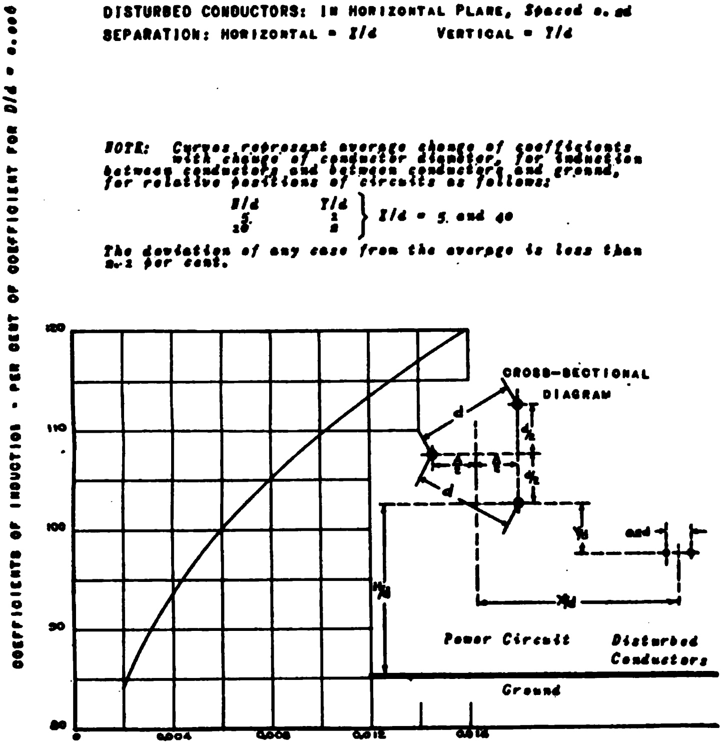
DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, Spaced 0.2d

SEPARATION: HORIZONTAL = X/d   VERTICAL = Y/d

NOTE: Curves represent average change of coefficients with change of conductor diameter, for induction between conductors and between conductors and ground, for relative positions of circuits as follows:

H/d	Y/d	
5	2	X/d = 5 and 40
20	8	

The deviation of any case from the average is less than 0.2 per cent.



DIAMETER OF POWER CONDUCTORS - D/d

10/27/16
J.C.I.I.
T. B. No. 55.

Curve Sheet No. 35

INDUCTION FROM BALANCED THREE-PHASE VOLTAGES

Effect of Varying Size of Power Conductors.

POWER CIRCUIT { Equilateral Triangle, Base Vertical, Vertex Toward Disturbed Conductors. Spacing of Conductors = d. Diameter of Conductors = D/d. Height of Lowest Conductor = H/d

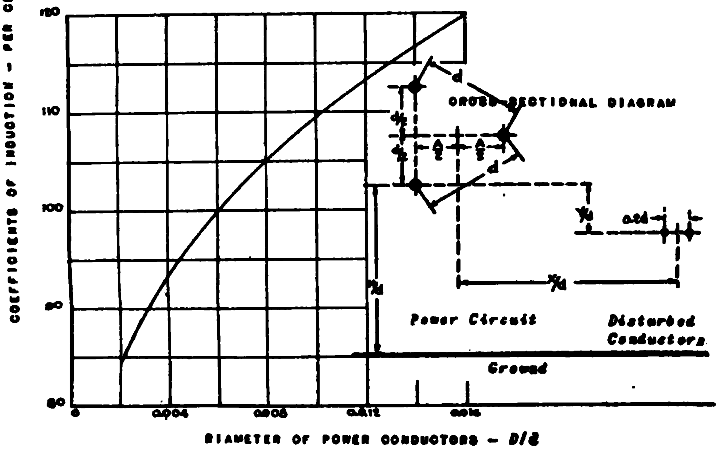
DISTURBED CONDUCTORS: In Horizontal Plane, Spaced 0.2d

SEPARATION: HORIZONTAL = X/d VERTICAL = Y/d

NOTE: Curves represent average change of coefficients with change of conductor diameter, for induction between conductors and between conductors and ground, for relative positions of circuits as follows:

H/d	Y/d	
3	0.5	X/d = 5 and 40
5	1	
10	2	
50	4	

The deviation of any case from the average is less than 0.1 per cent.



DIAMETER OF POWER CONDUCTORS - D/d

10/28/16
J.C.I.I.
T. R. No. 65.

CURVE SHEET NO. 35

INDUCTION FROM BALANCED THREE-PHASE VOLTAGES

EFFECT OF VARYING SIZE OF POWER CONDUCTORS.

POWER CIRCUIT:
- Equilateral Triangle, Base Vertical
- Vertex Toward Disturbed Conductors.
- Spacings of Conductors = d
- Diameter of Conductors = D/d
- Height of Lowest Conductor = H/d

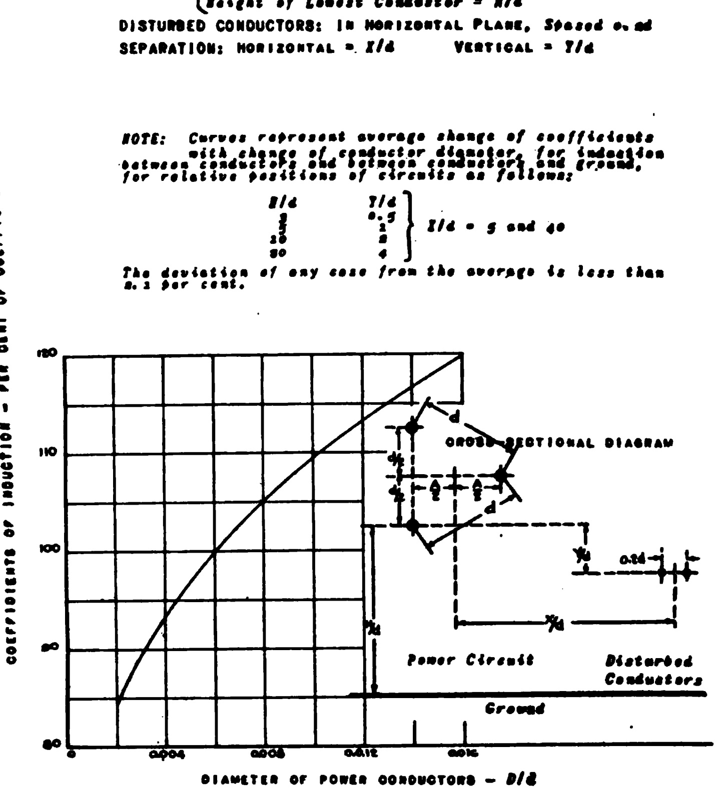
DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, Spaced 0.2d

SEPARATION: HORIZONTAL = X/d VERTICAL = Y/d

NOTE: Curves represent average change of coefficients with change of conductor diameter, for induction between conductors and between conductors and ground, for relative positions of circuits as follows:

H/d	Y/d	
2	0.5	X/d = 5 and 40
5	1	
10	2	
20	4	

The deviation of any case from the average is less than 0.1 per cent.



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J.C.I.I.
T. R. No. 65.

CURVE SHEET NO. 41

INDUCTION FROM BALANCED THREE-PHASE VOLTAGES

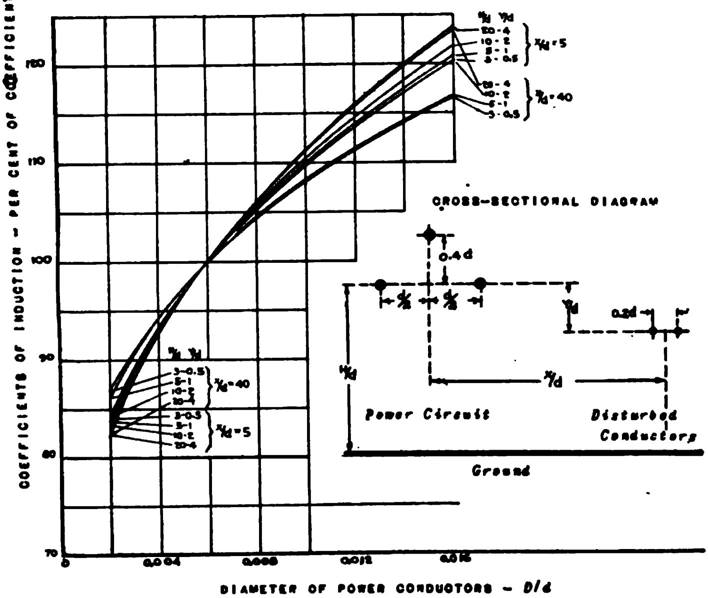
EFFECT OF VARYING SIZE OF POWER CONDUCTORS.

POWER CIRCUIT { ISOSCELES TRIANGLE, BASE HORIZONTAL = d, ALTITUDE = 0.4d
Diameter of Conductors = D/d
Height of Lower Conductors = H/d }

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, Spaced 0.2d

SEPARATION: HORIZONTAL = X/d   VERTICAL = Y/d

NOTE: Curves represent average change of coefficients with change of conductor diameter, for induction between conductors and between conductors and ground. The deviation of any case from the average is less than 0.8 per cent.



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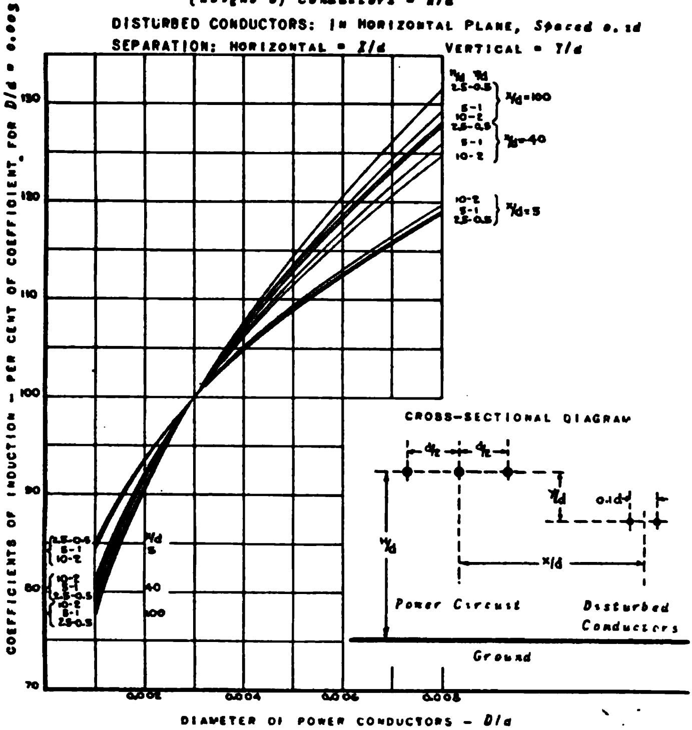
J.C.I.I.
T. R. No. 65.

CURVE SHEET NO. 55

INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
EFFECT OF VARYING SIZE OF POWER CONDUCTORS
ON THE
INDUCED VOLTAGES TO GROUND.

POWER CIRCUIT { Horizontal, Symmetrical; Spacings of Conductors = d, d/s, d/s; Diameter of Conductors = D/d; Height of Conductors = H/d }

DISTURBED CONDUCTORS: In Horizontal Plane, Spaced 0.1d
SEPARATION: HORIZONTAL = X/d   VERTICAL = Y/d



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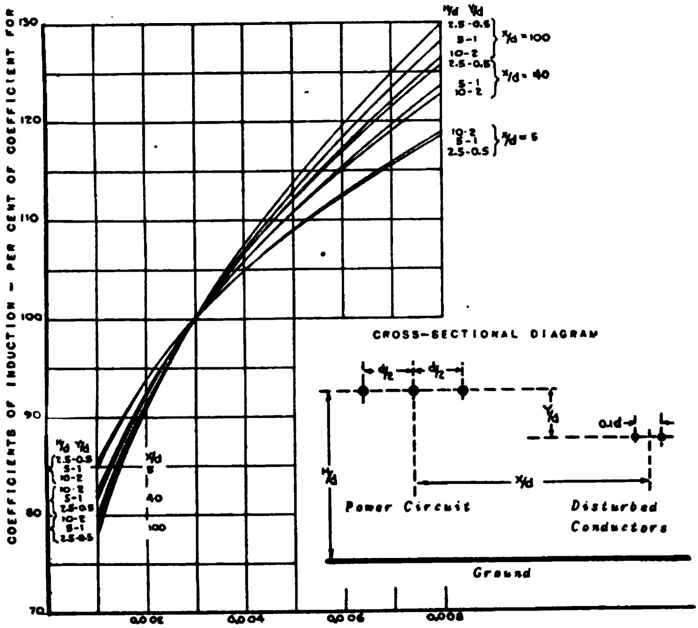
J.C.I.I.
T.R. NO. 65.

CURVE SHEET NO. 59

INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
EFFECT OF VARYING SIZE OF POWER CONDUCTORS
ON THE
INDUCED VOLTAGES BETWEEN CONDUCTORS

POWER CIRCUIT { HORIZONTAL, SYMMETRICAL
Spacings of Conductors = *d, d/2, d/2*
Diameter of Conductors = *D/d*
Height of Conductors = *H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.1d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*



DIAMETER OF POWER CONDUCTORS - *D/d*

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T.R. NO. 66.

Curve Sheet No. 62

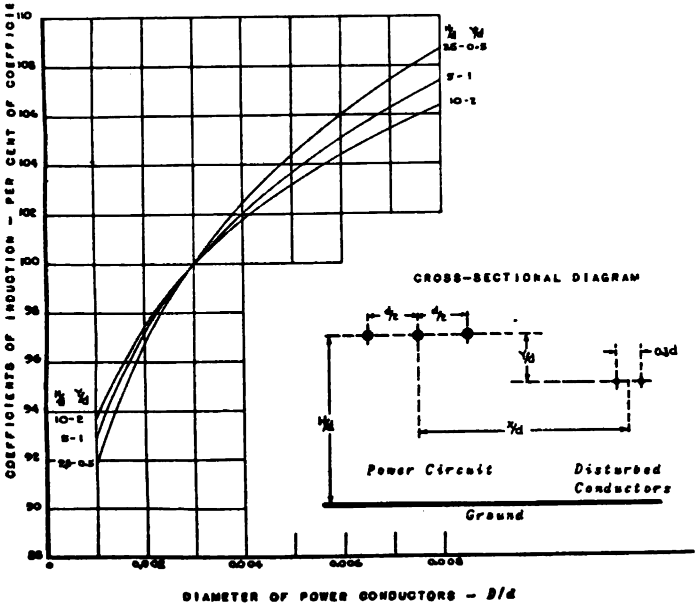
INDUCTION FROM RESIDUAL VOLTAGE OF THREE-PHASE CIRCUITS.
Effect of Varying Size of Power Conductors.

POWER CIRCUIT { Horizontal, Symmetrical
Spacings of Conductors = d/2, d/2
Diameter of Conductors = D/d
Height of Conductors = H/d

DISTURBED CONDUCTORS: In Horizontal Plane, Spaced 0.1d

SEPARATION: Horizontal = X/d   Vertical = Y/d

NOTE: Curves represent average change of coefficients with change of conductor diameter, for induction between conductors and between conductors and ground, for X/d = 5 and 40. The deviation of any case from the average is less than 0.2 per cent.



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J.C.I.I.
T. R. No. 65.

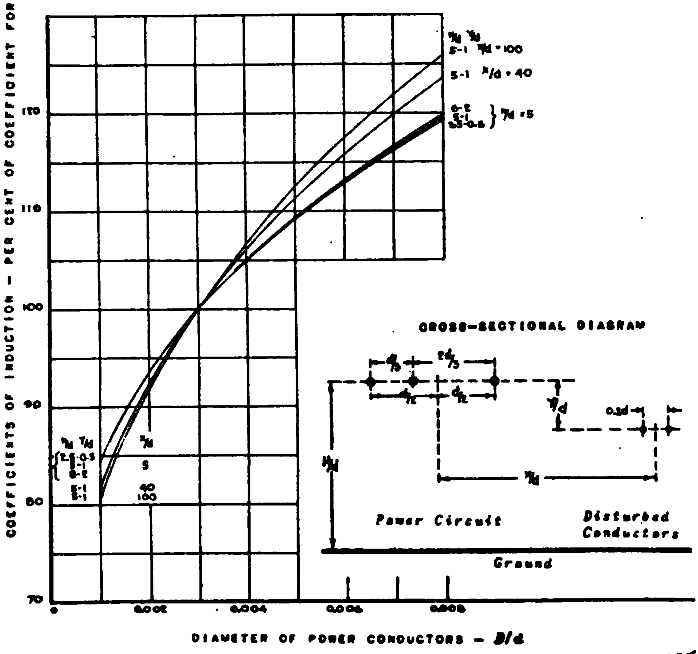
CURVE SHEET NO. 75

INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
EFFECT OF VARYING SIZE OF POWER CONDUCTORS ON THE INDUCED VOLTAGES TO GROUND

POWER CIRCUIT:
- Horizontal, Unsymmetrical
- Inside Conductor Displaced Away from Disturbed Conductors.
- Spacings of Conductors = d, d/3, 2d/3
- Diameter of Conductors = D/d
- Height of Conductors = H/d

DISTURBED CONDUCTORS: In Horizontal Plane, Spaced 0.1d

SEPARATION: Horizontal = X/d   Vertical = Y/d



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J. C. I. I.
T. R. No. 65

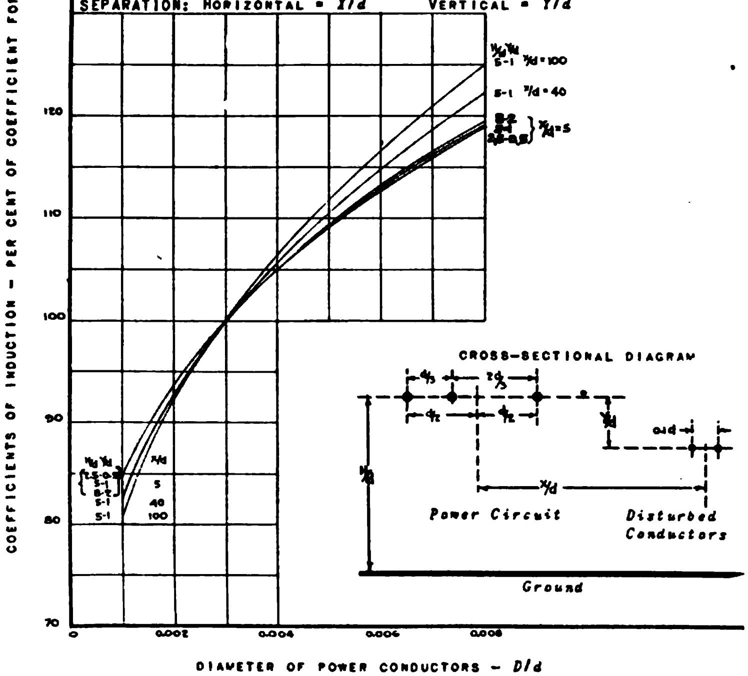
CURVE SHEET NO. 78

INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
EFFECT OF VARYING SIZE OF POWER CONDUCTORS
ON THE
INDUCED VOLTAGES BETWEEN CONDUCTORS.

POWER CIRCUIT: Horizontal, Unsymmetrical. Inside Conductor Displaced Away from Disturbed Conductors. Spacings of Conductors = d, d/3, 2d/3. Diameter of Conductors = D/d. Height of Conductors = H/d

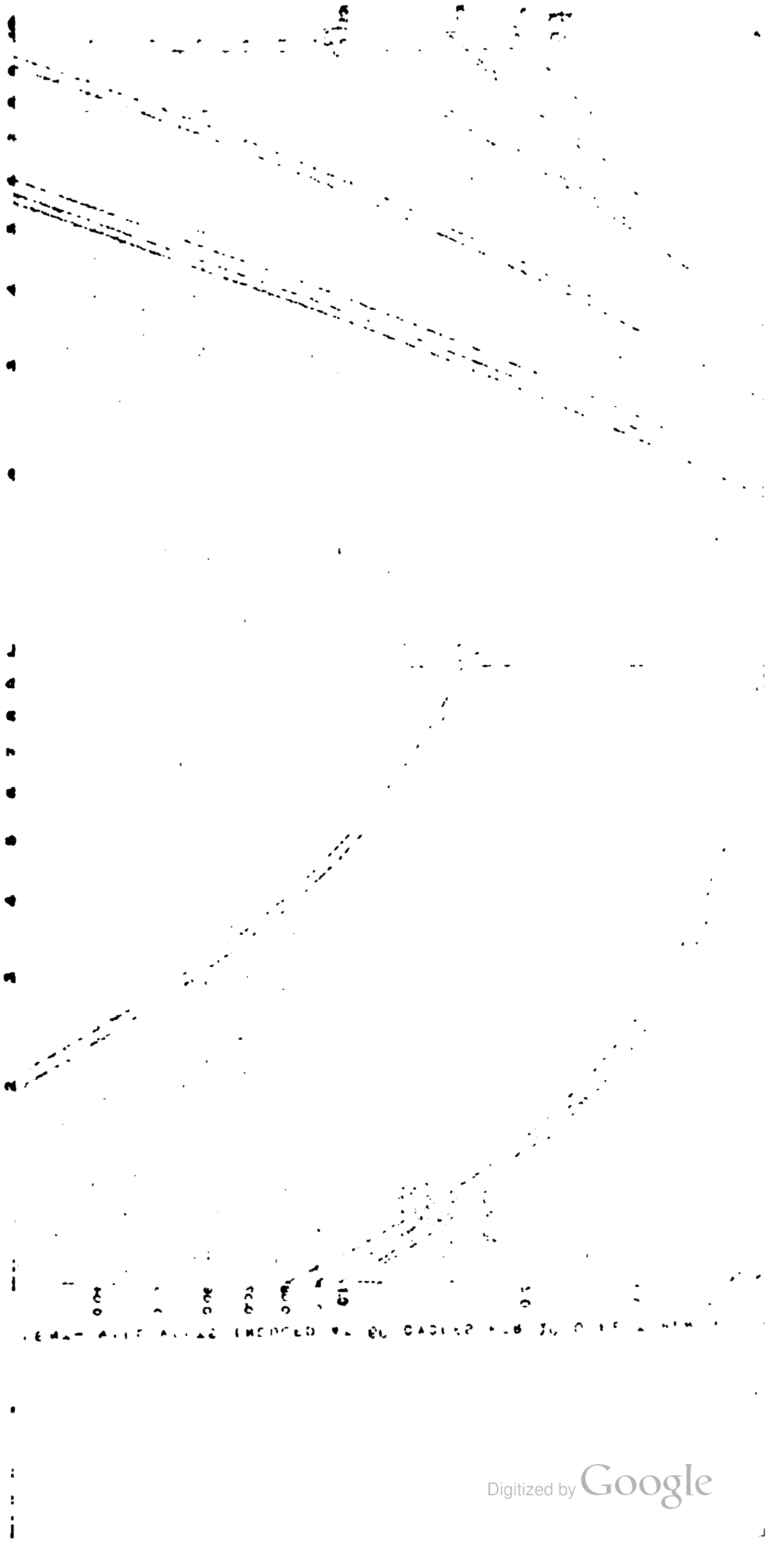
DISTURBED CONDUCTORS: In Horizontal Plane, Spaced 0.1d

SEPARATION: Horizontal = X/d   Vertical = Y/d



10/27/16

J.C.I.I.
T. R. No. 65

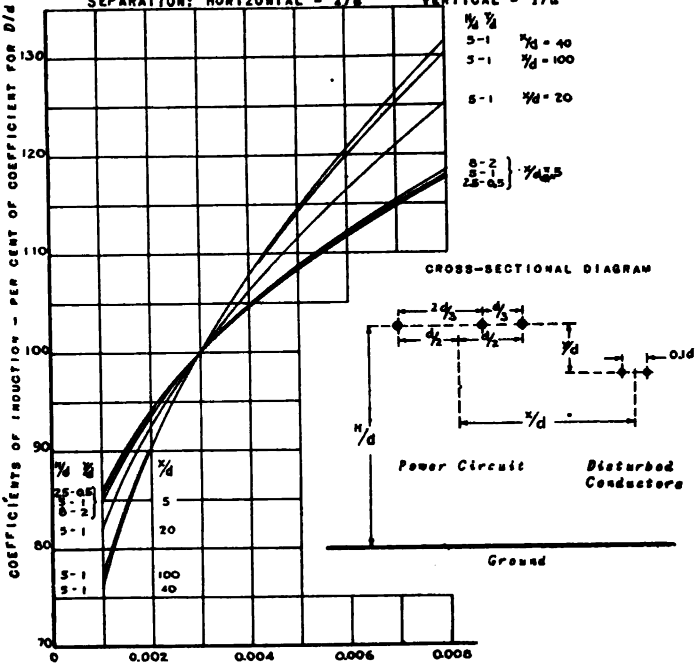


CURVE SHEET NO. 84

INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
EFFECT OF VARYING SIZE OF POWER CONDUCTORS
ON THE
INDUCED VOLTAGES TO GROUND

POWER CIRCUIT { Horizontal, Unsymmetrical
Inside Conductor Displaced Toward Disturbed Conductors
Spacings of Conductors = d, d/3, 2d/3
Diameter of Conductors = D/d
Height of Conductors = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, Spaced 0.1d
SEPARATION: HORIZONTAL = X/d VERTICAL = Y/d



11/2/16
J.C.I.I.
T. R. NO. 65.

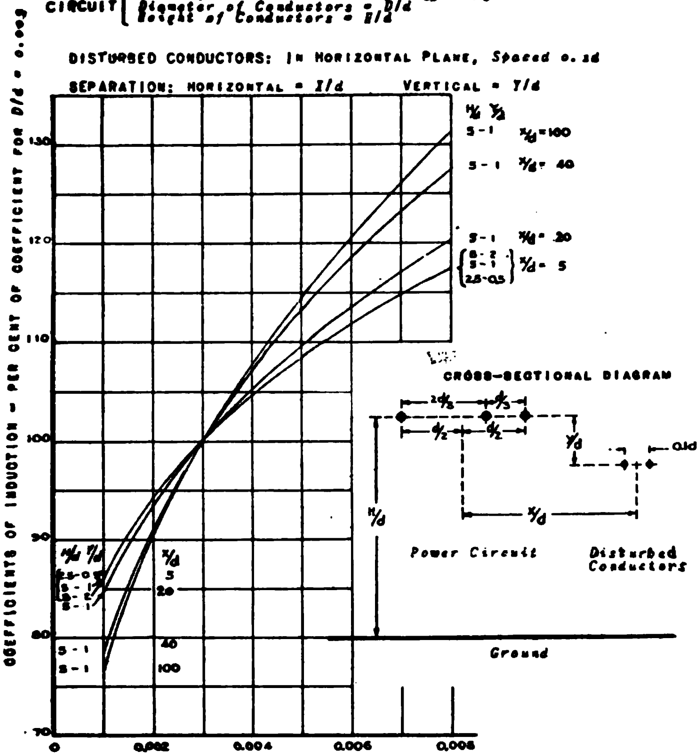
CURVE SHEET NO. 87

INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
EFFECT OF VARYING SIZE OF POWER CONDUCTORS
ON THE
INDUCED VOLTAGES BETWEEN CONDUCTORS

POWER CIRCUIT { Horizontal, Unsymmetrical
Inside Conductor Displaced Toward Disturbed Conductors
Spacings of Conductors = d, d/3, 2d/3
Diameter of Conductors = D/d
Height of Conductors = H/d

DISTURBED CONDUCTORS: In Horizontal Plane, Spaced 0.1d

SEPARATION: Horizontal = X/d Vertical = Y/d



11/2/16
J.C.I.I.
T. R. No. 65.

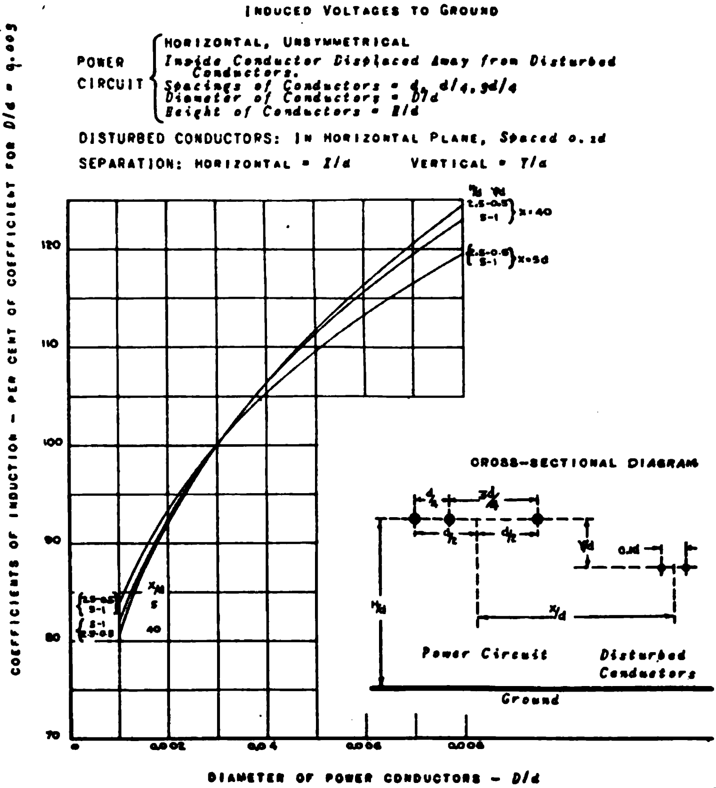
CURVE SHEET NO. 90

INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
EFFECT OF VARYING SIZE OF POWER CONDUCTORS
ON THE
INDUCED VOLTAGES TO GROUND

POWER CIRCUIT { HORIZONTAL, UNSYMMETRICAL
Inside Conductor Displaced Away from Disturbed Conductors.
Spacings of Conductors = d, d/4, 3d/4
Diameter of Conductors = D/d
Height of Conductors = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.1d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*



10/28/16.
J.C.I.I.
T. R. No. 65.

798

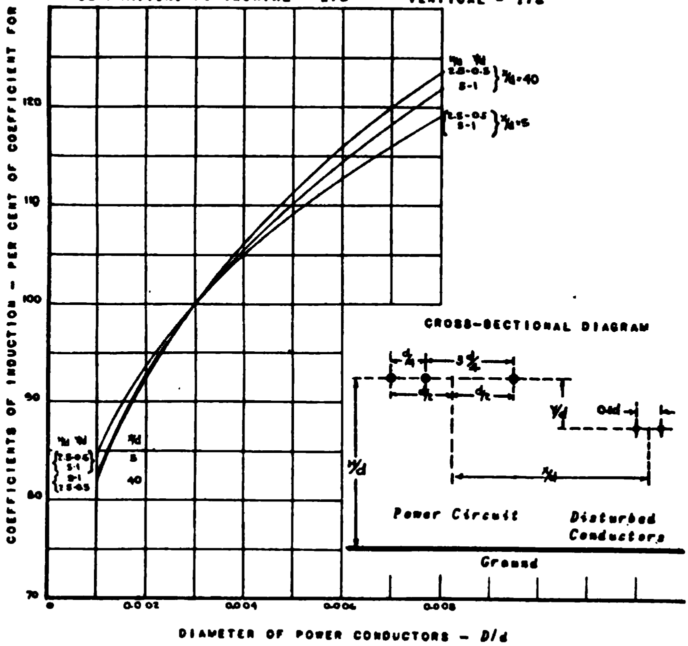
CURVE SHEET NO. 94

INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
EFFECT OF VARYING SIZE OF POWER CONDUCTORS
ON THE
INDUCED VOLTAGES BETWEEN CONDUCTORS

POWER CIRCUIT
- HORIZONTAL, UNSYMMETRICAL
- *Inside Conductor Displaced Away from Disturbed Conductors.*
- *Spacings of Conductors = d, d/4, 3d/4*
- *Diameter of Conductors = D/d*
- *Height of Conductors = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.1d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

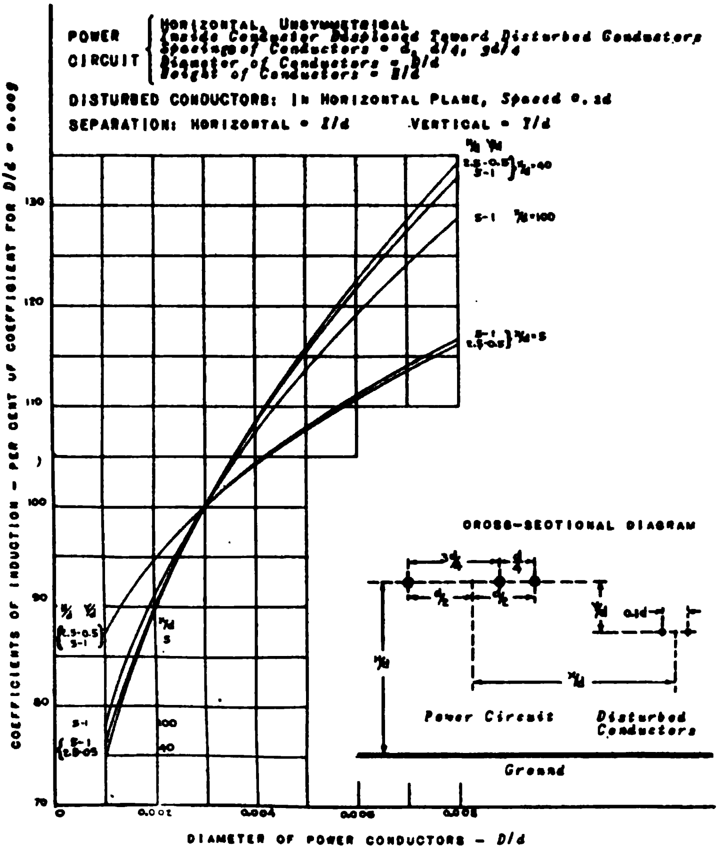


10/27/16
J.C.I.I.
T. R. No. 65.

CURVE SHEET NO. 97

INDUCTION FROM BALANCED THREE-PHASE VOLTAGES

EFFECT OF VARYING SIZE OF POWER CONDUCTORS ON THE INDUCED VOLTAGES TO GROUND



10/31/16

J. C. I. I.
T. R. NO. 65.

CURVE SHEET NO. 101

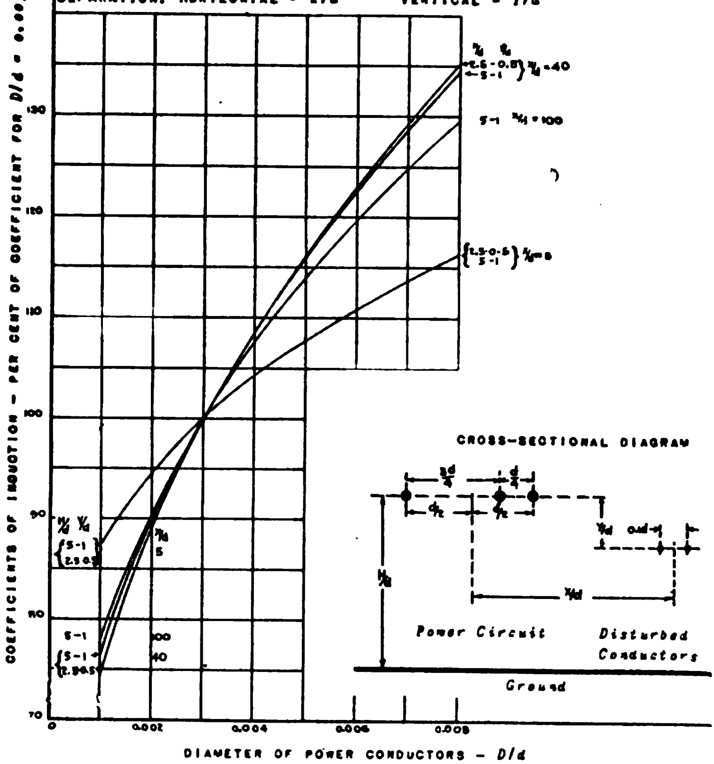
INDUCTION FROM BALANCED THREE-PHASE VOLTAGES

EFFECT OF VARYING SIZE OF POWER CONDUCTORS
ON THE INDUCED VOLTAGES BETWEEN CONDUCTORS.

POWER CIRCUIT { Horizontal, Unsymmetrical
Inside Conductor Displaced Toward Disturbed Conductors
Spacings of Conductors = d, d/4, 9d/4
Diameter of Conductors = D/d
Height of Conductors = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, Spaced 0.1d

SEPARATION: HORIZONTAL = X/d VERTICAL = Y/d



11/3/16
J.C.I.I.
T. R. NO. 65.

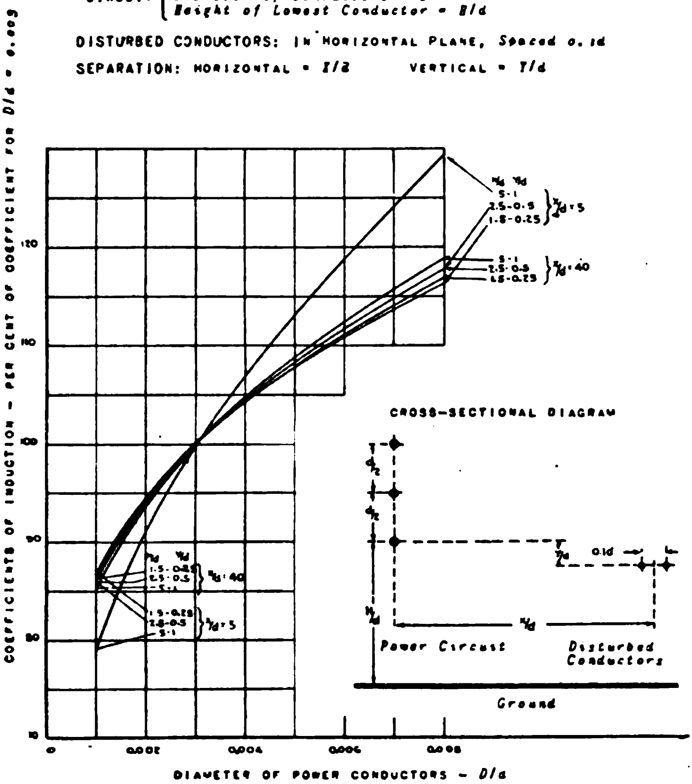
CURVE SHEET NO. 104

INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
EFFECT OF VARYING SIZE OF POWER CONDUCTORS
ON THE
INDUCED VOLTAGES TO GROUND

POWER CIRCUIT { Vertical; Spacings of Conductors = d, d/2, d/2; Diameter of Conductors = D/d; Height of Lowest Conductor = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, Spaced 0.1d

SEPARATION: HORIZONTAL = X/d   VERTICAL = Y/d



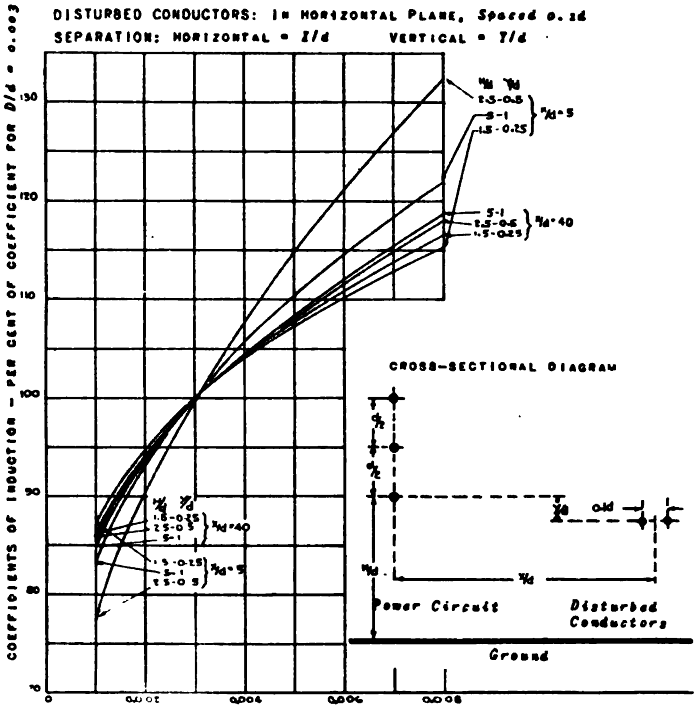
11/1/16
J. C. I. I.
T. R. NO. 65.

CURVE SHEET NO. 108

INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
EFFECT OF VARYING SIZE OF POWER CONDUCTORS
ON THE
INDUCED VOLTAGES BETWEEN CONDUCTORS

POWER CIRCUIT { VERTICAL
Spacings of Conductors = d, d/2, d/2
Diameter of Conductors = D/d
Height of Lowest Conductor = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, Spaced 0.1d
SEPARATION: HORIZONTAL = X/d   VERTICAL = Y/d



11/1/16
J.C.I.I.
T. R. No. 65.

CURVE SHEET NO. III

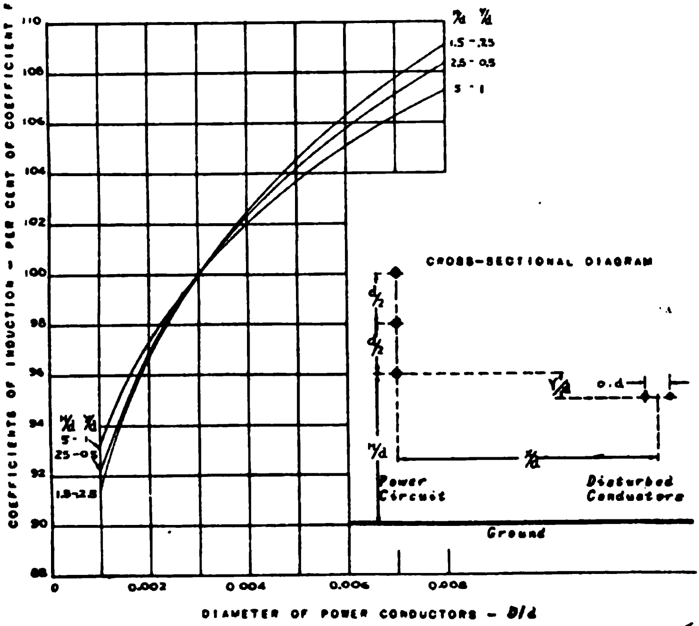
INDUCTION FROM RESIDUAL VOLTAGE OF THREE-PHASE CIRCUITS.
EFFECT OF VARYING SIZE OF POWER CONDUCTORS.

POWER CIRCUIT
- VERTICAL
- *Spacings of Conductors = d, d/a, d/s*
- *Diameter of Conductors = D/d*
- *Height of Lowest Conductor = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.1d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

NOTE: Curves represent average change of coefficients with change of conductor diameter, for induction between conductors and between conductors and ground, for X/d = 5 and 40. The deviation of any case from the average is less than 0.1 per cent.



11/1/16

J.C.I.I.
T. R. No. 65.

CURVE SHEET No. 130

J.C.I.I. T. R. No. 65.

COEFFICIENTS OF INDUCTION FROM RESIDUAL CURRENT OF THREE-PHASE CIRCUITS.

EFFECT OF VARYING ALTITUDE OF POWER-CIRCUIT TRIANGLE.

POWER CIRCUIT { ISOSCELES TRIANGLE, BASE HORIZONTAL = d, ALTITUDE = A
Height of Lower Conductors = 40d

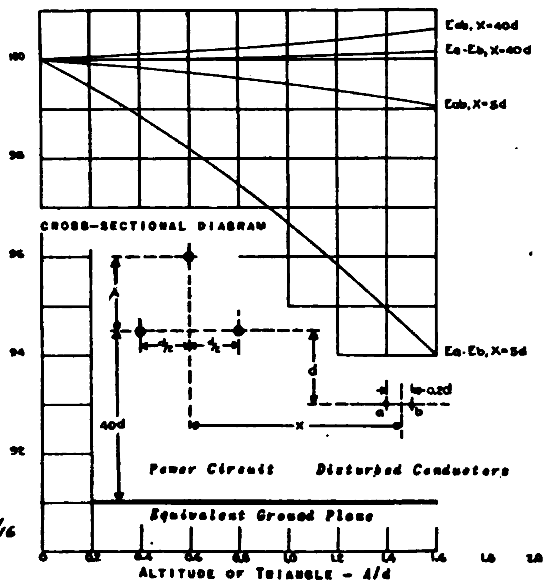
DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.2d*

SEPARATION: HORIZONTAL = X   VERTICAL = d

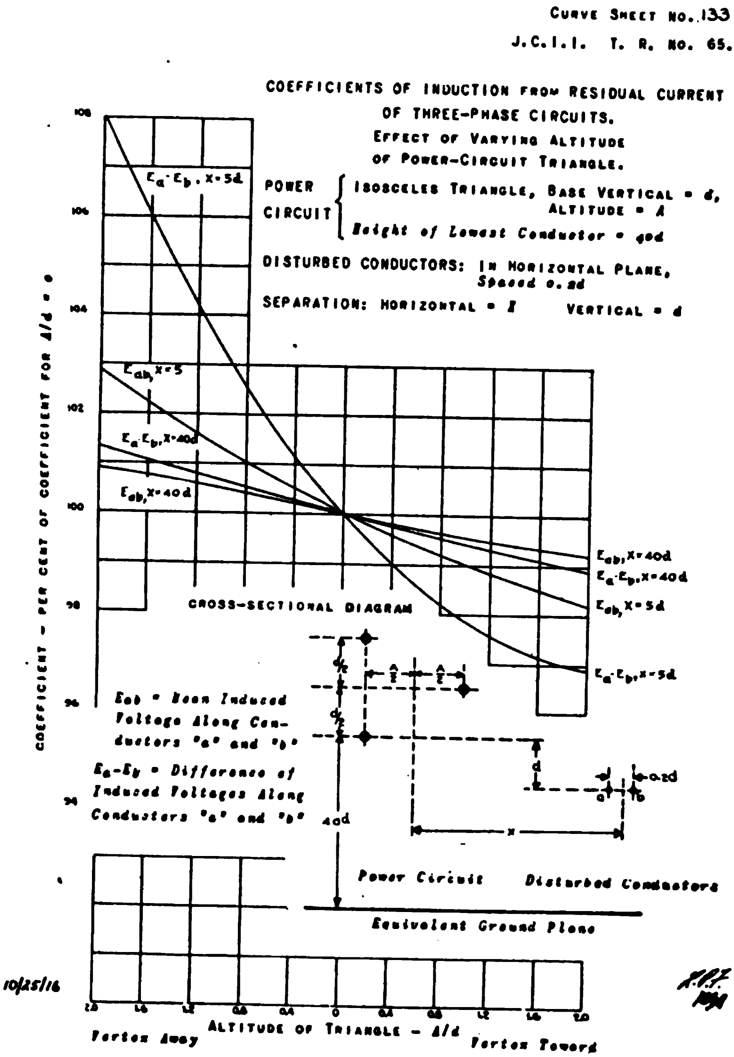
E_a = Mean Induced Voltage along Conductors "a" and "b"

E_a-E_b = Difference of Induced Voltages along Conductors "a" and "b"

COEFFICIENT - PER CENT OF COEFFICIENT FOR A/d = 0



10/25/16


CURVE SHEET NO. 133
J.C.I.I. T. R. NO. 65.
COEFFICIENTS OF INDUCTION FROM RESIDUAL CURRENT
OF THREE-PHASE CIRCUITS.
EFFECT OF VARYING ALTITUDE
OF POWER-CIRCUIT TRIANGLE.
POWER CIRCUIT
ISOSCELES TRIANGLE, BASE VERTICAL = d, ALTITUDE = A
Height of Lowest Conductor = 40d
DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, Spaced 0.2d
SEPARATION: HORIZONTAL = X VERTICAL = d
108
106
104
102
100
98
96
94
COEFFICIENT - PER CENT OF COEFFICIENT FOR A/d = 0
Ea-Eb, X=5d
Eab, X=5
Ea-Eb, X=40d
Eab, X=40d
Eab, X=40d
Ea-Eb, X=40d
Eab, X=5d
Ea-Eb, X=5d
CROSS-SECTIONAL DIAGRAM
Eab = Mean Induced Voltage Along Conductors "a" and "b"
Ea-Eb = Difference of Induced Voltages Along Conductors "a" and "b"
d/2
d/2
A/2
A/2
d
0.2d
a
b
x
40d
Power Circuit
Disturbed Conductors
Equivalent Ground Plane
2.0
1.6
1.2
0.8
0.4
0
0.4
0.8
1.2
1.6
2.0
Vertex Away
ALTITUDE OF TRIANGLE - A/d
Vertex Toward
10/25/16

CURVE SHEET NO. 136

J.C.I.I. T. R. NO. 65.

COEFFICIENTS OF INDUCTION FROM RESIDUAL CURRENT OF THREE-PHASE CIRCUITS.

EFFECT OF VARYING POSITION OF INSIDE CONDUCTOR OF HORIZONTAL POWER CIRCUIT.

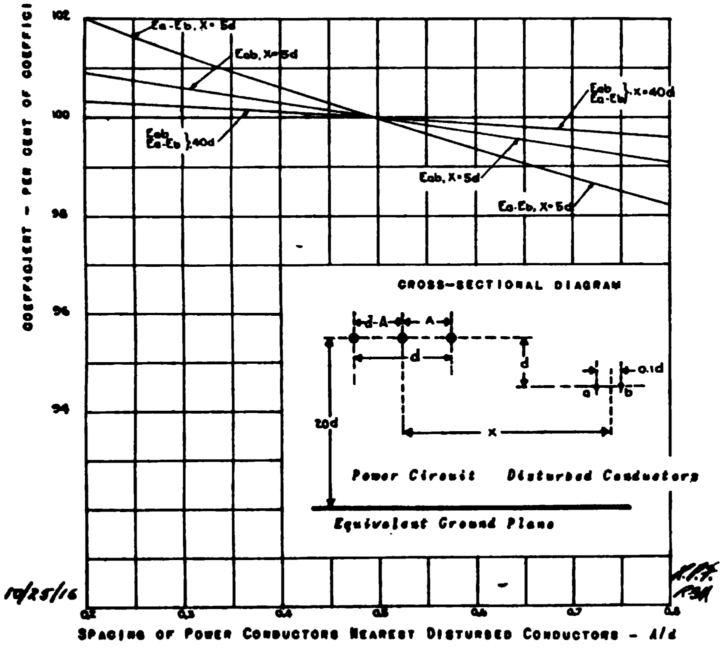
POWER CIRCUIT { HORIZONTAL; *Spacings of Conductors = d, d-A, A*; *Height of Conductors = 20d* }

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.1d*

SEPARATION: HORIZONTAL = X VERTICAL = d

Eab = Mean Induced Voltage along Conductors "a" and "b"

Ea-Eb = Difference of Induced Voltages along Conductors "a" and "b"



CURVE SHEET NO.137

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
VARIATION WITH SPACING OF POWER CONDUCTORS

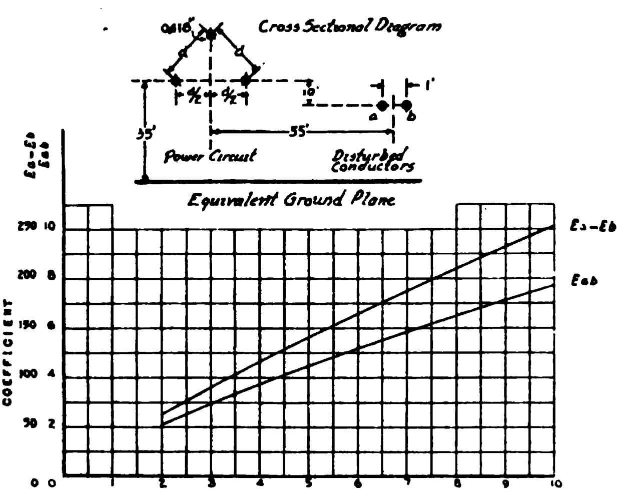
POWER CIRCUIT { Equilateral Triangle, Base Horizontal, Vertex Upward; Spacings of Conductors = d; Diameter of Conductors = 0.418 inches; Height of Lower Conductors = 35 feet

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, Spaced 1 foot

SEPARATION: HORIZONTAL = 55 feet VERTICAL = 10 feet

Eab = Mean induced voltage to ground of Conductors (a) and (b) - Volts per kilovolt between Power Conductors.

Ea-Eb = Induced voltage between Conductors (a) and (b) - Millivolts per kilovolt between Power Conductors.



12/28/16
J.C.I.I. T.R.No.65.

CURVE SHEET NO. 138

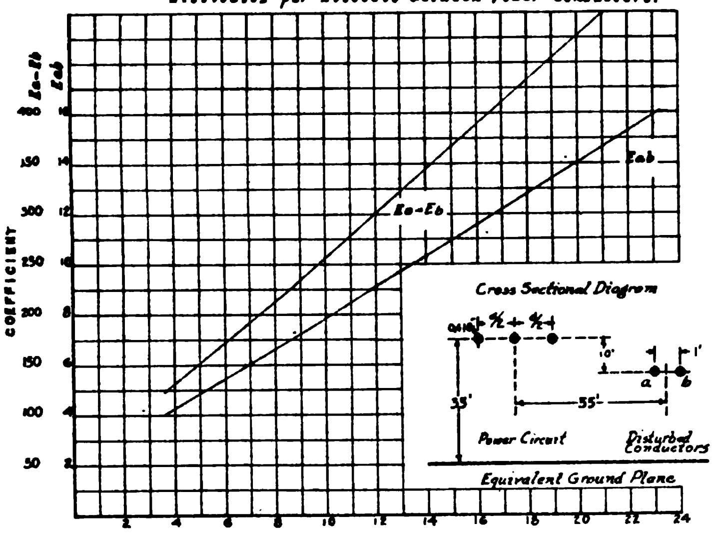
COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
VARIATION WITH SPACING OF POWER CONDUCTORS

POWER CIRCUIT { Horizontal Symmetrical
Spacings of Conductors d, d/2, d/2
Diameter of Conductors = 0.418 inches
Height of Conductors = 35 feet

DISTURBED CONDUCTORS: In Horizontal Plane, Spaced 1 foot
SEPARATION: Horizontal = 55 feet Vertical = 10 feet

Eab = Mean induced voltage to ground of Conductors (a) and (b) - Volts per kilovolt between Power Conductors.

Ea-Eb = Induced voltage between Conductors (a) and (b) - Millivolts per kilovolt between Power Conductors.



SPACING OF POWER CONDUCTORS, d - feet

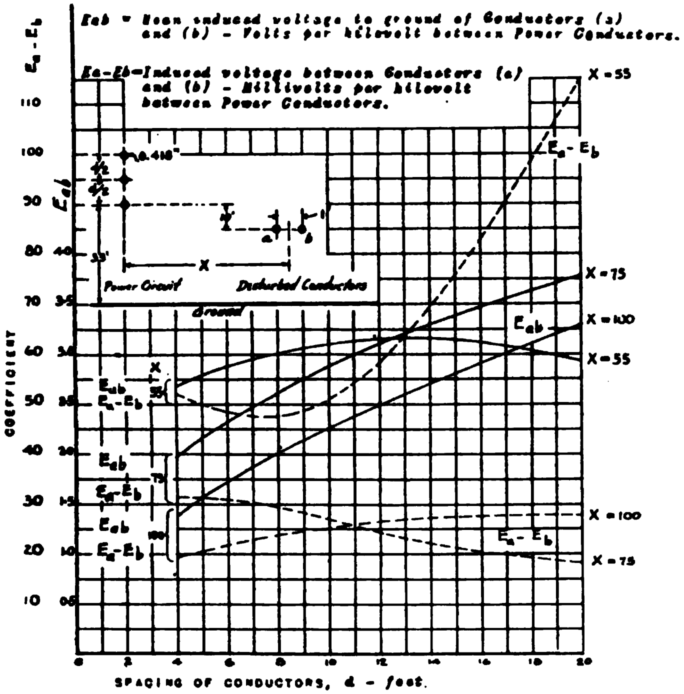
12/28/16
J.C.I.I. T.R. No. 65.

CURVE SHEET NO. 139

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
VARIATION WITH SPACING OF POWER CONDUCTORS

POWER CIRCUIT { VERTICAL
Spacings of Conductors = d, d/2, d/2
Diameter of Conductors = 0.428 inches
Height of Lowest Conductor = 35 feet

DISTURBED CONDUCTORS: In Horizontal Plane, Spaced 1 foot
SEPARATION: Horizontal = X feet   Vertical = 10 feet



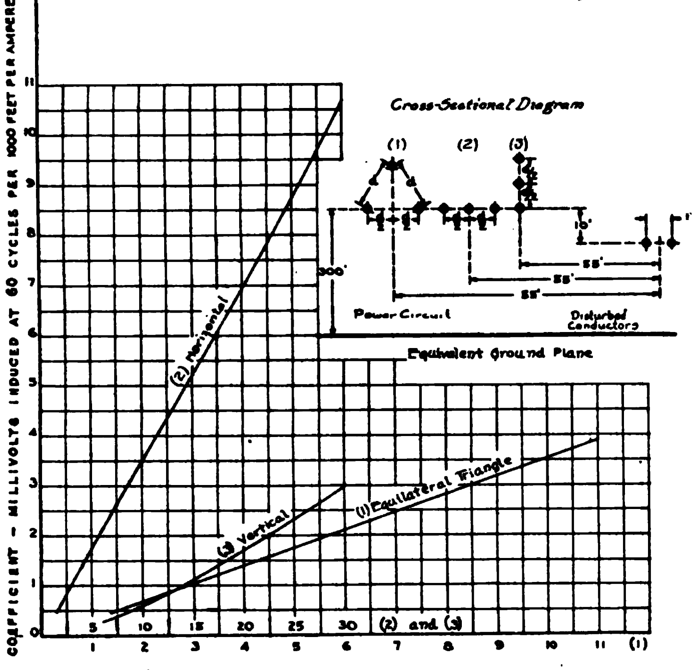
1/5/17
J.C.I.I. T.R. No. 55.

Curve Sheet No 140

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE CURRENTS
VARIATION WITH SPACING OF POWER CONDUCTORS
MEAN INDUCED VOLTAGE ALONG PAIR OF CONDUCTORS

POWER CIRCUIT
(1) Equilateral Triangle — Base Horizontal, Vertex Upward
(2) Horizontal, Symmetrical
(3) Vertical
Spacings of Conductors, (1) d, (2) and (3) d, d/2, d/2
Height of Lowest Conductor = 300 feet

DISTURBED CONDUCTORS: In Horizontal Plane, *Spaced 1 foot*
SEPARATION: Horizontal = *55 feet* Vertical = *10 feet*



SPACING OF POWER CONDUCTORS, d - feet.

11/27/16
J.C.I.I. T.R. No. 65.

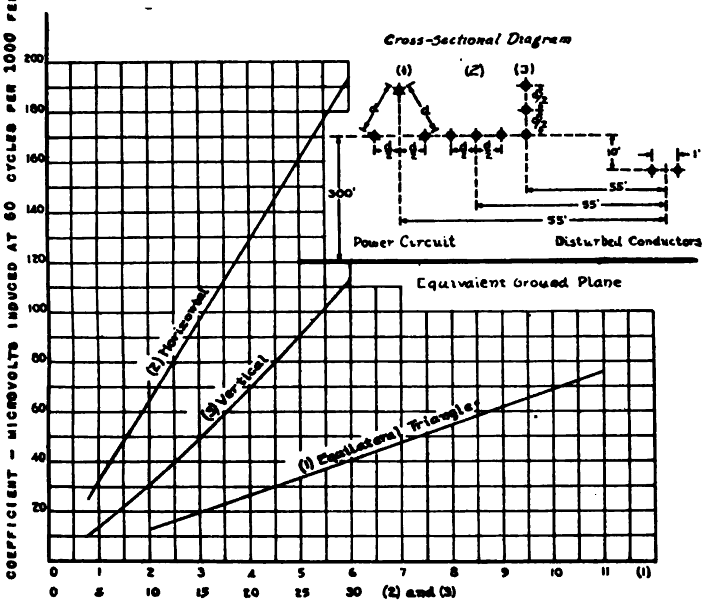
Curve Sheet No. 141

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE CURRENTS
VARIATION WITH SPACING OF POWER CONDUCTORS
DIFFERENCE OF INDUCED VOLTAGES ALONG PAIR OF CONDUCTORS

POWER CIRCUIT
- (1) Equilateral Triangle – Base Horizontal, Vertex Upward
- (2) Horizontal Symmetrical
- (3) Vertical
- *Spacings of Conductors, (1)d, (2) and (3) d, d/2, d/2*
- *Height of lowest Conductor = 300 feet*

DISTURBED CONDUCTORS: in Horizontal Plane, *Spaced 1 foot*

SEPARATION: Horizontal = *55 feet* Vertical = *10 feet*



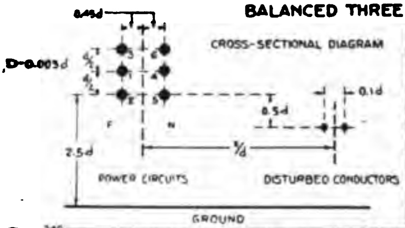
Spacing of Power Conductors, *d – feet*

J.C.1.1, No. 68.

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES

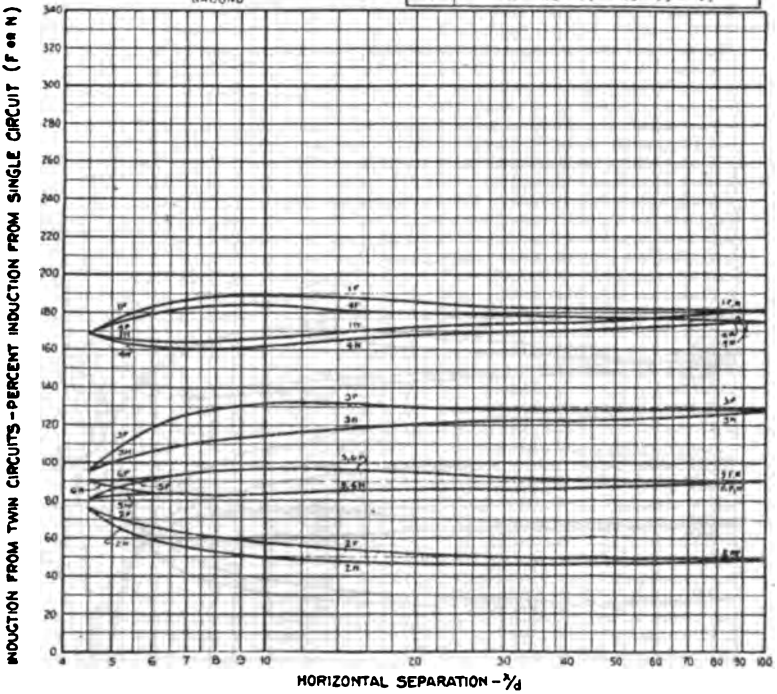
CURVE SHEET NO 179

COMPARISON OF SINGLE AND TWIN POWER CIRCUITS
WITH RESPECT TO
MEAN INDUCED VOLTAGE TO GROUND OF TWO ISOLATED CONDUCTORS
BY
BALANCED THREE-PHASE VOLTAGES



LEGEND

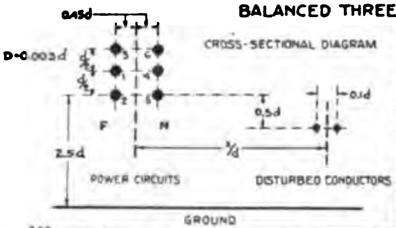
NO.	METHOD OF PARALLEL OPERATION OF TWIN CIRCUITS
1	$V_1 = V_4 \quad V_2 = V_5 \quad V_3 = V_6$
2	$V_1 = V_4 \quad V_2 = V_6 \quad V_3 = V_5$
3	$V_1 = V_6 \quad V_2 = V_5 \quad V_3 = V_4$
4	$V_1 = V_5 \quad V_2 = V_4 \quad V_3 = V_6$
5	$V_1 = V_5 \quad V_2 = V_6 \quad V_3 = V_4$
6	$V_1 = V_6 \quad V_2 = V_4 \quad V_3 = V_5$



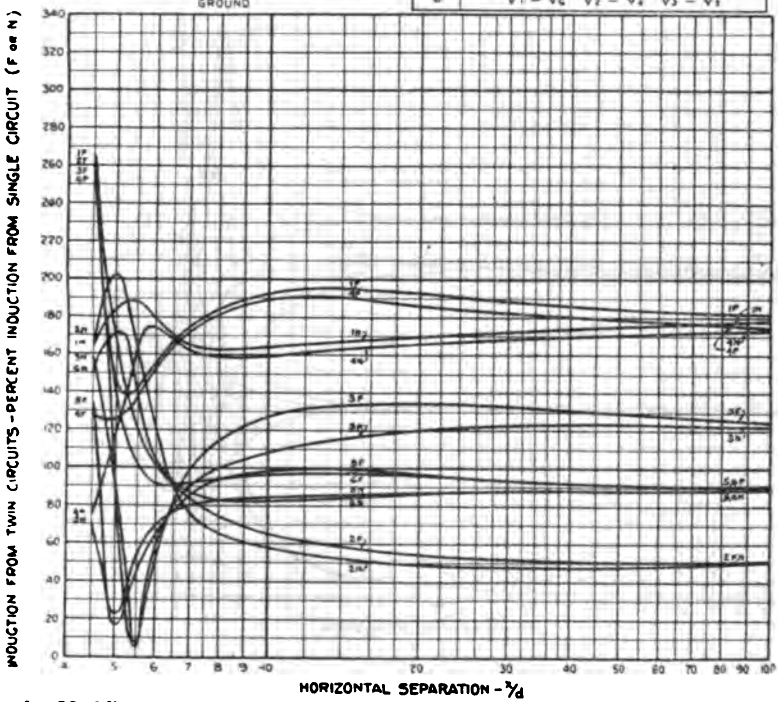
J.C.I.I. T.P. NO.65

CURVE SHEET NO. 180

COMPARISON OF SINGLE AND TWIN POWER CIRCUITS
WITH RESPECT TO
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS
BY
BALANCED THREE-PHASE VOLTAGES



LEGEND	
NO	METHOD OF PARALLEL OPERATION OF TWIN CIRCUITS
1	$V_1 = V_4$ $V_2 = V_5$ $V_3 = V_6$
2	$V_1 = V_4$ $V_2 = V_6$ $V_3 = V_5$
3	$V_1 = V_6$ $V_2 = V_5$ $V_3 = V_4$
4	$V_1 = V_5$ $V_2 = V_4$ $V_3 = V_6$
5	$V_1 = V_5$ $V_2 = V_6$ $V_3 = V_4$
6	$V_1 = V_6$ $V_2 = V_4$ $V_3 = V_5$



JCII TR NO 65

CURVE SHEET NO. 181

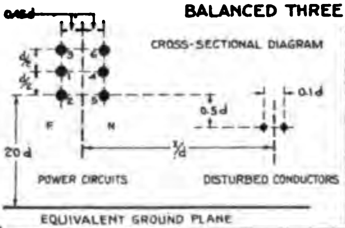
COMPARISON OF SINGLE AND TWIN POWER CIRCUITS

WITH RESPECT TO

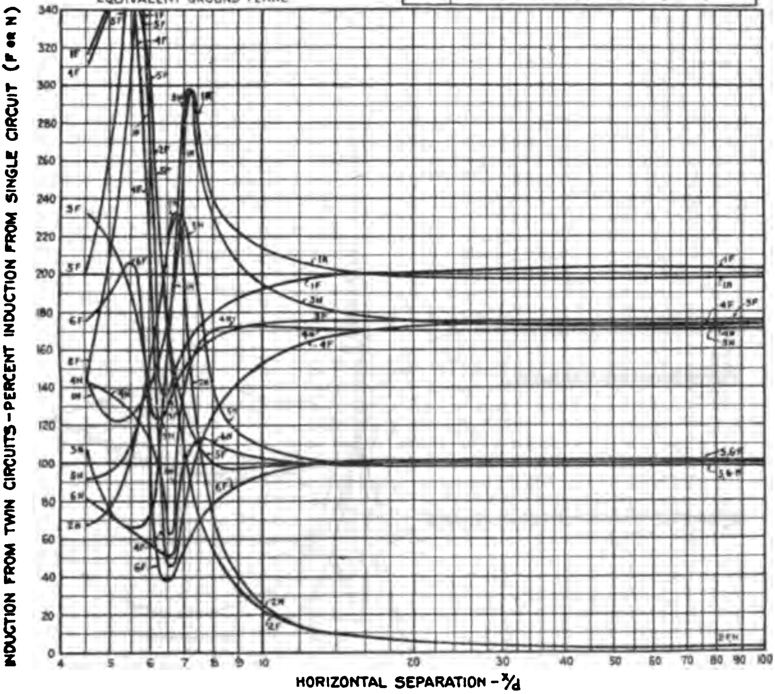
MEAN INDUCED VOLTAGE ALONG PAIR OF CONDUCTORS

BY

BALANCED THREE-PHASE CURRENTS



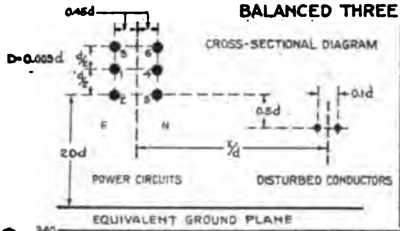
LEGEND	
NO.	METHOD OF PARALLEL OPERATION OF TWIN CIRCUITS
1	$I_1 = I_4 \quad I_2 = I_5 \quad I_3 = I_6$
2	$I_1 = I_4 \quad I_2 = I_6 \quad I_3 = I_5$
3	$I_1 = I_6 \quad I_2 = I_5 \quad I_3 = I_4$
4	$I_1 = I_5 \quad I_2 = I_4 \quad I_3 = I_6$
5	$I_1 = I_5 \quad I_2 = I_6 \quad I_3 = I_4$
6	$I_1 = I_6 \quad I_2 = I_4 \quad I_3 = I_5$



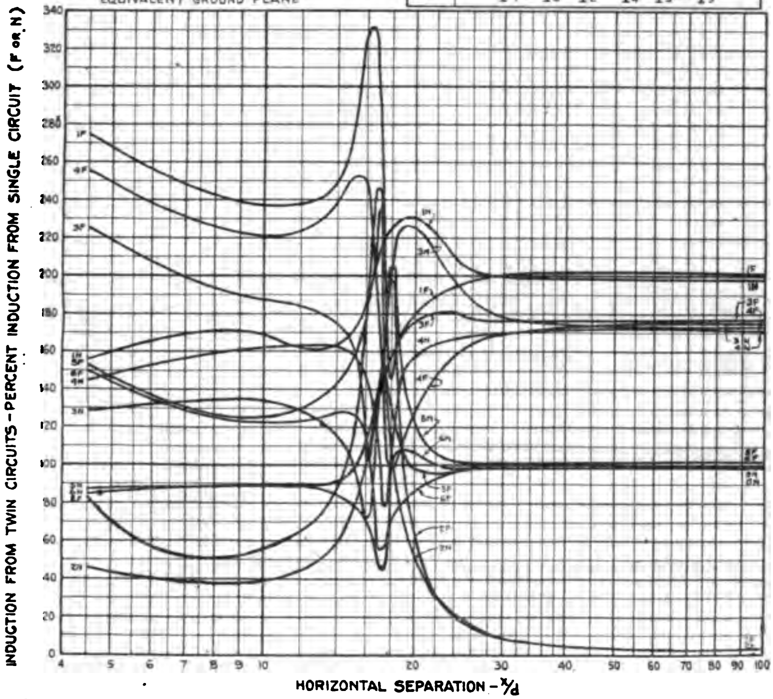
J.C.I.I. T.R. NO.65

CURVE SHEET NO. 182

COMPARISON OF SINGLE AND TWIN POWER CIRCUITS
WITH RESPECT TO
DIFFERENCE OF INDUCED VOLTAGES ALONG PAIR OF CONDUCTORS
BY
BALANCED THREE-PHASE CURRENTS.



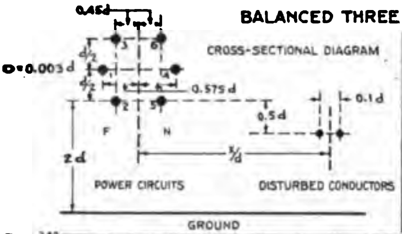
LEGEND	
NO.	METHOD OF PARALLEL OPERATION OF TWIN CIRCUITS
1	$I_1 = I_4 \quad I_2 = I_5 \quad I_3 = I_6$
2	$I_1 = I_4 \quad I_2 = I_6 \quad I_3 = I_5$
3	$I_1 = I_6 \quad I_2 = I_5 \quad I_3 = I_4$
4	$I_1 = I_5 \quad I_2 = I_4 \quad I_3 = I_6$
5	$I_1 = I_5 \quad I_2 = I_6 \quad I_3 = I_4$
6	$I_1 = I_6 \quad I_2 = I_4 \quad I_3 = I_5$



J.C.I.I. T.R. NO. 65

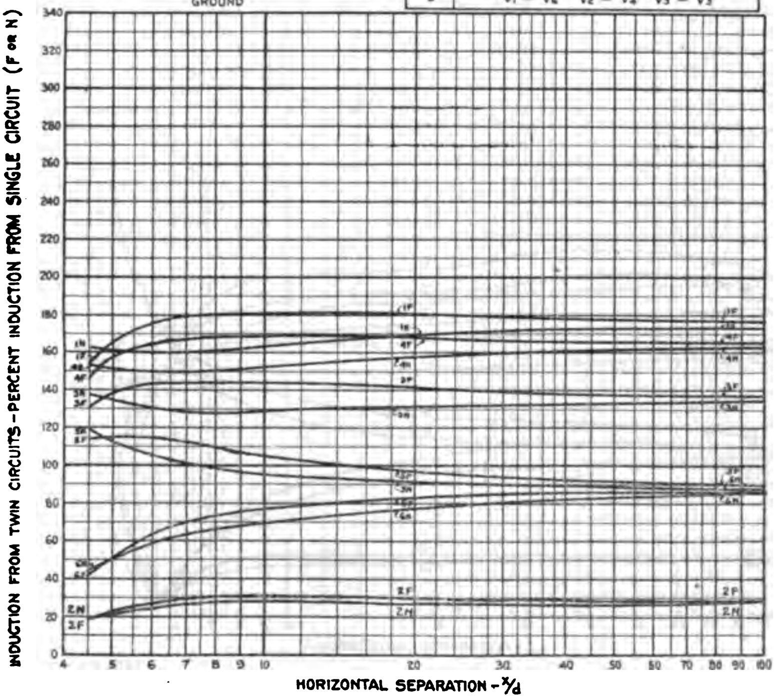
CURVE SHEET NO. 183

COMPARISON OF SINGLE AND TWIN POWER CIRCUITS
WITH RESPECT TO
MEAN INDUCED VOLTAGE TO GROUND OF TWO ISOLATED CONDUCTORS
BY
BALANCED THREE-PHASE VOLTAGES



LEGEND

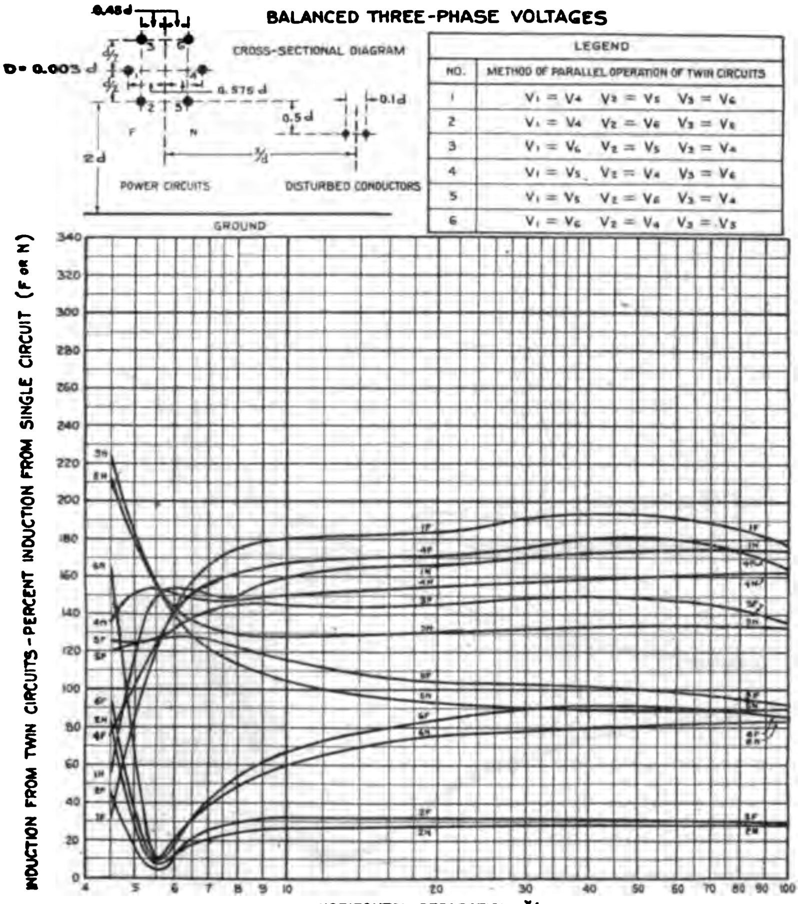
NO.	METHOD OF PARALLEL OPERATION OF TWIN CIRCUITS
1	$V_1 = V_4$ $V_2 = V_5$ $V_3 = V_6$
2	$V_1 = V_4$ $V_2 = V_6$ $V_3 = V_5$
3	$V_1 = V_6$ $V_2 = V_5$ $V_3 = V_4$
4	$V_1 = V_5$ $V_2 = V_4$ $V_3 = V_6$
5	$V_1 = V_5$ $V_2 = V_6$ $V_3 = V_4$
6	$V_1 = V_6$ $V_2 = V_4$ $V_3 = V_5$



J.C.I.I. T.R. NO.65

CURVE SHEET NO. 184

COMPARISON OF SINGLE AND TWIN POWER CIRCUITS
WITH RESPECT TO
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS
BY
BALANCED THREE-PHASE VOLTAGES

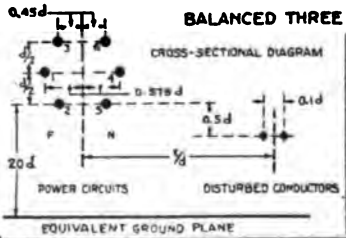


LEGEND	
NO.	METHOD OF PARALLEL OPERATION OF TWIN CIRCUITS
1	$V_1 = V_4$ $V_2 = V_5$ $V_3 = V_6$
2	$V_1 = V_4$ $V_2 = V_6$ $V_3 = V_5$
3	$V_1 = V_6$ $V_2 = V_5$ $V_3 = V_4$
4	$V_1 = V_5$ $V_2 = V_4$ $V_3 = V_6$
5	$V_1 = V_5$ $V_2 = V_6$ $V_3 = V_4$
6	$V_1 = V_6$ $V_2 = V_4$ $V_3 = V_5$

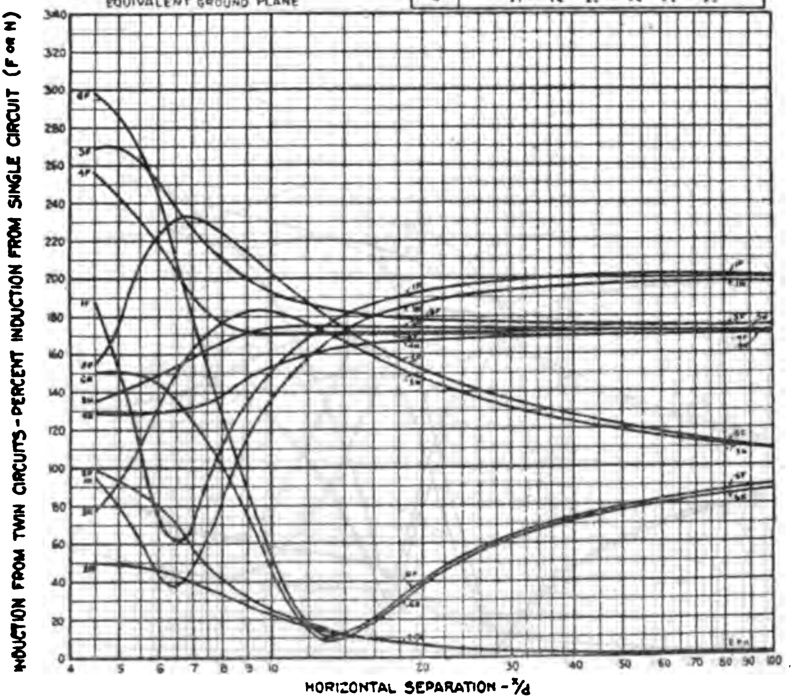
J.C.I.I. T.R. NO.65

CURVE SHEET NO. 185

COMPARISON OF SINGLE AND TWIN POWER CIRCUITS
WITH RESPECT TO
MEAN INDUCED VOLTAGE ALONG PAIR OF CONDUCTORS
BY
BALANCED THREE-PHASE CURRENTS



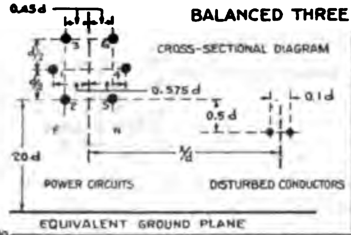
LEGEND	
NO	METHOD OF PARALLEL OPERATION OF TWIN CIRCUITS
1	$I_1 = I_4 \quad I_2 = I_5 \quad I_3 = I_6$
2	$I_1 = I_4 \quad I_2 = I_6 \quad I_3 = I_5$
3	$I_1 = I_6 \quad I_2 = I_5 \quad I_3 = I_4$
4	$I_1 = I_5 \quad I_2 = I_4 \quad I_3 = I_6$
5	$I_1 = I_5 \quad I_2 = I_6 \quad I_3 = I_4$
6	$I_1 = I_6 \quad I_2 = I_4 \quad I_3 = I_5$



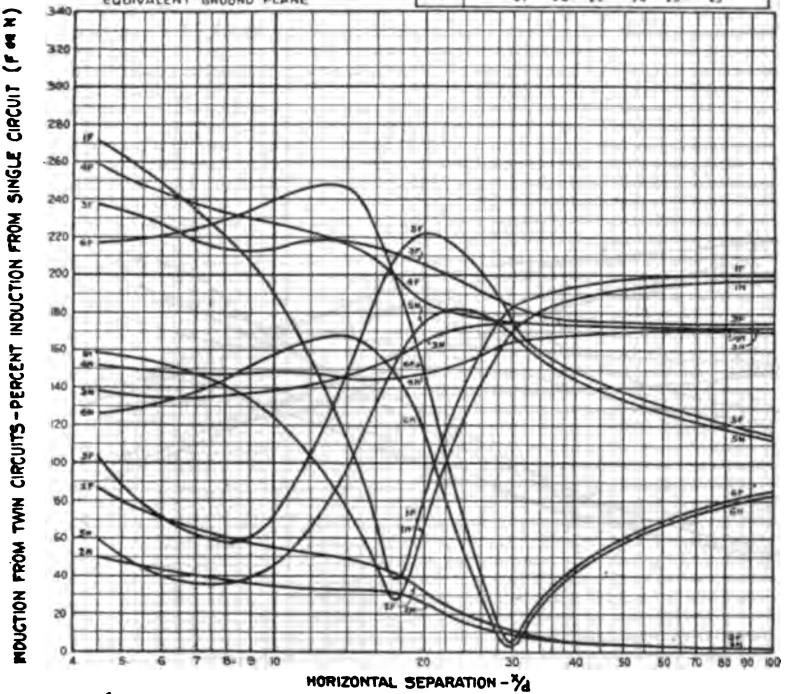
J.C.I.I. T.R. NO. 65

CURVE SHEET NO. 186

COMPARISON OF SINGLE AND TWIN POWER CIRCUITS
WITH RESPECT TO
DIFFERENCE OF INDUCED VOLTAGES ALONG PAIR OF CONDUCTORS
BY
BALANCED THREE-PHASE CURRENTS



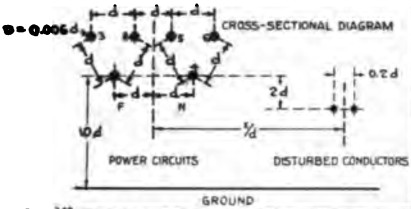
LEGEND	
NO.	METHOD OF PARALLEL OPERATION OF TWIN CIRCUITS
1	$I_1 = I_4$ $I_2 = I_5$ $I_3 = I_6$
2	$I_1 = I_4$ $I_2 = I_6$ $I_3 = I_5$
3	$I_1 = I_6$ $I_2 = I_5$ $I_3 = I_4$
4	$I_1 = I_5$ $I_2 = I_4$ $I_3 = I_6$
5	$I_1 = I_5$ $I_2 = I_6$ $I_3 = I_4$
6	$I_1 = I_6$ $I_2 = I_4$ $I_3 = I_5$



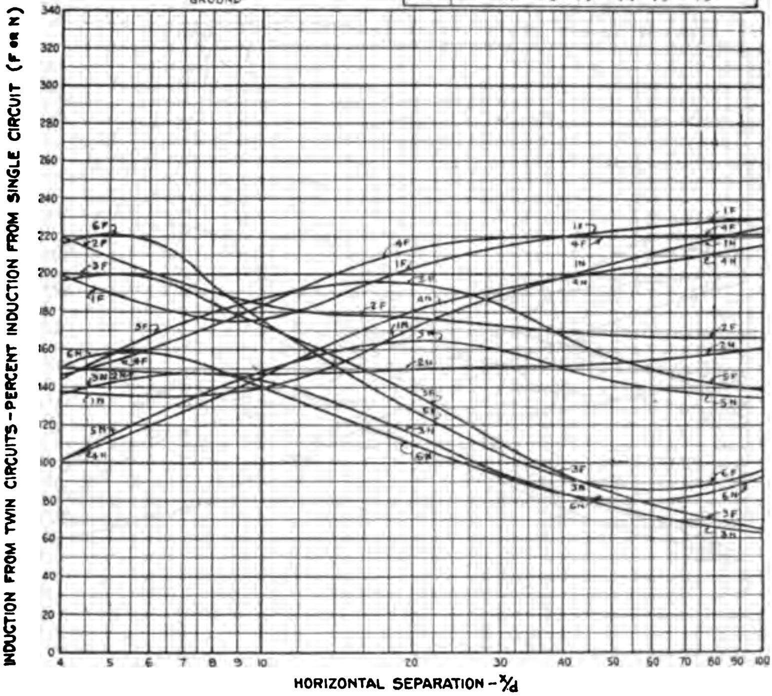
J.C.I.I. T.R. NO. 65

CURVE SHEET NO. 187

COMPARISON OF SINGLE AND TWIN POWER CIRCUITS
WITH RESPECT TO
MEAN INDUCED VOLTAGE TO GROUND OF TWO ISOLATED CONDUCTORS
BY
BALANCED THREE-PHASE VOLTAGES



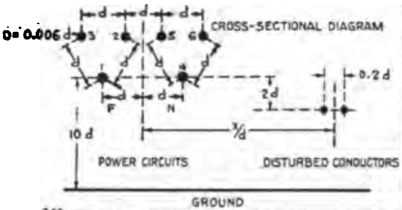
LEGEND	
NO.	METHOD OF PARALLEL OPERATION OF TWIN CIRCUITS
1	$V_1 = V_4 \quad V_2 = V_5 \quad V_3 = V_6$
2	$V_1 = V_4 \quad V_2 = V_6 \quad V_3 = V_5$
3	$V_1 = V_6 \quad V_2 = V_5 \quad V_3 = V_4$
4	$V_1 = V_5 \quad V_2 = V_4 \quad V_3 = V_6$
5	$V_1 = V_5 \quad V_2 = V_6 \quad V_3 = V_4$
6	$V_1 = V_6 \quad V_2 = V_4 \quad V_3 = V_5$



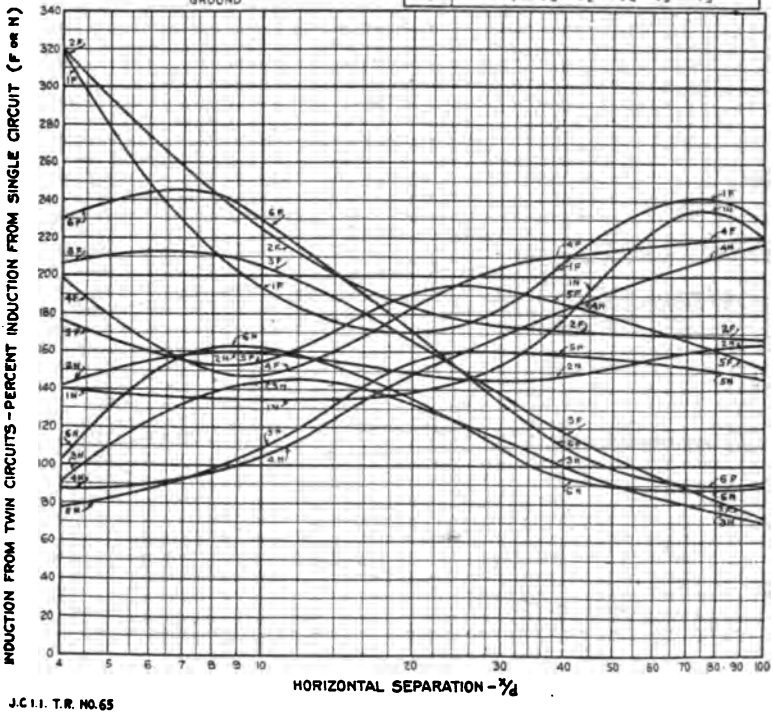
J.C.I.I. T.R. NO. 65

CURVE SHEET NO. 188

COMPARISON OF SINGLE AND TWIN POWER CIRCUITS
WITH RESPECT TO
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS
BY
BALANCED THREE-PHASE VOLTAGES



LEGEND	
NO.	METHOD OF PARALLEL OPERATION OF TWIN CIRCUITS
1	$V_1 = V_4$ $V_2 = V_5$ $V_3 = V_6$
2	$V_1 = V_4$ $V_2 = V_6$ $V_3 = V_5$
3	$V_1 = V_6$ $V_2 = V_5$ $V_3 = V_4$
4	$V_1 = V_5$ $V_2 = V_4$ $V_3 = V_6$
5	$V_1 = V_5$ $V_2 = V_6$ $V_3 = V_4$
6	$V_1 = V_6$ $V_2 = V_4$ $V_3 = V_5$

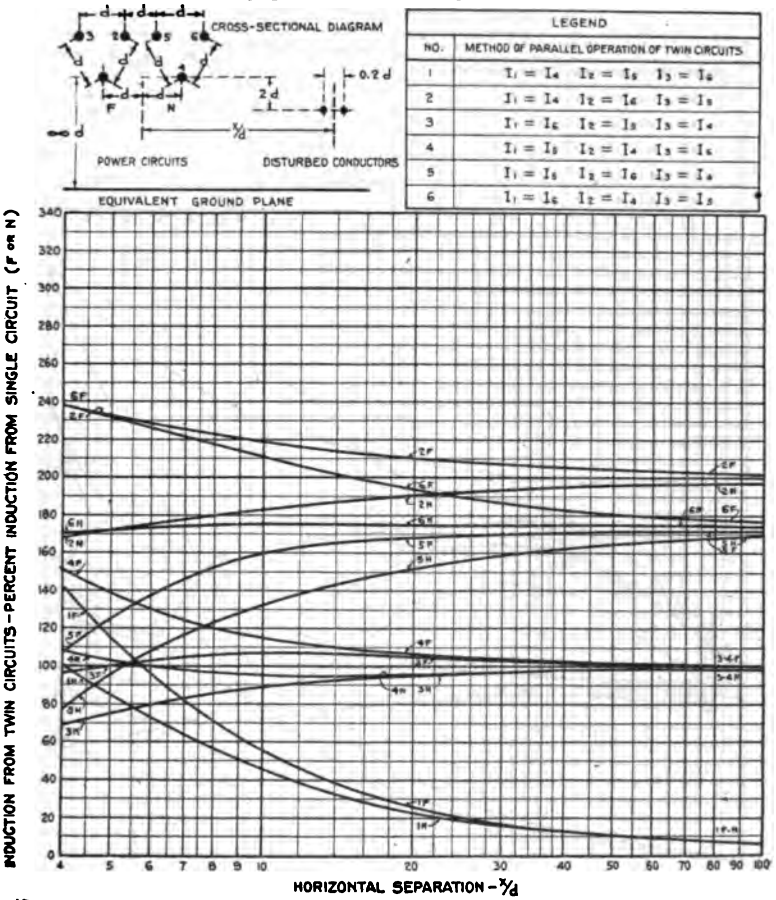


J.C.I.I. T.R. NO.65

1

CURVE SHEET NO. 189

COMPARISON OF SINGLE AND TWIN POWER CIRCUITS
WITH RESPECT TO
MEAN INDUCED VOLTAGE ALONG PAIR OF CONDUCTORS
BY
BALANCED THREE-PHASE CURRENTS

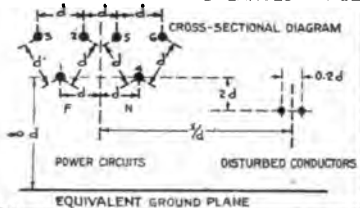


LEGEND	
NO.	METHOD OF PARALLEL OPERATION OF TWIN CIRCUITS
1	$I_1 = I_4$ $I_2 = I_5$ $I_3 = I_6$
2	$I_1 = I_4$ $I_2 = I_6$ $I_3 = I_5$
3	$I_1 = I_6$ $I_2 = I_5$ $I_3 = I_4$
4	$I_1 = I_5$ $I_2 = I_4$ $I_3 = I_6$
5	$I_1 = I_5$ $I_2 = I_6$ $I_3 = I_4$
6	$I_1 = I_6$ $I_2 = I_4$ $I_3 = I_5$

J.E.L.I. T.R. NO. 65

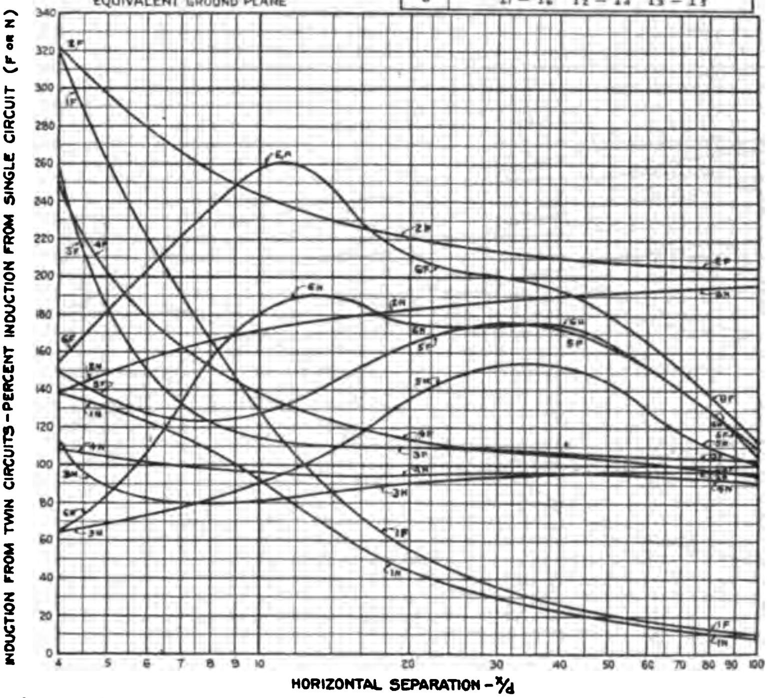
CURVE SHEET NO. 190

COMPARISON OF SINGLE AND TWIN POWER CIRCUITS
WITH RESPECT TO
DIFFERENCE OF INDUCED VOLTAGES ALONG PAIR OF CONDUCTORS
BY
BALANCED THREE-PHASE CURRENTS



LEGEND

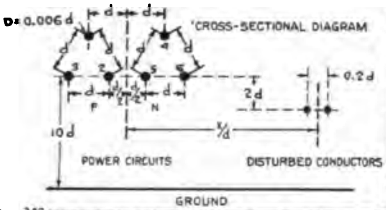
NO.	METHOD OF PARALLEL OPERATION OF TWIN CIRCUITS
1	$I_1 = I_4$ $I_2 = I_5$ $I_3 = I_6$
2	$I_1 = I_4$ $I_2 = I_6$ $I_3 = I_5$
3	$I_1 = I_6$ $I_2 = I_5$ $I_3 = I_4$
4	$I_1 = I_5$ $I_2 = I_4$ $I_3 = I_6$
5	$I_1 = I_5$ $I_2 = I_6$ $I_3 = I_4$
6	$I_1 = I_6$ $I_2 = I_4$ $I_3 = I_5$



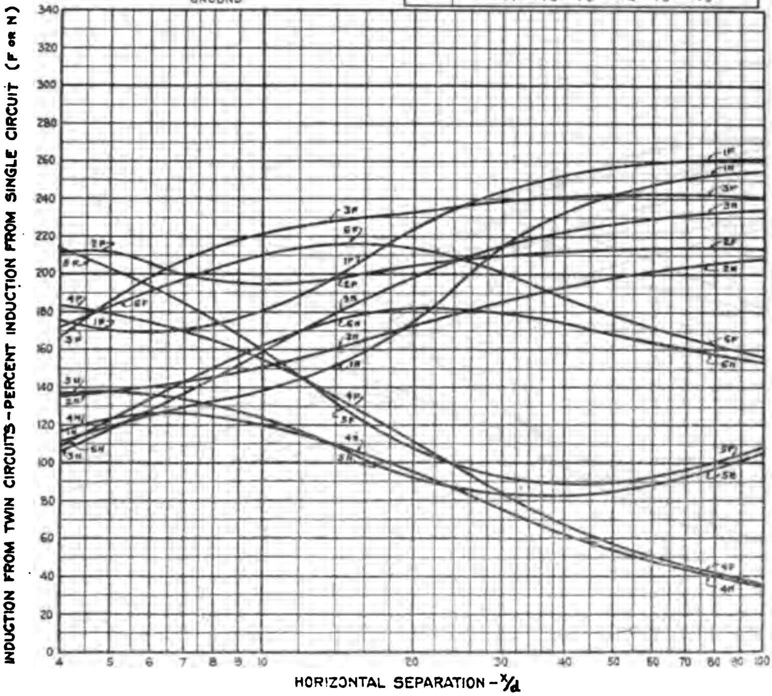
J.C.I.I. T.R. NO.65

CURVE SHEET NO. 191

COMPARISON OF SINGLE AND TWIN POWER CIRCUITS
WITH RESPECT TO
MEAN INDUCED VOLTAGE TO GROUND OF TWO ISOLATED CONDUCTORS
BY
BALANCED THREE-PHASE VOLTAGES



LEGEND	
NO.	METHOD OF PARALLEL OPERATION OF TWIN CIRCUITS
1	$V_1 = V_4 \quad V_2 = V_5 \quad V_3 = V_6$
2	$V_1 = V_4 \quad V_2 = V_6 \quad V_3 = V_5$
3	$V_1 = V_6 \quad V_2 = V_5 \quad V_3 = V_4$
4	$V_1 = V_5 \quad V_2 = V_4 \quad V_3 = V_6$
5	$V_1 = V_5 \quad V_2 = V_6 \quad V_3 = V_4$
6	$V_1 = V_6 \quad V_2 = V_4 \quad V_3 = V_5$

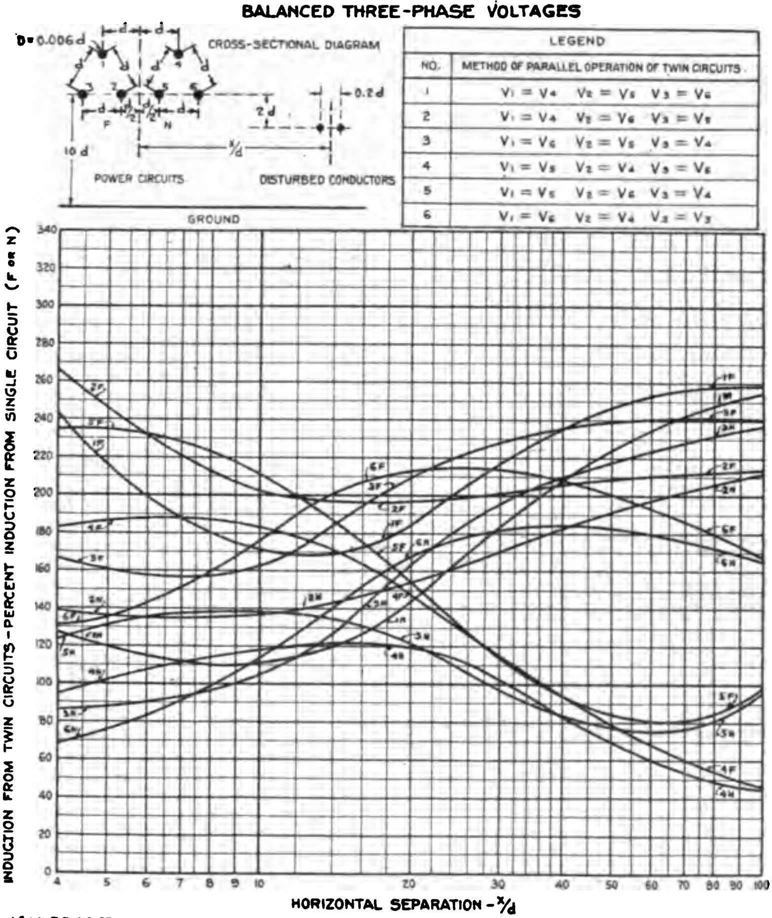


J.C.I.I. T.R. NO. 65

CURVE SHEET NO. 192

COMPARISON OF SINGLE AND TWIN POWER CIRCUITS
WITH RESPECT TO
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS
BY
BALANCED THREE-PHASE VOLTAGES

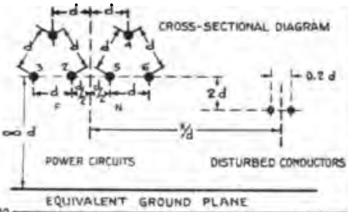
LEGEND	
NO.	METHOD OF PARALLEL OPERATION OF TWIN CIRCUITS.
1	$V_1 = V_4 \quad V_2 = V_5 \quad V_3 = V_6$
2	$V_1 = V_4 \quad V_2 = V_6 \quad V_3 = V_5$
3	$V_1 = V_6 \quad V_2 = V_5 \quad V_3 = V_4$
4	$V_1 = V_5 \quad V_2 = V_4 \quad V_3 = V_6$
5	$V_1 = V_5 \quad V_2 = V_6 \quad V_3 = V_4$
6	$V_1 = V_6 \quad V_2 = V_4 \quad V_3 = V_5$



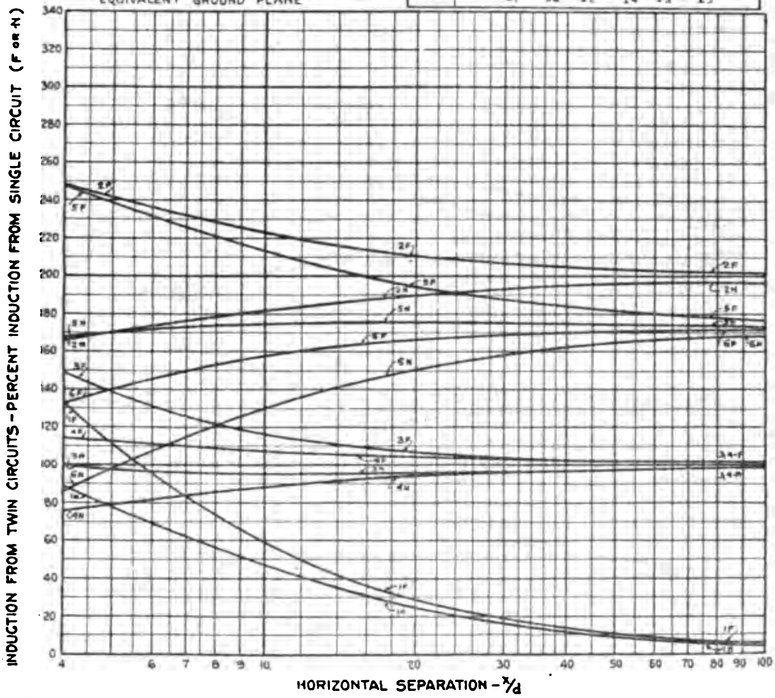
J.C.I.I. T.R. NO.65

CURVE SHEET NO. 193

COMPARISON OF SINGLE AND TWIN POWER CIRCUITS
WITH RESPECT TO
MEAN INDUCED VOLTAGE ALONG PAIR OF CONDUCTORS
BY
BALANCED THREE-PHASE CURRENTS



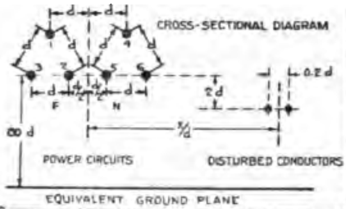
LEGEND	
NO.	METHOD OF PARALLEL OPERATION OF TWIN CIRCUITS
1	$I_1 = I_4 \quad I_2 = I_5 \quad I_3 = I_6$
2	$I_1 = I_4 \quad I_2 = I_6 \quad I_3 = I_5$
3	$I_1 = I_6 \quad I_2 = I_5 \quad I_3 = I_4$
4	$I_1 = I_5 \quad I_2 = I_4 \quad I_3 = I_6$
5	$I_1 = I_5 \quad I_2 = I_6 \quad I_3 = I_4$
6	$I_1 = I_6 \quad I_2 = I_4 \quad I_3 = I_5$



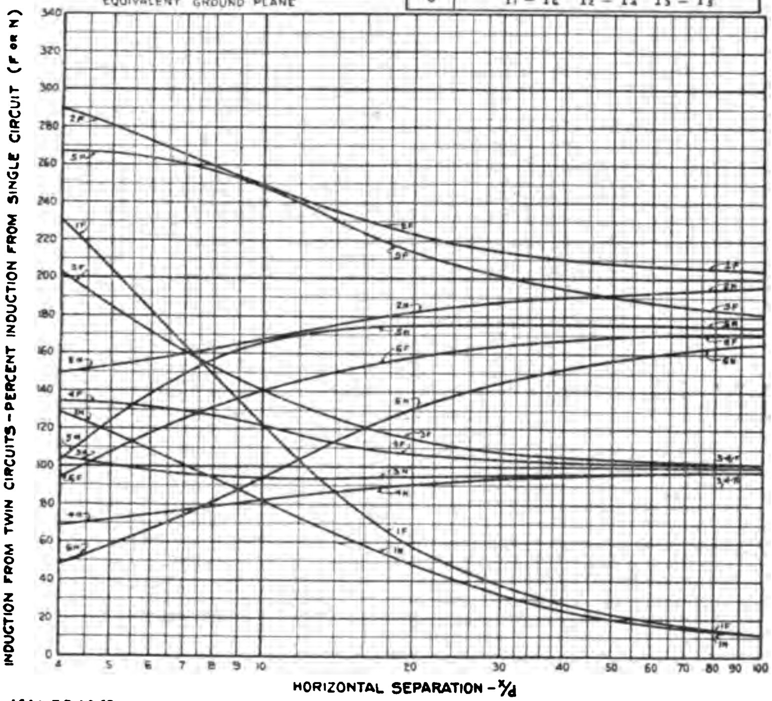
J.C.I.I. T.R. NO. 65

CURVE SHEET NO. [illegible]

COMPARISON OF SINGLE AND TWIN POWER CIRCUITS
WITH RESPECT TO
DIFFERENCE OF INDUCED VOLTAGES ALONG PAIR OF CONDUCTORS
BY
BALANCED THREE-PHASE CURRENTS



LEGEND	
NO.	METHOD OF PARALLEL OPERATION OF TWIN CIRCUITS
1	$I_1 = I_4 \quad I_2 = I_5 \quad I_3 = I_6$
2	$I_1 = I_4 \quad I_2 = I_6 \quad I_3 = I_5$
3	$I_1 = I_6 \quad I_2 = I_5 \quad I_3 = I_4$
4	$I_1 = I_5 \quad I_2 = I_4 \quad I_3 = I_6$
5	$I_1 = I_5 \quad I_2 = I_6 \quad I_3 = I_4$
6	$I_1 = I_6 \quad I_2 = I_4 \quad I_3 = I_5$



J.C.I.I. T.R NO.65

CURVE SHEET No. 195

RATIO OF MEAN TO DIFFERENCE OF INDUCED VOLTAGES

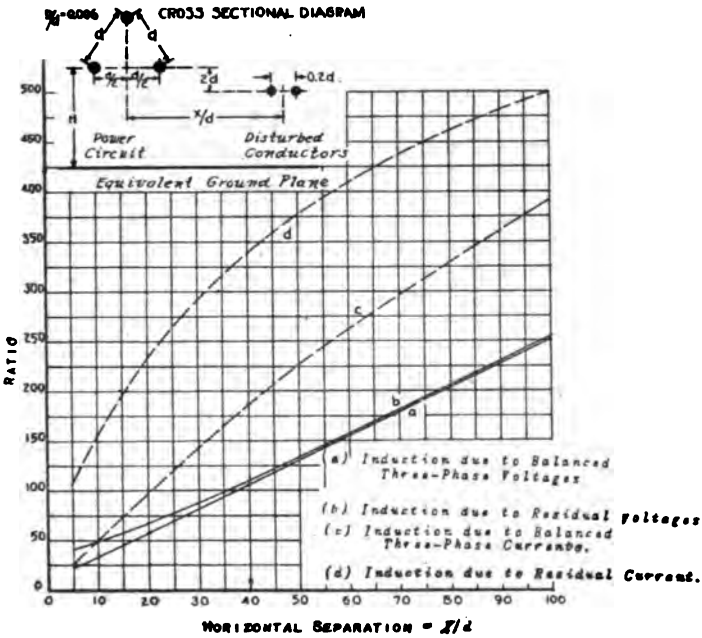
TO GROUND OR ALONG CONDUCTORS

VARIATION WITH HORIZONTAL SEPARATION

POWER CIRCUIT:
- Equilateral Triangle, Base Horizontal, Vertex Upward
- Spacings of Conductors = d
- Diameter of Conductors = 0.006d
- Height of Lower Conductors = {20d (a) and (b); 100d (c) and (d)} = H

DISTURBED CONDUCTORS: In Horizontal Plane, Spaced 0.2d

SEPARATION: Horizontal = X/d Vertical = 2d



1/4/17
J.O.I.I.
T.R.No.65.

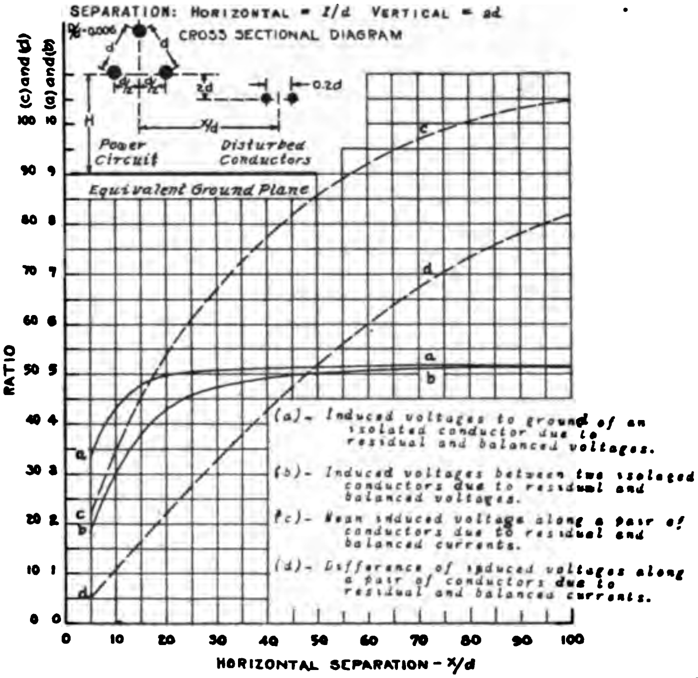
CURVE SHEET No. 196

RATIO OF INDUCED VOLTAGE FROM RESIDUAL VOLTAGE OR CURRENT
TO
INDUCED VOLTAGE FROM BALANCED THREE-PHASE VOLTAGES OR CURRENTS
FOR EQUAL MAGNITUDES OF RESIDUAL AND BALANCED VOLTAGES OR CURRENTS
VARIATION WITH HORIZONTAL SEPARATION

POWER CIRCUIT:
- Equilateral Triangle, Base Horizontal, Vertex Upward
- Spacing of Conductors = d
- Diameter of Conductors = 0.006d
- Height of Lower Conductors = 30d (a) and (b); 100d (c) and (d) = H

DISTURBED CONDUCTORS: In horizontal Plane, Spaced 0.2d

SEPARATION: Horizontal = X/d Vertical = 2d



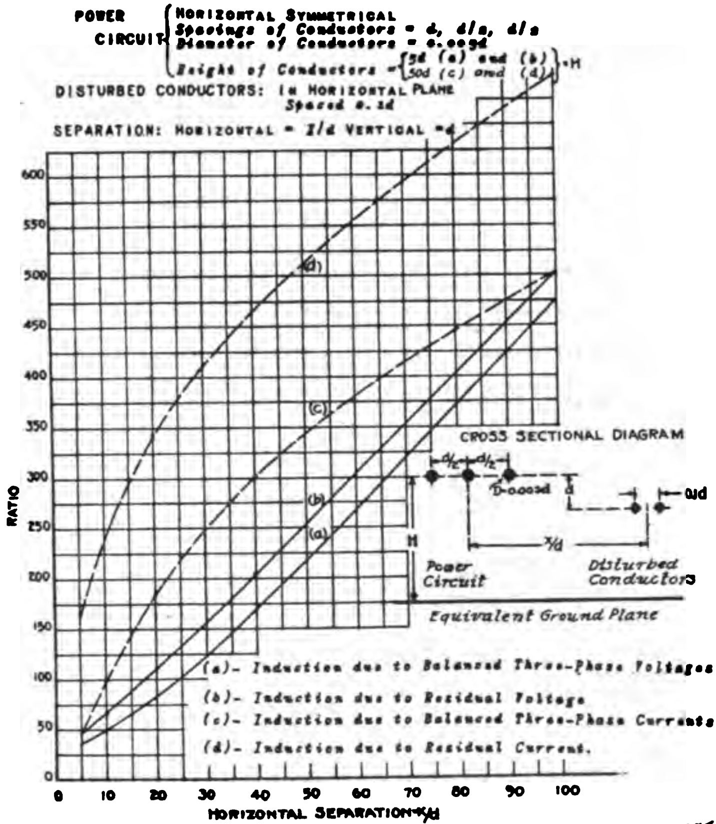
1/4/17
J.C.I.I.
T.R.No. 65.

CURVE SHEET No. 197

RATIO OF MEAN TO DIFFERENCE OF INDUCED VOLTAGES

TO GROUND OR ALONG CONDUCTORS

VARIATION WITH HORIZONTAL SEPARATION



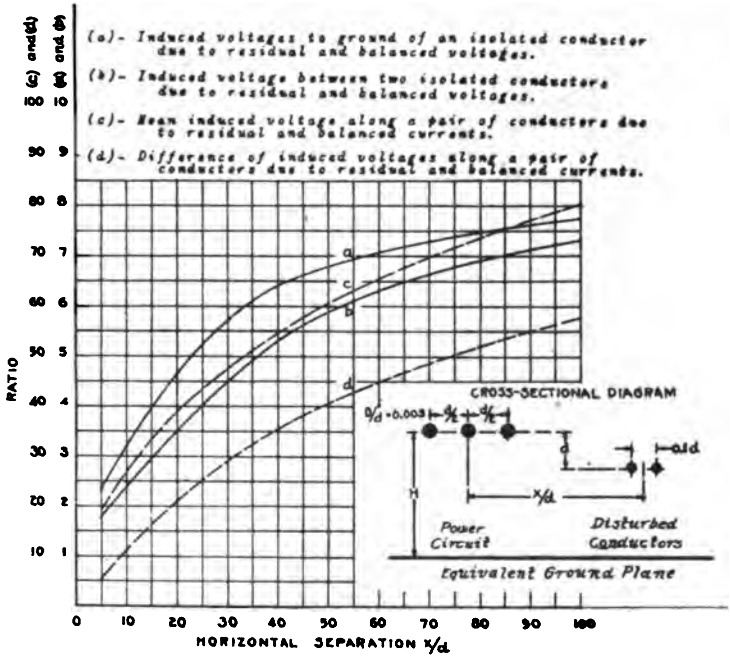
1/4/17
J.C.I.I.
T.R. No. 65

CURVE SHEET No. 198

RATIO OF INDUCED VOLTAGE FROM RESIDUAL VOLTAGE OR CURRENT
TO
INDUCED VOLTAGE FROM BALANCED THREE-PHASE VOLTAGES OR CURRENTS
FOR EQUAL MAGNITUDES OF RESIDUAL AND BALANCED VOLTAGES OR CURRENTS
VARIATION WITH HORIZONTAL SEPARATION

POWER CIRCUIT {
HORIZONTAL SYMMETRICAL
Spacings of Conductors = d, d/2, d/2
Diameter of Conductors = 0.003d
Height of Conductors = { 5d (a) and (b); 50d (c) and (d) } = H

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.5d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *d*



1/4/17
J.C.I.I.
T.R. No. 65.

CURVE SHEET NO. 199

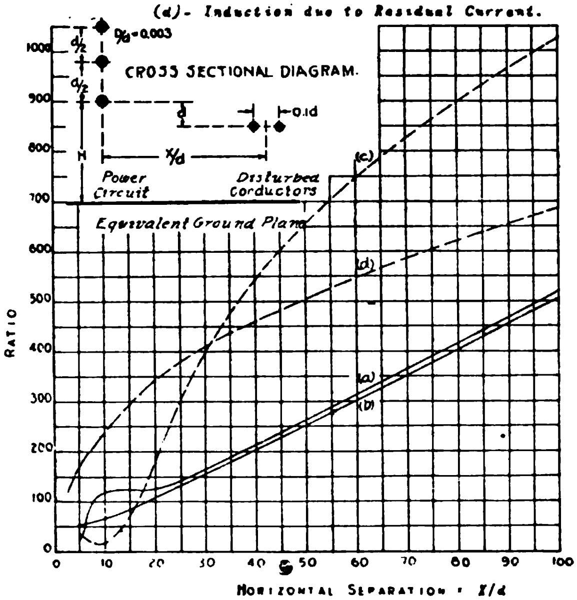
RATIO OF MEAN TO DIFFERENCE OF INDUCED VOLTAGES

TO GROUND OR ALONG CONDUCTORS
VARIATION WITH HORIZONTAL SEPARATION

POWER CIRCUIT
- VERTICAL Spacings of Conductors = d, d/2, d/2
- Diameter of Conductors = 0.003d
- Height of Conductors = 5d (a) and (b); 50d (c) and (d) = H

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.1d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *d*

(a)- Induction due to Balanced Three-Phase Voltages.
(b)- Induction due to Residual Voltage.
(c)- Induction due to Balanced Three-phase Currents.
(d)- Induction due to Residual Current.



J.C.I.I.
T.R. NO. 65.

Curve Sheet No. 200

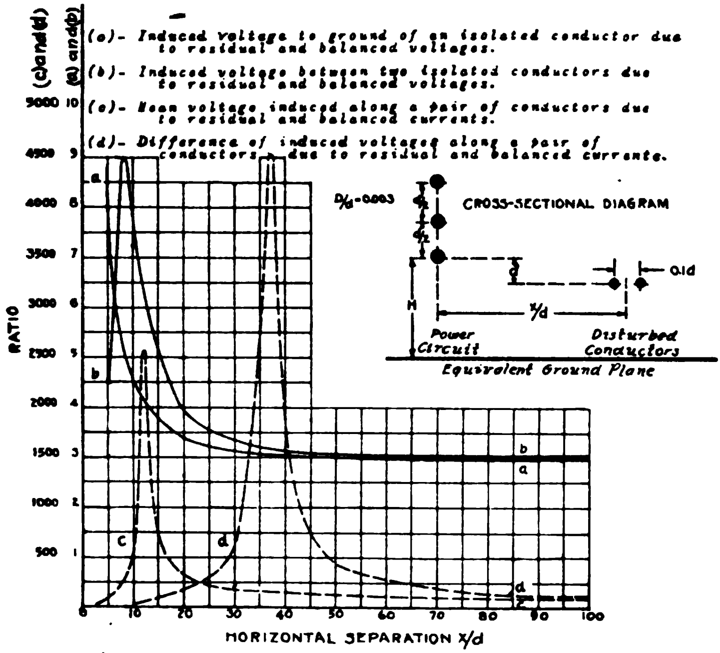
RATIO OF INDUCED VOLTAGE FROM RESIDUAL VOLTAGE OR CURRENT
TO
INDUCED VOLTAGE FROM BALANCED THREE-PHASE VOLTAGES OR CURRENTS

FOR EQUAL MAGNITUDES OF RESIDUAL AND BALANCED VOLTAGES OR CURRENTS
VARIATION WITH HORIZONTAL SEPARATION

POWER CIRCUIT { Vertical Spacings of Conductors = d, d/2, d/2; Diameter of Conductors = 0.003d; Height of Conductors = {5d (a) and (b); 50d (c) and (d)} = H

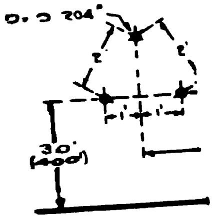
DISTURBED CONDUCTORS: In Horizontal Plane, Spaced 0.1d

SEPARATION: Horizontal = X/d Vertical = d



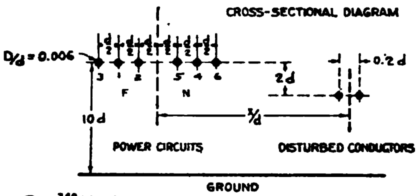
9/4/17
T. R. No. 65.

(1)

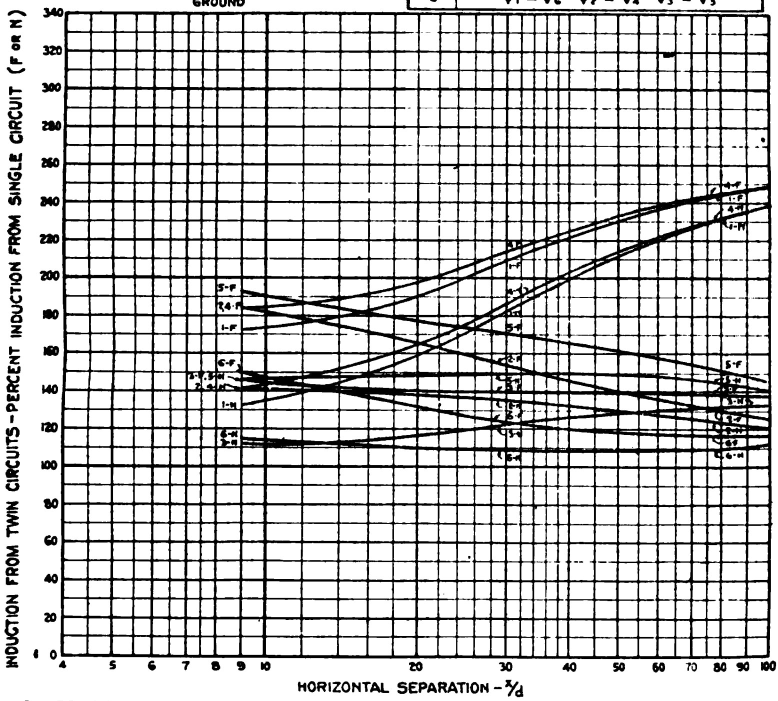


CURVE SHEET NO. 209

COMPARISON OF SINGLE AND TWIN POWER CIRCUITS
WITH RESPECT TO
MEAN INDUCED VOLTAGE TO GROUND OF TWO ISOLATED CONDUCTORS
BY
BALANCED THREE-PHASE VOLTAGES



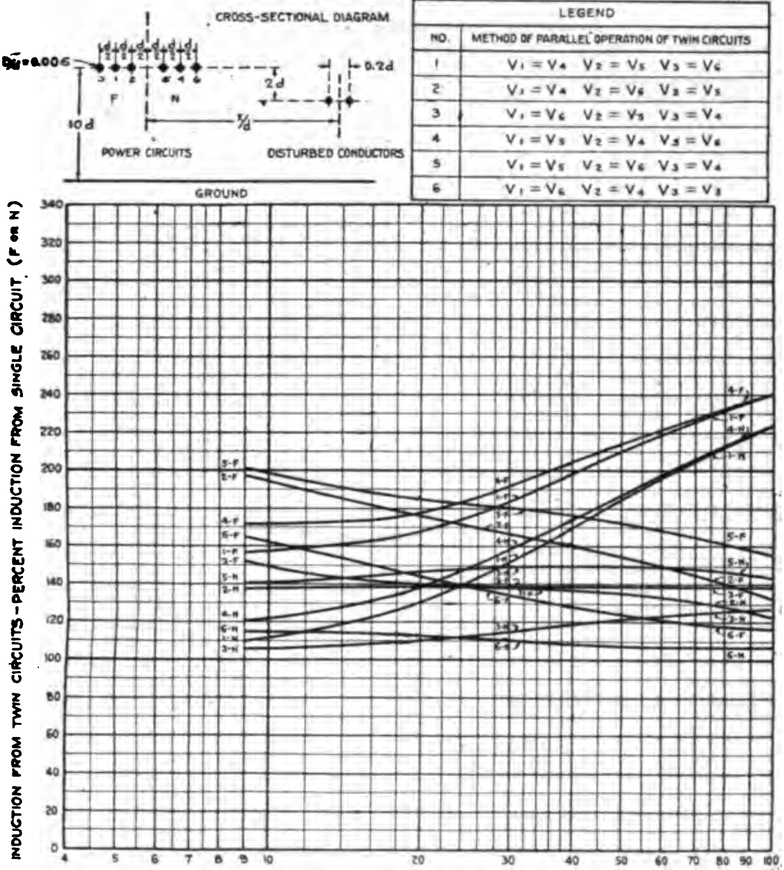
LEGEND	
NO.	METHOD OF PARALLEL OPERATION OF TWIN CIRCUITS
1	$V_1 = V_4$ $V_2 = V_5$ $V_3 = V_6$
2	$V_1 = V_4$ $V_2 = V_6$ $V_3 = V_5$
3	$V_1 = V_6$ $V_2 = V_5$ $V_3 = V_4$
4	$V_1 = V_5$ $V_2 = V_4$ $V_3 = V_6$
5	$V_1 = V_5$ $V_2 = V_6$ $V_3 = V_4$
6	$V_1 = V_6$ $V_2 = V_4$ $V_3 = V_5$



J.C.I.I. T.R. NO.65

CURVE SHEET NO. 210

COMPARISON OF SINGLE AND TWIN POWER CIRCUITS
WITH RESPECT TO
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS
BY
BALANCED THREE-PHASE VOLTAGES

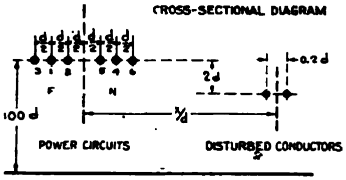


NO.	METHOD OF PARALLEL OPERATION OF TWIN CIRCUITS
1	$V_1 = V_4$ $V_2 = V_5$ $V_3 = V_6$
2	$V_1 = V_4$ $V_2 = V_6$ $V_3 = V_5$
3	$V_1 = V_6$ $V_2 = V_5$ $V_3 = V_4$
4	$V_1 = V_5$ $V_2 = V_4$ $V_3 = V_6$
5	$V_1 = V_5$ $V_2 = V_6$ $V_3 = V_4$
6	$V_1 = V_6$ $V_2 = V_4$ $V_3 = V_5$

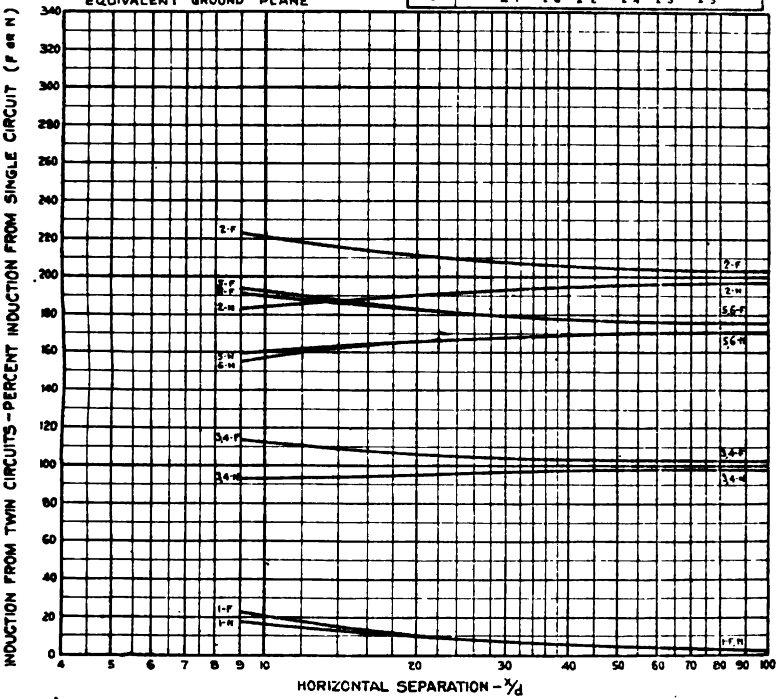
J.C.I.I. T.R. NO. 65

CURVE SHEET NO. 211

COMPARISON OF SINGLE AND TWIN POWER CIRCUITS
WITH RESPECT TO
MEAN INDUCED VOLTAGE ALONG PAIR OF CONDUCTORS
BY
BALANCED THREE-PHASE CURRENTS



LEGEND	
NO.	METHOD OF PARALLEL OPERATION OF TWIN CIRCUITS
1	$I_1 = I_4 \quad I_2 = I_5 \quad I_3 = I_6$
2	$I_1 = I_4 \quad I_2 = I_6 \quad I_3 = I_5$
3	$I_1 = I_6 \quad I_2 = I_5 \quad I_3 = I_4$
4	$I_1 = I_5 \quad I_2 = I_4 \quad I_3 = I_6$
5	$I_1 = I_5 \quad I_2 = I_6 \quad I_3 = I_4$
6	$I_1 = I_6 \quad I_2 = I_4 \quad I_3 = I_5$

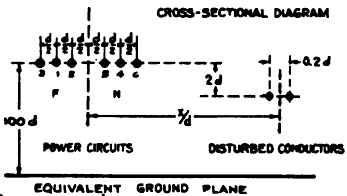


J.C.I.I. T.R. NO.65

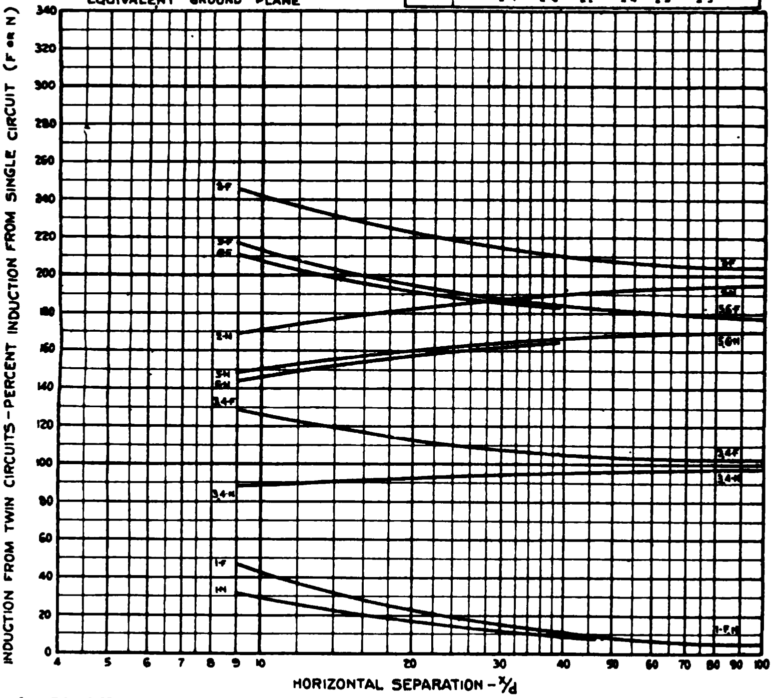
CURVE SHEET NO. 212

COMPARISON OF SINGLE AND TWIN POWER CIRCUITS
WITH RESPECT TO
DIFFERENCE OF INDUCED VOLTAGES ALONG PAIR OF CONDUCTORS
BY
BALANCED THREE-PHASE CURRENTS

CROSS-SECTIONAL DIAGRAM



LEGEND	
NO.	METHOD OF PARALLEL OPERATION OF TWIN CIRCUITS
1	$I_1 = I_4 \quad I_2 = I_5 \quad I_3 = I_6$
2	$I_1 = I_4 \quad I_2 = I_6 \quad I_3 = I_5$
3	$I_1 = I_6 \quad I_2 = I_5 \quad I_3 = I_4$
4	$I_1 = I_5 \quad I_2 = I_4 \quad I_3 = I_6$
5	$I_1 = I_5 \quad I_2 = I_6 \quad I_3 = I_4$
6	$I_1 = I_6 \quad I_2 = I_4 \quad I_3 = I_5$



J.C.I.I. T.R. NO. 65

INDEX OF TABLES.

COEFFICIENTS OF INDUCTION FROM SINGLE-CIRCUIT THREE-PHASE POWER LINES

CASE Nº	CONFIGURATION	BALANCED VOLTAGES.	RESIDUAL VOLTAGE	BALANCED CURRENTS	RESIDUAL CURRENT
12		74	74	75	76
13		77	77	78	79
14	A-VARIABLE	80	80	81	82
1, 2, 3, 4, 6, 7, 8, 9, 10, 11.	SINGLE-CIRCUIT POWER LINES.	83 (MISCELLANEOUS DATA)			
15	PHANTOM TELEPHONE CIRCUITS HORIZONTAL SQUARE VERTICAL POLE PAIR	84	86	85	86

COEFFICIENTS OF INDUCTION FROM TWIN-CIRCUIT THREE-PHASE POWER LINES.

CASE Nº	CONFIGURATION	BALANCED VOLTAGES.	BALANCED CURRENTS.
100,*† 102, 103		87 89*	88 90*
200, 202, 203			
400, 402, 403			
500, 502, 503			
600 602		91 92*	

J.C.I.I.T.R. Nº 65.

* INDUCTION FROM TWIN CIRCUITS EXPRESSED AS PER CENT OF INDUCTION FROM SINGLE CIRCUIT.
† SEE ALSO CASES 101, 104, 108, 110, 111, 112, 120.

W.J.D.

TABLE NO. 1

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR
Volts per Kilovolt between Power Conductors

POWER CIRCUIT: EQUILATERAL TRIANGLE, BASE HORIZONTAL, VERTEX UPWARD
Spacings of Conductors = d
Diameter of Conductors = 0.006d
Height of Lower Conductors = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.2d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

D/d	H/d	Y/d	X/d 4.9	X/d 9.9	X/d 19.9	X/d 39.9	X/d 99.9	CURVE SHEET NO.
0.006	3	0	11.67	3.780	1.071	0.2797	0.0452	1
		0.6	10.31	3.21	0.906	0.2337	0.0377	2
		1	8.68	2.61	0.724	0.1869	0.0302	3
	5	0	14.58	5.422	1.694	0.4623	0.07609	
		1	12.93	4.560	1.379	0.3709	0.06091	
		3	7.10	2.443	0.7042	0.1872	0.03049	
	10	0	16.65	7.33	2.785	0.868	0.1521	
		2	14.43	6.40	2.340	0.7072	0.1221	
		5	8.16	4.30	1.642	0.461	0.0786	
	20	0	17.39	8.26	3.697	1.417	0.2928	
		2	15.90	7.83	3.478	1.285	0.2651	
		10	5.23	4.033	2.132	0.783	0.1502	
0.002	3	0.5	8.710			0.1987		3
	5	1	10.88			0.3156		4
	10	2	12.12			0.5986		
	20	4	10.45			1.004		
0.010	3	0.5	11.29			0.2544		
	5	1	14.17			0.4056		
	10	2	15.54			0.7729		
	20	4	13.66			1.302		
0.016	3	0.5	12.37			0.2773		
	5	1	15.55			0.4429		
	10	2	17.39			0.8452		
	20	4	15.00			1.428		

TABLE NO. 2

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS
Millivolts per Kilovolt between Power Conductors

POWER CIRCUIT { EQUILATERAL TRIANGLE, BASE HORIZONTAL, VERTEX UPWARD
Spacings of Conductors = d
Diameter of Conductors = 0.006d
Height of Lower Conductors = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.2d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

s/d	*D/d*	*H/d*	*Y/d*	*X/d* 6	10	20	40	100	CURVE SHEET NO.
0.2	0.006	3	0	760.	134.5	19.4	2.76	0.180	5, 6, 7
			0.5	711.	117.0	17.2	2.90	0.160	
			1	631.	97.3	13.9	1.65	0.125	
		5	0	789.	178.1	31.1	4.47	0.290	
			1	762.7	160.	25.79	3.60	0.243	
			3	411.	92.6	13.54	1.83	0.122	
		10	0	752.	193.	45.4	8.03	0.588	
			2	652.6	185.8	39.9	6.670	0.4827	
			5	287.	128.	28.2	4.34	0.3024	
		20	0	726.	182.7	48.25	11.51	1.11	
			2	636.	181.1	47.47	10.91	1.015	
			10	114.	75.4	31.33	7.09	0.586	
	0.002	3	0.5	597.6			1.95		3, 4
		5	1	640.1			3.07		
		10	2	547.7			5.63		
		20	4	329.6			8.56		
	0.010	3	0.5	780.6			2.49		
		5	1	837.4			3.94		
		10	2	713.8			7.29		
		20	4	430.6			11.13		
	0.016	3	0.5	857.3			2.71		
		5	1	920.0			4.31		
		10	2	786.6			7.97		
		20	4	472.7			12.21		
0.1	0.006	10	2	326.2			3.3		
0.8				1633.			16.7		

J.C.I.I. – T.F. NO. 65.

TABLE NO. 3

COEFFICIENTS OF INDUCTION FROM RESIDUAL VOLTAGE OF THREE-PHASE CIRCUITS
INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR
Volts per Kilovolt Residual

POWER CIRCUIT:
- EQUILATERAL TRIANGLE, BASE HORIZONTAL, VERTEX UPWARD
- *Spacings of Conductors = d*
- *Diameter of Conductors = 0.006d*
- *Height of Lower Conductors = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.2d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

D/d	H/d	Y/d	X/d 4.9	X/d 9.9	X/d 19.9	X/d 39.9	X/d 99.9	Curve Sheet No.
0.006	3	0	42.26	14.71	4.130	1.065	0.1716	8, 9
		0.5	36.97	12.62	3.464	0.8888	0.1431	
		1	30.77	10.19	2.786	0.7120	0.1145	
	5	0	65.35	28.29	9.173	2.493	0.4081	
		1	56.59	23.54	7.477	2.005	0.3271	
		3	30.89	12.74	3.835	1.010	0.1638	
	10	0	97.27	55.36	23.93	7.722	1.357	
		2	84.24	48.13	20.12	6.295	1.090	
		5	53.82	32.54	13.29	4.016	0.6836	
	20	0	124.9	84.66	48.13	20.77	4.453	
		2	116.2	80.32	45.36	19.19	4.035	
		10	58.55	47.45	28.43	11.54	2.287	
0.002	3	0.5	33.74			0.8114		10
	5	1	52.14			1.948		
	10	2	78.45			5.862		
	20	4	95.87			16.41		
0.010	3	0.5	36.69			0.9299		
	5	1	58.92			2.087		
	10	2	87.23			6.518		
	20	4	105.3			18.02		
0.016	3	0.5	40.42			0.9713		
	5	1	61.25			2.169		
	10	2	90.19			6.739		
	20	4	108.4			18.55		

J.C.I.I. – T.R. NO. 65. W.J.D.

TABLE No. 4

COEFFICIENTS OF INDUCTION FROM RESIDUAL VOLTAGE OF THREE-PHASE CIRCUITS
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS
Millivolts per Kilovolt Residual

POWER CIRCUIT:
- Equilateral Triangle, Base Horizontal, Vertex Upward
- *Spacings of Conductors = d*
- *Diameter of Conductors = 0.006d*
- *Height of Lower Conductors = H/d*

DISTURBED CONDUCTORS: In Horizontal Plane, *Spaced 0.2d*
SEPARATION: Horizontal = *X/d* Vertical = *Y/d*

s/d	*D/d*	*H/d*	*Y/d*	*X/d* 5	10	20	40	100	Curve Sheet No.
0.2	0.006	3	0	2129.	491.	77.8	10.5	0.684	11, 12
			0.5	1919.	426.4	65.6	8.76	0.585	
			1	1632.	362.3	53.26	7.01	0.457	
		5	0	2502.	795.	163.2	24.03	1.621	
			1	2220.	686.	134.4	19.43	1.300	
			3	1177.	387.1	70.60	9.85	0.650	
		10	0	2628.	1111.	340.6	68.73	5.313	
			2	2040.	968.	297.2	56.98	4.279	
			5	1010	648.	203.9	37.04	26.93	
		20	0	2322.	1113.	474.1	148.8	16.61	
			2	1918.	1048.	456.7	140.4	15.05	
			10	990.	458.1	288.0	89.37	8.693	
	0.002	3	0.5	1751.			7.99		10
		5	1	2046.			17.90		
		10	2	1899.			53.06		
		20	4	1238.			122.1		
	0.010	3	0.5	2008.			9.15		
		5	1	2312.			20.23		
		10	2	2112.			59.00		
		20	4	1360.			134.1		
	0.016	3	0.5	2099.			9.56		
		5	1	2403.			21.02		
		10	2	2184.			60.99		
		20	4	1400.			138.1		
0.1	0.006	10	2	1020.			28.6		
0.5				5100.			142.5		

J.C.I.I. – T.R. No. 65.

TABLE No. 5

COEFFICIENTS OF INDUCTION FROM BALANCED AND RESIDUAL VOLTAGES OF THREE-PHASE CIRCUITS

INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR

AND

INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS

(Percent of Coefficient for D/d = 0.006d)

POWER CIRCUIT { EQUILATERAL TRIANGLE, BASE HORIZONTAL VERTEX UPWARD; *Spacings of Conductors = d*; *Diameter of Conductors = 0.006d*; *Height of Power Conductors = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.2d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

	H/d	*Y/d*	*D/d*	To Ground, *X/d* = 4.9	To Ground, *X/d* = 39.9	Between Conductors, *X/d* = 5	Between Conductors, *X/d* = 40	Curve Sheet No.
Balanced Voltages	3	0.5	0.002	84.48	85.02	84.05	84.78	3, 4
			0.010	109.5	108.9	109.8	108.3	
			0.016	120.0	118.7	120.6	117.8	
	5	1	0.002	84.05	85.09	83.93	85.28	
			0.010	109.5	109.4	109.8	109.4	
			0.016	120.2	119.4	120.6	119.7	
	10	2	0.002	83.99	84.64	83.87	84.42	
			0.010	109.6	109.3	109.3	109.3	
			0.016	120.5	119.5	120.5	119.6	
	20	4	0.002	83.93	84.44	83.98	84.25	
			0.010	109.7	109.5	109.7	109.6	
			0.016	120.5	119.9	120.4	120.2	
Residual Voltage	3	0.5	0.002	91.26	91.27	91.24	91.21	10
			0.010	104.6	104.0	104.6	104.4	
			0.016	109.3	109.3	109.4	109.1	
	5	1	0.002	92.14	92.22	92.12	92.13	
			0.010	104.1	104.1	104.1	104.1	
			0.016	108.2	108.2	108.2	108.2	
	10	2	0.002	93.13	93.12	93.09	93.12	
			0.010	103.6	103.5	103.5	103.6	
			0.016	107.1	107.0	107.1	107.0	
	20	4	0.002	93.90	93.93	93.88	93.85	
			0.010	103.1	103.1	103.1	103.1	
			0.016	106.2	106.2	106.1	106.1	

J.C.I.I. T.R. No. 45.

TABLE No. 6

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE CURRENTS
MEAN INDUCED VOLTAGE ALONG PAIR OF CONDUCTORS

Millivolts at 60 cycles per 1000 feet per Ampere per Phase

POWER CIRCUIT { Equilateral Triangle, Base Horizontal, Vertex Upward
Spacings of Conductors = d
Height of Lower Conductors = H/d

DISTURBED CONDUCTORS: In Horizontal Plane, *Spaced 0.2d*
SEPARATION: Horizontal = *X/d* Vertical = *Y/d*

H/d	Y/d	X/d: 5	10	20	40	100	Curve Sheet No.
20	0	3.934	1.907	0.8784	0.3462	0.07351	
	0.5	3.907					
	1	3.839	1.857	0.8493	0.3342	0.07006	
	2	3.551	1.786	0.8191	0.3196	0.06659	
	3	3.167	1.697	0.7860	0.3052	0.06302	
	5	2.385	1.479	0.7118	0.2745	0.05597	
40	0	3.985	1.970	0.9588	0.4415	0.1236	
	0.5	3.982					
	1	3.928	1.942	0.9461	0.4348	0.1210	
	2	3.688	1.897	0.9306	0.4276	0.1185	
	3	3.338	1.834	0.9130	0.4201	0.1159	
	5	2.614	1.665	0.8707	0.4048	0.1106	13 14 15
100	0	4.009	1.987	0.9879	0.4865	0.1774	
	1	3.983	1.978	0.9831	0.4840	0.1764	
	2	3.760	1.948	0.9758	0.4812	0.1753	
	3	3.430	1.899	0.9660	0.4809	0.1741	
	5	2.782	1.754	0.9396	0.4708	0.1718	
200	0				0.4935	0.1928	
	2				0.4917	0.1918	
	5				0.4852	0.1900	
∞	0	4.021	1.995	0.9957	0.4979	0.1990	
	0.5	4.040					
	1	4.008	1.994	0.9959	0.4977	0.1990	
	2	3.804	1.976	0.9933	0.4974	0.1990	
	3	3.485	1.933	0.9886	0.4968	0.1990	
	5	2.606	1.804	0.9715	0.4946	0.1988	

J.C.I.I. T.R. No. 65.

TABLE NO. 7

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE CURRENTS
DIFFERENCE OF INDUCED VOLTAGES ALONG PAIR OF CONDUCTORS
Microvolts at 60 Cycles per 1000 Feet per Ampere per Phase

POWER CIRCUIT { EQUILATERAL TRIANGLE, BASE HORIZONTAL, VERTEX UPWARD
Spacings of Conductors = d
Height of Lower Conductors = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

t/d	*H/d*	*Y/d*	*X/d* 5	10	20	40	100	CURVE SHEET No.
0.2	20	0	167.1	42.11	11.21	2.741	0.2778	16 17 18
		0.5	170.5					
		1.	168.1	42.3	11.11	2.678	0.264	
		2.	145.0	41.27	10.93	2.547	0.256	
		3.	114.9	39.10	10.64	2.498	0.2381	
		5.	69.69	32.62	9.869	2.302	0.2145	
	40	0	166.5	40.72	10.37	2.805	0.4154	
		0.5	169.0					
		1.	166.	41.06	10.50	2.794	0.4100	
		2.	144.0	40.23	10.49	2.775	0.401	
		3.	114.4	38.34	10.32	2.749	0.403	
		5.	70.4	32.46	9.869	2.685	0.3789	
	100	0	165.0	40.27	10.06	2.577	0.4498	
		1	166.6	40.64	10.11	2.576	0.450	
		2	143.7	39.82	10.05	2.574	0.4471	
		3	114.3	37.99	9.983	2.56	0.4471	
		5	70.51	32.3	9.547	2.55	0.4425	
	200	0				2.516	0.4180	
		2				2.514	0.4155	
		5				2.491	0.4178	
	∞	0	164.9	40.2	10.08	2.491	0.4022	
		0.5	168.5					
		1	165.5	40.53	10.00	2.492	0.3995	
		2.	143.6	39.74	9.988	2.491	0.398	
		3.	114.2	37.91	9.89	2.484	0.3979	
		5.	70.41	32.1	9.549	2.466	0.39785	
0.1	20	1	83.63			1.339		
0.5			418.1			6.678		

J.C.I.I. T.R. NO. 65.

WJD.

TABLE NO. 8

COEFFICIENTS OF INDUCTION FROM RESIDUAL CURRENT OF THREE-PHASE CIRCUITS
MEAN INDUCED VOLTAGE ALONG PAIR OF CONDUCTORS
Millivolts at 60 Cycles per 1000 Feet per Ampere Residual

POWER CIRCUIT { EQUILATERAL TRIANGLE, BASE HORIZONTAL, VERTEX UPWARD
Spacing of Conductors = d
Height of Lower Conductors = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.2d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

H/d	Y/d	X/d: 5	10	20	40	100	CURVE SHEET NO.
20	0	48.10	32.70	18.63	8.047	1.728	
	0.5	47.57					
	1	46.85	31.98	18.12	7.749	1.648	
	2	44.80	31.03	17.56	7.436	1.566	
	3.	42.26	29.87	16.92	7.109	1.483	
	5.	36.51	27.03	15.47	6.411	1.316	
40	0	63.81	48.03	32.63	18.56	5.716	
	0.5	63.43					
	1	62.83	47.58	32.31	18.32	5.602	
	2	61.09	46.90	31.93	18.05	5.486	19
	3	58.85	46.01	31.50	17.79	5.369	
	5	53.75	43.76	30.37	17.20	5.128	
100	0	84.78	68.90	53.06	37.47	18.52	
	1	83.97	68.60	52.90	37.35	18.42	
	2	82.41	68.09	52.68	37.21	18.33	
	3	80.34	67.38	52.41	37.06	18.23	
	5	76.60	65.49	51.71	36.71	18.03	
200	0				53.13	32.57	
	2				52.89	32.47	
	5				52.56	32.27	

TABLE NO. 5

COEFFICIENTS OF INDUCTION FROM RESIDUAL CURRENT OF THREE-PHASE CIRCUITS
DIFFERENCE OF INDUCED VOLTAGES ALONG PAIR OF CONDUCTORS.
Microvolts at 60 Cycles per 1000 Feet per Ampere Residual

POWER CIRCUIT { EQUILATERAL TRIANGLE, BASE HORIZONTAL, VERTEX UPWARD; *Spacings of Conductors = d*; *Height of Lower Conductors = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

t/d	*H/d*	*Y/d*	*X/d* 5	10	20	40	100	CURVE SHEET NO.
0.2	20	0	902.7	432.6	184.3	57.86	6.419	20
		0.5	863.5					
		1	847.9	424.2	181.6	56.3	5.807	
		2	745.6	407.5	177.6	54.57	5.861	
		3	625.9	384.0	172.4	52.65	5.559	
		5	415.7	325.0	158.9	48.31	5.033	
	40	0	913.1	452.2	216.3	92.	18.0	
		0.5	894					
		1	859.4	444.9	215.1	91.47	17.73	
		2	757.2	429.4	212.7	90.74	17.4	
		3	636.3	407.3	209.3	88.19	17.14	
		5	429.7	352.1	197.7	87.63	16.50	
	100	0	916.0	458.1	220.	111.	36.79	
		1	864.5	451.0	226.6	110.3	36.71	
		2	760.4	435.6	224.5	110.0	36.62	
		3	641.5	413.6	221.5	104.3	36.52	
		5	433.2	358.0	212.4	108.3	36.28	
	200	0				113.8	43.26	
		2				113.4	43.21	
		5				111.6	43.07	
0.1 0.5	20	1	424.2 2118.			28.15 14.08		

J.C.I.I.
T.R. No.65.

W.J.D.

TABLE. No. 10

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR.
Volts per Kilovolt between Power Conductors

POWER CIRCUIT:
- EQUILATERAL TRIANGLE, BASE VERTICAL
- *Vertex Away from Disturbed Conductors*
- *Spacings of Conductors = d*
- *Diameter of Conductors = D/d*
- *Height of Lowest Conductor = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.2d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

D/d	*H/d*	*Y/d*	*X/d* 4.9	9.9	19.9	39.9	99.9	CURVE SHEET NO.
0.006	3	0	11.73	4.069	1.133	0.2868	0.0456	21 22
		0.5	10.08	3.453	0.9498	0.2394	0.0380	
		1	8.270	2.803	0.7639	0.1918	0.0304	
	5	0	14.06	5.584	1.773	0.4748	0.0787	
		1	11.93	4.641	1.443	0.3819	0.06144	
		3	6.429	2.439	0.7392	0.1923	0.0308	
	10	0	15.89	7.263	2.836	0.8885	0.1538	
		2	13.42	6.181	2.369	0.7239	0.1234	
		5	7.926	4.075	1.560	0.4815	0.0775	
	20	0	16.60	8.109	3.687	1.431	0.2956	
		2	14.99	7.549	3.441	1.319	0.2678	
		10	5.314	3.642	2.076	0.786	0.1518	
0.002	5	1	10.03			0.3242		23 4
	10	2	11.26			0.6125		
0.010	5	1	13.08			0.4167		
	10	2	14.73			0.7910		
0.016	5	1	14.37			0.4550		
	10	2	16.16			0.8663		

J.C.I.I. – T.R. NO.65. ag.D.

TABLE NO. 11

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS
Millivolts per Kilovolt between Power Conductors

POWER CIRCUIT:
- EQUILATERAL TRIANGLE, BASE VERTICAL
- *Vertex Away from Disturbed Conductors*
- *Spacings of Conductors = d*
- *Diameter of Conductors = D/d*
- *Height of Lowest Conductor = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = Y/d

t/d	*D/d*	*H/d*	*Y/d*	*X/d* 5	10	20	40	100	CURVE SHEET NO.
0.2	0.006	3	0	686.6	142.1	22.01	2.86	0.184	24 25 26
			0.5	630.0	122.6	18.56	2.40	0.152	
			1	551.0	101.1	15.00	1.92	0.122	
		5	0	717.7	175.0	32.6	4.669	0.306	
			1	690.5	152.6	27.1	3.77	0.245	
			3	407.9	86.3	14.18	1.915	0.122	
		10	0	686.1	186.0	45.0	8.251	0.610	
			2	647.0	173.3	39.6	6.825	0.488	
			5	309.7	123.4	27.32	4.45	0.309	
		20	0	661.8	175.6	47.5	11.6	1.127	
			2	636.4	172.6	45.46	10.87	1.030	
			10	110.9	79.3	30.7	6.980	0.594	
	0.002	5	1	566.8			3.203		23 4
		10	2	542.8			5.78		
	0.010	5	1	758.4			4.122		
		10	2	710.0			7.466		
	0.016	5	1	833.7			4.503		
		10	2	780.0			8.190		
0.1	0.006	10	2	536.1			3.42		
0.5				1620.			17.11		

TABLE NO. 12

COEFFICIENTS OF INDUCTION FROM RESIDUAL VOLTAGE OF THREE-PHASE CIRCUITS
INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR
Volts per Kilovolt Residual

POWER CIRCUIT:
- EQUILATERAL TRIANGLE, BASE VERTICAL
- *Vertex Away from Disturbed Conductors*
- *Spacings of Conductors = d*
- *Diameter of Conductors = D/d*
- *Height of Lowest Conductor = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.[illegible]d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

D/d	*H/d*	*Y/d*	*X/d* 4.9	9.9	19.9	39.9	99.9	CURVE SHEET NO.
0.006	3	0	44.64	16.61	4.372	1.122	0.1802	
		0.5	38.94	13.29	3.667	0.9368	0.1502	
		1	32.31	10.82	2.949	0.7606	0.1202	
	6	0	67.60	29.46	9.531	2.584	0.4222	
		1	58.14	24.72	7.769	2.078	0.3380	
		3	31.40	13.19	3.984	1.0466	0.1692	
	10	0	99.13	56.43	24.43	7.887	1.383	
		2	84.90	48.90	20.63	6.425	1.110	
		5	53.69	32.92	13.56	4.099	0.6964	
	20	0	126.4	85.56	48.63	20.99	4.600	
		2	116.6	80.95	45.79	19.40	4.077	
		10	58.12	47.44	28.62	11.66	2.311	
0.002	6	1	63.61			1.917		10
	10	2	79.08			5.085		
0.010	6	1	60.62			2.163		
	10	2	87.90			6.682		
0.016	6	1	62.87			2.247		
	10	2	90.87			6.876		

J.C.I.I. - T.R. NO.65.

TABLE No. 13.

COEFFICIENTS OF INDUCTION FROM RESIDUAL VOLTAGE OF THREE-PHASE CIRCUITS
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS
Millivolts per Kilovolt Residual

POWER CIRCUIT:
- EQUILATERAL TRIANGLE, BASE VERTICAL
- *Vertex Away from Disturbed Conductors*
- *Spacings of Conductors = d*
- *Diameter of Conductors = D/d*
- *Height of Lowest Conductor = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

t/d	D/d	H/d	Y/d	X/d: 5	10	20	40	100	CURVE SHEET NO.
0.2	0.006	3	0	2228.	522.3	82.91	11.06	0.719	
			0.5	1991.	452.2	69.97	9.25	0.600	
			1	1679.	373.5	56.56	7.41	0.479	
		5	0	2573.	825.2	166.6	24.96	1.680	
			1	2237.	715.9	139.8	20.18	1.345	
			3	1153.	397.4	73.31	10.23	0.675	
		10	0	2578.	1099.	347.3	70.19	5.416	
			2	2004.	975.9	302.6	58.21	4.363	
			5	962.8	644.9	207.2	38.33	2.748	
		20	0	2367.	1127.	478.7	160.4	16.69	
			2	1884.	1051.	459.7	141.7	15.22	
			10	368.1	447.7	287.5	90.13	8.791	
	0.002	5	1	2062.			18.61		
		10	2	1867.			54.22		
	0.010	5	1	2329.			21.00		10
		10	2	2075.			60.26		
	0.016	5	1	2420.			21.82		
		10	2	2146.			62.28		
0.1	0.006	10	2	687.7			29.10		
0.5				5010.			153.5		

J.C.I.J. - T.R. No. 65.

TABLE NO. 14

COEFFICIENTS OF INDUCTION FROM BALANCED AND RESIDUAL VOLTAGES OF THREE-PHASE CIRCUITS
INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR AND INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS.

(Percent of Coefficient for D/d = 0.006d)

POWER CIRCUIT:
- Equilateral Triangle, Base Vertical
- Vertex Away from Disturbed Conductors
- Spacings of Conductors = d
- Diameter of Conductors = 0.006d
- Height of Lowest Conductor = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.2d*

SEPARATION: HORIZONTAL = X/d VERTICAL = Y/d

	H/d	Y/d	D/d	To Ground, X/d 4.9	To Ground, X/d 39.9	Between Conductors, X/d 5	Between Conductors, X/d 40	Curve Sheet No.
Balanced Voltages	5	1	0.002	84.07	84.89	82.08	84.96	23
			0.010	109.6	109.1	109.8	109.3	
			0.016	120.5	119.1	120.2	119.4	
	10	2	0.002	83.90	84.61	83.90	84.69	
			0.010	109.7	109.3	109.7	109.4	
			0.016	120.4	119.5	120.6	120.0	
Residual Voltage	5	1	0.002	92.20	92.25	92.18	92.22	10
			0.010	104.1	104.1	104.1	104.1	
			0.016	108.1	108.1	108.1	108.1	
	10	2	0.002	93.14	93.15	93.16	93.15	
			0.010	103.6	103.5	103.5	103.6	
			0.016	107.0	107.0	107.1	107.0	

J.C.I.I. T.R. No. 65.

TABLE No. 15

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE CURRENTS.
MEAN INDUCED VOLTAGE ALONG PAIR OF CONDUCTORS.
Millivolts at 60 cycles per 1000 feet per Ampere per Phase

POWER CIRCUIT { Equilateral Triangle, Base Vertical; Vertex Away from Disturbed Conductors; Spacings of Conductors = d; Height of Lowest Conductor = H/d }

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, Spaced 0.2d

SEPARATION: HORIZONTAL = X/d VERTICAL = Y/d

H/d	Y/d	X/d 5	X/d 10	X/d 20	X/d 40	X/d 100	Curve Sheet No.
20	0	3.758	1.875	0.8744	0.3500	0.07420	27 28 29
	0.5	3.676					
	1	3.591	1.803	0.8444	0.3364	0.07075	
	2	3.332					
	3	3.019	1.628	0.7759			
	5	2.348	1.418	0.6985	0.2764	0.05651	
40	0	3.810	1.930	0.9518	0.4412	0.1242	
	5	2.594	1.613	0.8538	0.4018	0.09319	
100	0		1.951	0.9800	0.4848	0.1774	
	1	3.747					
200	0				0.4926	0.1926	
	5			0.9415		0.1897	
∞	0	3.848	1.959	0.9875	0.4957	0.1987	
	0.5	3.825					
	1	3.782	1.945	0.9854	0.4954	0.1987	
	2	3.622					
	3	3.377					
	5	2.802	1.762	0.9566	0.4913	0.1984	

TABLE NO. 16

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE CURRENTS.

DIFFERENCE OF INDUCED VOLTAGES ALONG PAIR OF CONDUCTORS.

(Microvolts at 60 Cycles per 1000 Feet per Ampere per Phase)

POWER CIRCUIT { EQUILATERAL TRIANGLE, BASE VERTICAL
Vertex Away from Disturbed Conductors
Spacings of Conductors = d
Height of Lowest Conductor = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

t/d	*H/d*	*Y/d*	*X/d* 5	10	20	40	100	CURVE SHEET NO.
0.2	20	0	152.1	40.45	11.01	2.741	0.2810	30 31 32
		0.5	154.5					
		1	155.2	40.03	10.83	2.670	0.2685	
		2	144.8					
		3	122.1	37.63	10.31			
		5	75.91	32.65	9.559	2.286	0.2171	
	40	0	150.6	39.08	10.30	2.806	0.4164	
		5	76.69	32.77	9.646	2.649	0.3812	
	100	0		38.65	9.894	2.556	0.4486	
		1	153.8					
	200	0				2.493	0.4176	
		5			9.491		0.4117	
	∞	0	150.0	38.56	9.792	2.467	0.3969	
		0.5	152.7					
		1	153.7	38.39	9.767	2.463	0.3968	
		2	144.0					
		3	121.9					
		5	76.8	32.60	9.326	2.429	0.3940	
0.1	20	1	77.58			1.334		
	∞		76.83			1.234		
0.5	20	1	328.8			6.667		
	∞		385.1			6.167		

J.C.I.I. - T.R. NO. 65. W.T.D.

TABLE NO. 17

COEFFICIENTS OF INDUCTION FROM RESIDUAL CURRENT OF THREE-PHASE CIRCUITS.
MEAN INDUCED VOLTAGE ALONG PAIR OF CONDUCTORS
Millivolts at 60 cycles per 1000 feet per ampere residual

POWER CIRCUIT { Equilateral Triangle, Base Vertical
Vertex Away from Disturbed Conductors
Spacings of Conductors = d
Height of Lowest Conductor = H/d

DISTURBED CONDUCTORS: In Horizontal Plane, Spaced 0.8d
SEPARATION: Horizontal = X/d Vertical = Y/d

H/d	Y/d	X/d 5	10	20	40	100	Curve Sheet No.
20	0	48.78	33.11	18.85	8.149	1.750	
	0.5	48.15					
	1	47.29	32.33	18.33	7.846	1.668	
	2	45.06					
	3	42.36	30.11	17.10			
	5	36.45	27.19	15.63	6.495	1.333	
40	0	64.45	48.41	32.84	18.67	5.763	
	5	53.63	43.87	30.61	17.29	5.161	
100	0		69.24	53.26	37.57	18.57	
200	0				53.14	32.61	
	5			47.92		32.31	

J.C.I.I. – T.R. No. 65. W.J.D.

Table No. 18

COEFFICIENTS OF INDUCTION FROM RESIDUAL CURRENT OF THREE-PHASE CIRCUITS.
DIFFERENCE OF INDUCED VOLTAGES ALONG PAIR OF CONDUCTORS.
(Microvolts at 60 Cycles per 1000 Feet per Ampere Residual)

POWER CIRCUIT:
- EQUILATERAL TRIANGLE, BASE VERTICAL
- *Vertex Away from Disturbed Conductors*
- *Spacings of Conductors = d*
- *Height of Lowest Conductor = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*

SEPARATION: HORIZONTAL = *X/d*   VERTICAL = *Y/d*

t/d	*H/d*	*Y/d*	*X/d* 5	10	20	40	100	Curve Sheet No.
0.1	20	0	921.9	439.0	186.5	58.57	6.501	
		0.5	893.4					
		1	849.6	428.4	183.5	56.98	6.217	
		2	733.7					
		3	607.8	384.0	173.8			
		5	397.5	322.2	159.8	48.87	5.025	
	40	0	931.9	458.2	218.0	92.59	18.13	
		5	411.0	347.8	200.0	88.03	16.60	
	100	0		463.9	229.1	110.9	36.88	
	200	0				114.2	43.33	
		5			214.4		43.13	
0.1	20	1	424.8			28.49		
0.5	20	1	2125.			142.4		

J.C.I.I. - T.R. NO.65.   W.J.D.

TABLE NO. 19

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR
Volts per Kilovolt between Power Conductors

POWER CIRCUIT:
- EQUILATERAL TRIANGLE, BASE VERTICAL
- *Vertex Toward Disturbed Conductors*
- *Spacings of Conductors = d*
- *Diameter of Conductors = 0.006d*
- *Height of Lowest Conductor = H/d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*
DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.9d*

D/d	H/d	Y/d	X/d 4.9	X/d 9.9	X/d 19.9	X/d 39.9	X/d 99.9	CURVE SHEET NO.
0.006	3	0	12.49	3.870	1.054	0.2739	0.04466	33, 34
		0.5	10.93	3.298	0.8840	0.2286	0.03725	
		1	9.081	2.687	0.7108	0.1831	0.02979	
	5	0	15.16	5.622	1.696	0.4561	0.07530	
		1	12.85	4.733	1.383	0.3666	0.06029	
		3	6.550	2.530	0.7092	0.1847	0.03018	
	10	0	16.96	7.508	2.832	0.8677	0.1511	
		2	13.35	6.436	2.381	0.7072	0.1213	
		5	7.228	4.186	1.570	0.4511	0.07612	
	20	0	17.60	8.375	3.661	1.429	0.2920	
		2	14.68	7.746	3.509	1.319	0.2645	
		10	4.919	3.794	2.109	0.7903	0.1499	
0.002	3	0.5	9.230			0.1946		35
	5	1	10.82			0.3115		
	10	2	11.22			0.5985		
	20	4	9.378			1.012		
0.010	3	0.5	11.97			0.2490		
	5	1	14.09			0.3992		
	10	2	14.65			0.7728		
	20	4	12.26			1.313		
0.016	3	0.5	13.11			0.2715		
	5	1	15.45			0.4369		
	10	2	16.08			0.8462		
	20	4	13.47			1.438		

J.C.I.I. T.R. NO.65.

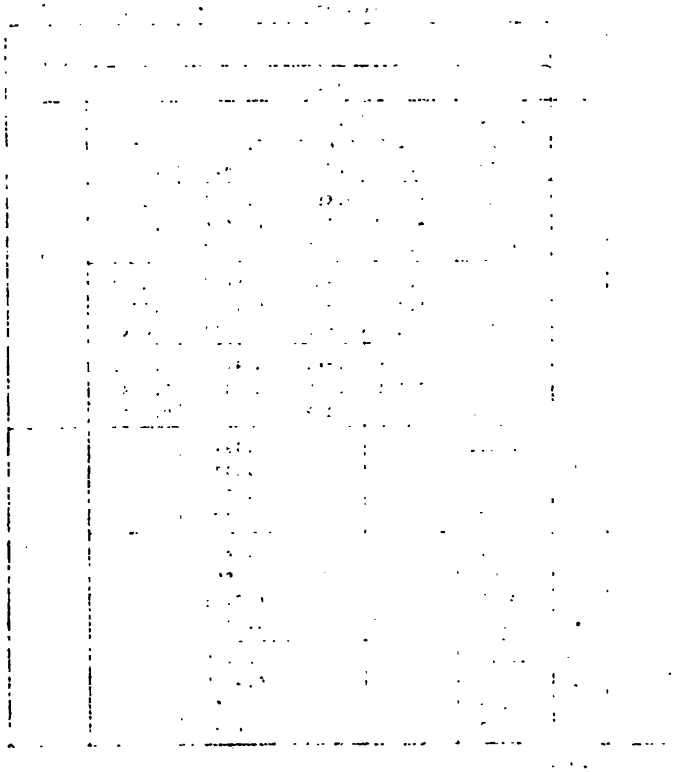


TABLE No. 20

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS
Millivolts per Kilovolt between Power Conductors

POWER CIRCUIT:
- EQUILATERAL TRIANGLE, BASE VERTICAL
- *Vertex Toward Disturbed Conductors.*
- *Spacings of Conductors = d*
- *Diameter of Conductors = 0.006d*
- *Height of Lowest Conductor = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

t/d	*D/d*	*H/d*	*Y/d*	*X/d* 5	10	20	40	100	CURVE SHEET NO.
0.2	0.006	3	0	776.4	144.1	20.38	2.69	0.18	36 37 38
			0.5	697.0	126.	17.3	2.26	0.15	
			1	587.	104.3	13.9	1.80	0.12	
		5	0	787.1	185.9	31.88	4.44	0.300	
			1	667.	163.4	26.52	3.59	0.239	
			3	318.6	92.0	14.0	1.82	0.120	
		10	0	745.7	196.1	46.47	8.123	0.594	
			2	521.7	174.3	40.96	6.75	0.479	
			5	260.4	112.3	28.34	4.40	0.301	
		20	0	721.9	185.2	48.75	11.65	1.116	
			2	507.8	170.1	47.2	11.03	1.017	
			10	117.2	67.3	29.50	6.51	0.5973	
	0.002	3	0.5	586.7			1.91		35 4
		5	1	559.6			3.04		
		10	2	437.5			5.70		
		20	4	290.9			8.645		
	0.010	3	0.5	764.8			2.45		
		5	1	731.5			3.91		
		10	2	572.7			7.38		
		20	4	379.8			11.25		
	0.016	3	0.5	839.3			2.66		
		5	1	803.5			4.27		
		10	2	629.1			8.08		
		20	4	417.2			12.33		
0.1	0.006	10	2	260.8			3.38		
0.5				1305.			16.88		

J.C.I.I. T.R. No. 65. WJD.

TABLE NO. 21

COEFFICIENTS OF INDUCTION FROM RESIDUAL VOLTAGE OF THREE-PHASE CIRCUITS
INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR.

Volts per Kilovolt Residual

POWER CIRCUIT:
- EQUILATERAL TRIANGLE, BASE VERTICAL
- *Vertex Toward Disturbed Conductors*
- *Spacings of Conductors = d*
- *Diameter of Conductors = 0.006d*
- *Height of Lowest Conductor = H/d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*
DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.2d*

D/d	*H/d*	*Y/d*	*X/d*					CURVE SHEET NO.
			4.9	9.9	19.9	39.9	99.9	
0.006	3	0	41.55	14.88	4.254	1.106	0.1792	
		0.5	36.17	12.65	3.566	0.9236	0.1493	
		1	29.98	10.29	2.867	0.7399	0.1195	
	5	0	63.63	28.29	9.291	2.548	0.4198	
		1	54.99	23.71	7.670	2.049	0.3361	
		3	29.75	12.63	3.880	1.032	0.1683	
	10	0	95.60	54.86	23.93	7.781	1.375	
		2	82.03	47.61	20.10	6.341	1.104	
		5	52.29	31.90	13.25	4.045	0.6925	
	20	0	123.1	83.05	47.94	20.78	4.476	
		2	113.9	79.44	45.13	19.19	4.055	
		10	57.69	46.78	28.20	11.53	2.298	
0.002	3	0.5	33.07			0.8444		
	5	1	60.71			1.890		
	10	2	76.41			5.907		
	20	4	93.76			16.41		
0.010	3	0.5	37.84			0.9662		
	5	1	67.24			2.133		10
	10	2	84.93			6.565		
	20	4	103.2			18.01		
0.016	3	0.5	39.50			1.008		
	5	1	69.47			2.2[illegible]6		
	10	2	87.79			6.786		
	20	4	106.2			18.54		

J.C.I.I. T.R. NO. 65.

TABLE NO. 22

COEFFICIENTS OF INDUCTION FROM RESIDUAL VOLTAGE OF THREE-PHASE CIRCUITS
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS
Millivolts per Kilovolt Residual

POWER CIRCUIT:
- EQUILATERAL TRIANGLE, BASE VERTICAL
- *Vertex Toward Disturbed Conductors*
- *Spacings of Conductors = d*
- *Diameter of Conductors = 0.006d*
- *Height of Lowest Conductor = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

t/d	*D/d*	*H/d*	*Y/d*	*X/d*					CURVE SHEET NO.
				5	10	20	40	100	
0.2	0.006	3	0	2025.	488.2	79.68	10.84	0.71	10
			0.5	1806.	422.	67.2	9.04	0.59	
			1	1522.	348.0	53.2	7.26	0.46	
		5	0	2388.	780.6	162.5	24.45	1.665	
			1	2081.	676.1	134.6	19.76	1.333	
			3	1088.	374.7	70.54	10.03	0.669	
		10	0	2427.	1067.	337.6	68.89	5.371	
			2	1918.	939.1	293.9	57.11	4.326	
			5	955.9	624.1	201.0	37.10	2.724	
		20	0	2239.	1092.	469.2	148.2	16.56	
			2	1815.	1021.	450.9	139.7	15.10	
			10	380.6	445.8	282.7	88.62	8.722	
	0.002	3	0.5	1651.			8.28		
		5	1	1919.			18.23		
		10	2	1787.			53.20		
		20	4	1177.			121.5		
	0.010	3	0.5	1889.			9.48		
		5	1	2166.			20.56		
		10	2	1987.			59.12		
		20	4	1293.			133.4		
	0.016	3	0.5	1972.			9.88		
		5	1	2251.			21.37		
		10	2	2054.			61.11		
		20	4	1331.			137.3		
0.1	0.006	10	2	959.3			28.55		
0.5				4797.			142.8		

TABLE No. 23

COEFFICIENTS OF INDUCTION FROM BALANCED AND RESIDUAL VOLTAGES OF THREE-PHASE CIRCUITS

INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR

AND

INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS

(Percent of Coefficient for D/d = 0.006d

POWER CIRCUIT: Equilateral Triangle, Base Vertical, *Vertex Toward Disturbed Conductors. Spacings of Conductors = d. Diameter of Conductors = 0.006d. Height of Lowest Conductor = H/d*

DISTURBED CONDUCTORS: In Horizontal Plane, *Spaced 0.2d*

SEPARATION: Horizontal = *X/d* Vertical = *Y/d*

	H/d	V/d	D/d	To Ground X/d 4.9	To Ground X/d 39.9	Between Conductors X/d 5	Between Conductors X/d 40	Curve Sheet No.
Balanced Voltages	3	0.6	0.002	84.45	85.13	84.18	84.51	35 4
			0.010	109.6	108.9	109.7	108.4	
			0.016	119.9	118.8	120.4	117.7	
	6	1	0.002	84.20	85.11	83.90	84.88	
			0.010	109.6	109.1	109.7	108.9	
			0.016	120.2	119.4	120.6	118.9	
	10	2	0.002	84.04	84.63	83.92	84.44	
			0.010	109.7	109.3	109.8	108.3	
			0.016	120.4	119.5	120.6	119.7	
	20	4	0.002	83.96	84.33	84.00	84.18	
			0.010	109.8	109.4	109.7	109.5	
			0.016	120.6	119.8	120.5	120.1	
Residual Voltages	3	0.6	0.002	91.43	91.42	91.42	91.59	10
			0.010	104.6	104.6	104.6	104.9	
			0.016	109.2	109.1	109.2	109.3	
	6	1	0.002	92.22	92.24	92.22	92.26	
			0.010	104.1	104.1	104.1	104.0	
			0.016	108.2	108.2	108.2	108.2	
	10	2	0.002	93.15	93.17	93.12	93.15	
			0.010	103.5	103.5	103.5	103.5	
			0.016	107.0	107.0	107.0	107.0	
	20	4	0.002	93.76	93.93	93.86	93.90	
			0.010	103.2	103.1	103.1	103.1	
			0.016	106.2	106.1	106.1	106.1	

J.C.I.I. T.R. No. 65. WJD

TABLE No. 24

COEFFICIENTS OF INDUCTION FROM BALANCED AND RESIDUAL CURRENTS OF THREE-PHASE CIRCUITS.

POWER CIRCUIT: Equilateral Triangle, Base Vertical; *Vertex Toward Disturbed Conductors*; *Spacings of Conductors = d*; *Height of Lowest Conductor = H/d*

DISTURBED CONDUCTORS: In Horizontal Plane, *Spaced t/d*

SEPARATION: Horizontal = *X/d* Vertical = *Y/d*

			t/d	*H/d*	*Y/d*	*X/d* = 5	10	20	40	100
Mean Induced Voltage Along Pair of Conductors	*Millivolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.2	20	0	3.991	1.935	0.8871	0.3491	0.07337
Mean Induced Voltage Along Pair of Conductors	*Millivolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.2	∞	0	4.060	2.015	1.002	0.4993	0.1994
Mean Induced Voltage Along Pair of Conductors	*Millivolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.2	∞	0.5	3.977				
Mean Induced Voltage Along Pair of Conductors	*Millivolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.2	∞	1	3.851	1.991	0.9989	0.4989	
Mean Induced Voltage Along Pair of Conductors	*Millivolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.2	∞	2	3.523	1.945	0.9935	0.4963	0.1992
Mean Induced Voltage Along Pair of Conductors	*Millivolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.2	∞	3	3.170	1.882			
Mean Induced Voltage Along Pair of Conductors	*Millivolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.2	∞	5	2.565	1.729	0.9623	0.4944	0.1990
Mean Induced Voltage Along Pair of Conductors	*Millivolts at 60 Cycles per 1000 Feet per Ampere*	*Residual Current*	0.2	20	0	47.51	32.49	18.58	8.065	1.740
Difference of Induced Voltages Along Pair of Conductors	*Microvolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.2	20	0	166.4	42.68	11.32	2.778	0.2775
Difference of Induced Voltages Along Pair of Conductors	*Microvolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.2	∞	0	164.2	40.73	9.953	2.490	0.3993
Difference of Induced Voltages Along Pair of Conductors	*Microvolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.2	∞	0.5	154.8				
Difference of Induced Voltages Along Pair of Conductors	*Microvolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.2	∞	1	141.8	39.48	10.02	2.500	
Difference of Induced Voltages Along Pair of Conductors	*Microvolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.2	∞	2	114.3	37.26	9.887	2.492	0.3989
Difference of Induced Voltages Along Pair of Conductors	*Microvolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.2	∞	3	93.37	34.45			
Difference of Induced Voltages Along Pair of Conductors	*Microvolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.2	∞	5	67.18	28.73	9.192	2.452	0.3979
Difference of Induced Voltages Along Pair of Conductors	*Microvolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.1	20	1	71.82			1.350	
Difference of Induced Voltages Along Pair of Conductors	*Microvolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.1	∞	1	70.88			1.251	
Difference of Induced Voltages Along Pair of Conductors	*Microvolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.5	20	1	359.6			6.756	
Difference of Induced Voltages Along Pair of Conductors	*Microvolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.5	∞	1	354.9			6.254	
Difference of Induced Voltages Along Pair of Conductors	*Microvolts at 60 Cycles per 1000 Feet per Ampere*	*Residual Current*	0.2	20	0	872.1	425.3	182.8	57.73	6.450
Difference of Induced Voltages Along Pair of Conductors	*Microvolts at 60 Cycles per 1000 Feet per Ampere*	*Residual Current*	0.1	20	1	404.5			28.06	
Difference of Induced Voltages Along Pair of Conductors	*Microvolts at 60 Cycles per 1000 Feet per Ampere*	*Residual Current*	0.5	20	1	2023.			140.4	

J.C.I.I. – T.R. No. 65.

W.G.D.

TABLE NO. 25

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR
(Volts per Kilovolt between Power Conductors)

POWER CIRCUIT { Isosceles Triangle, Base Horizontal = d, Altitude = 0.4d; Diameter of Conductors = 0.006d; Height of Lower Conductors = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.2d*

SEPARATION: HORIZONTAL = X/d   VERTICAL = Y/d

D/d	H/d	Y/d	X/d 4.9	9.9	19.9	39.9	99.9	CURVE SHEET NO.
0.006	3	0	10.66	2.610	0.4955	0.1061	0.0159	39, 40
		0.5	9.710	2.168	0.4166	0.08857	0.0132	
		1	8.342	1.787	0.3357	0.0710	0.01062	
	6	0	14.31	4.418	0.9337	0.1720	0.02231	
		1	13.11	3.863	0.7722	0.1387	0.01763	
		3	7.302	2.184	0.4032	0.0898	0.00882	
	10	0	17.14	7.140	2.186	0.4342	0.0357	
		2	15.13	6.591	1.920	0.3601	0.0279	
		5	8.424	4.687	1.327	0.2342	0.0181	
	20	0	18.45	8.918	3.828	1.196	0.1397	
		2	17.08	8.812	3.747	1.131	0.1273	
		10	5.843	4.807	2.623	0.7269	0.0733	
0.002	3	0.6	8.167			0.07693		41
	6	1	10.98			0.1206		
	10	2	12.59			0.3044		
	20	4	10.97			0.8634		
0.010	3	0.5	10.66			0.0956		
	6	1	14.41			0.1494		
	10	2	16.69			0.3944		
	20	4	14.80			1.168		
0.016	3	0.5	11.69			0.1031		
	6	1	15.85			0.1612		
	10	2	18.43			0.4326		
	20	4	16.46			1.300		

TABLE No. 26

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS.
(Millivolts per Kilovolt between Power Conductors)

POWER CIRCUIT { Isosceles Triangle, Base Horizontal = *d*, Altitude = 0.4*d*; *Diameter of Conductors = D/d*; *Height of Lower Conductors = H/d*
DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

t/d	*D/d*	*H/d*	*Y/d*	*X/d* 5	10	20	40	100	CURVE SHEET NO.
0.2	0.006	3	0	754.7	114.	11.7	1.15	0.0646	42 43 44
			0.5	722.	101.6	9.85	0.957	0.053	
			1	634.0	85.6	7.97	0.763	0.043	
		5	0	798.0	177.6	22.7	2.09	0.0953	
			1	748.5	159.1	19.2	1.69	0.0756	
			3	370.	103.	10.4	0.87	0.038	
		10	0	758.	201.	43.9	5.61	0.197	
			2	578.5	190.	40.4	4.75	0.181	
			5	203.	124.	29.2	3.18	0.101	
		20	0	731.6	192.2	52.94	11.85	0.730	
			2	552.	186.5	53.25	11.66	0.670	
			10	93.9	62	34.6	7.97	0.411	
	0.002	3	0.5	602.0			0.83		41
		5	1	625.2			1.44		
		10	2	480.			3.99		
		20	4	230.3			9.04		
	0.010	3	0.5	788.9			1.033		
		5	1	823.8			1.82		
		10	2	639.1			5.22		
		20	4	314.3			12.12		
	0.018	3	0.5	867.8			1.119		
		5	1	907.9			1.97		
		10	2	707.1			5.74		
		20	4	350.9			13.45		
0.1	0.006	10	2	293.6			2.37		
0.5				1447.			11.87		

J.C.I.I. - T.R. No. 65. WGD

TABLE NO. 27

COEFFICIENTS OF INDUCTION FROM RESIDUAL VOLTAGE OF THREE-PHASE CIRCUITS
INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR.
(Volts per Kilovolt Residual)

POWER CIRCUIT { Isosceles Triangle, Base Horizontal = d, Altitude = 0.4d; *Diameter of Conductors = 0.006d*; *Height of Lower Conductors = H/d* }

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.2d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

D/d	*H/d*	*Y/d*	*X/d* 4.9	9.9	19.9	39.9	99.9	CURVE SHEET NO.
0.006	3	0	39.75	13.64	3.801	0.9775	0.1576	
		0.5	34.87	11.61	3.188	0.8160	0.1312	
		1	29.10	9.458	2.564	0.6537	0.1050	
	5	0	62.31	26.79	8.606	2.332	0.3817	
		1	54.23	22.54	7.016	1.865	0.3057	
		3	29.80	12.07	3.599	0.9444	0.1530	
	10	0	93.66	53.08	22.85	7.350	1.290	
		2	81.65	46.27	19.23	5.992	1.035	
		5	52.38	31.37	12.73	3.824	0.6496	
	20	0	130.9	81.79	46.42	19.99	4.278	
		2	113.0	77.75	43.77	18.48	3.8763	
		10	57.09	46.12	27.50	11.12	2.197.	
0.002	3	0.5	31.93			0.7473		
	5	1	50.11			1.733		
	10	2	76.21			5.594		
	20	4				15.83		
0.010	3	0.5	36.43			0.8524		
	5	1	56.39			1.950		
	10	2	84.46			6.197		
	20	4				17.33		
0.016	3	0.5				0.8888		
	5	1	58.53			2.023		
	10	2	87.22			6.399		
	20	4				17.82		

J.C.I.I. – T.R. No.65. W.J.D.

TABLE No. 28

COEFFICIENTS OF INDUCTION FROM RESIDUAL VOLTAGE OF THREE-PHASE CIRCUITS
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS
(Millivolts per Kilovolt Residual)

POWER CIRCUIT { ISOSCELES TRIANGLE, BASE HORIZONTAL = *d*, ALTITUDE = 0.4*d*; *Diameter of Conductors = D/d*; *Height of Lower Conductors = H/d* }
DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

t/d	*D/d*	*H/d*	*Y/d*	*X/d* 5	10	20	40	100	CURVE SHEET No.
	0.006	3	0	2038.	461.	72.0	9.62	0.627	
			0.5	1849.	399.	60.83	8.04	0.523	
			1	1580.	330.3	49.2	6.45	0.419	
		5	0	2420.	759.7	152.6	22.60	1.517	
			1	2171.	662.9	126.6	18.19	1.216	
			3	1167.	369.	66.46	9.23	0.609	
		10	0	2456.	1042.	326.5	83.4	5.049	
			2	2022.	939.	285.4	54.34	4.067	
			5	1006.	633.	196.2	36.34	2.561	
		20	0	2265.	1073.	458.4	143.6	15.87	
			2	1903.	1022.	442.4	136.3	14.46	
			10	386.1	451.	280.4	86.3	8.36	
	0.002	3	0.5	1692.			7.37		
		5	1	2006.			16.82		
		10	2	1883.			50.73		
		20	4				118.1		
	0.010	3	0.5				8.40		
		5	1	2257.			18.92		
		10	2	2089.			56.20		
		20	4				129.3		
	0.016	3	0.5				8.76		
		5	1	2344.			19.63		
		10	2	2157.			58.04		
		20	4				133.0		
0.1	0.006	10	2	1007.			27.17		
0.5				5043.			136.9		

J.C.I.I. - T.R. No. 65. W.J.P.

TABLE No. 29

COEFFICIENTS OF INDUCTION FROM BALANCED AND RESIDUAL VOLTAGES OF THREE-PHASE CIRCUITS

INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR

AND

INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS.

(Percent of Coefficient for D/d = 0.006d)

POWER CIRCUIT { Isosceles Triangle, Base Horizontal =d, Altitude =0.4d; Diameter of Conductors = D/d; Height of Lower Conductors = H/d

DISTURBED CONDUCTORS IN HORIZONTAL PLANE, Spaced 0.2d

SEPARATION: HORIZONTAL = X/d VERTICAL = Y/d

	H/d	Y/d	D/d	To Ground X/d 4.9	To Ground X/d 39.9	Between Conductors X/d 5	Between Conductors X/d 40	Curve Sheet No.
Balanced Voltages	3	0.5	0.002	84.11	86.83	83.66	86.78	41
			0.010	109.7	107.8	109.3	107.9	
			0.016	120.3	116.4	120.2	116.9	
	5	1	0.002	83.75	86.95	83.58	85.71	
			0.010	109.9	107.7	110.1	108.3	
			0.016	120.9	116.2	121.3	117.3	
	10	2	0.002	83.21	84.53	82.97	84.00	
			0.010	110.3	109.6	110.5	109.9	
			0.016	121.8	120.1	122.2	120.8	
	20	4	0.002	82.30	82.15	81.70	82.63	
			0.010	111.0	111.3	111.5	110.8	
			0.016	123.5	123.7	124.5	123.0	
Residual Voltages	3	0.5	0.002	91.55	91.58	91.51	87.74	
			0.010		104.5			
			0.016		108.9			
	5	1	0.002	92.40	92.92	92.40	92.47	
			0.010	104.0	104.6	104.0	104.0	
			0.016	107.9	108.5	108.0	107.9	
	10	2	0.002	93.34	93.36	93.13	93.36	
			0.010	103.4	103.4	103.3	103.4	
			0.016	106.8	106.8	106.7	106.9	
	20	4	0.002		94.00		94.10	
			0.010		102.9		103.0	
			0.016		105.8		106.0	

J.C.I.I. T.R. No. 65.

ugD.

TABLE No. 30

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE CURRENTS.
MEAN INDUCED VOLTAGE ALONG PAIR OF CONDUCTORS.
(Millivolts at 60 cycles per 1000 feet per ampere per phase)

POWER CIRCUIT { ISOSCELES TRIANGLE, BASE HORIZONTAL = *d*, ALTITUDE = 0.4*d*
Height of Lower Conductors = *H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced* 0.8*d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

H/d	*Y/d*	*X/d* 5	10	20	40	100	CURVE SHEET NO.
20	0	3.934	1.883	0.8146	0.2729	0.04180	45, 46, 47
	0.5	3.895					
	1	3.785					
	2	3.408	1.786	0.7808	0.2555	0.03795	
	3	2.924					
	5	2.019	1.450	0.6964	0.2241	0.03199	
40	0	3.981	1.963	0.9418	0.4078	0.08948	
	5	2.123	1.681	0.8707	0.3851	0.08123	
100	0		1.987	0.9860	0.4802	0.1635	
200	0				0.4931	0.1885	
	5				0.4849	0.1873	
∞	0	3.999	1.993	0.9954	0.4976	0.1990	
	0.5	3.972					
	1	3.874	1.977	0.9935	0.4974		
	2	3.517	1.928	0.9871	0.4965		
	3	3.054	1.850	0.9766	0.4952		
	5	2.189	1.639	0.9441	0.4909	0.1986	

J.C.I.I. - T.R. No.65. W.J.B.

Table No. 31

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE CURRENTS.
DIFFERENCE OF INDUCED VOLTAGES ALONG PAIR OF CONDUCTORS.
(Microvolts at 60 Cycles per 1000 Feet per Ampere per Phase)

POWER CIRCUIT { Isosceles Triangle, Base Horizontal = *d*, Altitude = 0.4d
Height of Lower Conductors = H/d

DISTURBED CONDUCTORS: In Horizontal Plane, *Spaced t/d*

SEPARATION: Horizontal = *X/d* Vertical = *Y/d*

t/d	*H/d*	*Y/d*	*X/d* = 5	10	20	40	100	Curve Sheet No.
0.2	20	0	164.0	42.02	11.16	2.646	0.1845	48, 49, 50
		0.5	160.9					
		1	149.9					
		2	112.8	38.42	10.89	2.446	0.1690	
		3	73.92					
		5	32.70	24.40	9.591	2.215	0.1441	
	40	0	162.2	40.56	10.47	2.790	0.3609	
		5	32.94	23.07	9.080	2.683	0.3368	
	100	0		40.06	10.06	2.576	0.4465	
	200	0				2.513	0.4188	
		5				2.418	0.4164	
	∞	0	161.6	39.96	9.96	2.490	0.3980	
		0.5	168.6					
		1	147.6	39.14	9.91	2.486		
		2	110.6	36.35	9.73	2.475		
		3	72.04	32.22	9.434	2.463		
		5	32.99	22.54	8.562	2.393	0.3956	
0.1	20	1	74.93			1.251		
	∞	1	73.77			1.243		
0.5	20	1	375.4			6.264		
	∞	1	369.6			6.215		

J.C.I.I. - T.R. No. 65. wgD.

TABLE NO. 32

COEFFICIENTS OF INDUCTION FROM RESIDUAL CURRENT OF THREE-PHASE CIRCUITS.
MEAN INDUCED VOLTAGE ALONG PAIR OF CONDUCTORS
(Millivolts at 60 Cycles per 1000 Feet per Ampere Residual)

POWER CIRCUIT { ISOSCELES TRIANGLE, BASE HORIZONTAL = d, ALTITUDE = $0.4d$
Height of Lower Conductors = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced* $0.2d$

SEPARATION: HORIZONTAL = X/d VERTICAL = Y/d

H/d	Y/d	X/d 5	10	20	40	100	CURVE SHEET NO.
20	0	48.10	32.64	18.56	8.004	1.716	
	0.5	47.64					
	1	46.96					
	2	45.01	31.03	17.50	7.397	1.555	
	3	42.50					
	5	36.77	27.09	15.44	6.384	1.307	
40	0	63.86	48.02	32.60	18.53	5.699	
	5	54.06	43.87	30.47	17.17	5.113	
100	0		68.90	53.05	37.45	18.51	
	5		65.63	51.74	36.71	18.01	
200	0				53.04	32.56	
	5				52.57	32.26	

 WJB.

TABLE NO. 33

COEFFICIENTS OF INDUCTION FROM RESIDUAL CURRENT OF THREE-PHASE CIRCUITS.
DIFFERENCE OF INDUCED VOLTAGES ALONG PAIR OF CONDUCTORS.
(Microvolts at 60 Cycles per 1000 Feet per Ampere Residual)

POWER CIRCUIT { Isosceles Triangle, Base Horizontal = *d*, Altitude = 0.4*d*; *Height of Lower Conductors = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

t/d	*H/d*	*Y/d*	*X/d* = 5	10	20	40	100	CURVE SHEET NO.
0.2	20	0	909.5	433.3	184.2	57.65	6.376	
		0.5	895.1					
		1	863.5					
		2	763.8	410.5	177.7	54.39	5.814	
		3	643.5					
		5	428.0	329.4	159.4	48.19	4.929	
	40	0	920.0	463.1	216.4	91.99	17.97	
		5	442.2	355.8	200.4	87.67	16.46	
	100	0		459.0	227.6	110.5	36.78	
		5		362.6	213.3	108.4	36.28	
	200	0				113.8	43.26	
		5				111.9	43.07	
0.1	20	1	431.7			28.06		
0.5	20	1	2160.			140.3		

J.C.I.I. – T.R. NO.65. WJD.

TABLE No. 34

COEFFICIENTS OF INDUCTION FROM BALANCED AND RESIDUAL VOLTAGES OF THREE-PHASE CIRCUITS

POWER CIRCUIT { ISOSCELES TRIANGLE, BASE HORIZONTAL = *d*, ALTITUDE = 1.25
Diameter of Conductors = 0.006d
Height of Lower Conductors = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.2d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

				H/d	Y/d	X/d = 4.9	9.9	19.9	39.9	99.9
INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR	*Volts induced per kilovolt*	* *Balanced Voltage Between Conductors*		5	0	15.20	6.589	2.363	0.6808	0.1164
					1	13.05	5.086	1.919	0.5476	0.0913
					3	7.063	2.811	0.9782	0.2754	0.0456
				10	0	16.91	8.236	3.693	1.303	0.2406
					1	15.82	7.532	3.372	1.210	0.2176
					3	12.28	5.993	2.685	0.9325	0.1697
		Residual Voltage		5	0	67.42	29.46	9.612	3.619	0.4297
					1	58.15	24.78	7.834	2.107	0.3441
					3	28.55	13.22	4.016	1.060	0.1722
				10	0	99.72	57.00	24.74	8.007	1.409
					1	94.01	53.59	22.85	7.275	1.270
					3	76.26	44.73	18.58	5.754	0.9912

			t/d	H/d	Y/d	X/d = 5	10	20	40	100
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS	*Millivolts induced per kilovolt*	* *Balanced Voltage Between Conductors*	0.2	5	0	791.4	186.9	40.13	6.977	0.485
					1	791.0	163.4	32.3	5.24	0.3613
					3	453.6	94.69	16.95	2.67	0.174
			0.2	10	0	760.0	193.4	51.10	10.86	0.9267
					1	800.3	190.4	47.64	10.16	0.8453
					3	592.1	173.2	39.26	7.97	0.75
		Residual Voltage	0.2	5	0	2547	822.6	169.2	25.05	1.75
					1	2439	714.3	140.	20.4	1.367
					3	1179	395.	73.	10.36	0.682
			0.2	10	0	2563	1107	350.8	72.	5.730
					1	2371	1059	331.4	65.2	4.979
					3	1671	890	277.2	52.4	3.8

J. C. I. I. T R. No. 66.. * SEE CURVE SHEETS NO. 51, 52 WJB

TABLE NO. 37

COEFFICIENTS OF INDUCTION FROM RESIDUAL VOLTAGE OF THREE-PHASE CIRCUITS
INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR
Volts per Kilovolt Residual

POWER CIRCUIT: Horizontal Symmetrical; *Spacings of Conductors = d, d/a, d/a*; *Diameter of Conductors = 0.009d*; *Height of Conductors = H/d*

DISTURBED CONDUCTORS: In Horizontal Plane, *Spaced 0./d*
SEPARATION: Horizontal = *X/d* Vertical = Y/D

D/d	H/d	Y/d	X/d 4.95	9.95	19.95	39.95	99.95	Curve Sheet No.
0.003	1.5	0	14.76	4.063	1.041	0.2614	0.04185	60 61
		0.25	12.58	3.408	0.8686	0.2179	0.03488	
		0.5	10.25	2.741	0.6959	0.1744	0.02791	
	2.5	0	28.98	9.224	2.488	0.6343	0.1020	
		0.5	24.43	7.521	2.001	0.5081	0.0818	
		1.5	13.11	3.860	1.008	0.2545	0.0408	
	5	0	57.04	24.44	7.834	2.123	0.3478	
		1.0	49.83	20.58	6.387	1.707	0.2785	
		2.5	33 85	13.61	4.076	1.073	0.1742	
	10	0	87.20	49.38	21.23	6.821	1.197	
		1	83.02	46.59	19.62	6.201	1.080	
		5	49.34	29.32	11.82	3.549	0.6030	
0.001	2.5	0.5	22.41			0.4663		62
	5	1	46.27			1.585		
	10	2	71.68			5.211		
0.005	2.5	0.5	25.50			0.5303		
	5	1	51.68			1.771		
	10	2	78.97			5.741		
0.008	2.5	0.5	26.56			0.5525		
	5	1	53.51			1.833		
	10	2	81.39			5.917		

J. C. I. I.
T. R. No. 65.

TABLE No. 38

COEFFICIENTS OF INDUCTION FROM RESIDUAL VOLTAGE OF THREE-PHASE CIRCUITS.
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS
Millivolts per Kilovolt Residual

POWER CIRCUIT:
- HORIZONTAL SYMMETRICAL
- *Spacings of Conductors = d, d/a, d/a*
- *Diameter of Conductors = 0.009d*
- *Height of Conductors = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced a./d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

t/d	*D/d*	*H/d*	*Y/d*	*X/d* 5	10	20	40	100	CURVE SHEET No.
0.1	0.003	1.5	0	505.	77.4	10.1	1.301	.0836	63 64
			0.25	439.5	65.3	8.56	1.08	.072	
			0.5	384.0	53.	6.88	.868	.0557	
		2.5	0	830.	165.	24.1	3.140	0.203	
			0.5	728.	136.5	19.5	2.52	0.168	
			1.5	410.0	71.7	9.8	1.264	0.062	
		5	0	1128.	350.4	70.	10.28	0.691	
			1.0	1022	307.1	58.04	8.308	0.553	
			2.5	690.4	211.3	37.76	5.253	0.347	
		10	0	1157.	489.1	152.6	30.51	2.346	
			1	1101.	472.6	144.1	27.99	2.119	
			5	486.1	300.6	92.0	16.46	1.190	
		2.5	0.5	667.6			2.32		62
		5	1	949.2			7.71		
		10	2	906.2			23.71		
	0.005	2.5	0.5	759.5			2.63		
		5	1	1060.			8.61		
		10	2	998.4			26.1		
	0.008	2.5	0.5	791.4			2.74		
		5	1	1098.			8.922		
		10	2	1029.			26.92		
0.05	0.003	5	1	511.1			4.154		
0.25				2556.			207.7		

J.C.I.I.
I.D.No.68.

WJB.

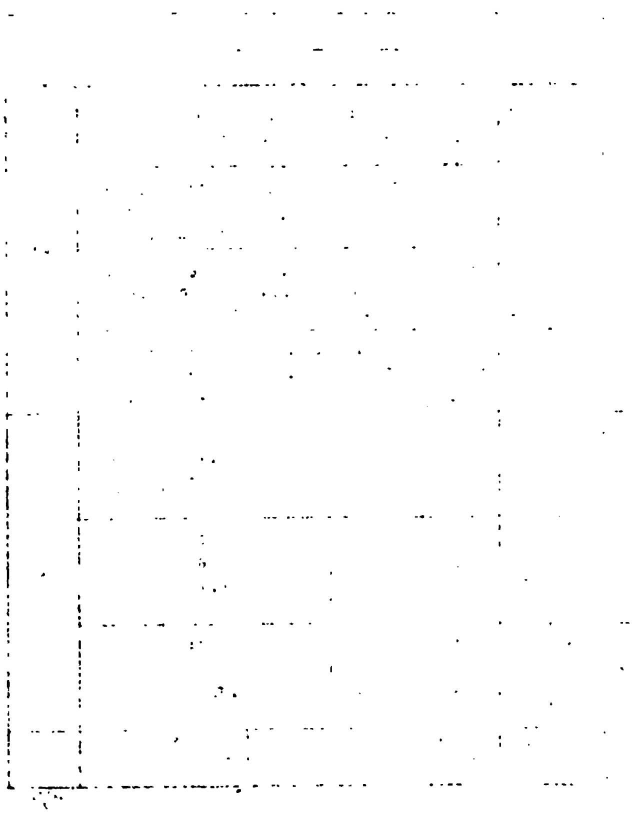


TABLE NO. 39

COEFFICIENTS OF INDUCTION FROM BALANCED AND RESIDUAL
VOLTAGES OF THREE-PHASE CIRCUITS
INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR
AND
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS
(Percent of Coefficient for D/d = 0.003d)

POWER CIRCUIT { Horizontal, Symmetrical; Spacings of Conductors = d, d/2 d/2; Diameter of Conductors : D/d; Height of Conductors = H/d }

DISTURBED CONDUCTORS IN HORIZONTAL PLANE, *Spaced 0.1d*
SEPARATION · HORIZONTAL = *X/d* VERTICAL = *Y/d*

	H/d	Y/d	D/d	To Ground X/d 4.95	To Ground X/d 39.95	Between Conductor X/d 5	Between Conductor X/d 40	Curve Sheet No.
Balanced Voltages	2.5	0.5	0.001	84.87	79.63	85.15	81.28	55 59
			0.005	102.1	113.0	109.0	112.0	
			0.008	119.	127.7	118.6	125.5	
	5	1	0.001	84.70	80.57	85.01	81.75	
			0.005	109.1	112.2	106.9	110.7	
			0.008	119.1	126.9	118.6	123.6	
	10	2	0.001	84.34	81.21	84.77	82.50	
			0.005	109.5	111.7	109.0	110.7	
			0.008	119.9	124.8	119.0	122.9	
Residual Voltages	2.5	0. 0.5	0.001	91.73	91.77	91.70	92.06	62
			0.005	104.4	104.4	104.3	104.4	
			0.008	108.7	108.7	108.7	108.7	
	5	1	0.001	92.86	92.85	92.88	92.80	
			0.005	103.7	103.8	103.7	103.6	
			0.008	107.4	107.4	107.4	107.4	
	10	2	0.001	93.70	93.71	93.70	93.72	
			0.005	103.2	103.2	103.2	103.2	
			0.008	106.4	106.4	106.4	106.4	

J.C.I.I. T.R. NO. 65. *W.J.B.*

TABLE NO. 40

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE CURRENTS
MEAN INDUCED VOLTAGE ALONG PAIR OF CONDUCTORS
(Millivolts at 60 cycles per 1000 feet per Ampere per Phase)

POWER CIRCUIT { HORIZONTAL SYMMETRICAL
Spacings of Conductors = d, d/2, d/2
Height of Conductors = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.2d*
SEPARATION: HORIZONTAL = *X/d*. VERTICAL = *Y/d*

H/d	*Y/d*	*X/d* 4.95	9.95	19.95	39.95	99.95	CURVE SHEET NO.
10	0	3.806	1.606	0.5003	0.09985	.007666	
	0.25	3.789					
	0.5	3.752					
	1.	3.619	1.552	0.4723	0.09161		
	1.5	3.424				.006551	
	2.5	2.908	1.395	0.4187	0.07828	.005798	
20	0		1.866	0.7992	0.2494	.02749	
	1	3.811			0.2428		65 66 67
	2.5	3.775	1.751	0.7524	0.2314	.02445	
50	0		1.983	0.9597	0.4296	.09961	
	2.5		1.906			.09697	
100	0	4.0341			0.4791	.1593	
	2.5			0.9723		.1584	
∞	0.	4.036	2.002	0.9979	0.4982	.1991	
	0.25	4.026					
	0.5	3.995		0.4981			
	1.	3.875	1.982		0.4972		
	1.5	3.692					
	2.5	3.206	1.883	0.9824	0.4963	.1990	

J.C.I.I.
T.R. NO. 65. W.J.B.

TABLE NO. 41

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE CURRENTS

DIFFERENCE OF INDUCED VOLTAGES ALONG PAIR OF CONDUCTORS

Microvolts at 60 Cycles per 1000 Feet per Ampere per Phase

POWER CIRCUIT { HORIZONTAL, SYMMETRICAL; *Spacings of Conductors = d, d/2, d/2*; *Height of Conductors = H/d* }

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

t/d	H/d	Y/d	X/d: 4.95	9.95	19.95	39.95	99.95	CURVE SHEET No.
0.1	10	0	86.36	22.57	5.008	0.6498	0.02242	68, 69, 70
		0.25	85.80					
		0.50	83.99					
		1.	77.05	22.02	4.843	0.6030		
		1.5	66.90				0.01923	
		2.5	43.38	19.21	4.408	0.5233	0.01703	
	20	0		21.19	5.607	1.248	0.07492	
		1	72.49			1.225		
		2.5	39.58	17.84	5.390	1.131	0.06791	
	50	0		20.35	5.181	1.372	0.1993	
		2.5		16.90			0.196	
	100	0	82.24			1.291	0.2231	
		2.5			4.623		0.223	
	∞	0	82.21	20.18	5.003	1.247	0.1992	
		0.25	81.57					
		0.5	79.67					
		1.	72.57	19.56	4.966	1.238		
		1.5	62.20					
		2.5	36.25	16.69	4.776	1.232	0.1989	
0.05	10	0.5	41.99			0.3138		
	∞		39.83			0.6236		
0.25	10	0.5	210.1			1.568		
	∞		199.3			3.116		

J.C.I.I.
T.R. NO. 65.

W.G.D.

TABLE NO. 42

COEFFICIENTS OF INDUCTION FROM RESIDUAL CURRENT OF THREE-PHASE CIRCUITS
MEAN INDUCED VOLTAGE ALONG PAIR OF CONDUCTORS
Millivolts at 60 Cycles per 1000 Feet per Ampere Residual

POWER CIRCUIT { HORIZONTAL SYMMETRICAL; *Spacings of Conductors = d, d/2, d/2*; *Height of Conductors = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced, o./d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

H/d	*Y/d*	*X/d*					CURVE SHEET NO.
		4.95	9.95	19.95	39.95	99.95	
10	0	32.86	18.61	7.998	2.570	0.4511	
	0.25	32.55					
	0.5	32.19					
	1	31.28	17.56	7.394	2.336		
	1.5	30.15				0.3845	
	2.5	27.34	15.51	6.383	1.973	0.3398	
20	0	48.27	32.68	18.54	7.980	1.707	
	1	47.21			7.615		
	2.5	44.16	30.59	17.19	7.217	1.506	71
50	0	69.8	53.16	37.50	22.79	7.971	
	2.5		51.88				
100	0	85.09			37.47	18.50	
	2.5	82.15		52.62		18.26	

J.C.I.I.
T.R. No. 65.

W.J.D.

TABLE NO. 43

COEFFICIENTS OF INDUCTION FROM RESIDUAL CURRENT OF THREE-PHASE CIRCUITS'
DIFFERENCE OF INDUCED VOLTAGES ALONG PAIR OF CONDUCTORS.
Microvolts at 60 Cycles per 1000 Feet per Ampere Residual

POWER CIRCUIT { HORIZONTAL SYMMETRICAL
Spacing of Conductors = d, d/a, d/a
Height of Conductors = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

t/d	*H/d*	*Y/d*	*X/d* 4.95	9.96	19.95	39.95	99.95	CURVE SHEET NO.
0.1	10	0	440.7	185.6	57.80	11.53	0.8852	72
		0.25	438.8					
		0.5	434,5					
		1.	419.1	179.3	54.56	10.58		
		1.5	396.0				0.7565	
		2.5	385.9	161.1	48.37	9.041	0.6695	
	20	0	460.5	217.9	92.30	28.80	3.174	
		1	436.9			27.97		
		2.5	362.3	202.3	88.05	26.73	2.823	
	50	0		229.1	119.8	49.61	11.50	
		2.5		215.1				
	100	0	467.2			55.32	18.40	
		2.5	370.0		112,3		18.29	
0.05	10	0.5	217.2			5.53		
0.25	10	0.5	1086.			27.66		

J.C.I.I.
T.R. NO. 85.

W.J.D.

TABLE NO. 44

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR
(Volts per Kilovolt between Power Conductors)

POWER CIRCUIT:
- HORIZONTAL, UNSYMMETRICAL
- *Inside Conductor Displaced Away from Disturbed Conductors.*
- *Spacings of Conductors = d, d/3, 2d/3*
- *Diameter of Conductors = D/d*
- *Height of Conductors = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.1d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

D/d	*H/d*	*Y/d*	*X/d* 4.95	9.95	19.95	39.95	89.95	CURVE SHEET NO.
0.003	2.5	0	10.23	2.220	0.4116	0.07911	0.01036	73 74
		0.5	8.678	1.832	0.3319		0.00829	
		1.5	4.847	0.9663	0.1678	0.031779	0.004148	
	5	0	16.56	5.6324	1.3521	0.2786	0.03651	
		1	14.87	4.8712	1.11287	0.2245	0.02924	
		2.5	10.06	3.3124	0.71770	0.1414	0.01829	
	8	0	19.75	8.502	2.679	0.6332	0.08587	
		1	18.59	7.980	2.431		0.07527	
		4	10.33	5.209	1.506	0.3283	0.04318	
0.001	2.5	0.5	7.319					75
	5	1	12.53			0.1832	0.02342	
	8	2	13.66					
0.005	2.5	0.5	9.488					
	5	1	16.26			0.2497	0.03262	
	8	2	17.79					
0.008	2.5	0.5	10.37					
	5	1	17.78			0.2776	0.03662	
	8	2	19.47					

J.C.I.I. - T.R. No. 65. W.J.D.

TABLE NO. 45

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS.
(Millivolts per Kilovolt between Power Conductors)

POWER CIRCUIT {
HORIZONTAL, UNSYMMETRICAL
Inside Conductor Displaced Away from Disturbed Conductors.
Spacings of Conductors = d, d/3, 8d/3
Diameter of Conductors = D/d
Height of Conductors = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

t/d	*D/d*	*H/d*	*Y/d*	*X/d* 5	10	20	40	100	CURVE SHEET NO.
0.1	0.003								76 77
		2.5	0	371.7	51.8	4.98	0.4647	0.022	
			0.5	337.4	43.7	4.04		0.017	
			1.5	197.1	23.6	2.05	0.184	0.009	
		5	0	427.7	101.7	15.02	1.59	0.0795	
			1	385.4	92.0	12.63	1.287	0.06608	
			2.5	231.3	65.0	8.36	0.81	0.039	
		8	0	416.1	118.8	25.58	3.45	0.184	
			1	377.5	114.5	23.82		0.162	
			4	112.8	73.94	15.58	1.848	0.0941	
	0.001	2.5	0.5	285.8					76
		5	1	326.1			1.060	0.05336	
		8	4	241.0					
	0.005	2.5	0.5	368.0					
		5	1	420.5			1.423	0.07390	
		8	4	312.0					
	0.008	2.5	0.5	401.5					
		5	1	458.7			1.575	0.08259	
		8	4	340.7					
0.05	0.003	5	1	192.6			0.6432		
0.25				963.8			3.216		

J.C.I.I. - T.R. NO.65. *wgB.*

TABLE No. 46

COEFFICIENTS OF INDUCTION FROM BALANCED VOLTAGES
OF THREE-PHASE CIRCUITS
INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR
AND
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS
(Percent of Coefficient for D/d = 0.003d)

POWER CIRCUIT { HORIZONTAL, UNSYMMETRICAL
Inside Conductor Displaced Away from and Toward Disturbed Conductors
Spacings of Conductors = d, d/3, 2d/3,
Height of Conductors = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.2d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*.

	H/d	Y/d	D/d	To Ground X/d 4.95	To Ground X/d 39.95	Between Conductors X/d 5	Between Conductors X/d 40	Curve Sheet No.
Inside Conductor Away From Disturbed Conductors	2.5	0.5	0.001	84.34		84.71		75 78
			0.005	109.3		109.1		
			0.008	119.5		119.0		
	5	1	0.001	84.26	81.60	84.61	82.36	
			0.005	109.3	111.2	109.1	110.6	
			0.008	119.6	123.6	119.0	122.4	
	8	2	0.001	84.16		84.56		
			0.005	109.8		109.5		
			0.008	120.0		119.5		
Inside Conductor Toward Disturbed Conductors	2.5	0.5	0.001	86.09		86.07		84 87
			0.005	108.4		108.3		
			0.008	117.6		117.4		
	5	1	0.001	85.88	76.04	86.04	78.60	
			0.005	108.5	114.7	108.4	113.3	
			0.008	118.0	131.6	117.5	128.4	
	8	2	0.001	84.89		85.71		
			0.005	108.3		108.4		
			0.008	118.3		117.7		

J.C. I.I. T.R. No. 65.

[illegible]

TABLE NO. 47

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE CURRENTS
MEAN INDUCED VOLTAGE ALONG PAIR OF CONDUCTORS
(Millivolts at 60 Cycles per 1000 Feet per Ampere per Phase)

POWER CIRCUIT:
- HORIZONTAL, UNSYMMETRICAL
- *Inside Conductor Displaced Away from Disturbed Conductors*
- *Spacings of Conductors = d, d/9, 2d/9*
- *Height of Conductors = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.1d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

H/d	*Y/d*	*X/d* 4.95	9.95	19.95	39.95	99.95	CURVE SHEET NO.
10	0	3.698	1.641	0.5108	0.1018	0.007813	79-80
	0.25	3.878					
	0.5	3.840					
	1	3.705	1.586	0.4822	0.09344		
	1.5	3.502					
	2.5	2.972	1.425	0.4274	0.07986	0.005909	
20	0		1.925	0.8152	0.2543	0.02801	
	2.5	3.204	1.788	0.7777	0.2360	0.02491	
50	0		2.024	0.9786	0.4379	0.1015	
100	0				0.4882	0.1623	
∞	0	4.134	2.044	1.017	0.5077	0.2028	
	0.25	4.119					
	0.5	4.087					
	1	3.963					
	1.5	3.774					
	2.5	3.274	1.921	1.002	0.5057	0.2027	

J.C.I.I. - T.R. No. 65. W.J.D.

TABLE NO. 48

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE CURRENTS.
DIFFERENCE OF INDUCED VOLTAGES ALONG PAIR OF CONDUCTORS.
(Microvolts at 60 Cycles per 1000 Feet per Ampere per Phase)

POWER CIRCUIT: HORIZONTAL, UNSYMMETRICAL
Inside Conductor Displaced Away from Disturbed Conductors
Spacings of Conductors = d, d/3, 2d/3
Height of Conductors = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

t/d	*H/d*	*Y/d*	*X/d* 4.95	9.95	19.95	39.95	99.95	CURVE SHEET NO.
0.1	10	0	88.68	23.09	5.121	0.6632	0.02286	81
		0.25	88.11					
		0.5	86.23					
		1	79.07	22.53	4.946	0.6153		
		1.5	68.56					
		2.5	44.28	19.62	4.504	0.5342	0.01736	
	20	0		21.68	5.723	1.273	0.07636	
		2.5	40.42	18.24	5.501	1.214	0.06861	
	50	0		20.83	5.288	1.398	0.2031	
	100	0				1.317	0.2273	
	∞	0	84.46	20.63	5.108	1.272	0.2030	
		0.25	83.80					
		0.5	81.83					
		1	74.47					
		1.5	63.76					
		2.5	39.05	17.07	4.875	1.257	0.2026	
0.05	10	0.5	43.12			0.3201		
	∞		40.91			0.6358		
0.25	10	0.5	215.7			1.600		
	∞		204.7			3.177		

J.C.I.I. – T.R. NO.65. *dJD.*

TABLE NO. 49

COEFFICIENTS OF INDUCTION FROM RESIDUAL CURRENT OF THREE-PHASE CIRCUITS
MEAN INDUCED VOLTAGE ALONG PAIR OF CONDUCTORS.
(Millivolts at 60 Cycles per 1000 Feet per Ampere Residual)

POWER CIRCUIT:
- HORIZONTAL, UNSYMMETRICAL
- *Inside Conductor Displaced Away from Disturbed Conductors*
- *Spacings of Conductors = d, d/3, 2d/3*
- *Height of Conductors = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.1d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

H/d	*Y/d*	*X/d* 4.95	9.95	19.95	39.95	99.95	CURVE SHEET NO.
10	0	32.62	18.51	7.947	2.564	0.4506	
	0.25	32.32					
	0.5	31.95					
	1	31.05	17.46	7.364	2.331		
	1.5	29.93					
	2.5	27.15	15.42	6.356	1.968	0.3394	
20	0		32.56	18.49	7.964	1.705	
	2.5	43.96	30 48	17.14	7.202	1.505	
50	0		53.04	37.44	22.76	7.964	
100	0				37.44	18.49	

J.C.I.I. - T.R. NO. 65.

TABLE NO. 50

COEFFICIENTS OF INDUCTION FROM RESIDUAL CURRENT OF THREE-PHASE CIRCUITS.
DIFFERENCE OF INDUCED VOLTAGES ALONG PAIR OF CONDUCTORS.
(Microvolts at 60 Cycles per 1000 Feet, per Ampere Residual)

POWER CIRCUIT { HORIZONTAL, UNSYMMETRICAL
Inside Conductor Displaced Away from Disturbed Conductors
Spacings of Conductors = d, d/3, 2d/3
Height of Conductors = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

t/d	*H/d*	*Y/d*	*X/d* 4.95	9.95	19.95	39.95	99.95	CURVE SHEET NO.
0.1	10	0	435.4	184.1	57.47	11.49	0.8838	
		0.25	433.5					
		0.5	429.3					
		1	414.3	178.0	54.25	10.54		
		1.5	391.8					
		2.5	333.1	159.9	48.08	9.007	0.6686	
	20	0		216.6	91.94	28.73	3.169	
		2.5	359.8	201.2	87.70	26.65	2.819	
	50	0		227.8	110.5	49.52	11.40	
	100	0				65.24	18.38	
0.05	10	0.5	214.7			5.512		
0.2	10	0.5	1073.			27.56		

W.J.D.

J.C.I.I. – T.R. NO. 65.

TABLE No. 50

COEFFICIENTS OF INDUCTION FROM RESIDUAL CURRENT OF THREE-PHASE CIRCUITS.
DIFFERENCE OF INDUCED VOLTAGES ALONG PAIR OF CONDUCTORS.
(Microvolts at 60 Cycles per 1000 Feet, per Ampere Residual)

POWER CIRCUIT { HORIZONTAL, UNSYMMETRICAL
Inside Conductor Displaced Away from Disturbed Conductors
Spacings of Conductors = d, d/3, 2d/3
Height of Conductors = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

t/d	*H/d*	*Y/d*	*X/d* 4.95	9.95	19.95	39.95	99.95	CURVE SHEET NO.
0.1	10	0	436.4	184.1	57.47	11.49	0.8838	
		0.25	433.5					
		0.5	429.3					
		1	414.3	178.0	54.25	10.64		
		1.5	391.8					
		2.5	333.1	159.9	48.08	9.007	0.6686	
	20	0		216.6	91.94	28.73	3.169	
		2.5	359.8	201.2	87.70	26.65	2.819	
	50	0		227.8	110.5	49.52	11.40	
	100	0				55.24	18.38	
0.05	10	0.5	214.7			5.512		
0.2	10	0.5	1073.			27.56		

J.C.I.I. - T.R. No. 65.

W.J.D.

TABLE NO. 51

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR
(Volts per Kilovolt between Power Conductors)

POWER CIRCUIT:
- HORIZONTAL, UNSYMMETRICAL
- *Inside Conductor Displaced Toward Disturbed Conductors.*
- *Spacing of Conductors = d, d/3, 2d/3*
- *Diameter of Conductors = 0.003d*
- *Height of Conductors = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.1d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

D/d	*H/d*	*Y/d*	*X/d* 4.95	9.95	19.95	39.95	99.95	CURVE SHEET NO.
0.003	2.5	0	6.634	1.219	0.1867	0.04249	0.007818	82 83
		0.5	5.851	1.014	0.1507		0.006257	
		1.5	3.322	0.5345	0.07619	0.01704	0.003130	
	5	0	10.10	2.998	0.6193	0.1428	0.02653	
		1	9.207	2.637	0.5110	0.1148	0.02124	
		2.5	6.200	1.824	0.3305	0.07216	0.01329	
	8	0	11.16	4.253	1.221	0.3184	0.06080	
		1	10.54	4.039	1.110		0.06327	
		4	5.4298	2.676	0.6924	0.1635	0.03052	
0.001	2.5	0.5	5.037					84
	5	1	7.907			0.08729	0.01641	
	8	2	7.742			0.1884		
0.005	2.5	0.5	6.340					
	5	1	9.993		0.5710	0.1317	0.02426	
	8	2	9.876			0.2771		
0.008	2.5	0.5	6.878				0.008255	
	5	1	10.88		0.6404	0.1511	0.02769	
	8	2	10.79			0.3166	0.05925	

J.C.I.I. – T.R. NO. 65.

W.J.S

TABLE NO. 52

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS.
(Millivolts per Kilovolt between Power Conductors)

POWER CIRCUIT:
- HORIZONTAL, UNSYMMETRICAL
- *Inside Conductor Displaced Toward Disturbed Conductors*
- *Spacing of Conductors = d, d/3, 2d/3*
- *Diameter of Conductors = 0.003d*
- *Height of Conductors = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

t/d	*D/d*	*H/d*	*Y/d*	*X/d* 5	10	20	40	100	CURVE SHEET NO.
0.1	0.003	2.5	0	273.0	33.0	2.49	0.211	0.015	85, 86
			0.5	250.5	28.2	2.02		0.012	
			1.5	147.9	15.2	1.02	0.085	0.0068	
		5	0	295.8	62.0	7.66	0.730	0.050	
			1	265.7	54.4	6.48	0.5874	0.03972	
			2.5	151.6	40.8	4.33	0.373	0.02487	
		8	0	308.7	68.0	12.67	1.53	0.112	
			1	249.7	66.0	11.8		0.093	
			4	52.1	42.0	7.95	0.84	0.057	
	0.001	2.5	0.5	215.6					87
		5	1	228.6		5.49	0.462	0.03052	
		8	2	151.7			0.979		
	0.005	2.5	0.5	271.4					
		5	1	287.9		7.10	0.666	0.04556	
		8	2	191.8			1.378		
	0.008	2.5	0.5	294.1				0.01565	
		5	1	312.3		7.812	0.755	0.05216	
		8	2	208.3			1.558	0.1107	
0.05	0.003	5	1	146.6			0.293		
0.25				664.6			1.468		

J.C.I.I. — T.R. NO. 65.

TABLE NO. 53

COEFFICIENTS OF INDUCTION FROM BALANCED AND RESIDUAL CURRENTS OF THREE-PHASE CIRCUITS.

POWER CIRCUIT { HORIZONTAL UNSYMMETRICAL
Inside Conductor Displaced Toward Disturbed Conductors.
Spacings of Conductors = d, d/3, 2d/3
Height of Lowest Conductor = E/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

			t/d	E/d	Y/d	X/d 4.95	X/d 9.95	X/d 19.95	X/d 39.95	X/d 99.95
MEAN INDUCED VOLTAGE ALONG PAIR OF CONDUCTORS	*Millivolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.1	10	0	3.851	1.630	0.5084	0.1015	0.007802
					2.5	2.951	1.415			
				∞	0	4.090	2.034	1.015	0.5071	0.2027
					1	3.929				
					1.5	3.744				
					2.5	3.265	1.913			
		Residual Current	0.1	10	0	33.11	16.71	8.031	2.577	0.4516
					2.5	27.53	15.60			
DIFFERENCE OF INDUCED VOLTAGE ALONG PAIR OF CONDUCTORS	*Microvolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.1	10	0	87.07	22.87	5.086	0.6604	0.02282
					2.5	44.08	19.49			
				∞	0	82.86	20.43	5.084	1.268	0.2028
					1	73.23				
					1.5	62.88				
					2.5	38.87	16.93			
			0.05	10	0.5	42.36			0.3190	
				∞		40.16			0.6344	
			0.25	10	0.5	211.9			1.593	
				∞		200.9			3.170	
		Residual Current	0.1	10	0	446.4	187.0	58.13	11.57	0.8867
					2.5	338.7	162.4			
			0.05	10	0.5	220.0			5.562	
			0.25			1100.			27.76	

J.C.I.I. - T.R. NO. 65. W.J.D.

TABLE NO. 54

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR
(Volts per Kilovolt between Power Conductors)

POWER CIRCUIT:
- HORIZONTAL, UNSYMMETRICAL
- *Inside Conductor Displaced Away from Disturbed Conductors.*
- *Spacings of Conductors = d, d/4, 9d/4*
- *Diameter of Conductors = D/d*
- *Height of Conductors = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.2d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

D/d	*H/d*	*Y/d*	*X/d* 4.95	9.95	19.95	39.95	99.95	CURVE SHEET NO.
0.003	1.5	0	5.736	1.053	0.1900	0.03757	0.005065	88 89
		0.25	4.962	0.8863	0.1587	0.03132	0.004219	
		0.5	4.095	0.7152	0.1272	0.02506	0.003377	
	2.5	0	10.81	2.491	0.4809	0.09550	0.01275	
		0.5	9.415	2.064	0.3877	0.0765	0.01020	
		1.5	5.243	1.070	0.1959	0.03836	0.005102	
	5	0	18.28	6.354	1.574	0.3346	0.045	
		1	16.38	5.481	1.294	0.2696	0.03586	
		2.5	11.07	3.717	0.8335	0.1656	0.02243	
	8	0	22.08	9.694	3.126	0.7591	0.10522	
		1	20.77	9.111	2.832	0.6715	0.09223	
		4	11.68	5.912	1.751	0.3943	0.05290	
0.001	2.5	0.5	7.926			0.06191		90
	5	1	13.78			0.2204		
0.005	2.5	0.5	10.30			0.08546		
	5	1	17.92			0.2992		
0.008	2.5	0.5	11.26			0.09531		
	5	1	19.59			0.3318		

J.C.I.I. - T.R. No.65. WJD.

TABLE NO. 55

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS.
(Millivolts per Kilovolt between Power Conductors)

POWER CIRCUIT:
- HORIZONTAL, UNSYMMETRICAL
- *Inside Conductor Displaced Away from Disturbed Conductors.*
- *Spacings of Conductors = d, d/4, 9d/4*
- *Diameter of Conductors = D/d*
- *Height of Conductors = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

t/d	*D/d*	*H/d*	*Y/d*	*X/d* 5	10	20	40	100	CURVE SHEET NO.
0.1	0.003	1.5	0	258.7	26.0	2.27	0.216	0.010	91 92 93
			0.25	229.3	22.0	1.91	0.177	0.0086	
			0.5	193.1	18.0	1.54	0.141	0.00718	
		2.5	0	396.9	56.8	5.68	0.540	0.027	
			0.5	359.5	48.0	4.60	0.434	0.0217	
			1.5	209.6	25.8	2.37	0.218	0.010	
		5	0	462.1	112.4	17.07	1.88	0.096	
			1	416.8	101.2	14.34	1.52	0.0768	
			2.5	252.5	71.4	9.46	0.95	0.049	
		8	0	454.4	132.7	29.23	4.06	0.223	
			1	411.2	127.5	27.17	3.624	0.197	
			4	130.1	82.6	17.70	2.17	0.114	
	0.001	2.5	0.5	304.2			0.355		94
		5	1	352.3			1.25		
	0.005	2.5	0.5	392.5			0.483		
		5	1	454.9			1.676		
	0.008	2.5	0.5	428.0			0.536		
		5	1	496.2			1.852		
0.05	0.003	5	1	208.4			0.77		
0.25				1042.			3.79		

J.C.I.I. – T.R. NO.65. W.J.D.

Table No. 56

COEFFICIENTS OF INDUCTION FROM RESIDUAL VOLTAGE OF THREE-PHASE CIRCUITS INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR.

(Volts per Kilovolt Residual)

POWER CIRCUIT { Horizontal, Unsymmetrical
Inside Conductor Displaced Away from Disturbed Conductors.
Spacings of Conductors = d, 2/4, 9d/4
Diameter of Conductors = D/d
Height of Conductors = H/d

DISTURBED CONDUCTORS: In Horizontal Plane, *Spaced 0.2d*

SEPARATION: Horizontal = *X/d*   Vertical = *Y/d*

D/d	*H/d*	*Y/d*	*X/d* 4.95	9.95	19.95	39.95	99.95	Curve Sheet No.
0.003	1.5	0	14.31	3.966	1.020	0.2569	0.04119	
		0.25	12.19	3.326	0.8515	0.2142	0.03433	
		0.5	9.938	2.675	0.6822	0.1714	0.02746	
	2.5	0	28.23	9.027	2.445	0.6246	0.1006	
		0.5	23.78	7.360	1.986	0.5004	0.08047	
		1.5	12.76	3.776	0.9903	0.2507	0.04025	
	5	0	55.89	24.01	7.716	2.094		
		1	48.81	20.21	6.290	1.686	0.2752	
		2.5	33.16	13.36	4.014	1.069	0.1722	
	8	0	76.423	40.04	15.58	4.328	0.7957	
		1	71.482	36.85	13.98	4.123	0.6973	
		4	45.64	23.45	8.452	2.400	0.3997	
0.001	2.5	0.5	21.81			0.4593		
	5	1	45.30			1.565		
0.005	2.5	0.5	24.83			0.5221		
	5	1	50.64			1.747		
0.008	2.5	0.5	25.89			0.5439		
	5	1	52.45			1.809		

J.C.I.I. - T.R. No.65.   W.J.D.

TABLE NO. 57

COEFFICIENTS OF INDUCTION FROM RESIDUAL VOLTAGE OF THREE-PHASE CIRCUITS
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS
(Millivolts per Kilovolt Residual)

POWER CIRCUIT:
- HORIZONTAL, UNSYMMETRICAL
- *Inside Conductor Displaced Away from Disturbed Conductors.*
- *Spacings of Conductors = d, d/4, 9d/4*
- *Diameter of Conductors = D/d*
- *Height of Conductors = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*

SEPARATION: HORIZONTAL = X/d VERTICAL = Y/d

t/d	D/d	H/d	Y/d	X/d 5	X/d 10	X/d 20	X/d 40	X/d 100	Curve Sheet No.
0.1	0.003	1.5	0	487.4	75.2	10.03	1.277	0.082	
			0.25	423.5	63.60	8.38	1.064	0.069	
			0.5	350.8	51.4	8.67	0.852	0.055	
		2.5	0	805.0	160.3	23.58	3.089	0.200	
			0.5	705.1	133.1	19.07	2.478	0.160	
			1.5	397.0	69.9	9.68	1.243	0.081	
		5	0	1101.4	343.6	68.77	10.12	0.682	
			1	998.1	300.7	57.03	8.19	0.547	
			2.5	675.0	206.9	37.09	5.18	0.342	
		8	0	1144.	450.5	122.3	21.65	1.570	
			1	1081.	427.4	111.8	19.25	1.378	
			4	576.9	283.2	70.92	11.40	0.793	
	0.001	2.5	0.5	646.1			2.274		
		5	1	928.4			7.604		
	0.005	2.5	0.5	736.5			2.585		
		5	1	1036.			8.493		
	0.008	2.5	0.5	768.1			2.693		
		5	1	1073.			8.793		
0.05	0.003	5	1	499.1			4.10		
0.25				2496.			20.47		

J.C.I.I. - T.R. NO. 65.

W.J.B.

TABLE NO. 58

COEFFICIENTS OF INDUCTION FROM BALANCED AND RESIDUAL VOLTAGES OF THREE-PHASE CIRCUITS

INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR

AND

INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS.

(Percent of Coefficient for D/d = 0.003)

POWER CIRCUIT:
- HORIZONTAL, UNSYMMETRICAL
- *Inside conductor Displaced Away from and Toward Disturbed Conductors*
- *Spacings of Conductors: = d, d/4, 9d/4,*
- *Height of Conductors = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.1d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

		H/d	Y/d	D/d	To Ground X/d 4.95	To Ground X/d 39.95	Between Conductors X/d 5	Between Conductors X/d 40	Curve Sheet No.
Inside Conductor Away From Disturbed Conductors	Balanced Voltages	2.5	0.5	0.001	84.18	80.90	84.62	81.60	90, 94
				0.005	109.4	111.7	109.2	111.3	
				0.008	119.6	124.5	119.0	123.5	
		5	1	0.001	84.13	81.75	84.52	82.24	
				0.005	109.4	111.0	109.1	110.3	
				0.008	119.6	123.1	119.0	121.6	
	Residual Voltages	2.5	0.5	0.001	91.72	91.79	91.63	91.77	
				0.005	104.4	104.3	104.4	104.3	
				0.008	108.9	108.7	108.9	108.7	
		5	1	0.001	92.81	92.88	93.02	92.84	
				0.005	103.8	103.7	103.8	103.7	
				0.008	107.5	107.4	107.5	107.4	
Inside Conductor Toward Disturbed Conductors	Balanced Voltages	2.5	0.5	0.001	87.27	74.80	86.93	74.21	97, 101
				0.005	107.7	116.0	107.8	116.4	
				0.008	116.2	134.2	116.3	135.2	
		5	1	0.001	87.05	76.38	87.04	76.26	
				0.005	108.2	115.4	107.8	116.3	
				0.008	116.8	132.6	116.4	134.6	
	Residual Voltages	2.5	5	0.001	91.92	91.85	91.98	91.83	
				0.005	104.24	104.3	104.2	104.3	
				0.008	108.5	108.6	108.4	108.6	
		5	1	0.001	92.98	92.91	92.02	92.92	
				0.005	103.6	103.7	103.6	103.7	
				0.008	107.2	107.3	107.2	107.3	

J.C.I.I.. T.R.NO. 65. W.J.B.

TABLE No. 59

COEFFICIENTS OF INDUCTION FROM BALANCED AND RESIDUAL CURRENTS OF THREE-PHASE CIRCUITS.

POWER CIRCUIT:
- HORIZONTAL, UNSYMMETRICAL
- *Inside Conductor Displaced Away from Disturbed Conductors*
- *Spacings of Conductors = d, d/4, 9d/4*
- *Height of Conductors = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced s/d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

			s/d	*H/d*	*Y/d*	*X/d* 4.95	9.95	19.95	39.95	99.95
MEAN INDUCED VOLTAGE ALONG PAIR OF CONDUCTORS	*Millivolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.1	10	0 2.5	3.984 3.040	1.678	0,5223	0.1041	0.007986
				∞	0 2.5	4.224 3.348	2.090	1.040	0.5189	0.2073
		Residual Current	0.1	10	0 2.5	32.50 27.06	18.46	7.951	2.561	0.4504
DIFFERENCE OF INDUCED VOLTAGES ALONG PAIR OF CONDUCTORS	*Microvolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.1	10	0 2.5	90.76 46.53	23.61	5.233	0.6781	0.02336
				∞	0 2.5	86.43 39.94	21.10	5.223	1.300	0.2076
			0.05	10 ∞	0.5	44.12 41.86			0.3274 0.6500	
			0.25	10 ∞	0.5	220.7 209.5			1.636 3.248	
		Residual Current	0.1	10	0 2.5	432.9 331.8	183.4	57.32	11.47	0.8831
			0.05	10	0.5	213.4			5.502	
			0.25	10	0.5	1067.			27.51	

J.C. i. i. – T.R. No. 65. W.J.B.

TABLE No. 60

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR
(Volts per Kilovolt between Power Conductors)

POWER CIRCUIT:
- Horizontal, Unsymmetrical
- *Inside Conductor Displaced Toward Disturbed Conductors;*
- *Spacing of Conductors = d, d/4, 9d/4*
- *Diameter of Conductors = D/d*
- *Height of Conductors = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE; *Spaced 0.2d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

D/d	*H/d*	*Y/d*	*X/d* 4.95	9.95	19.95	39.95	99.95	CURVE SHEET No.
0.003	1.5	0	3.106	0.3819	0.06153	0.01964	0.003890	95, 96
		0.25	2.717	0.3231	0.05136	0.01637	0.003242	
		0.5	2.262	0.2617	0.04114	0.0131	0.002594	
	2.5	0	5.654	0.9364	0.1514	0.04709	0.009531	
		0.5	5.021	0.7822	0.1217	0.03770	0.007625	
		1.5	2.874	0.4146	0.06137	0.01887	0.003815	
	5	0	8.227	2.268	0.4885	0.1521	0.03208	
		1	7.559	2.011	0.3985	0.1221	0.02568	
		2.5	5.083	1.403	0.2561	0.07660	0.01636	
	8	0	9.705	3.149	0.9752	0.3336	0.07324	
		1	8.239	2.998	0.8779	0.2935	0.06416	
		4	4.081	1.995	0.5370	0.1697	0.03676	
0.001	2.5	0.5	4.382			0.0282	0.005882	97
	5	1	6.580			0.09326	0.02003	
0.005	2.5	0.5	5.406			0.04373		
	5	1	8.176			0.1409	0.02918	
0.008	2.5	0.5	5.832			0.05058		
	5	1	8.832			0.1621	0.03309	

J.C.I.I. – T.R. No. 65. W.J.B.

TABLE NO. 61

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS.
(Millivolts per Kilovolt between Power Conductors)

POWER CIRCUIT:
- HORIZONTAL, UNSYMMETRICAL
- *Inside Conductor Displaced Toward Disturbed Conductors*
- *Spacing of Conductors = d, d/4, 9d/4*
- *Diameter of Conductors = D/d*
- *Height of Conductors = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

t/d	*D/d*	*H/d*	*Y/d*	*X/d* = 5	10	20	40	100	CURVE SHEET NO.
0.1	0.003	1.5	0	165.6	12.4	0.702	0.082	0.008	98, 99, 100
			0.25	148.3	10.6	0.59	0.070	0.00623	
			0.5	126.7	8.7	0.474	0.056	0.00483	
		2.5	0	244.1	27.6	1.86	0.200	0.017	
			0.5	225.0	23.6	1.50	0.169	0.01407	
			1.5	133.6	12.8	0.758	0.0808	0.008	
		5	0	250.0	50.6	5.6	0.63	0.057	
			1	230.8	46.9	4.9	0.514	0.0486	
			2.5	128.5	33.8	3.49	0.328	0.0290	
		8	0	240.3	53.37	9.365	1.37	0.131	
			1	212.4	52.1	8.88	1.21	0.1162	
			4	37.5	32.9	5.913	0.713	0.0664	
	0.001	2.5	0.5	195.6			0.118	0.01075	101
		5	1	200.9			0.392	0.0376	
	0.005	2.5	0.5	242.5			0.166		
		5	1	248.7			0.5976	0.0564	
	0.008	2.5	0.5	261.4			0.215		
		5	1	268.7			0.6920	0.06305	
0.05	0.003	5	1	115.4			0.256		
0.25				577.2			1.291		

J.C.J.I. - T.R. NO.65.

W.J.D.

TABLE NO. 62

COEFFICIENTS OF INDUCTION FROM RESIDUAL VOLTAGE OF THREE-PHASE CIRCUITS.
INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR.
(Volts per Kilovolt Residual)

POWER CIRCUIT:
- HORIZONTAL, UNSYMMETRICAL
- *Inside Conductor Displaced Toward Disturbed Conductors*
- *Spacing of Conductors = d, d/4, 3d/4*
- *Diameter of Conductors = D/d*
- *Height of Conductors = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.2d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

D/d	*H/d*	*Y/d*	*X/d* 4.95	9.95	19.95	39.95	99.95	CURVE SHEET NO.
0.003	1.5	0	14.82	4.044	1.030	0.2582	0.04127	
		0.25	12.63	3.392	0.8601	0.2163	0.03440	
		0.5	10.30	2.728	0.6891	0.1723	0.02702	
	2.5	0	29.06	9.191	2.463	0.6278	0.1008	
		0.5	24.51	7.498	1.986	0.5029	0.080632	
		1.5	13.17	3.848	1.000	0.2519	0.04033	
	5	0	57.01	24.35	7.785	2.105	0.3444	
		1	49.83	20.50	6.347	1.693	0.2767	
		2.5	33.85	13.57	4.051	1.064	0.1725	
	8	0	77.58	40.49	15.71	4.695	0.7973	
		1	72.58	37.29	14.10	4.143	0.6987	
		4	45.22	23.74	8.524	2.407	0.4016	
0.001	2.5	0.5	22.53			0.4619		
	5	1	46.33			1.573		
0.005	2.5	0.5	25.55			0.5246		
	5	1	51.84			1.755		
0.008	2.5	0.5	26.59			0.5463		
	5	1	53.42			1.817		

J.C.I.I. - T.R. NO.65. W.J.D.

TABLE No. 63

COEFFICIENTS OF INDUCTION FROM RESIDUAL VOLTAGE OF THREE-PHASE CIRCUITS
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS
(Millivolts per Kilovolt Residual)

POWER CIRCUIT:
- HORIZONTAL, UNSYMMETRICAL
- *Inside Conductor Displaced Toward Disturbed Conductors.*
- *Spacing of Conductors = d, d/4, 5d/4*
- *Diameter of Conductors = D/d*
- *Height of Conductors = H/d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

t/d	D/d	H/d	Y/d	X/d 5	X/d 10	X/d 20	X/d 40	X/d 100	Curve Sheet No.
0.1	0.003	1.5	0	485.6	77.5	10.18	1.286	0.082	
			0.25	445.1	65.3	8.50	1.074	0.06910	
			0.5	369.0	52.8	6.827	0.858	0.055	
		2.5	0	837.4	164.5	23.92	3.111	0.201	
			0.5	734.7	136.7	19.62	2.497	0.161	
			1.5	414.2	71.9	9.81	1.253	0.080	
		5	0	1132.	360.5	69.7	10.21	0.684	
			1	1026.	307.1	57.8	8.248	0.549	
			2.5	691.6	208.4	37.66	5.215	0.345	
		8	0	1171.	457.6	123.7	21.80	1.575	
			1	1129.	434.4	113.3	19.38	1.382	
			4	582.9	287.7	71.77	11.48	0.796	
	0.001	2.5	0.5	675.8			2.293		
		5	1	954.4			7.664		
	0.005	2.5	0.5	785.6			2.604		
		5	1	1063.			8.550		
	0.008	2.5	0.5	796.7			2.711		
		5	1	1100.			8.861		
0.05	0.003	5	1	513.0			4.124		
0.25				2586.			20.62		

J.C.I.I. – T.R. No.65.

TABLE NO. 64

COEFFICIENTS OF INDUCTION FROM BALANCED AND RESIDUAL CURRENTS OF THREE-PHASE CIRCUITS.

POWER CIRCUIT { HORIZONTAL, UNSYMMETRICAL
Inside Conductor Displaced Toward Disturbed Conductors
Spacings of Conductors = d, d/4, 3d/4
Height of Conductors = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

			t/d	*H/d*	*Y/d*	*X/d* 4.95	9.95	19.95	39.95	99.95
MEAN INDUCED VOLTAGE ALONG PAIR OF CONDUCTORS	*Millivolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.1	10	0 2.5	3.932 3.013	1.664	0.5192	0.1037	0.007973
				∞	0 2.5	4.176 3.326	2.078	1.037	0.5182	0.2072
		Residual Current	0.1	10	0 2.5	33.23 27.62	18.76	8.047	2.580	0.4519
DIFFERENCE OF INDUCED VOLTAGES ALONG PAIR OF CONDUCTORS.	*Microvolts at 60 Cycles per 1000 Feet per Ampere*	*Per Phase Balanced Currents*	0.1	10	0 2.5	88.75 45.03	23.35	6.194	0.6747	0.02331
				∞	0 2.5	84.46 39.71	20.86	5.193	1.296	0.2072
			0.05	10	0.5	43.18 40.93			0.3258 0.6483	
			0.25	10	0.5	216.0 204.8			1.628 3.239	
		Residual Current	0.1	10	0 2.5	449.3 340.2	187.8	58.29	11.60	0.8874
			0.05	10	0.5	221.4			5.562	
			0.25	10	0.5	1107.			27.81	

I.I. – T.R. NO. 65. W.J.D.

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TABLE No. 67

COEFFICIENTS OF INDUCTION FROM RESIDUAL VOLTAGE OF THREE-PHASE CIRCUITS
INDUCED VOLTAGE *TO GROUND* OF A SINGLE ISOLATED CONDUCTOR.
Volts per Kilovolt, Residual

POWER CIRCUIT {VERTICAL
Spacings of Conductors = d, d/a, d/a
Diameter of Conductors = .003d
Height of Lowest Conductor = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.1d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL *Y/d*

D/d	*H/d*	*Y/d*	*X/d* 4.95	9.95	19.95	39.95	99.95	CURVE SHEET No
0.003	1.5	0	16.81	4.887	1.276	0.3218	0.0516	109 110
		0.25	14.27	4.098	1.084	0.2683	0.0430	
		0.5	11.59	3.295	0.8523	0.2147	0.03440	
	2.5	0	30.74	10.27	2.830	0.7262	0.1170	
		0.25	25.68	8.363	2.276	0.5817	0.09360	
		1.5	13.60	4.284	1.146	0.2914	0.04681	
	5	0	57.76	25.47	8.338	2.280	0.3747	
		1	49.66	21.36	6.794	1.833	0.3000	
		2.5	33.17	14.05		1.153	0.1877	
	8	0	77.85	41.40	16.35	4.943	0.8440	
		1	71.87	37.95	14.65	4.361	0.7396	
		4	43.86	23.89	8.840	2.538	0.4240	
0.001	1.5	0.25	13.03			0.2452		111
	2.5	0.50	23.65			0.5358		
	5.0	1	46.18			1.705		
0.005	1.5	0.25	14.92			0.2806		
	2.5	0.5	26.75			0.6059		
	5	1	51.46			1.900		
0.008	1.5	0.25	15.58			0.2930		
	2.5	0.50	27.81			0.6300		
	5.0	1	53.25			1.986		

J.C.I.I.
T.R. No. 65.

W.J.D.

TABLE No. 68

COEFFICIENTS OF INDUCTION FROM RESIDUAL VOLTAGE OF THREE-PHASE CIRCUITS
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS
Millivolts per Kilovolt Residual

POWER CIRCUIT { VERTICAL
Spacings of Conductors = d, d/2, d/2
Diameter of Conductors = .009d
Height of Lowest Conductor = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL *Y/d*

t/d	D/d.	H/d	Y/d	X/d 5	X/d 10	X/d 20	X/d 40	X/d 100	CURVE SHEET NO.
0.1	0.003	1.5	0	539.	91.0	12.5	1.629	0.104	112 113
			0.25	463.6	76.75	10.43	1.334	0.085	
			0.5	381.3	62.0	8.34	1.067	0.069	
		2.5	0	828.7	178.3	27.1	3.582	0.233	
			0.5	711.6	147.4	21.93	2.878	0.187	
			1.5	389.8	77.12	11.10	1.443	0.094	
		5	0	1094	356.0	73.7	11.00	0.744	
			1	954.3	308.5	61.0	8.88	0.596	
			2.5	622.	209.8	39.60	5.62	0.373	
		8	0	1131.	457.0	126.9	22.80	1.664	
			1	1031	429.0	116.0	20.28	1.459	
			4	523.5	277.0	73.0	12.02	0.832	
	0.001	1.5	0.25	423.5			1.218		111
		2.5	0.50	655.1			2.651		
		5.0	1	887.3			8.27		
	0.005	1.5	0.25	485.			1.395		
		2.5	0.5	741.5			3.0		
		5	1	989.1			9.2		
	0.008	1.5	0.25	506.5			1.456		
		2.5	0.50	771.			3.12		
		5.0	1	1023.			9.53		
0.05	0.003	5	1	477.1			4.445		
0.25				2386.			22.23		

T. R. No. 65. W.G.D.

TABLE NO. 69

COEFFICIENTS OF INDUCTION FROM BALANCED AND RESIDUAL
VOLTAGES OF THREE-PHASE CIRCUITS
INDUCED VOLTAGE TO GROUND OF A SINGLE ISOLATED CONDUCTOR
AND
INDUCED VOLTAGE BETWEEN TWO ISOLATED CONDUCTORS.
(Percent of Coefficient for D/d = 0.003d)

POWER CIRCUIT { VERTICAL
Spacings of Conductors = d, d/2, d/2
Diameter of Conductors = D/d
Height of Lowest Conductor = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.1d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

	H/d	Y/d	D/d	To Ground		Between Conductors		Curve Sheet No.
				X/d				
				4.95	39.95	5	40	
Balanced Voltages	1.5	0.25	0.001 0.005 0.008	86.81 107.9 116.4	86.31 108.1 116.9	87.43 107.5 115.4	86.02 108.0 116.6	104 106
	2.5	0.5	0.001 0.005 0.008	85.80 108.4 117.7	85.88 108.3 117.4	77.70 114.9 132.5	85.95 108.1 118.2	
	5	1	0.001 0.005 0.008	78.46 113.8 129.4	85.10 109.0 118.8	83.23 110.4 121.8	84.76 108.2 118.9	
Residual Voltage	1.5	0.25	0.001 0.005 0.008	91.31 104.6 109.2	91.39 104.6 109.2	91.36 104.6 109.2	91.30 104.6 109.2	111
	2.5	0.5	0.001 0.005 0.008	92.10 104.9 108.3	92.11 104.9 108.3	92.06 104.9 108.4	92.11 104.2 108.4	
	5	1	0.001 0.005 0.008	92.99 103.6 107.2	93.02 103.7 107.3	92.98 103.6 107.2	93.13 103.6 107.8	

J.C.I.I.
T.R.No.65.

W.J.B.

TABLE No. 72.

COEFFICIENTS OF INDUCTION FROM RESIDUAL CURRENT OF THREE-PHASE CIRCUITS.
MEAN INDUCED VOLTAGE ALONG PAIR OF CONDUCTORS.
Millivolts at 60 Cycles per 1000 feet per Ampere Residual

POWER CIRCUIT { VERTICAL
Spacings of Conductors = d, d/2, d/2
Height of Lowest Conductor = H/d }

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.1d*
SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

H/d	*Y/d*	*X/d* 2.5	5	10	20	40	100	CURVE SHEET NO.
10	0	47.80	32.90	18.60	8.240	2.677	0.4727	
	0.5	46.07	32.03	18.36	7.931	2.556	0.4491	
	1	43.75	30.96	17.76	7.608	2.433	0.4282	
	1.5	41.07	29.69	17.07	7.271	2.308	0.4029	
	2.5	35.35	26.73	15.59	6.558	2.064	0.3560	
20	0	63.33	48.06	32.77	18.74	8.106	1.744	
	0.5	61.87	47.45	31.64	18.46			
	1	59.83	46.63	32.01	18.19	7.803	1.663	
	1.5	57.45	45.64	31.54	17.91			
	2.5	52.36	43.24	30.44	17.30	7.324	1.539	126
60	0	84.18	68.80	53.09	37.53	22.88	8.021	
	0.5			52.40	37.40			
	1	81.02		52.64		22.65	7.91	
	1.5			52.33	37.09			
	2.5	74.09		51.56	36.73	22.29	7.724	
100	0		84.87	68.88	53.07	37.49	18.54	
	0.5		84.26					
	1		83.85		52.96	37.36	44.90	
	1.5		82.91					
	2.5		80.99		52.54	37.15	18.30	

J.C.I.I. ; T.R. No. 65 — WJ.D

TABLE No. 73

COEFFICIENTS OF INDUCTION FROM RESIDUAL CURRENT OF THREE-PHASE CIRCUITS.
DIFFERENCE OF INDUCED VOLTAGES ALONG PAIR OF CONDUCTORS.
Microvolts at 60 Cycles per 1000 Feet per Ampere Residual.

POWER CIRCUIT { VERTICAL
Spacings of Conductors = d, d/2, d/2
Height of Lowest Conductor = H/d

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced t/d*

SEPARATION: HORIZONTAL = *X/d* VERTICAL = *Y/d*

t/d	H/d	Y/d	X/d 2.5	5	10	20	40	100	Curve Sheet No.
0.1	10	0	852.	426.4	184.6	58.74	11.9	0.926	127
		0.5	770.	412.5	181.1	57.11	11.45	0.176	
		1	661.4	391.6	176.5	55.29	10.95	0.837	
		1.5	549.6	365.4	170.7	53.32	10.42	0.7914	
		2.5	364.9	305.1	156.4	48.04	9.35	0.700	
	20	0	861.8	445.3	215.6	92.25	29.07	3.247	
		0.5	780.	432.5	209.8	91.6			
		1	672.6	412.7	210.6	90.77	28.28	3.097	
		1.5	561.	390.9	206.5	89.79			
		2.5	379.	330.2	195.7	87.40	26.92	2.679	
	50	0	864.7	451.1	226.6	110.	49.57	11.55	
		0.5			224.9	110.			
		1	675.7		222.1		49.37	11.43	
		1.5			218.4	109			
		2.5	381.7		208.2	108	48.92	11.24	
	100	0		451.9	228.3	113.6	56	18.42	
		0.5		439.3					
		1.		419.9		111.8	55	18.36	
		1.5		394.9					
		2.5		337.7		111.2	55	18.19	
0.05	10	0.5		206.2			5.72		
0.25				1030.			28.62		

J.C. T.R. No. 65. W.J.B.

TABLE No. 74

COEFFICIENTS OF INDUCTION FROM BALANCED AND RESIDUAL VOLTAGES OF THREE-PHASE CIRCUITS.

EFFECT OF VARYING ALTITUDE OF POWER-CIRCUIT TRIANGLE.

POWER CIRCUIT { Isosceles Triangle, Base Horizontal = d, Altitude = A; Diameter of Conductors = 0.006d; Height of Lower Conductors = 5d

DISTURBED CONDUCTORS: In Horizontal Plane, Spaced 0.2d

SEPARATION { Mean horizontal (from plane of vertex) = X; Vertical (from plane of lower power conductors) = d

	A/d	E_a $X/d = 5$	E_a $X/d = 40$	$E_a - E_b$ $X/d = 5$	$E_a - E_b$ $X/d = 40$
BALANCED VOLTAGES	0	13.82	0.1803	736.5	2.09
	0.2	13.46	0.1087	744.9	1.54
	0.4	13.11	0.1387	748.5	1.68
	0.6	12.94	0.2369	752.3	2.46
	0.866	12.94	0.3708	763.	3.60
	1.25	13.05	0.6476	791.	5.19
	1.6	13.14	0.6913	830.5	6.50
RESIDUAL VOLTAGE	0	62.92	1.799	2149	17.47
	0.2	63.34	1.823	2151	17.75
	0.4	64.23	1.866	2171	18.19
	0.6	65.27	1.930	2194	18.73
	0.866	66.44	2.000	2220	19.43
	1.25	68.15	2.107	2240	20.39
	1.6	69.24	2.193	2244	21.21

See Curve Sheet No. 128

E_a = Induced voltage to ground of conductor nearer power circuit in volts per kilovolt between power conductors or per kilovolt residual

$E_a - E_b$ = Induced voltage between disturbed conductors in millivolts per kilovolt between power conductors or per kilovolt residual

R.4.11

TABLE No. 75

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE CURRENTS.

EFFECT OF VARYING ALTITUDE OF POWER-CIRCUIT TRIANGLE.

POWER CIRCUIT { Isosceles Triangle, Base Horizontal = *d*, Altitude = *A*; *Height of Lower Conductors = H*

DISTURBED CONDUCTORS: In Horizontal Plane, *Spaced 0.2d*

SEPARATION { Mean horizontal (from plane of vertex) = *Z*; Vertical (from plane of lower power conductors) = *d*

H/d	*A/d*	E_{ab}		$E_a - E_b$	
		Z/d = 5	*Z/d = 40*	*Z/d = 5*	*Z/d = 40*
40	0	3.826	0.3958	143.3	2.786
	0.2	3.831	0.3979	144.8	2.786
	0.4	3.844	0.4045	148.2	2.789
	0.6	3.870	0.4261	154.1	2.791
	0.866	3.928	0.4348	166.0	2.794
	1.25	4.076	0.4719	191.1	2.799
	1.6	4.290	0.5121	220.3	2.802
∞	0	3.840	0.4973	142.6	2.482
	0.2	3.850	0.4973	144.1	2.484
	0.4	3.874	0.4974	147.8	2.484
	0.6	3.916	0.4974	153.6	2.484
	0.866	4.008	0.4977	165.5	2.492
	1.25	4.228	0.4982	190.7	2.500
	1.6	4.530	0.4991	220.0	2.519

See Curve Sheet No. 120

E_{ab} = *Mean induced voltage along disturbed conductors in millivolts at 60 cycles per 1000 feet per ampere per phase.*

$E_a - E_b$ = *Difference of induced voltages along disturbed conductors in microvolts at 60 cycles per 1000 feet per ampere per phase.*

R44.

TABLE No. 76

COEFFICIENTS OF INDUCTION FROM RESIDUAL CURRENTS OF THREE-PHASE CIRCUITS.

EFFECT OF VARYING ALTITUDE OF POWER-CIRCUIT TRIANGLE.

POWER CIRCUIT $\begin{cases}\text{Isosceles Triangle, Base Horizontal} = d, \text{ Altitude} = A \\ \text{Height of Lower Conductors} = 40d\end{cases}$

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced* $0.2d$

SEPARATION: $\begin{cases}\text{Mean horizontal (from plane of vertex)} = X \\ \text{Vertical (from plane of lower power conductors)} = d\end{cases}$

A/d	E_{ab}		$E_a - E_b$	
	$X/d = 5$	$X/d = 40$	$X/d = 5$	$X/d = 40$
0	63.09	18.26	885.1	91.39
0.2	63.05	18.27	880.2	91.41
0.4	62.99	18.28	874.6	91.44
0.6	62.93	18.30	868.4	91.46
0.866	62.83	18.32	859.4	91.47
1.25	62.66	18.34	845.3	91.49
1.6	62.48	18.37	831.7	91.51

See Curve Sheet No. 130

E_{ab} = *Mean induced voltage along disturbed conductors in millivolts at 60 cycles per 1000 feet per ampere.*

$E_a - E_b$ = *Difference of induced voltages along disturbed conductors in microvolts at 60 cycles per 1000 feet per ampere.*

AAM.

TABLE NO. 77

COEFFICIENTS OF INDUCTION FROM BALANCED AND RESIDUAL VOLTAGES OF THREE-PHASE CIRCUITS.

EFFECT OF VARYING ALTITUDE OF POWER-CIRCUIT TRIANGLE.

POWER CIRCUIT { Isosceles Triangle, Base Vertical = d, Altitude = A; Diameter of Conductors = 0.006d; Height of Lowest Conductor = 9d }

DISTURBED CONDUCTORS: In Horizontal Plane, Spaced 0.2d

SEPARATION { Mean horizontal (from midpoint of altitude) = X; Vertical (from plane of lowest power conductor) = d }

	A/d	E_a		E_a-E_b	
		$X/d = 5$	$X/d = 40$	$X/d = 5$	$X/d = 40$
BALANCED VOLTAGES	-2.0	14.31	0.1939	1038.	2.09
	-1.4	11.01	0.1918	775.3	1.95
	-0.866	8.27	0.1916	551.0	1.92
	-0.5	6.447	0.1938	394.1	1.92
	-0.2	4.720	0.1958	242.5	1.92
	0	3.356	0.1940	127.8	1.89
	0.2	2.871	0.1886	109.0	1.81
	0.5	5.300	0.1822	330.0	1.76
	0.866	9.081	0.1831	587.	1.80
	1.4	14.09	0.1940	915.8	1.97
	2.0	19.25	0.2108	1256.	2.24
RESIDUAL VOLTAGE	-2.0	37.73	0.8225	2014	8.15
	-1.4	34.88	0.7886	1834	7.81
	-0.866	32.31	0.7506	1679	7.41
	-0.5	30.52	0.7205	1575	7.13
	-0.2	29.20	0.6985	1499	6.89
	0	28.68	0.6919	1470	6.81
	0.2	28.61	0.6968	1459	6.85
	0.5	29.11	0.7141	1480	7.01
	0.866	29.98	0.7399	1522	7.26
	1.4	31.25	0.7721	1588	7.57
	2.0	32.61	0.8002	1667	7.83

See Curve Sheet No. 131

E_a = *Induced voltage to ground of conductor nearer power circuit in volts per kilovolt between conductors or per kilovolt residual.*

E_a-E_b = *Induced voltage between disturbed conductors in millivolts per kilovolt between conductors or per kilovolt residual.*

Vertex toward + *Vertex away -*

J.C.I.L T.R. No. 65 RSM

TABLE No. 76

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE CURRENTS.
EFFECT OF VARYING ALTITUDE OF POWER-CIRCUIT TRIANGLE.

POWER CIRCUIT { ISOSCELES TRIANGLE, BASE VERTICAL = d, ALTITUDE = A; *Height of Lowest Conductor* = H

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.2d*

SEPARATION: { MEAN HORIZONTAL (FROM MIDPOINT OF ALTITUDE) = X; VERTICAL (FROM PLANE OF LOWEST POWER CONDUCTOR) = d

H/d	A/d	E_{ab}		E_a-E_b	
		$X/d = 5$	$X/d = 40$	$X/d = 5$	$X/d = 40$
40	-2.0	8.461	0.9316	311.7	6.398
	-1.4	5.913	0.6641	225.3	4.474
	-0.866	3.689	0.4340	154.2	2.764
	-0.5	2.217	0.2904	111.7	1.593
	-0.2	1.156	0.2022	86.60	0.661
	0	0.8430	0.1812	79.33	0.232
	0.2	1.223	0.2038	82.15	0.698
	0.5	2.307	0.2932	102.7	1.640
	0.866	3.783	0.4374	142.3	2.806
	1.4	6.006	0.6676	211.3	4.616
	2.0	8.552	0.9350	296.5	6.437
∞	-2.0	8.541	1.148	312.8	5.706
	-1.4	5.993	0.8017	226.3	3.987
	-0.866	3.782	0.4954	153.7	2.468
	-0.5	2.342	0.2867	111.6	1.423
	-0.2	1.367	0.1146	86.68	0.582
	0	1.091	0.01872	79.4	0.186
	0.2	1.402	0.1180	82.16	0.620
	0.5	2.407	0.2889	102.6	1.464
	0.866	3.881	0.4990	141.6	2.502
	1.4	6.061	0.8052	211.1	4.027
	2.0	8.606	1.150	294.1	5.741

See Curve Sheet No. 132

E_{ab} = *Mean induced voltage along disturbed conductors in millivolts at 60 cycles per 1000 feet per ampere per phase.*

E_a-E_b = *Difference of induced voltages along disturbed conductors in microvolts at 60 cycles per 1000 feet per ampere per phase.*

Vertex toward + *Vertex away*

RHR

TABLE NO. 79

COEFFICIENTS OF INDUCTION FROM RESIDUAL CURRENT OF THREE-PHASE CIRCUITS.
EFFECT OF VARYING ALTITUDE OF POWER-CIRCUIT TRIANGLE.

POWER CIRCUIT { ISOSCELES TRIANGLE, BASE VERTICAL = d, ALTITUDE = A
Height of Lowest Conductor = 40d

DISTURBED CONDUCTORS. IN HORIZONTAL PLANE, *Spaced 0.2d*

SEPARATION { MEAN HORIZONTAL (FROM MIDPOINT OF ALTITUDE) = X
VERTICAL (FROM PLANE OF LOWEST POWER CONDUCTOR) = d

A/d	E_{ab}		$E_a - E_b$	
	$X/d = 5$	$X/d = 40$	$X/d = 5$	$X/d = 40$
-2.0	64.82	18.52	903.9	92.66
-1.4	63.72	18.47	878.9	92.30
-0.866	63.24	18.43	860.1	91.99
-0.5	62.94	18.40	849.1	91.79
-0.2	62.71	18.38	841.1	91.62
0	62.57	18.36	836.3	91.51
0.2	62.44	18.34	832.0	91.41
0.5	62.24	18.32	826.1	91.25
0.866	62.04	18.30	820.2	91.07
1.4	61.77	18.26	814.0	90.81
2.0	61.53	18.22	810.2	90.52

See Curve Sheet No. 135

E_{ab} = *Mean induced voltage along disturbed conductors in millivolts at 60 cycles per 1000 feet per ampere.*

$E_a - E_b$ = *Difference of induced voltages along disturbed conductors in microvolts at 60 cycles per 1000 feet per ampere.*

Vertex toward + *Vertex away −*

REN

Table No. 80

COEFFICIENTS OF INDUCTION FROM BALANCED AND RESIDUAL VOLTAGES OF THREE-PHASE CIRCUITS.

Effect of Varying Position of Inside Conductor of Horizontal Power Circuit.

POWER CIRCUIT {Horizontal; Spacings of Conductors = d, $d-\Delta$, Δ; Diameter of Conductors = $0.009d$; Height of Conductors = $5d$}

DISTURBED CONDUCTORS: In Horizontal Plane, Spaced $0.1d$

SEPARATION: {Mean horizontal (from plane midway between outside power conductors) = X, Vertical = d}

	Δ/d	E_a		E_a-E_b	
		$X/d = 5$	$X/d = 40$	$X/d = 5$	$X/d = 40$
Balanced Voltages	0.2	6.428	0.1391	206.8	0.53
	0.25	7.659	0.1271	230.8	0.514
	0.333	9.21	0.1148	265.7	0.578
	0.4	10.40	0.1241	290.	0.71
	0.5	12.09	0.1534	326.8	0.899
	0.6	13.74	0.1934	361.6	1.13
	0.667	14.87	0.2245	385.4	1.29
	0.75	16.38	0.2696	416.8	1.52
	0.8	17.38	0.3014	437.4	1.68
Residual Voltage	0.2	49.63	1.684	1023	8.21
	0.25	49.84	1.693	1026	8.25
	0.333	49.97	1.702	1027	8.29
	0.4	49.97	1.706	1027	8.30
	0.5	49.83	1.707	1022	8.308
	0.6	49.54	1.708	1015	8.28
	0.667	49.26	1.697	1008	8.27
	0.75	48.81	1.686	998.1	8.19
	0.8	48.45	1.674	990.5	8.14

See Curve Sheet No. 134

E_a = Induced voltage to ground of conductor nearer power circuit in volts per kilovolt between conductors or per kilovolt residual.

E_a-E_b = Induced voltage between disturbed conductors in millivolts per kilovolt between conductors or per kilovolt residual.

R.S.A.

TABLE NO. 81

COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE CURRENTS.
EFFECT OF VARYING POSITION OF INSIDE CONDUCTOR OF HORIZONTAL POWER CIRCUITS.

POWER CIRCUIT: Horizontal; Spacings of Conductors = d, $d-\Delta$, Δ; Height of Conductors = H

DISTURBED CONDUCTORS: In Horizontal Plane, Spaced 0.2d

SEPARATION: Mean horizontal (from plane midway between outside power conductors) = X, Vertical = d

H/d	Δ/d	E_{ab}		E_a-E_b	
		$X/d = 5$	$X/d = 40$	$X/d = 5$	$X/d = 40$
20	0.2	3.972	0.2560	75.86	1.296
	0.25	3.906	0.2517	74.64	1.273
	0.333	3.826	0.2464	73.18	1.246
	0.4	3.789	0.2436	72.65	1.233
	0.5	3.775	0.2422	72.49	1.225
	0.6	3.811	0.2440	73.37	1.233
	0.667	3.861	0.2469	74.41	1.249
	0.75	3.950	0.2525	76.15	1.276
	0.8	4.015	0.2567	77.40	1.298
∞	0.2	4.040	0.5259	74.59	1.310
	0.25	3.974	0.5172	73.37	1.289
	0.333	3.892	0.5061	71.94	1.259
	0.4	3.853	0.5004	71.35	1.246
	0.5	3.839	0.4973	71.27	1.238
	0.6	3.876	0.5008	72.14	1.249
	0.667	3.926	0.5067	73.15	1.263
	0.75	4.016	0.5180	74.88	1.292
	0.8	4.083	0.5266	76.10	1.314

See Curve Sheet No. 135

E_{ab} = Mean induced voltage along disturbed conductors in millivolts at 60 cycles per 1000 feet per ampere per phase.

E_a-E_b = Difference of induced voltages along disturbed conductors in microvolts at 60 cycles per 1000 feet per ampere per phase.

R.A.R.

TABLE No. 82

COEFFICIENTS OF INDUCTION FROM RESIDUAL CURRENT OF THREE-PHASE CIRCUITS.
EFFECT OF VARYING POSITION OF INSIDE CONDUCTOR OF HORIZONTAL POWER CIRCUIT.

POWER CIRCUIT: Horizontal; *Spacings of Conductors = d, d-A, A*; *Height of Conductors = 20d*

DISTURBED CONDUCTORS: IN HORIZONTAL PLANE, *Spaced 0.1d*

SEPARATION: MEAN HORIZONTAL (FROM PLANE MIDWAY BETWEEN OUTSIDE POWER CONDUCTORS) = *X*, VERTICAL = *d*

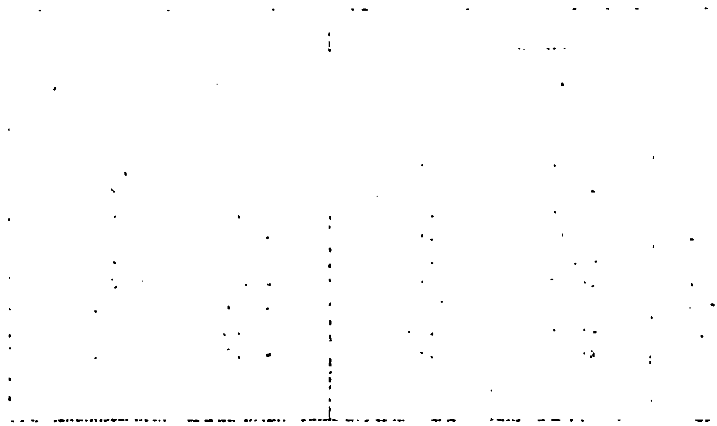
A/d	E_{ab}		E_a-E_b	
	$X/d = 5$	$X/d = 40$	$X/d = 5$	$X/d = 40$
0.2	47.46	7.699	446.7	28.11
0.25	47.38	7.695	444.2	28.09
0.333	47.26	7.687	441.7	28.05
0.4	47.11	7.680	439.7	28.02
0.5	47.01	7.671	436.9	27.97
0.6	46.87	7.662	434.2	27.92
0.667	46.77	7.656	432.4	27.89
0.75	46.66	7.648	430.3	27.85
0.8	46.59	7.643	429.0	27.83

See Curve Sheet No. 136

E_{ab} = *Mean induced voltage along disturbed conductors in millivolts at 60 cycles per 1000 feet per ampere.*

E_a-E_b = *Difference of induced voltages along disturbed conductors in microvolts at 60 cycles per 1000 feet per ampere.*

R.W.R.

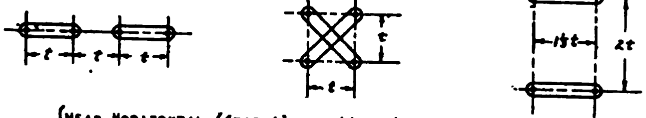


COMPARISON OF CONDUCTOR ARRANGEMENTS FOR METALLIC TELEPHONE CIRCUITS
WITH RESPECT TO
COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE VOLTAGES

POWER CIRCUIT { *Greatest Spacing of Conductors = d* / *Diameter of Conductors = D* / *Height of Lowest Conductor = H* }

TELEPHONE CONDUCTORS

HORIZONTAL PHANTOM — SQUARE PHANTOM — VERTICAL PHANTOM



SEPARATION { MEAN HORIZONTAL *(from plane midway between outside power conductors) = X* / MEAN VERTICAL *(from plane of lowest power conductor) = Y* }

Power-Circuit Configuration	H/d	D/d	t/d	Y/d	X/d	Phantom Circuit: Horizontal	Phantom Circuit: Square	Phantom Circuit: Vertical	Side Circuit of Phantom: Average Horizontal	Side Circuit of Phantom: Square	Side Circuit of Phantom: Square	Side Circuit of Phantom: Average Vertical
Vertical	2.5	0.003	0.1	0.6	5	58.6	2.52	470	30.9	253	220	41.2
					40	1.52	0.0208	17.1	0.799	9.34	7.75	1.06
Horizontal Symmetrical	5	0.003	0.1	1.1	5	606	3.68	406	320	521	119	427
					40	1.66	0.0107	7.44	0.876	4.60	2.88	1.17
Equilateral Triangle-Base Horizontal	10	0.006	0.2	2.2	5	1186	19.9	1007.	625	920	656	634
					40	12.4	0.073	33.1	6.52	32.8	10.8	8.69

J.C.I.I.-T.R.No.65.
R.A.A.

NOTE: *Coefficients are expressed in millivolts per kilovolt between power conductors.*

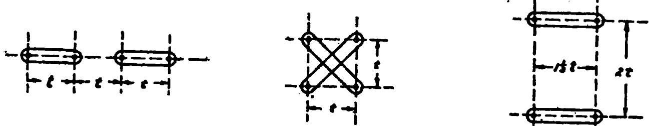
TABLE No. 84.

COMPARISON OF CONDUCTOR ARRANGEMENTS FOR METALLIC TELEPHONE CIRCUITS
WITH RESPECT TO
COEFFICIENTS OF INDUCTION FROM BALANCED THREE-PHASE CURRENTS

POWER CIRCUIT { *Greatest Spacing of Conductors* = *d*; *Height of Lowest Conductor* = *H* }

TELEPHONE CONDUCTORS

HORIZONTAL PHANTOM — SQUARE PHANTOM — VERTICAL PHANTOM.



SEPARATION { MEAN HORIZONTAL (*from plane midway between outside power conductors*) = *X*; MEAN VERTICAL (*from plane of lowest power conductor*) = *Y* }

Power-Circuit Configuration	H/d	t/d	Y/d	X/d	Phantom Circuit: Horizontal	Phantom Circuit: Square	Phantom Circuit: Vertical	Side Circuit of Phantom: Average Horizontal	Side Circuit of Phantom: Square	Side Circuit of Phantom: Square	Side Circuit of Phantom: Average Vertical
Vertical	∞	0.1	0.6	5 40	63.0 0.138	1.17 0.00310	137 2.48	31.5 0.0689	37.6 1.17	99.7 1.31	42.0 0.0918
Horizontal Symmetrical	∞	0.1	1.1	5 40	139 2.48	0.929 0.000254	65.1 0.137	69.5 1.24	102 1.31	37.0 1.18	92.6 1.65
Equilateral Triangle-Base Horizontal	∞	0.12	2.12	5 40	276 4.98	4.68 0.0123	237 4.94	138 2.49	194 3.46	169 3.55	184 3.32

J.C.I.I. - T.R.No.65.
R.A.M.

NOTE: Coefficients are expressed in microvolts induced at 60 cycles per 1000 feet per ampere per phase.

TABLE No. 85

TABLE NO. 86.

COMPARISON OF CONDUCTOR ARRANGEMENTS FOR METALLIC TELEPHONE CIRCUITS

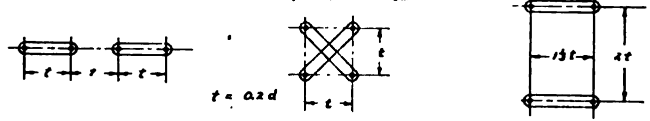
WITH RESPECT TO

COEFFICIENTS OF INDUCTION FROM RESIDUAL VOLTAGE AND CURRENT OF THREE-PHASE POWER CIRCUITS

POWER CIRCUIT: Equilateral Triangle, Base Horizontal, Vertex Upward
Spacing of Conductors = d
Diameter of Conductors = D
height of Lower Conductors = H

TELEPHONE CONDUCTORS

HORIZONTAL PHANTOM — SQUARE PHANTOM — VERTICAL PHANTOM



SEPARATION: Mean Horizontal (from plane of vertex of power triangle) = X
Mean Vertical (from plane of lower power conductors) = 6.6 feet

	H/d	D/d	X/d	Phantom Circuit: Horizontal	Phantom Circuit: Square	Phantom Circuit: Vertical	Side Circuit of Phantom: Average Horizontal	Side Circuit of Phantom: Square	Side Circuit of Phantom: Square	Side Circuit of Phantom: Average Vertical
Residual Voltage	10	0.006	5	3730	36.4	3520	1970	3730	205	2620
			40	106	0.626	294	55.7	203	91.2	74.3
Residual Current	100	any	5	1470	11.8	780	737	1127	347	982
			40	220	0.0398	58.8	110	139	80.5	147

J.C.T.I. - T.R. No. 66.
R4 M

NOTE: Coefficients for residual voltage are expressed in millivolts induced per kilovolt residual and for residual current in microvolts induced at 60 cycles per 1000 feet per ampere residual.

Technical Report No. 66.

January 6, 1917.

CO-ORDINATION OF TRANSPOSITION SYSTEMS FOR POWER AND TELEPHONE CIRCUITS.

OUTLINE.

I. Introduction.

Transpositions in power and telephone circuits constitute a very important factor in determining the interference experienced from parallelism of the two classes of circuits. It is the purpose of this report to review the work done upon the transposition problem in connection with the general investigation by the committee, with particular attention to the development of co-ordinated systems of transpositions,

designed to render the power and telephone circuits mutually noninductive; and to present some examples illustrating the best present practice in this regard. Brief reference will be made to other phases of the problem which are treated in detail in other reports. Part of the matter herein has appeared in previous reports, but is collected here for the sake of convenience and completeness.

A new transposition system for "exposed" telephone lines, developed by the American Telephone and Telegraph Company, and designed, among other features, to meet the need of such a system made manifest by the earlier work of the committee, and in accordance with its recommendations of July 7, 1914, to the Railroad Commission, is described and its use shown by the examples given.

The discussion of power transpositions in this report applies particularly to three-phase circuits. For single-phase circuits the power transposition problem is relatively simple, since a transposition alters the phase of induction by 180°, as in the telephone circuits. The same is true of a 4-wire quarter-phase circuit which can be treated as two single-phase circuits.

II. General Principles.

A—Function of Transpositions.

The function of transpositions is to equalize the relations of the several members of a circuit comprised of two or more parallel conductors to each other and to nearby influences, by interchanging the conductors so that they experience similar conditions in a given total length.

With respect to parallelism, transpositions of the power and telephone circuits are important by reason of their effect upon the mutual relationship of the two classes of circuits, expressed by their mutual inductances and capacitances. The effect of power transpositions is to produce neutralizing effects in adjacent lengths of the telephone circuit, from balanced voltages and currents, while the telephone transpositions tend to equalize the effects on the two sides of the circuit of induction from all sources.

In a metallic telephone circuit, the interchange of position of the two conductors serves to balance their admittances to other circuits and to ground, and may help to balance their series impedances. An example of the latter is the case of a horizontal pole-pair phantom, since the two sides have different series constants. It is assumed that the conditions of the surroundings of the transposed circuit remain unchanged, and that the lengths of circuit on either side of such transpositions are equal. Similarly, in a three-phase circuit, by dividing the given section into three equal parts by means of transpositions at its third-points, the

admittances of the conductors to ground and to other circuits, and the series impedances, are equalized. The effect of power transpositions in reducing the residual voltages and currents due to line unbalances of three-phase circuits is discussed in technical report No. 51.

In Appendix III of the "Report by the Joint Committee on Inductive Interference to the Railroad Commission of the State of California," of July 7, 1914, a more extended discussion is given of the underlying principles of the subject.*

B—Unbalanced Exposures.

The object of co-ordinated systems of power and telephone transpositions is to provide such relative location of transpositions in the two classes of circuits that maximum effectiveness in reducing the unbalanced exposures, for both balanced and residual voltages and currents, is secured.

By "unbalanced exposure" is meant that length of parallel between a telephone and a power circuit, neither of which is transposed, which will produce inductive effects of the same magnitude as those produced in the actual parallel of the same cross-sectional dimensions with the given transpositions in the power and telephone circuits.

Table I and Drawing No. 137† indicate the fundamental basis of calculation of unbalanced exposures for induction between conductors (transverse) and from the conductors to ground or along the circuit (longitudinal) from balanced and residual voltages and currents of a three-phase power circuit.

TABLE I.

Unbalanced Exposures.

Type of induction	Currents and voltages in power circuit	Length of unbalanced exposure
Longitudinal	Residual	$a + b + c + a' + b' + c'$ (See note below.)
Longitudinal	Balanced	$(a + a') \angle 0° + (b + b') \angle 120° + (c + c') \angle 240°$ (Vec.)
Transverse	Residual	$(a + b + c) - (a' + b' + c')$
Transverse	Balanced	$(a - a') \angle 0° + (b - b') \angle 120° + (c - c') \angle 240°$ (Vec.)

NOTE.—Independent of all transpositions.

a, a′, b, b′, c, c′, represent lengths of the several types of exposure, and are proportional to inductive effects in uniform parallels.

Unbalanced exposure lengths serve to indicate the relative effectiveness of different transpositions schemes for a given circuit, and the relative severity of disturbance in different circuits for a given scheme.

*Attention is called to two corrections to Section I of Appendix III. In the tenth line from the bottom of page 32, after the words "capacity and leakage to ground," the words "and series impedance" should be inserted.

In the sentence beginning on the twentieth line of page 33 the word "not" was omitted, after the words "it can." The sentence should read "If the communication circuit has a ground return, it can not be transposed. * * *"

†These are taken from Technical Report No. 39.

Where power and telephone transpositions are not carefully co-ordinated, large unbalanced exposures frequently result. The maximum induction occurs where there are no transpositions at all. Induction from conductors to ground (or longitudinal) from balanced components is reduced in some degree by the power transpositions, however located. If the power and telephone transpositions are not co-ordinated, induction between sides of telephone circuits may be greater with both power and telephone transpositions present than when either kind is removed.

In the above discussion uniformity of exposure within the parallel has been assumed. However, changes in separation of the power and communication circuits, changes in spacing or arrangement of wires, cross-overs, junctions, loads, or in general, any agencies that alter the phase or magnitude of the induction (other than transpositions) are points of discontinuity, and must be allowed for. Lengths of unbalanced exposure have different significance on opposite sides of such points and must be reckoned independently. Sections of line between points of discontinuity are termed "uniform." Ordinarily, it is desirable to secure a balance (zero unbalanced exposure) within each such uniform section.

Appendix I of this report describes the details of a procedure for the calculation of unbalanced exposures in an actual parallel, where there is usually a complex system of telephone transpositions, besides points of discontinuity and power transpositions.

Even though the transpositions are so co-ordinated as to result in balanced exposures as thus calculated, they are not perfectly effective. The reasons are briefly discussed below:

1. Nonuniformity. Only the major irregularities occurring in actual lines can be taken into account in planning transposition schemes, and numerous minor ones must be disregarded.

2. The neutralization and equalization effects discussed above are based on the assumption that there is no attenuation or phase change in the induction or in the power-circuit voltages and currents. The unbalances introduced by phase change and attenuation may be considerable and are discussed in detail below.

3. It is impracticable to perfectly balance the series impedances and the admittances to ground of the sides of a telephone circuit, hence the induction from wires to ground and along the circuit (longitudinal) causes current in the metallic circuits.*

C—Phase-Change and Attenuation.

The speed of electric waves, though great, is finite, and they are attenuated in their travel along the power and telephone circuits. The limiting speed is 186,000 miles per second, but they may be retarded so that the actual speed is much less. Thus, in the time required to

*A discussion of this subject is given in Technical Report No. 55

complete one cycle at 1000 cycles per second, an electric wave can not travel more than 186 miles and may travel only a small fraction of that distance. Attenuation varies greatly with the character of the circuit.

The principal practical interest of these phenomena in the present study lies in their bearing on the spacing and character of power transpositions in uniform parallels. In order to determine the extent of the unbalanced exposures due to this cause, consider first a very simple parallel, a uniform exposure of a telephone conductor to a three-phase power circuit with power transpositions at the third-points, so that there are equal exposures of the three different phase arrangements. See Fig. 1, drawing No. 354.

Let L = length of the barrel in miles.
V = speed of propagation in miles per second.
α = attenuation per mile.
β = phase-change per mile, in radians.
f = frequency, cycles per second.
U = unbalanced exposure in miles.

Assume first no phase-change or attenuation in the balanced voltages and currents along the power circuit, and that reflection from the ends of the telephone circuit is negligible. These balanced components in the power circuit induce waves in the telephone conductor, which are 120° apart in phase, at points L/3 apart, as for instance at P_1, P_2, P_3. That is, the induction at P_2 differs in phase by 120° in opposite directions from that at P_1 and P_3, respectively. If the wave travelled along the telephone conductor without attenuation at infinite speed there would be at every point three waves exactly neutralizing each other and the resultant would be zero.

The time for a wave to travel L/3 miles along the telephone circuit is $t = L/3V$ seconds. One second corresponds to 360f degrees of time-phase change. So the change of time-phase represented by t is

$$\left.\begin{aligned}\phi &= \frac{2\pi fL}{3V} \text{ in radians} \\ &= \frac{120\, fL}{V} \text{ in degrees}\end{aligned}\right\} \qquad (1)$$

For a fixed value of ϕ and V, the length and frequency vary reciprocally, that is, from equation (1)

$$Lf = \text{constant} \qquad (2)$$

Considering point P_1, the waves from sections B and C arrive there $120° + \phi$ and $240° + 2\phi$ out of phase with the wave from A. The

same relationship of the three waves holds for all points to the left of P_1, since the three waves travel equal distances from P_1 and are distorted alike.

Fig. 2, drawing No. 354, shows the resultant unbalance, for the condition that attenuation is zero, and thus shows the effect of phase change only. OAB is an equilateral triangle, representing the case of infinite wave-speed. AD represents the wave from Section B, and DE the wave from Section C, giving, with OA, representing the wave from Section A, the resultant OE. For practical purposes a graphical solution for OE is amply accurate where ϕ is several degrees in magnitude. (In Fig. 2, the value of ϕ is 8°.) There is, however, a simple solution which involves the triangle OCE. OC = BD by construction. The lengths OA, AB and DC are made equal to L/3. Strictly, in place of L/3, the chord of an arc of length L/3 subtended by an angle of ϕ degrees should be used. For small values of ϕ the difference is negligible.

$$BD = 2\ AB \sin \tfrac{1}{2}\phi = 2\ \frac{L}{3}\ \sin \tfrac{1}{2}\phi \tag{3}$$

For small values of ϕ, $\sin \phi = \phi$, so

$$BD = \frac{L}{3}\ \phi = U_b \tag{4}$$

Expressing ϕ in degrees, as in (1), it is

$$U_b = \frac{L\phi}{3}\,\frac{\pi}{180} = \frac{L\phi}{172} \tag{5}$$

Similarly
$$CE = \frac{2\,L\phi}{3} = 2\,U_b \tag{6}$$

U is obtained by combining OC and CE, with phase difference of $120° + \tfrac{1}{2}\phi$. That is

$$U = -U_b\,[1 + 2\,\underline{/120° + \tfrac{1}{2}\phi}]\ \underline{/150° - \tfrac{1}{2}\phi} \tag{7}$$

or substituting the value of U_b from (1) and (5),

$$U = \frac{2\pi f}{V}\left[\frac{L}{3}\right]^2\left[1 + 2\underline{/120 + \tfrac{1}{2}\phi}\right] = \frac{2\pi f\,(L)^2}{9V}\left[1 + 2\underline{/120 + \tfrac{1}{2}\phi}\right] \tag{8}$$

whence it is evident that the unbalance varies directly as the frequency, as the square of the length of barrel, and inversely as the wave-speed, provided that ϕ is small. The term in parentheses is nearly constant for small values of ϕ, being 1.732 at $\phi = 0$ and decreasing nearly uniformly to 1.558 at $\phi = 20°$. See Fig. 3, Drawing No. 354.

The discussion above (and Fig. 2 of Drawing No. 354) is based on the assumption that, traveling from the point of observation P_1, each transposition causes a lag of 120°. When, as in considering the same

barrel from the opposite end, the power transposition causes a lead of 120°, the last term of equation (8) becomes $1 + 2 \angle 120° - \frac{1}{2}\phi$, that is the sign of $\frac{1}{2}\phi$ is changed. Hence the values are increased by a factor $\frac{1 + 2\angle 120 - \frac{1}{2}\phi}{1 + 2\angle 120 + \frac{1}{2}\phi}$, which amounts to 1.80 for the extreme case of $\phi/2 = 30°$.

An alternative form of expression of U may be derived from consideration of the similar isosceles triangles O E F and A D F, whence

$$U = \frac{2\pi f\,(L)^2}{9V}\left[\sqrt{3} + \frac{180}{\pi}\;\frac{\cos\phi - 1}{\phi}\right] \tag{8-a}$$

for ϕ in degrees, when ϕ is small. This expression is simpler for calculation than (8).

The impedance measurements on the Santa Cruz-Watsonville telephone line* for circuits consisting of one and of four wires, respectively, with ground for the other side, and for frequencies of 240 to 1200 cycles per second, gave results as follows:

α varies from 0.004 to 0.008 and
β varies from 0.011 to 0.056

From these data an average value of

$V\ (= \frac{2\pi f}{\beta})$ of 135000 miles per second is obtained.

For $\phi = 60°$, the figure ADEO of Drawing No. 354 collapses on the straight line OA. If the attenuation is nil, the resultant is

$$U = \frac{L}{3} \times \frac{3}{\pi} = \frac{L}{\pi} \tag{9}$$

From equation (1), at a frequency of 1000 cycles† this would take place at

$$L = \frac{60 \times 135000}{120 \times 1000} = 67.5 \text{ miles,}$$

Then $U = 21.5$ miles.

Considering the effect of attenuation for this same case of $\phi = 60°$; let $x = \alpha L/3$. Then with close approximation

$$U = e^{1x}\frac{L}{\pi}(1 - e^{x} + e^{2x}) \tag{10}$$

*See technical report No. 29.

†1,000 cycles is chosen, owing to the ease of reckoning for other frequencies, the values of ϕ and U being nearly directly proportional to the frequency.

Using L = 67.5 miles, and $\alpha = 0.010$, U = 16.1 miles, so that in this extreme case, the large unbalance due to phase-change is not greatly affected by even the large attenuation assumed.

At the other extreme, considering only unbalance due to attenuation, with close approximation

$$U = \frac{Le^{ix}}{3} \quad (1 + e^{x} \angle 120^\circ + e^{2x} \angle 240^\circ) \qquad (11)$$

or, in the case given above of L = 67.5 miles, U = 6.2 miles, to be compared with 21.5 miles. For moderate lengths of barrel it varies approximately as the square of the length and is nearly independent of frequency since α varies but slowly therewith.

Consider next a 6-mile barrel at 1000 cycles.

From (1), $\phi = 5^\circ .33$

From (7), U = 0.313 mi.

The exact solution of Fig. 2, Drawing No. 354, is

for $\alpha = 0$. U = 0.314 mi.

and for $\alpha = 0.010$ U = 0.312

The instances given indicate that for the range of frequencies and distances involved in practice, the attenuation effect is of minor importance, and the approximate formula of equation (7) is amply accurate for all ordinary purposes. For the rather extreme case of a 24-mile barrel at 1500 cycles, equation (7) indicates an unbalance of 6.5 miles, which is about 6 per cent too high.

The speed of propagation is generally from 170000 to 180000 miles per second in power circuits and in nonloaded metallic telephone circuits. The attenuation along power circuits is also much less than the values given above (0.001 in a typical case). The effect on the unbalanced exposures of attenuation and phase-change of the waves in the power circuit may be either additive or subtractive with respect to the effect of the corresponding quantities in the parallel telephone circuit, depending on the position chosen on the telephone line. For example, suppose the wave-speed the same in both power and telephone circuits. Then at one end of the telephone line (Fig. 1, Drawing No. 354) the waves arrive 120° apart. At the other end the angular distortion is double that which would occur, were there no phase change along the power circuit. Thus, at opposite ends of a parallel, the effects of phase-change and attenuation along the power circuit are opposite. Moreover, it has been shown that the attenuation is of minor importance. Hence, the discussion first given, considering the phase-change along the telephone circuit only, gives a satisfactory view of the average effects, while the worst will be roughly double this value. Throughout this discussion it

is assumed that there is no reflection of waves from the terminals of the communication circuit.

On Drawing No. 355 are presented some curves showing the unbalances for various frequencies and lengths of barrel. Except for the extreme cases, where graphical construction was used, the curves were obtained by equation (7). The curves are for frequencies of 60 cycles and its 5th, 15th and 25th harmonics. Values for other frequencies and the given wave-speed of 135000 miles per second are obtainable by direct interpolation. For a given frequency and length of barrel the unbalances will be smaller with higher wave-speeds. The curves show that unbalances of 0.1 mile occur as follows:

Frequency Cycles per second	Length of barrel miles
300	6.1
900	3.6
1500	2.8

In illustration of the effect of considering the attenuation and phase change in the power circuit, as well as in the telephone circuits, Table I-A is presented, showing results of some computations on the basis of (1) power line infinite in length, (2) open-circuited at end of parallel.

TABLE I-A.

Unbalance Due to Phase-Change and Attenuation in Both Power and Telephone Lines. Single Barrel—500 and 1500 Cycles per Second.

Power line	Infinite		Open-circuited at end of parallel	
Length of barrel—miles \ Frequency cycles per second	500	1500	500	1500
3	0.06	0.18	0.05	0.13
18	2.2	6.8	1.2	5.7
30	6.1	17.6	3.7	168
42	11.8	30.0	8.2	28

The results for the infinite power line are somewhat higher than those shown on P. I. C. No. 355, and would be about double those values if allowances were made for difference in assumed speed of propagation.

The enormous unbalance at 1500 cycles for the 30-mile barrel with the power line open-circuited, is due to the fact that the line is practically a quarter-wave length at this frequency. At other frequencies the difference between the two conditions is small.

Phase-change and attenuation effects determine the frequency of occurrence of telephone transpositions for the reduction of cross-talk. The close spacing of telephone transpositions for this purpose is in general adequate as regards like effects due to parallels. This is particularly true in the case of the telephone transposition systems designed for co-ordination with power transpositions (see Section V).

The discussion hitherto given has been concerned with the effects of one barrel in the power circuit. Owing to the large unbalance with long barrels it becomes necessary to use more than one barrel in long uniform parallels. Hence, a question arises concerning the arrangement of the transpositions where there are two or more barrels. In order to study the effects of various arrangements, a 24-mile parallel will be considered. For convenience, a phase change of 2.5° per mile is used, corresponding to $f = 1000$ cycles and $V = 144000$ miles per second (or 940 cycles and $V = 135000$ miles per second). Considering the effect at one end of the 24-mile section and disregarding attenuation and reflection, there is an unbalanced exposure of 22.9 miles with no transpositions, and with one barrel, 4.3 miles. When there are two or more barrels three simple arrangements may be used, having barrels of equal length. With two of the arrangements every third power transposition is omitted. Let A, B, C represent unit lengths of the three different phase arrangements. Using like transpositions equally spaced, the sequence of phase arrangements is ABC, ABC, ABC, etc., designated as "continuous" barreling (abbreviated as "C"). The same, with every third transposition omitted, gives ABC, CAB, BCA, ABC, CAB, etc., designated as "halting" (H). If, besides omitting every third transposition, those of alternate barrels be reversed, it is ABC, CBA, ABC, CBA, ABC, etc., designated as "halting-reversing" (H-R).

The first repeats itself in every barrel; the second in every three barrels; the third in every two barrels. The unbalances for the three types of barrelling are given in Table II for 0, 1, 2, 3, 4, 6, and 8 barrels, corresponding to no transpositions, 24, 12, 8, 6, 4 and 3-mile barrels.

With continuous or C barrelling the unbalances of successive barrels add nearly in common phase, lagging successively by an amount due to the phase change per barrel-length.

With H barrelling, the unbalances of three successive barrels nearly balance each other, being successively out of phase by 120° + the phase change per barrel.

With the H-R barrelling, the effects of two successive barrels are almost exactly opposite in phase and the magnitudes are in the ratio $\frac{1 + 2\,\underline{/120 + \frac{1}{2}\phi}}{1 + 2\,\underline{/120 - \frac{1}{2}\phi}}$, as shown above. For example, with 4-mile barrels the values are 0.132 and 0.137 miles, respectively. See Table II for other values.

TABLE II.

Unbalanced Exposures in a 24-mile Parallel.

Different Lengths and Types of Barrelling.

V = 144000 miles per second; f = 1000 cycles per second.

Barrels		Unbalanced exposure—miles				
		One barrel		Total unbalance		
Number	Length, miles	ABC	CBA	C	H	H-R
0	—	—	—	22.9	—	—
1	24	4.3	5.1	4.3	—	—
2	12	1.15	1.27	2.2	1.6	0.6
3	8	0.52	0.55	1.49	0.82	0.43
4	6	0.294	0.310	1.13	0.19	0.15
6	4	0.132	0.137	0.76	0.08	0.07
8	3	0.074	0.076	0.57	0.10	0.04

For 2, 4 and 8 barrels in all, the H-R barrelling has an integral number of cycles in the 24 miles and is superior to the H type of barrelling, which has not. Similarly for three barrels, the H is better than the H-R. With six barrels, both H-R and H types have an integral number of cycles and are nearly equally good, the H-R being a little better. Nine H barrels would give an unbalance practically equal to that of eight H-R barrels. The advantage of both types of halting barrels over continuous barrels is clearly shown.

Consideration of the effect of attenuation (0.010 per mile) with 4-mile barrels, indicates a reduction of the unbalances given in the table, by 10 per cent or less.

As noted in the case of single barrels, with a finite length of power line the unbalances may greatly exceed the values indicated in Table II, at lengths of parallel and frequencies approaching a quarter wave length. If the length of one barrel is a quarter wave-length the unbalance may be still greater.

Many combinations of transpositions could be devised, but the figures given illustrate the simple schemes, by which the unbalances are reducible to a point beyond which it is probable that minor irregularities will control the result.

It would be well to determine by more rigid analysis or by experiment the effect of the factors which have been neglected in the above discussion, and to extend the scope of the investigation.

D—Types of Power Transpositions.*

A power transposition of the ordinary type changes the phase of induction from a three-phase circuit by 120°. At such a transposition the several phase-wires interchange positions in cyclic order. If the

*See also T. R. No. 67.

direction of rotation be reversed, the direction of change of phase is also reversed. The ability to change the phase of induction by either + or — 120° at a transposition in a three-phase line is frequently of value in securing longitudinal balance, especially where the power line is alternately on one side, then on the other, of the telephone line; and also where transverse balance to balanced components is obtained by both power and telephone transpositions. Such power transpositions may be distinguished as "Normal" or "Reversed," "Normal" being conveniently defined for existing lines by a transposition already installed. (It should be observed that the designation "left over right" or "right over left," while definite for a circuit of triangular configuration, has no significance for flat or vertical lines.) Examples of such use will be found in Section VI.

Where there are three or more power conductors it is possible to interchange pairs, leaving other conductors undisturbed. For a three-phase line this procedure causes the phase angle of the induction from balanced components to be reversed in sign with respect to a reference line parallel to the vector representing the wave on the undisturbed conductor. However, the principal use of such special transpositions is to annul the effect of cross-overs. The longitudinal induction into a given telephone conductor from a power circuit having vertical configuration suffers no change due to a cross-over, if the separation of each power conductor and the given telephone conductor remains the same. With a power line having two conductors in a horizontal plane and its third conductor symmetrically placed with respect to the first two, the same condition of constant distances from each conductor of the power circuit to the telephone conductor is fulfilled if the first two conductors of the power circuit are interchanged at the cross-over. For any unsymmetrical power circuit, the configuration must be "reversed," that is, the arrangement on one side of the cross-over must be the image of that on the other. Thus a "wishbone" line would need to have the vertex toward (or away from) the telephone conductor, on both sides of the cross-over.

When the separation of the power and telephone circuits is large compared to the length of the telephone cross-arms, and the same on both sides of the cross-over, the effect of the cross-over on longitudinal induction may be approximately nullified for all the telephone conductors.

To secure the effect of an ordinary transposition it is only necessary to superpose a cyclic transposition upon the rearrangement necessary as discussed above. Hence it is sometimes feasible either to eliminate the discontinuity, for longitudinal induction, occasioned by a cross-over; or to use it as an ordinary transposition point; which is con-

venient, because cross-overs are usually made neutral points in the telephone transposition system. An objection to the use of cross-over transpositions is that for small separations between lines the change in the magnitude and phase of the induction within the space of a telephone cross-arm may quite appreciably affect the balance for the circuits on the ends of the cross-arm.

The methods of accomplishing these results at a cross-over are shown on drawing No. 364.

III. Studies of Particular Cases.

During the early part of the Joint Committee's investigation several studies were made of schemes to decrease the unbalances existing between power and telephone circuits which were under experimental investigation. A brief review of these studies is here given.

A—Morgan Hill—Gilroy.

The transpositions shown as now existing in the 22-kV. circuit between Morgan Hill and Gilroy (see Section VI-A, and drawing No. 358) were placed while the Joint Committee was investigating this parallel.* These transpositions reduced the longitudinal unbalances from 13400 and 30300 feet to 1600 and 100 feet, north and south of the cross-over, respectively, but left large transverse unbalances. A large part of the 22-kV. line outside the parallel remained untransposed; the residual voltage was not noticeably affected.

B—Salinas—North.

A study was made during the tests at Salinas** to determine upon modifications of the existing transposition system§ which would reduce the transverse unbalances. The plan thus worked out required ten power transpositions and extra transpositions in the telephone circuits, and would have necessitated the special design of transpositions for any new telephone circuits added to the line.

C—Santa Cruz—Watsonville.

It was planned to conduct rather extensive tests to compare various transposition systems on the Santa Cruz-Watsonville parallel.† Accordingly a study was undertaken‡ to determine the unbalanced exposures:

1. With existing transpositions.
2. With power transpositions removed.

*See technical reports Nos. 16-20.
**See technical report No. 15, page 19.
§See Section VI-C, and drawing No. 359 for description of existing arrangement and unbalanced exposures.
†See Section VI-E, for description.
‡See technical report No. 39.

3. With telephone transpositions unchanged and, in the power line, the following:
   (a) two barrels.
   (b) 6-mile barrels.
   (c) 2-mile barrels.
4. With "Standard" A and Z sections§ in the telephone line, and in the power line,
   (a) no transpositions.
   (b) two barrels.

Meanwhile an "Experimental Modification of Transpositions of the 'Standard' System for Balance with Power-Line Transpositions" had been developed by the American Telephone & Telegraph Company for one cross-arm (ten wires) which provides balance to 6, 3, 1 and ½-mile barrels.

The unbalances were then determined* for several schemes in which this modified system was used, using nominal 6-mile power barrels and various arrangements of the telephone transposition sections.

These studies demonstrated the large unbalances resulting from the haphazard location of the telephone transpositions with respect to points of discontinuity (in this case the cross-over and ends of parallel) and the unsuitability of the "Standard" transposition system for co-ordination with power transpositions. It proved impracticable to carry out the test as planned at Santa Cruz,** but observations of the effectiveness of transpositions were made at San Fernando, and are given in technical reports Nos. 54–57.

IV. Characteristics of "Standard" Telephone Transposition System.

In connection with the studies discussed in III the transverse unbalances for induction from balanced three-phase voltages and currents, occasioned by using a "Standard" 8-mile telephone transposition section in a uniform parallel, with ½, 1, 1½, 2, 3, 4 and 6-mile barrels in the power line, were determined for the physical and the end horizontal phantom circuits of a 40-wire telephone line. Phase change and attenuation were neglected, and it was assumed that all transpositions were located exactly at the theoretical positions. The power barrels are of the continuous type, with certain transposition points directly opposite mile-points of the telephone transposition section.

Table III gives the transverse unbalances in miles per 8-mile section. Table IV shows the distance in which balance is obtained for those circuits whose unbalance is given as zero in Table III.

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§"A"—"Standard" "8-mile" section. "Z"—"Standard" "4-mile" section.
*See technical report No. 46.
**See technical reports Nos. 45, 48, 49.

TABLE III.

Unbalanced Exposures—Miles in One "Standard" 8-mile Section.

Transverse Induction from Balanced Three-Phase Power Circuit.

Circuit	Length of power barrel (miles)						
	½	1	1½	2	3	4	6
1–4	0	0	0.87	0	1.73	4.62	3.46
7–10	0	0	1.50	0	3.00	4.00	0
1–2	0	2.31	0.87	1.15	0	1.15	0
3–4	0	2.31	0.87	1.15	0	1.15	0
5–6	0	4.00	0	2.00	0	1.00	0
7–8	0	0	0	0	0	0	3.00
9–10	0	0	1.50	0	3.00	0	3.00
11–14	0	0	0.87	4.62	1.73	2.31	0
17–20	0	0	1.50	4.00	0	2.00	0
11–12	0	0	1.50	0	0	0.58	0
13–14	0	0	1.50	0	0	0.58	0
15–16	0	0	2.60	0	5.19	0	0
17–18	0	0	1.50	0	1.50	0	3.00
19–20	0	0	3.00	0	1.50	0	3.00
21–24	0	0	4.50	0	0	0	0
27–30	0	0	1.50	0	0	2.00	0
21–22	0	0	0	2.00	1.50	1.04	3.00
23–24	0	0	1.50	2.00	1.50	1.04	3.00
25–26	0	0	0	0	0	1.00	0
27–28	2.31	1.15	0.87	0.58	0.87	2.89	1.73
29–30	2.31	1.15	0	0.58	0.87	2.31	1.73
31–34	0	0	2.60	0	5.19	0	0
37–40	0	0	2.60	0	0	0	0
31–32	2.31	1.15	1.50	2.08	0	1.04	0
33–34	2.31	1.15	1.50	2.08	0	1.04	0
35–36	0	0	2.60	0	0	1.15	0
37–38	0	0	0.87	2.31	0.87	0.87	0
39–40	0	0	0.87	2.31	0.87	1.44	0
No. circuits not balanced	4	7	23	13	14	20	9

TABLE IV.

Distances Required to Obtain Balance. "Standard" 8-mile Section.

Transverse Induction from Balanced Three-Phase Power Circuit.

Circuit	Length of power barrel (miles)						
	½	1	1½	2	3	4	6
1–4	½	1	*	2	*	*	*
7–10	⅛	1	*	4	*	*	2
1–2	1	*	*	*	1	*	1
3–4	1	*	*	*	1	*	1
5–6	1	*	½	*	½	*	½
7–8	1	2	8	8	4	8	*
9–10	1	4	*	5	*	6	*
11–14	⅛	1	*	*	*	*	2
17–20	½	2	*	*	1	*	1
11–12	1	8	*	7½	1	*	1
13–14	1	8	*	7½	1	*	1
15–16	⅛	1	*	4	*	8	2
17–18	2	8	*	3	*	7	*
19–20	1	8	*	6	*	8	*
21–24	½	2	*	4	1	6	1
27–30	½	4	*	4	1	*	1
21–22	2	8	8	*	*	*	*
23–24	2	8	*	*	*	*	*
25–26	4	4	½	3	½	*	½
27–28	*	*	*	*	*	*	*
29–30	*	*	8	*	*	*	*
31–34	⅛	1	*	4	*	8	2
37–40	½	2	*	8	1	6	1
31–32	*	*	*	*	1	*	1
33–34	*	*	*	*	1	*	1
35–36	½	2	*	4	1	*	1
37–38	2	8	*	*	*	*	2
39–40	2	8	*	*	*	*	2

*Balance not obtained within one 8-mile section.

Half-mile power barrels balance, usually in fairly short distances, to all except four physical circuits. Large unbalances on some circuits and long distances between balance-points on the others are characteristic of the other barrels. The ½, 1, 2, and 4-mile barrels provide longitudinal balance, while for 1½, 3 and 6-mile barrels the longitudinal unbalances per 8-mile section are ½, 1, and 2-miles, respectively; ¼-mile and 12-mile barrels would have zero transverse unbalances.

In order to show how the resultant unbalanced exposure varies throughout an 8-mile section, diagrams were drawn in polar co-ordinates, indicating the variation of the resultant unbalance in phase and magni-

tude from the beginning of the section to any other point in it. Such diagrams for a few typical circuits are reproduced here. See drawings Nos. 366 and 367. The numbered points are the successive telephone transposition poles ¼-mile apart.

Drawing No. 366 (Fig. 1) shows a case of large unbalance, with continuous 3-mile barrels. Fig. 2 shows a case where balance is secured in 8 miles (continuous 4-mile barrels), with large unbalance of a partial section. Figs. 3 and 4 show how a large unbalance is built up on a frequently transposed telephone circuit by one-mile barrels, and the excellent balancing of the same circuit to one-half mile barrels (continuous in both cases). A great variety of figures occur, of which those shown are fairly representative.

Drawing No. 367 shows transverse and longitudinal unbalance diagrams for one circuit, with 2-mile continuous, halting, and halting-reversing barrels. Longitudinal balance is obtained in all cases, but halting barrels reduce the transverse unbalances to ½ that for the other two cases. In three sections (24 miles) there would be exact balance, for halting 2-mile barrels. It must be remembered that no account is here taken of phase-change and attenuation, the effects of which are set forth above in III.

From such drawings one may obtain the resultant unbalance between any two points in the section; maximum length of unbalanced exposure between any two points; and distances required to obtain balance.

The unsuitability of the "Standard" system for coordination with power barrels is plainly shown by the results of these studies.

V. Development of "Exposed-Line" Telephone Transposition Systems.

Methods for transposing telephone lines involved in parallels have been developed by the American Telephone and Telegraph Company which are readily combined with power transpositions to secure coordinated systems, and which make it possible to take account of the discontinuities more economically than was possible with the "Standard" system. The several methods are described below and their uses and limitations discussed.

A—The "E" Section.

An elaborate study was made to develop a new section having a nominal length of eight miles (actually eight miles or less), which should preserve a high standard of balance with respect to cross-talk and telegraph induction, give balance within short lengths for all circuits with respect to the induction from both balanced and residual voltages and currents of parallel power circuits, and properly coordinate with power barrels of several lengths.

Investigation of $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{2}$, 2, 3, 4 and 6-mile continuous barrels in relation to the 64 fundamental types of telephone transpositions indicated that economical telephone transposition systems could not be developed to coordinate with barrels other than those $\frac{1}{2}$, 3 and 6 miles in length. Other variations of power barreling were considered, involving halting barrels, reversals, and special irregular arrangements of power transpositions. The only one that appeared to have advantages involves halting and reversing $\frac{1}{2}$-mile barrels as against two continuous barrels per mile, requiring four power transpositions between mile points instead of five. With the "four-transposition" scheme, the transpositions of the two component barrels are rotated oppositely. In either case, transpositions are omitted at even mile points.

Other considerations which involved extensive study are: use of fundamental transposition types on given pin positions or on given side circuits of phantoms; extent of occurrence of circuits in the various pin positions; economical limits to cross-talk and telegraphic induction; number of transposition poles per section; and the relation of the new system to existing telephone transposition systems. The chosen plan permits of 256 permutations giving about equal approaches to cross-talk balance, from which the most economical arrangement was determined.

The system evolved has the following characteristics: balance to non-transposed foreign circuits in each mile; balance to three-phase power circuits transposed for $\frac{1}{2}$ and 3-mile barrels (and to barrels whose length is any integral multiple of 3 miles) in each mile. The system of $\frac{1}{2}$-mile barrels, halting at mile points only, having five transpositions (with the same direction of rotation) between mile points, was chosen on account of the greater economy found to obtain therewith. The American Telephone & Telegraph Company's drawings 93-E-3 and 93-E-4 show the system for the commonly occurring circuits of four cross-arms. Copies of these drawings are attached. For horizontal pole-pair phantoms and some vertical phantoms 64 transposition poles per 8-mile section are required. Further design work is necessary for the circuits below the fourth crossarm. An L section of half the length of the E section (that is, suited for 4 miles and less) and having similar characteristics, is in preparation.*

It is desirable to use these transposition sections wherever possible in parallels on account of the combination of good cross-talk balance, frequent balance to foreign circuits and provision for coordination with power transpositions, and they are the normal means to be used where uniformity and length of parallel justify. It is frequently possible to

*See Proc. A. I. E. E., July, 1918, p. 739.

include points of discontinuity within sections by so placing the telephone transposition sections that the discontinuities occur at mile-points, which are neutral points with respect to external sources of induction.

B—Consecutive "X" Sections.

In the "Standard" system of telephone transposition the term "X section" designates a section ½ mile or less in length, containing three transposition poles at its quarter points.

The use of X sections in parallels often involves the use of several in succession, each "X section" having transpositions added so as to secure balance within itself to nontransposed parallel foreign circuits. Transpositions are introduced at the junction points of successive sections to reduce the cross-talk as far as possible. See American Telephone & Telegraph Company Drawings 92-E-75, 76, 77, attached.

Where parallels contain numerous points of discontinuity separated by short distances and where short lengths intervene between ends of longer sections and points of discontinuity, these consecutive X sections are very convenient. Junction points of sections are placed opposite power transpositions and points of discontinuity. It is not necessary, though desirable, that successive sections be of the same length. Where lengths greater than ½ mile occur, yet too short for an L section, two or more X sections are used. Consecutive X sections are decidedly inferior to the E and L sections on account of the relatively poor cross-talk balance and the greater cost for a given length.

C—"Whole-Line" Transpositions.

Where two-wire circuits only are concerned the cross-talk balance of the existing telephone transposition system is not disturbed by inserting transpositions in all the circuits at any point, that is, transposing the "whole-line." It is thus possible in such cases to place extra transpositions wherever required to balance the exposures to power circuits. Where there are phantom circuits on the telephone line the same is not generally true but by observing proper precautions the method may be employed in some cases without increasing the cross-talk to excessive values. Drawing 93-E-2 presents a method applicable to 8-mile sections of the Standard or ABC systems. All telephone circuits are transposed at points between the regular transposition points as follows:

1. *Quarter-mile units.* All telephone circuits are transposed at the ¼ and ¾ points of the distances between adjacent regular transposition points.

2. *Half-mile units.* All telephone circuits are transposed at points midway between an AA or BB pole and the adjacent regular transposition poles on either side. In addition, where a two-wire transposition occurs in one side of a phantom, but none in the other, at the AA or BB pole, such two-wire transposition must be moved to the adjacent pair of pins so that it remains in the side circuit where it was originally.

Power transpositions are placed opposite points midway between the "whole-line" transposition points.

The best use of this method is obtained in very short parallels or in short portions of long parallels where it is inconvenient to use the exposed-line sections, and where otherwise a long section of line outside the parallel would have to be retransposed. It is permissible to use it for distances greater than one mile on nonloaded lines, though considerable increase of cross-talk is occasioned thereby. The method is a very simple and economical solution of the problem of coordination of transpositions, where it is applicable.

D—Special Transpositions.

In some instances where there are but few telephone circuits and where the consideration of cross-talk balance will permit, additional transpositions may be inserted in a few telephone circuits, having great unbalanced exposures; and such specially located transpositions may provide the simplest solution. This is apt to involve the redesign of future telephone circuits and has the disadvantage of increasing the nonstandard construction in the telephone plant.

VI. Practical Examples.

In the design of coordinated transposition systems for parallels a wide variety of conditions is encountered. Particularly is this true of parallels which have been in existence for some time and where the lines were located with little or no thought of inductive interference. Thus unnecessary crossovers, irregularities of separation, power and telephone transpositions so located with respect to each other as to be very inefficient, and the like, are found, besides the unavoidable points of discontinuity. Hence nearly every case presents some unique features. It often requires the use of two or more of the general methods as hitherto discussed to meet these conditions.

In the case of a projected parallel, it is possible to secure greater uniformity and thus often lessen the transposition requirements. Towers can sometimes be advantageously spaced, to permit of securing the proper location of power transpositions.

In order to illustrate the application in practice of the methods hitherto discussed, description will be given of several parallels which

have been the subject of investigation by the Joint Committee, showing the existing transpositions and rearrangements involving the use of "exposed-line" telephone transpositions and the utilization of existing transpositions in so far as practicable. For evidently the practical problem is to determine a scheme which shall give maximum effectiveness with minimum alteration in either power or telephone circuits.

A summary is given of the more important characteristics of each parallel. Among these the item "induced volt miles" is included, being the product of the length of the parallel in miles by the computed induced voltage between isolated telephone conductors and ground due to the balanced voltages of fundamental frequency with the given configuration from an untransposed power circuit.

Unbalanced exposure figures are given which include, in some cases, circuits not now existent on the poles, but which will occur when the telephone circuits increase in number. These figures take account only of transpositions and do not indicate the differences among circuits due to position, distance between sides, loading and connected apparatus. Also the effects of phase-change and attenuation are neglected.

A—San Jose–Gilroy Parallel.

Communication Company—The Pacific Telephone and Telegraph Co.

Power company	S. & S. F. P.	C. C. G. & E.	P. G. & E.
Power circuit—			
Voltage—kV.	55	22	11
Configuration	Vertical	Flat	Flat
Length of parallel—miles	20.3	8.6	2.1
Average horizontal separation—feet	70	70	70
Induced volt—miles	4,200	400	70

For 20.3 miles between San Jose and Gilroy the single-circuit 55-kV., three-phase, 60-cycle transmission line of the Sierra and San Francisco Power Company parallels the main coast and San Joaquin Valley route circuits of The Pacific Telephone and Telegraph Company.* For 8.6 miles north from the Gilroy end, this parallel is coincident with the parallel involving the Coast Counties Gas & Electric Company's single-circuit, 22-kV., three-phase 60-cycle transmission line† from Morgan Hill to Gilroy, which is built under the 55-kV. line.

At the San Jose end, a single-circuit 11-kV., three-phase, 60-cycle distribution line of the Pacific Gas & Electric Company is built under the 55-kV. line for 2.1 miles.

The Western Union Telegraph Company's circuits and Southern Pacific railway signalling circuits are also paralleled by these power lines, for about 17 miles.

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*Technical report No. 38 summarizes the tests made on this and other parallels with the Sierra & San Francisco Power Company lines.

†Technical reports Nos. 2, 16, 17, 18, 19, 20, 21 present results of tests on the Coast Counties Gas & Electric Company's 22-kV. line and the telephone circuits.

On drawing No. 359 there are shown in the upper figure the lengths of parallels and existing transpositions of both power and communication lines. The parallels with the 22 and 11-kV. power circuits extend beyond the 55-kV. parallel, but are considered herein only in so far as they affect the plans within the limits of the latter.

Drawing No. 358 shows the relation of this and other parallels to the telephone system, and the connections of the transformers on the 55-kV line. A discussion of the residual voltages and currents of this line is given in technical report No. 61.

The 22-kV. line is isolated from ground. It is partly of triangular and partly of flat configuration, with some transpositions which reduce its residual voltage due to capacitance unbalance.

Table V describes the telephone circuits involved.

TABLE V. Telephone Circuits—San Jose-Gilroy Junction.

Pin position	Number	Type	Weight, lb. per mile	Terminals
1–4	*1445L	Physical	172	San Francisco–Fresno
	*1446L	Physical	172	San Francisco–Fresno
	*1440L	Phantom	172	San Francisco–Bakersfield
5-6	1343L	Physical	287	San Francisco–San Luis Obispo
7–10	*1450L	Physical	172	San Francisco–Los Angeles
	*1443L	Physical	172	San Francisco–Fresno
	*1438L	Phantom	172	San Francisco–Fresno
11–14	1767	Physical	172	San Jose–Hollister
	1776	Physical	172	San Jose–Hollister
	1690	Phantom	172	San Francisco–Hollister
15–16	1475	Physical	287	San Francisco–San Luis Obispo
17–20	295	Physical	172	San Jose–Salinas
	159	Physical	172	San Jose–Salinas
	1673	Phantom	172	San Jose–Salinas
21–24	151	Physical	172	San Francisco–Salinas
	269	Physical	172	San Francisco–Salinas
	1680	Phantom	172	San Francisco–Salinas
25–26	1441	Physical	435	San Francisco–Los Angeles
27–30	78	Physical	172	San Jose–Gilroy
	297	Physical	172	San Jose–Gilroy
	1537	Phantom	172	San Jose–Morgan Hill
35–36	1769	Physical	No. 9 iron	San Jose–Madrone
37–40	*1442	Physical	172	San Jose–Fresno
	*1434	Physical	172	San Jose–Fresno
	205	Phantom	172	San Jose–Fresno

L signifies a loaded circuit.
*These circuits branch at Gilroy Junction and are the ones involved in the short parallel from Gilroy Junction east.

The 55-kV. parallel begins 6½ miles south of San Jose at power pole 25/8 (telephone pole 6/20), and continues for 12 miles to the first crossover. There are seven power transpositions and two telephone load-points in this section. The power line then crosses over and parallels at a distance of 600 feet for ½ mile at Morgan Hill, then the close parallel continues for 2½ miles to a crossover. At Morgan Hill the parallel with the Coast Counties Gas and Electric Company's 22-kV. line begins. There are two transpositions in the 22-kV. line and one in the 55-kV. line in the section from Morgan Hill to the next crossover. From this crossover to Gilroy Junction there are two transpositions in the 22-kV. line and three in the 55-kV. line. The parallel is very uniform throughout. There are no transpositions in the 11-kV. line.

The lengths of unbalanced exposures have been computed for all the phantoms and for three physical circuits for the present transposition scheme and are given in Table VI. These lengths of unbalanced exposure apply accurately to the 55-kV. parallel. For the section with telephone line "west" the exposures of the 22-kV. line are identical with those of the 55-kV. line, except that some of the telephone circuits continue to parallel the 22-kV. line south of Gilroy Junction.

TABLE VI.

Lengths of Unbalanced Exposure—Feet.
San Jose-Gilroy Parallel—Present Transpositions.

Telephone line	East		West	
Pin position	Residual	Balanced	Residual	Balanced
	For Transverse Induction.			
1-4	6100	3800	—8300	5200
7-10	—2500	3500	1700	5500
11-14	2400	6600	—2100	5300
17-20	1100	2200	—1900	1000
21-24	—1200	8500	—2100	2500
27-30	1100	5300	2200	4100
31-34	—6000	8300	1800	6000
37-40	100	1200	1600	3000
5-16	—100	8800	1300	1300
25-36	5500	15900	1400	6000
5-6	2600	2400	—800	2200
7-8	2600	5000	—6000	8200
9-10	—100	13900	—3700	2800
Total*	28800	74800	29600	46500
Average	2200	5700	2300	3600
	For Longitudinal Induction.			
All	77000	1300	30300	900

*In finding the total, the physical circuit unbalances were given half the weight of the phantom circuit unbalances, (since the coefficients of induction for physical circuits are approximately half of those of phantom circuits) so the total and average values are in terms of phantom circuits.

The figures given are the unbalances for the sections of the parallel each way from the third crossover. The short portion at 600-foot separation has been neglected.

The longitudinal unbalance to balanced voltages and currents is very small compared to the length of the line (given by the longitudinal residual unbalances). This is due to the fact that the existing power transpositions were so placed as to provide an integral number of barrels within the exposure.

The transverse unbalances vary widely. Circuit 1-4 has transpositions only every two miles, and its unbalances are great. Circuit 5-6 is transposed every $\frac{1}{2}$ mile, yet its unbalances are considerable, due to the lack of co-ordination between the power and telephone transposition systems.

The prominent feature of the present transposition arrangement is the existence of four 6-mile barrels in the 55-kV. line and one 6-mile barrel in the 22-kV. line (from its crossover to the end of the parallel) which provide fair longitudinal balance. The 55-kV. line can not be relieved from service except by operating the auxiliary steam plants at Monterey and Salinas.

Accordingly, the plan of retransposition is based upon the retention of the existing power transpositions with only a few minor changes and the complete retransposition of the telephone line to provide balance between points of discontinuity and power transpositions. Advantage is taken of the opportunity to correct the slight inequality of the lengths of loading sections in the telephone line. The telegraph and signalling circuits have good longitudinal balance to the power circuit for induction from balanced currents and voltages by reason of the existing power transpositions.

The telephone load-point and the 11-kV. parallel at the San Jose end require the use of L sections and consecutive X sections in the telephone line as far as pole 8/15, with transpositions in the 11-kV. line opposite junctions of sections to provide longitudinal balance.

An E section extends from pole 8/15 to 15/25, having its two-mile points approximately coincident with three existing power transpositions. From the end of the E section to the following load point and throughout the remainder of the parallel short sections (L sections and consecutive X sections) are necessary, owing to the many discontinuities. At the north end of the exposed-line section an unbalance to the 11-kV. line exists for about 1300 feet. To eliminate this unbalance by means of whole-line transpositions is inadvisable, since the whole-line transposition scheme is not applicable to the E section where there are many telephone circuits; and moreover, if applied to the present condition it would be necessary to use quarter-mile units for a

distance of one mile on account of the presence of the 55-kV. parallel. It is therefore proposed to obtain partial balance by installing two transpositions in the 11-kV. line.

By moving one of the two transpositions in the 22-kV. line in the portion between poles 19/7 and 21/39, longitudinal balance is secured and a more satisfactory arrangement of the telephone transpositions made possible. Three changes are called for in the 55-kV. line transpositions as follows:

(1) Move transposition at power pole 27/8 north about 300 feet to coincide with the end of the telephone transposition section.

(2) Move transposition at power pole 37/14, just north of the first crossover, south to or beyond the crossover.

(3) Take out the transposition on power pole 41/6, 600 feet south of the third crossover.

The effect of these power transposition changes is to eliminate unbalances of from 500 to 1000 feet on every telephone circuit for transverse induction from balanced voltages and currents.

Under the proposed scheme the small remaining unbalances will be those due to the inexact location of the poles with respect to the theoretical transposition points and to the slight irregularities of construction, and should not amount in any case to more than a few hundred feet.

Three of the changes in power transpositions herein discussed, namely, those respecting the short barrel in the 11-kV. line and changes (1) and (2) in the 55-kV. line, designed to correct small unbalances otherwise left in the parallel, would be of relatively small benefit and it is questionable whether the expense of their installation is warranted, except in case of a favorable opportunity.

B—Gilroy–East Parallel.

Communication company .... The Pacific Tel. & Tel. Co.
Power company .... The Sierra & S. F. Power Co.
Power circuit—
  Voltage .... 55-kV.
  Configuration .... Vertical.
Length of parallel .... 0.4 mile.
Average horizontal separation .... 45 feet.
Induced volt-miles 80

This parallel is an extension of the San Jose-Gilroy parallel, since some of the telephone circuits there affected, and the same 55-kV. line, are involved. It is an example of a short uniform parallel occurring in a "Standard" telephone transposition section. For such a case, whole-line transpositions usually offer the simplest solution.

The telephone circuits involved are each indicated by an asterisk (*) in Table V.

The parallel extends from the S-pole (27/29) to a point near pole 1/17 between the first A-pole and the BF-pole of the first "Standard" telephone transposition section east of Gilroy Junction. There are no power transpositions.

It is proposed to install whole-line transpositions at poles 1/5 and 1/14 according to the "half-mile unit" system.

The unbalances, present and proposed, are given in Table VII for the existing circuits:

TABLE VII.
Lengths of Unbalanced Exposure—Feet.
Gilroy—East Parallel.

Pin position	Present	Proposed
	For transverse induction	
1–4, 7–10, 3–4, 7–8, 15–16	2300	50
1–2, 5–6, 9–10	700	100
	For longitudinal induction	
All	2300	2300

The exposures are the same for balanced and residual currents and voltages.

C—Salinas–North Parallel.

Communication company ....The Pacific Tel. & Tel. Co.
Power company ....The Sierra & S. F. Power Co.
Power circuit—
  Voltage ....55-kV.
  Configuration ....Triangle.
Length of parallel............7.3 miles.
Average horizontal separation............57 feet.
Induced volt-miles 3400

The power circuit is the same 55-kV. line of A and B. The configuration is, however, triangular with an average distance between wires of

8 feet. Likewise, some of the same telephone circuits are involved. The telephone circuits are listed in Table VIII:

TABLE VIII.

Telephone Circuits—Salinas-North.

Pin position	No.	Type	Weight, lb. per mile	Terminals
	151	Physical	172	San Francisco-Salinas.
1–4	269	Physical	172	San Francisco-Salinas.
	1680	Phantom	172	San Francisco-Salinas.
5–6	1343L	Physical	287	San Francisco–San Luis Obispo.
	295	Physical	172	San Jose–Salinas.
7–10	159	Physical	172	San Jose–Salinas.
	1673	Phantom	172	San Jose–Salinas.
13–14	1475	Physical	287	San Francisco–San Luis Obispo.
15–16	1441	Physical	485	San Francisco–Los Angeles.
17–18	1781	Physical	172	Hollister–Salinas.

L = Loaded. All others non-loaded.

Drawing No. 360 shows the present as well as proposed transposition schemes. There is one "Standard" telephone transposition section 38300 feet in length which includes all but 2300 feet of the parallel. The power and telephone lines exchange sides of the road about midway along the parallel. There are five power transpositions, one just at the north end of the parallel.

The lengths of unbalanced exposures have been computed for all the phantoms and three physical circuits of the two top cross-arms and are given in Table IX:

TABLE IX.

Lengths of Unbalanced Exposure—Feet.
Salinas-North Parallel—Present Transpositions.

Telephone line	East		West	
Pin position	Residual	Balanced	Residual	Balanced
	For transverse induction:			
1–4	—2200	8500	2300	9900
7–10	2200	11300	—2600	8100
11–14	—2400	7000	—2000	5800
17–20	2200	1100	—2000	4600
5–16	—3000	3100	—2700	4900
5–6	500	1900	—700	2700
7–8	2000	8700	—2000	6300
9–10	—200	2600	400	5800
Totals*	13400	37600	13200	40700
Average	1700	4700	1600	5100
	For longitudinal induction:			
All	21600	4800	16800	2800

*In finding the total, the physical circuit unbalances were given half the weight of the phantom circuit unbalances, so the total and average values are in terms of phantom circuits.

The transverse unbalanced lengths are large, expecially on phantom circuits and for balanced voltages and currents. The longitudinal unbalances for balanced components are rather small, compared with those for the residuals. The lengths of unbalanced exposures here given correct and supplement those given in technical report No. 15.

The plans for retransposition of the power circuit call for a few changes to be made under the present conditions and for further changes at such times as the addition of a second circuit or other reconstruction makes it convenient. The first plan utilizes three of the present transpositions, requires that one be relocated at a point 1600 feet south of its present position and a special transposition at the crossover. The transposition 400 feet from the north end of the parallel is to be taken out. In the telephone line, the present "Standard" section is replaced by an E section with its mid-point at the crossover and its Salinas end coincident with the south end of the parallel. A Y section extends thence to the load point at Salinas. At the other end, there are short sections between the end of the E section and the north end of the parallel and beyond to the load point, whose location is changed, involving further retransposition of the telephone circuits north of the parallel. The telephone transpositions specified are in accordance with the ultimate proposed scheme. The power transpositions coincide with mile-points of the E section within, at most, a few hundred feet. The crossover transposition is of the 120° type so that the crossover has the same effect as an ordinary transposition for longitudinal induction. The triangular configuration, with base horizontal, allows this to be readily done. The establishment of longitudinal balance is made dependent upon the combined effect of the sections on opposite sides of the crossover. The change in horizontal separation is neglected. The expense of transposing the telephone circuits would be greater if a crossover transposition were not used, and the cross-talk higher, due to the necessity of using short sections in order to provide balance on the two sides of the crossover independently.

In the ultimate scheme the number of power transpositions is increased by one, providing three halting barrels, and the locations altered to bring them as nearly as practicable opposite the mile-points of the E section. The slight change in horizontal separation is provided for, since the junction point of two barrels in the power line nearly coincides with the point of change. The section to the north of this point contains two barrels, the crossover acting as a transposition. The barrels in the power circuit are 2.4 miles in length. The plan is a good illustration of the use of a crossover transposition and halting power barrels.

D—SALINAS-SOLEDAD.

Communication company ------------------The Pacific Tel. & Tel. Co.
Power company ---------------------------Coast Valleys G. & E. Co.
Power circuit—
Voltage -----------------------------------33 kV.
Configuration ----------------------------Triangular.
Length of parallel---------------------------25.4 miles.
Average horizontal separation----------------57 feet.
Induced volt-miles --------------------------3400.

This parallel is the remaining portion of a longer parallel between Salinas and King City,* involving the Coast Valleys Gas & Electric Company's single-circuit, 33-kV., three-phase 60-cycle line and the toll lines of The Pacific Telephone and Telegraph Company besides telegraph and signalling circuits of the Western Union Telegraph Company and the Southern Pacific Railroad. The telephone circuits involved are described in Table X.

TABLE X.
Telephone Circuits—Salinas-Soledad.

Pin position	No.	Type	Weight, lb. per mile	Terminals
1-2	833	Physical	172	Salinas-King City.
3-4	1775	Physical	172	Salinas-King City.
5-6	1343L	Physical	287	San Francisco-San Luis Obispo.
7-10	299	Physical	172	Salinas-Paso Robles.
	1773	Physical	172	Salinas-San Luis Obispo.
	1475	Phantom	172	San Francisco-San Luis Obispo.
11-12	1777	Physical	No. 10 iron	Salinas-Gonzales.
13-14	821	Physical	No. 9 iron	Salinas-Greenfield.
15-16	1441	Physical	435	San Francisco-Los Angeles.
17-18	820	Physical	No. 10 iron	Salinas-Gonzales.

L = Loaded—all others non-loaded.

Drawing No. 361 shows the present and proposed transposition schemes. There are now three "Standard" sections and a Z section in the exposure. For 21.2 miles the power line is west of the telephone line, leaving 4.2 miles east of the telephone line. There are seven crossovers within the parallel, three points where leads are connected to the power circuit, and six power-circuit transpositions. Except for the numerous crossovers the parallel is very uniform.

Table XI shows the unbalanced exposures for all the phantom circuits of the two full cross-arms and three physical circuits.

*Tests thereon are described in technical report No. 38 (Exposure No. 3).

TABLE XI.

Lengths of Unbalanced Exposure*—Feet.

Salinas-Soledad Parallel—Present Transpositions.

Telephone line	East		West	
Pin position	Residual	Balanced	Residual	Balanced
	For transverse induction:			
1-4	—8200	21300	4600	6600
7-10	—8900	14400	5900	5900
11-14	—3900	21500	600	7600
17-20	4700	8100	—6900	4500
5-16	5300	4200	—8000	4800
5-6	100	3500	800	300
7-8	9300	15100	7700	11500
9-10	—500	7500	600	2100
Totals**	40000	82600	30600	35400
Average	5000	10300	3800	4600
	For longitudinal induction:			
All	111800	10700	22200	12200

*For whole exposure, neglecting the effect of the loads on the power circuit within the exposure.

**In finding the total, the physical circuit unbalances were given half the weight of the phantom circuit unbalances, so the total and average values are in terms of phantom circuits.

Large unbalanced exposures, for both longitudinal and transverse induction, are characteristic. This is to be expected by reason of the many crossovers within the parallel, and the lack of coordination of transpositions.

The telegraph and signalling circuits extend throughout the parallel and are subject to roughly the same longitudinal unbalance.

The great length of parallel broken into short sections of irregular length by the many points of discontinuity makes possible a number of different arrangements of power and telephone transpositions which will provide a more or less satisfactory balance. The scheme presented was evolved after a consideration of several such possibilities.

Considering first the section between Salinas and Chualar substation, it will be noticed that the two crossovers divide the distance between the Salinas end of the parallel and the load point (8/25) into three very nearly equal parts. An E section is therefore proposed to extend 8/9 of this distance, whereby its three and six-mile points will fall at the crossovers. Short sections extend to the load point. This arrangement makes it possible to obtain longitudinal balance by means of six power transpositions composing three barrels. Two existing power transpositions at power poles 1/18 and 5/8 are to be removed. The portion from the load point to the Chualar substation has three short telephone transposition sections and two power transpositions

opposite their junction points. This provides balance throughout this section. It would be possible to omit two of the power transpositions (at 11360 and 32900) and by reversing the two power transpositions (at 20010 and 24290) secure approximate longitudinal balance for the telephone circuits but such procedure would leave a large unbalance for the telegraph and signalling circuits.

Between Chualar substation and the south end of the parallel there are three short sections of the power line east of the telephone line, two of them coming between long sections of power line west of the telephone line. The proposed plan contemplates securing longitudinal balance for the exposures on the two sides independently, without the use of special crossover transpositions. It is found possible to use two E sections from the two load points (16/23 and 24/21) to the crossovers adjacent on the north. Power transpositions are placed opposite certain mile points of these exposed line sections. Other power transpositions are placed opposite the remaining portions of the telephone line so that longitudinal balance is secured between substations and the short sections of power line east of the telephone line have the proper phase relation to give approximate longitudinal balance. In the telephone line, short transposition sections are utilized between E sections, their junction points being placed opposite points of discontinuity and power transpositions. For the power circuit eleven new transpositions are required, one being "reversed"; also three of the existing transpositions are to be removed. The power transposition opposite Chualar substation is at a neutral point, hence can be left in place, or taken out as desired.

In placing the power transpositions the twofold object was sought of using the smallest number consistent with adequate longitudinal balance and distributing them as effectively along the line as practicable. The proposed transposition scheme requires complete retransposition of the telephone line from Salinas to Soledad. This parallel differs from those already considered in that the few existing power transpositions are irregularly located, and, owing partly to the many points of discontinuity caused by crossovers, substations, telephone load-points and the power-circuit loads, it is not practicable to use them in designing the coordinated scheme. For the whole parallel the plan requires the installation of 19 new power transpositions and the removal of 5 of the present transpositions, as shown on drawing No. 361.

It is of interest to consider here a plan which can be adopted as a temporary expedient in the event that it is not feasible to take the power line out of service for the length of time necessary to carry out the complete retransposition scheme at the time of retransposition of the telephone circuits. By taking out the power transposition at

power pole 1/18, moving the power transposition at power pole 14/7 south about 650 feet and the one at 21/2 south about 1640 feet, transverse balance is secured, except for the effect of the power transpositions immediately adjacent to crossovers, 5/8 and 25/22. This scheme leaves large longitudinal unbalances on both telephone and telegraph lines. Its advantage lies in that partial relief is secured with minimum interruption of the power service.

E—Napa-Sonoma Parallel.

Communication company .... The Pacific Tel. & Tel. Co.
Power company .... Great Western Power Co.
Power circuit—
  Voltage .... 22 kV.
  Configuration .... Flat.
Length of parallel .... 7.4 miles.
Average horizontal separation .... 50 feet.
Induced volt-miles .... 1100.

One of the circuits of the extensive isolated 22-kV. system of the Great Western Power Company parallels the telephone circuits listed in Table XII.

TABLE XII.

Telephone Circuits—Napa-Sonoma.

Pin position	No.	Type	Weight, lb. per mile	Terminals
	216	Physical	172	Santa Rosa-Oakland.
1-4	255	Physical	172	Santa Rosa-Oakland.
	1311	Phantom	172	Santa Rosa-Oakland.
5-6	18	Physical	No. 9	Napa-Sonoma.
	1194	Physical	172	Napa-Santa Rosa.
7-10	1196	Physical	172	Napa-Santa Rosa.
	1045	Phantom	172	Santa Rosa-Sacramento.

None of the circuits are loaded.

Drawing No. 362 shows the present and proposed transposition schemes. The telephone transpositions within the parallel are parts of two "Standard" sections. The parallel is broken into short sections of irregular length by four crossovers. For a small distance within the limits of the parallel the separation is about 1000 feet. Except for this the separation is quite uniform. There are three power transpositions. The lengths of unbalanced exposure have been computed for two phantom and two physical circuits and are given in Table XIII. Some of the values given are in error by a few hundred feet.

TABLE XIII.

Lengths of Unbalanced Exposure—Feet.

Napa-Sonoma Parallel—Present Transpositions.

Power line	North		South	
Pin position	Residual	Balanced	Residual	Balanced
For transverse induction:				
1–4	1700	5000	3700	6300
7–10	6100	3600	13400	14400
7–8	6100	3600	3600	6600
9–10	200	700	900	5200
For longitudinal induction:				
All	6100	3600	31000	3700

Road construction has rendered desirable the removal of the power line to a new location, at a separation of about ½ mile, between telephone poles 28/46 and 31/2. The length over the new route is practically the same as the present. The telephone line will remain in its present location, thereby eliminating two miles of parallel and creating two additional crossovers. In placing new telephone transpositions allowance has been made for two load points within the limits of the two present "Standard" sections, although there are no loaded circuits at the present time over this route. The retransposition plans provide for several short sections, one E section and one L section in the telephone line. One power transposition is eliminated by the moving of the power line and one of the remaining power transpositions, opposite 31/42, is to be taken out. By placing a new power transposition midway between the existing power transposition at pole 33/35 and the crossover at pole 34/25, the longitudinal unbalance is reduced to about six hundred feet for the portion of the close parallel with the power line north of the telephone line. Longitudinal balance for the several portions where the power line is south of the telephone line is obtained by means of four new power transpositions, as shown on the drawing, one being within the close parallel, one in the portion at great separation between poles 31/19 and 31/33 and two in the rerouted section of the power line. The first two mentioned are "reversed." These several power transpositions serve to provide the proper phase arrangement of the power conductors in the several sections of the parallel and also to provide approximate capacitance balance of the power conductors to ground within the outside limits, of the parallel. The telephone transpositions include one Y section from the end of the present "Standard" section, pole 36/41, to the beginning of the parallel at 35/35. Five X sections provide neutral points at the crossovers and opposite the first two power transpositions. The first load point is

coincident with the crossover at 33/18. The next power transposition occurs opposite the one-mile point of the L section which extends between crossovers at 33/18 and 31/33. Two X sections extend to 31/2. An E section with one end at 31/2 and the other at the next load-point has its third mile-point at the crossover, 28/46. The fourth mile-point is about 800 feet from the end of the parallel at 28/6. The resulting unbalance can be eliminated by using "whole-line" transpositions for ¼ mile, which is permissible in this particular case, with the E section. A Z section extends from the load-point to the end of the present "Standard" section at 22/37. It is of interest to note that, should it be necessary to continue the parallelism between 31/2 and 28/46, the power transpositions shown for that portion of the power line could be placed opposite the first and second mile points, and would then give accurate longitudinal balance while the capacitance balance would be only slightly disturbed.

F—Santa Cruz–Watsonville Parallel.

Communication company .... The Pacific Tel. & Tel. Co.
Power company .... Coast Counties Gas & Electric Co.
Power circuit—
  Voltage .... 22-kV.
  Configuration .... Triangle.*
Length of parallel .... 17.0 miles.
Average horizontal separation ...... 45 feet.
Induced volt-miles—
  Triangle 1200
  Flat 2600

The 22-kV. circuit between Santa Cruz and Watsonville, which is part of an isolated line extending from San Jose via Gilroy and Watsonville to Big Creek (Santa Cruz County) parallels the telephone circuits listed in Table XIV, between Santa Cruz and Watsonville.

TABLE XIV.

Telephone Circuits—Santa Cruz-Watsonville.

Pin position	No.	Type	Weight, lb. per mile	Terminals
	161	Physical	172	San Francisco-Watsonville.
1-4	286	Physical	172	San Francisco-Watsonville.
	1682	Phantom	172	San Francisco-Salinas.
5-6	1763	Physical	No. 12 (iron)	Santa Cruz-Watsonville.
	59	Physical	172	Santa Cruz-Salinas.
7-10	1761	Physical	172	Santa Cruz-Watsonville.
	1728	Phantom	172	San Jose-Watsonville.

*Being gradually changed to flat construction as line is rebuilt.

None of the circuits are loaded. Drawing No. 363 shows the present and the proposed transposition schemes. The parallel covers the greater part of two "Standard" telephone transposition sections, having a crossover near their junction. The separation is quite irregular and the configuration is triangular, of several sizes. It is being gradually replaced by unsymmetrical flat construction. There are three power transpositions, one about 600 feet from the south end of the parallel.

Technical reports Nos. 39 and 46 show the present unbalanced exposures. The unbalances, both longitudinal and transverse, are in general large.

In retransposing the telephone line the sections are to be so located as to provide for ultimate loading. The plan of retransposition shown is chosen from several of the same general character, involving two E sections with various arrangements of short sections. A Y section extends from the end of the existing "Standard" section at Santa Cruz to the beginning of the parallel. From the beginning of the parallel to the crossover an E section extends 8/9 of the distance, then an X section to the load-point, then another X section to the crossover. From the crossover to the second load-point there are two X sections for 1/9, then an E section for the remaining 8/9 of the distance. An L section extends from this load-point, having its third mile-point opposite the Watsonville end of the parallel. An X section extends to the end of the existing "Standard" section at Watsonville.

The power transpositions are placed opposite mile-points of the E and L sections to form halting "three-mile" barrels and their distribution is such as to minimize the unbalance due to phase-change along the lines.

VII. Resumé.

Transpositions in power and telephone circuits constitute a very important factor in determining the inductive interference experienced from parallelism. Their function is to equalize the relations of the several conductors to each other and to the influences of the neighboring circuits, by interchanging the positions of the several conductors of each circuit. In order that they may be effective in reducing the inductive interference it is necessary that the transpositions of the two classes of circuits be carefully coordinated.

The relative effectiveness of transposition schemes may be estimated from a study of the unbalanced exposures (equivalent lengths of parallel of nontransposed circuits). Lengths of unbalanced exposure also indicate the relative severity of induction into the different circuits of the same line. Convenient methods for the calculation of unbalanced exposure have been devised and are given in an appendix.

Studies of the "Standard" telephone transposition system as encountered in particular parallels and in general have shown its unsuitability for coordination with power transposition barrels of practicable lengths. To meet the need of a flexible system which should conserve the standards of cross-talk balance, more effectively balance the telephone circuits with respect to all foreign circuits and at the same time coordinate with power transposition barrels of several lengths, the American Telephone and Telegraph Company has developed three methods which, singly, or in combination, make it practicable to reduce the unbalanced exposures to small amounts:

1. E and L sections, nominally 8 and 4 miles in length, balancing to nominal ½, 3 and multiples of 3-mile barrels.
2. Consecutive X sections, that is, successive sections ½ mile or less in length.
3. "Whole-line" transpositions, whereby extra transpositions are inserted in all the circuits of the telephone line.

In considering the application of transpositions in parallels, points at which material changes in either the phase or magnitude of the induction occur must be considered as points of discontinuity. These include crossovers, load-points and changes of separation or configuration. Thus the problem becomes that of providing for balanced exposures of the telephone line to the power line, within the uniform sections between points of discontinuity. The length and character of transposition sections and barrels is usually determined by the location of the points of discontinuity.

It is practicable in some cases to nullify the effect of a crossover by special power transpositions, or to cause it to have the same effect as an ordinary transposition, on the induction between conductors and ground. Where a power line is alternately on one side and the other of a telephone line it is sometimes feasible to combine successive lengths on the same side in obtaining balance for induction between conductors and ground.

The direction of the cyclic rotation of power transpositions is of importance for three-phase circuits. It is often desirable to use "normal" and "reversed" transpositions in combination, thereby lessening the number of power transpositions required.

Phase-change and attenuation cause large unbalances for induction from balanced voltages and currents at telephonic frequencies with barrels more than a few miles in length. They occur in both power and telephone lines and affect the telephone line differently at different points along the parallel. The unbalance due to phase change is usually much greater than that due to attenuation and varies directly with the frequency and as the square of the length of barrel, approximately. In a

long uniform parallel the length of power transposition barrels required to obtain adequate balance is determined by this factor.

Where there are several power transposition barrels in succession it is advantageous to omit the transpositions at the junctions of successive barrels, since thereby the unbalance due to phase change may be very materially reduced, and shorter barrels are obtained with a given number of transpositions, which still further diminishes the unbalance. Where the barrels are thus "halted," and all transpositions have like rotation, the greatest advantage occurs when multiples of three barrels are used. If the transpositions of alternate barrels be reversed, the greatest advantage is obtained with multiples of two barrels. With barrels four miles and less in length the unbalances are reduced to a small fraction of a mile. Thus it may be expected that three-mile barrels in a long parallel will give a degree of balance such that the irregularities of the construction, even in the more uniform parallels, will control the unbalances. For large separations, usually giving small coefficients of induction, and with good wave-form, six-mile barrels should be sufficiently effective. It must be remembered, however, that consideration of discontinuities more often controls the lengths of barrel required.

The application of the principles and methods described in this report to typical practical cases is illustrated by plans of co-ordinated transposition systems for each of the several parallels investigated by the Joint Committee.

Respectfully submitted.

(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

ATTACHMENTS: Appendix—Method of Calculation of Lengths of Unbalanced Exposure. P. I. C. Drawings Nos. 137, 354, 355, 358, 359, 360, 361, 362, 363, 364, 366, 3677, 368; A. T. & T. Co. Drawings Nos.: 92-E-75, 92-E-76, 92-E-77, 93-E-2, 93-E-3, 93-E-4.

Approved: January 8, 1917.

(Signed) R. W. MASTICK,
Field Engineer.

Approved: January 8, 1917.

SUBCOMMITTEE ON TESTS,
(Signed) J. E. WOODBRIDGE,
Chairman.

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE,
(Signed) ARTHUR F. BRIDGE,
Secretary.

January 8, 1917.

APPENDIX.

METHOD OF CALCULATION OF LENGTHS OF UNBALANCED EXPOSURE.

In the study of parallels of power and telephone lines it is often desired to know the lengths of unbalanced exposure for given transposition schemes. The purpose of this appendix is to present a convenient method for doing the work.

A diagram of telephone transpositions is made up as shown on drawing No. 368, Fig. 1, which shows the "+" and "—" exposures of each circuit on the top crossarm for a complete section of the "Standard" system with phantoms on pins 1-4 and 7-10. The circuit numbers refer to pins occupied at the beginning of the first transposition section. This diagram applies to all sections transposed in this manner. Such diagrams should be made for all types of telephone transposition sections to be studied.

A "Transposition Distance Chart" is then prepared, whose purpose is to show the actual distances between successive telephone transposition poles, and the distances from the nearest telephone transposition poles to all power transpositions and points of discontinuity, such as ends of parallel, crossovers, or load-taps. This chart is not to scale, but the spaces between successive telephone transposition points are equal, and the same as the corresponding spaces on the transposition diagram. There is shown on drawing No. 368, Fig. 2, a "Transposition Distance Chart" showing part of the San Jose-Gilroy parallel. It is convenient to divide the parallel into zones, separated by points of discontinuity or telephone load-points.

In order to determine the unbalances for any circuit, it is convenient to make one of these two drawings on a series of stiff paper strips, which can be laid upon the other drawing in proximity to each zone as desired. For example, to determine the unbalances for circuit 7-8 in Zone 2, the transposition diagram for 7-8 should be drawn on a narrow strip of stiff paper, and laid over the "Transposition Distance Chart," close to the part representing Zone 2, and with corresponding points of the two drawings coincident. It is immaterial which of the drawings is made on the strips.

The three different phases of induction from a transposed three-phase power circuit of uniform construction, are designated A, B, C. B is assumed to lag by 120° behind A, and C, to lead A by 120°. Transverse induction is, in addition + or —, depending on the point in the telephone transposition section.

The sample record sheet attached shows the form used. The figures entered are for circuit 7-8, Zone 2, San Jose-Gilroy parallel. The sheet was designed for use with a listing adding-machine. First, of all the +A distances of the circuit in the Zone are printed and totaled. Then the —A's are printed and subtotaled. The +A total is now added and the grand total taken, which gives the numerical sum of +A's and —A's, or the "longitudinal" A-exposure. The same is done for "B" and "C." The "total" mark at the head of each column insures that the machine is clear to begin with. The differences of +A's and —A's, etc., give the transverse unbalances. The algebraic sum of the A, B and C transverse unbalances is the transverse residual unbalance. The three transverse and longitudinal unbalances are then combined in three-phase relation to determine the resultant unbalances to balanced currents and voltages.

To combine several zones requires computation to determine the relative magnitude of the induction per unit length, and the phase difference, where crossovers or changes in separation occur.

It should be observed that some telephone circuits have opposite exposures at corresponding points of successive transposition sections. Account must be taken of this fact in combining the unbalances of successive sections.

ATTACHED: Sample Record Sheet.

(SAMPLE RECORD SHEET.)

Lengths of Unbalanced Exposure.

Parallel—San Jose-Gilroy........................ {S. & S. F. P. Co. / T. P. T. & T. Co.}

Remarks—Present transpositions.

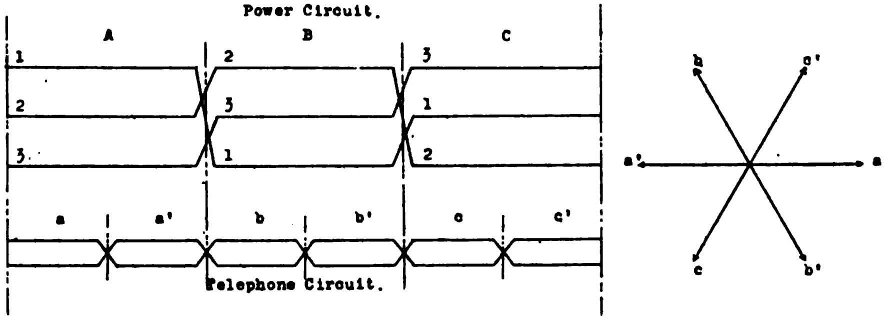
Circuit No. 7-8.			Zone No. 2.		
+A	—A	+B	—B	+C	—C
1330	1020	840	1330	1170	730
1340	1340	1210	1840	340	1250
500	1220	1340	390		1250
1340	1340	1180	810		1170
1350	1210	1360	1340		1360
	520	1210			1340
		1340			1230
		1210			320
		1330			
		1340			
		880			
5860	6650	132[illegible]0	5210	1510	8700
	5860		18240		1510
Long.	12510		18450		10210
Trans.	—790		+8030		—7190

Transverse	Residual
+	—
8030	790
	7190
8030	
7980	7980
+50	

Transverse	Balanced	Longitudinal
+8030	B	18540
—7190	C	10210
840	B+C	28660
—790	A	12510
—420	—½ (B+C)	—14330
—1210	x=A—½ (B+C)	—1820
+15220	B—C	+8240
+13180	y=0.866 (B—C)	+7140
13240	$U=(x^2+y^2)^{1/2}$	7360

39.

PHASE ANGLES OF TRANSVERSE INDUCTION FROM THREE PHASE CIRCUITS.
RELATIONSHIP TO POWER AND TELEPHONE TRANSPOSITIONS.



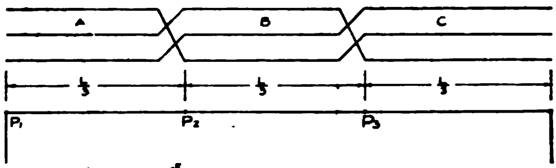
a, b, and c --Power circuit in three possible positions, telephone circuit in its untransposed position.

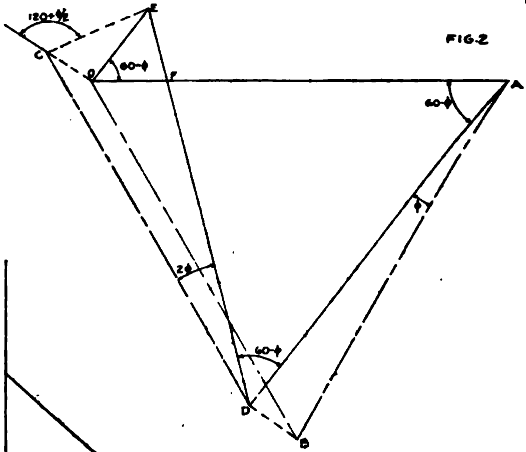
a',b',and c'--Power circuit in same three positions, telephone circuit in its transposed position.

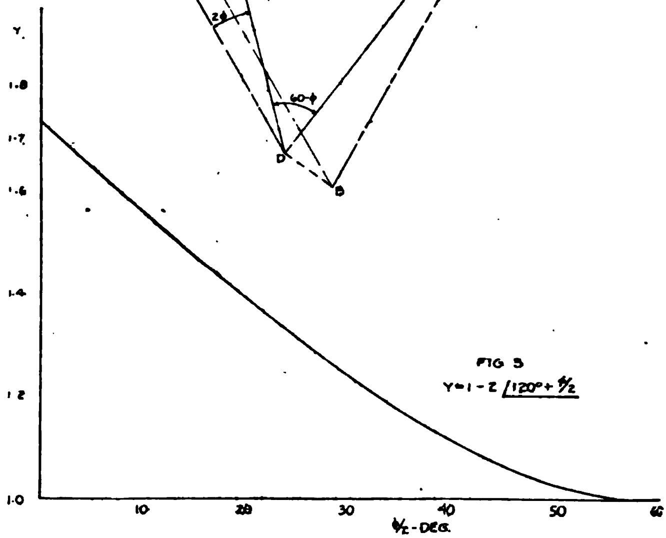
P.I.C. # 137
Nov. 19, 1913.

P.I.C. 364
9-4-16
REVISED
1-2-12

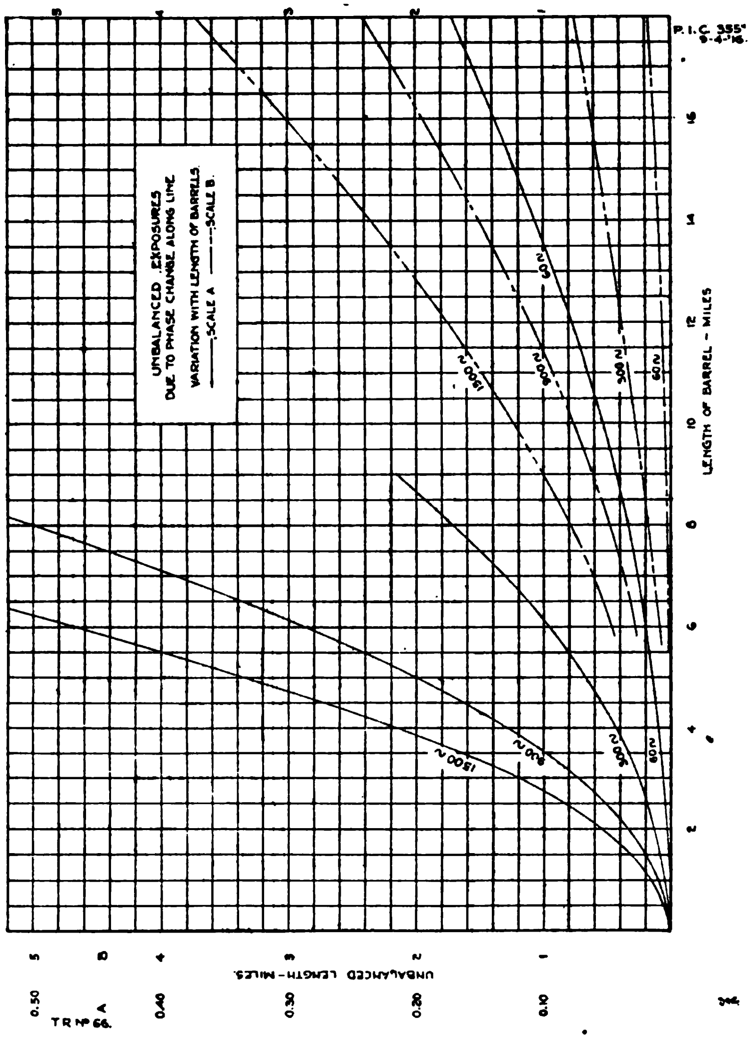
FIG. 1

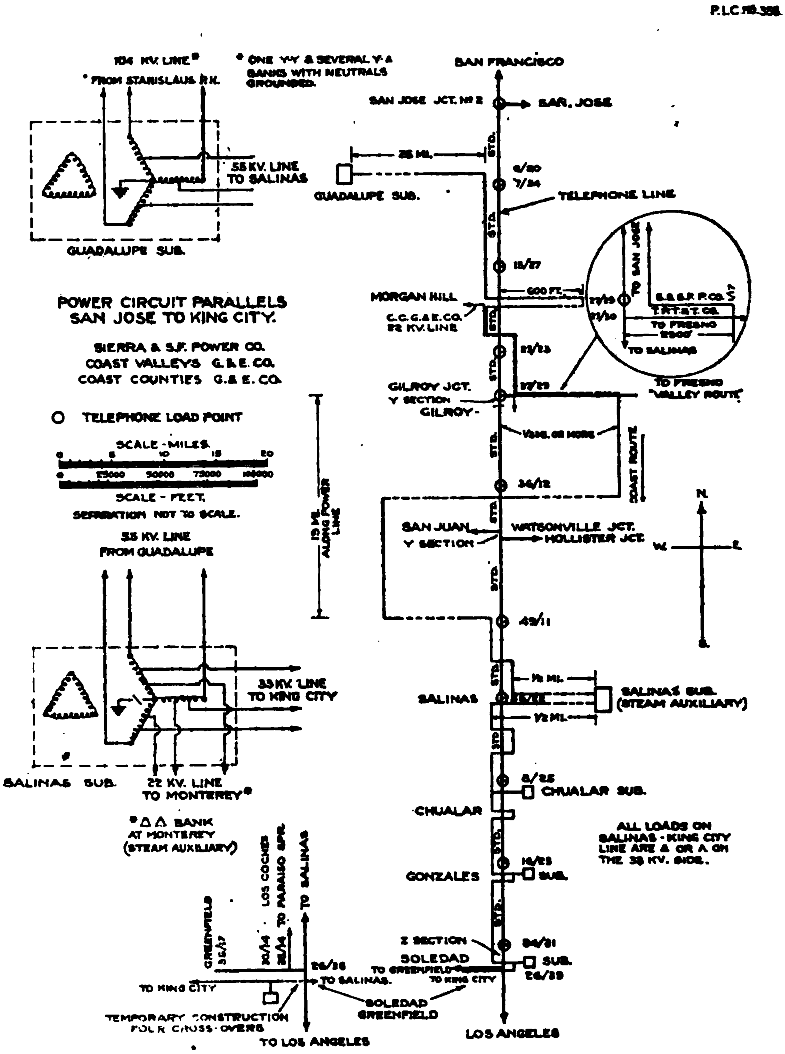






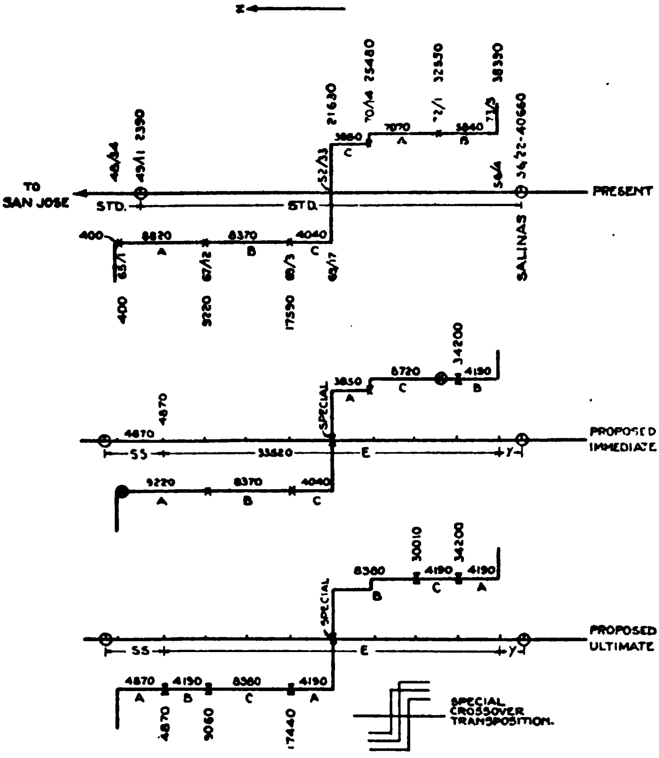
T.R.N° 66.


UNBALANCED EXPOSURES
DUE TO PHASE CHANGE ALONG LINE
VARIATION WITH LENGTH OF BARRELS
SCALE A
SCALE B
1500 ~
900 ~
300 ~
60 ~
LENGTH OF BARREL - MILES
UNBALANCED LENGTH - MILES
B
A
P.I.C. 355
9-4-'16
TR No 66.


P.L.C. NO. 388
104 KV. LINE*
* FROM STANISLAUS P.H.
* ONE Y-Y & SEVERAL Y-Δ BANKS WITH NEUTRALS GROUNDED.
55 KV. LINE TO SALINAS
GUADALUPE SUB.
POWER CIRCUIT PARALLELS SAN JOSE TO KING CITY.
SIERRA & S.F. POWER CO.
COAST VALLEYS G. & E. CO.
COAST COUNTIES G. & E. CO.
TELEPHONE LOAD POINT
SCALE - MILES.
SCALE - FEET.
SEPARATION NOT TO SCALE.
55 KV. LINE FROM GUADALUPE
55 KV. LINE TO KING CITY
SALINAS SUB.
22 KV. LINE TO MONTEREY*
*Δ Δ BANK AT MONTEREY (STEAM AUXILIARY)
SAN FRANCISCO
SAN JOSE JCT. NO. 2
SAN JOSE
TELEPHONE LINE
MORGAN HILL
C.C.G.&E.CO. 22 KV. LINE
GILROY JCT. Y SECTION
GILROY
TO FRESNO "VALLEY ROUTE"
COAST ROUTE
SAN JUAN Y SECTION
WATSONVILLE JCT.
HOLLISTER JCT.
SALINAS
SALINAS SUB. (STEAM AUXILIARY)
CHUALAR SUB.
CHUALAR
GONZALES
Z SECTION
SOLEDAD
LOS ANGELES
ALL LOADS ON SALINAS - KING CITY LINE ARE Δ OR Λ ON THE 55 KV. SIDE.
TO KING CITY
TEMPORARY CONSTRUCTION FOUR CROSS-OVERS
TO LOS ANGELES
SOLEDAD GREENFIELD
TO SALINAS
TO PARAISO SPR.
LOS COCHES
GREENFIELD

Pl.C.No 360.
9-5-16.

PRESENT AND PROPOSED TRANSPOSITION SCHEMES
FOR THE
SALINAS - NORTH PARALLEL.
S. & S.F. POWER CO. AND THE PAC. TEL. & TEL. CO.



LEGEND

X	POWER TRANSPOSITION.
⊗	POWER TRANSPOSITION TO BE REMOVED.
I	POWER TRANSPOSITION TO BE INSTALLED.
A, B, C	POWER CONDUCTOR ARRANGEMENTS.
	NEUTRAL POINT - TELEPHONE TRANSPOSITIONS.
	TELEPHONE LOAD POINT
TELEPHONE TRANSPOSITION SECTIONS.	
STD.	STANDARD SECTION
E	"8-MILE" EXPOSED LINE SECTION
SS	SHORT SECTIONS.

NOTES

TRANSPOSITIONS TO BE LOCATED ON POLES NEAREST TO POINTS INDICATED. DISTANCES ARE ALONG THE TELEPHONE LINE, FROM NORTH END OF PARALLEL. IN FEET

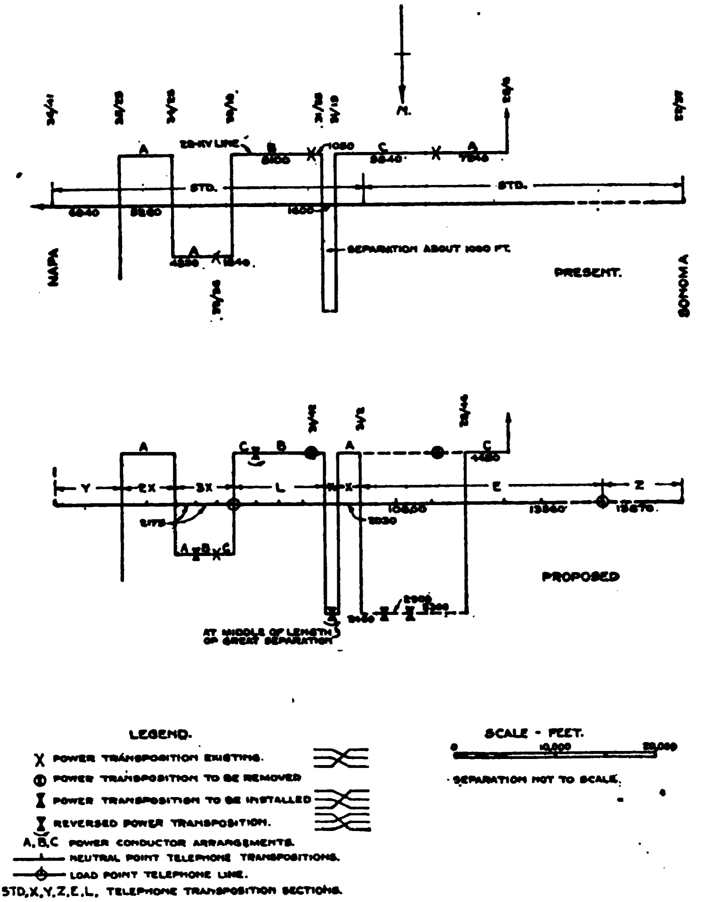
SCALE 0 10000 20000 FT

SEPARATION - NOT TO SCALE

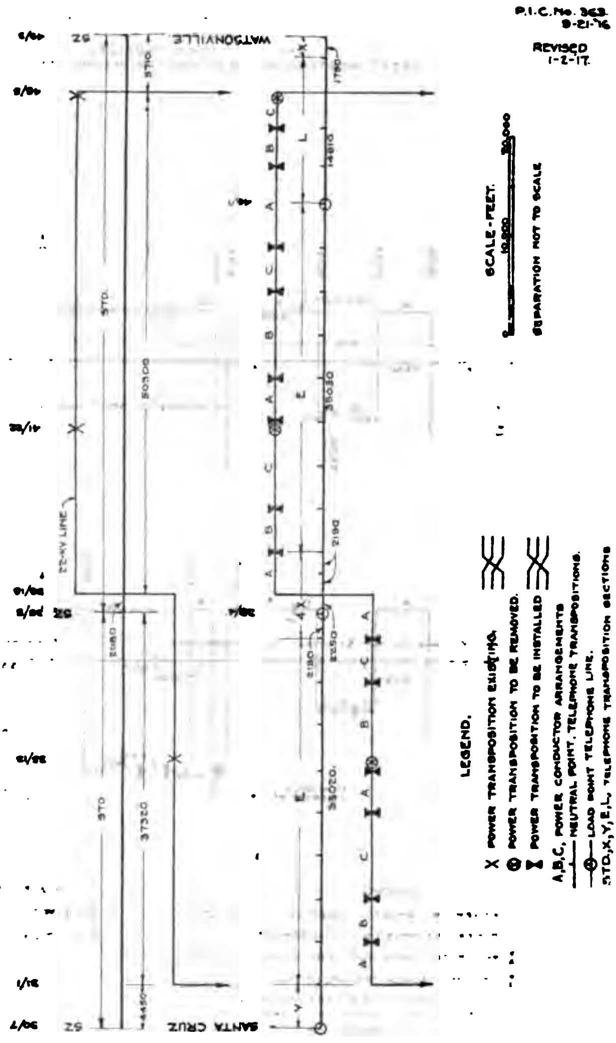
T R No 66.

P.I.C. No. 362.
9-21-16.

PRESENT AND PROPOSED TRANSPOSITION SCHEMES
FOR THE
NAPA-SONOMA PARALLEL.
GREAT WESTERN POWER CO. AND THE PACIFIC TEL. & TEL. CO.



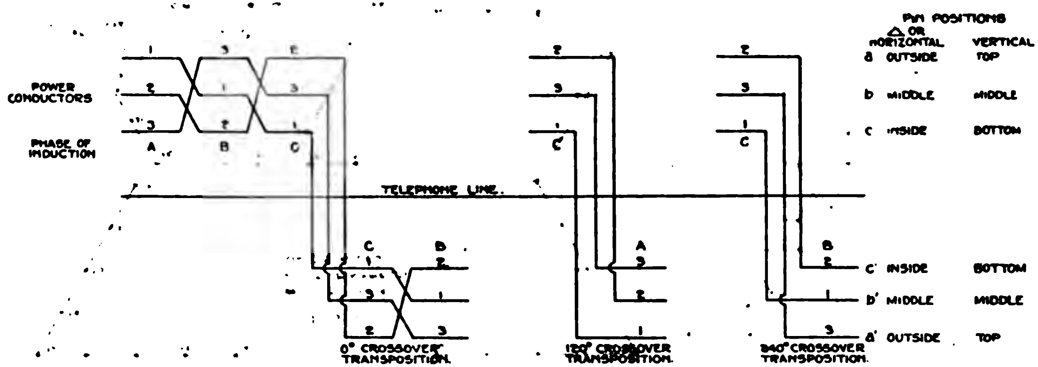
PRESENT AND PROPOSED TRANSPOSITION SCHEMES FOR THE SANTA CRUZ-WATSONVILLE PARALLEL. COAST COUNTIES GAS & ELEC. CO. AND THE PACIFIC TEL. & TEL. CO



T.R. No. 66.

T.R. No. 66.

CROSSOVER TRANSPOSITIONS FOR THREE PHASE CIRCUITS.

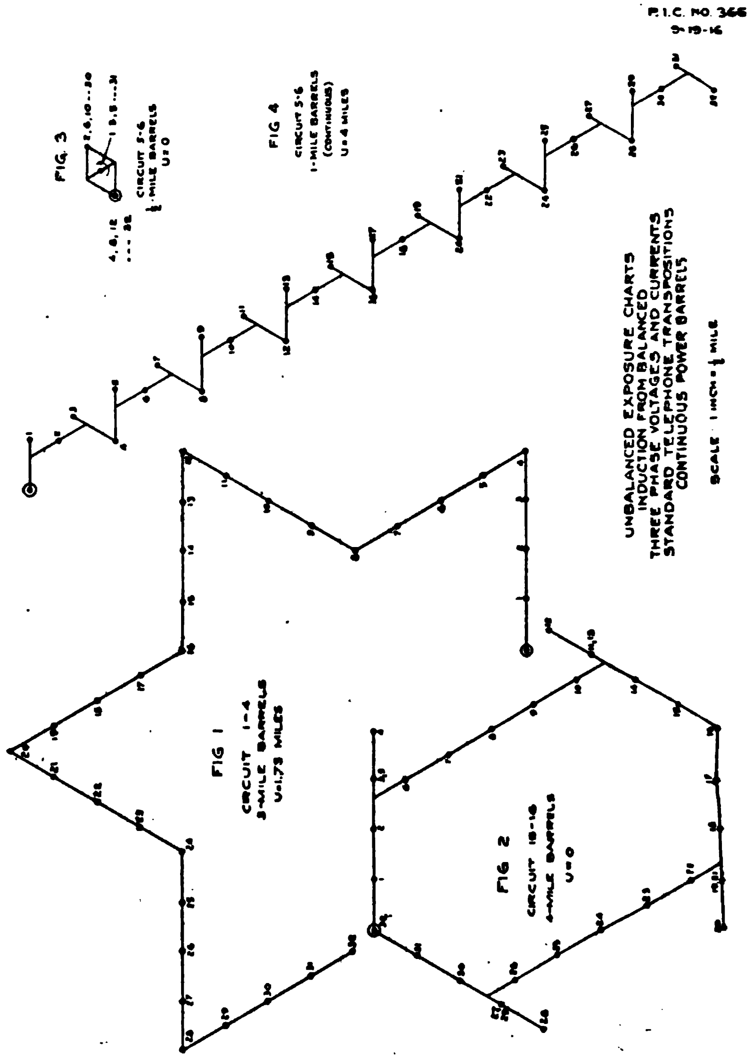


NOTES.-
THE CROSSOVER TRANSPOSITIONS ARE DESIGNATED BY THE DEGREES DIFFERENCE IN THE PHASE ANGLES OF THE INDUCTION BETWEEN THE TELEPHONE CONDUCTORS AND GROUND, IMMEDIATELY BEFORE AND AFTER THE CROSSOVER.

THE PHASE CHANGE OF THE INDUCTION BETWEEN THE CONDUCTORS OF A NONTRANSPOSED TELEPHONE CIRCUIT ON THE TWO SIDES OF A CROSSOVER TRANSPOSITION IS 180° PLUS THE CHANGE IN THE INDUCTION BETWEEN THE TELEPHONE CONDUCTORS AND GROUND.

THE CROSSOVER TRANSPOSITIONS ARE CORRECTLY SHOWN FOR ANY CONFIGURATION OF POWER CONDUCTORS IF THE HORIZONTAL AND VERTICAL SEPARATIONS FROM THE TELEPHONE LINE OF THE PIN POSITIONS a', b', c' ARE THE SAME AS THOSE OF PIN POSITIONS a, b, c, RESPECTIVELY.

P.I.C. 364
9-28-16.



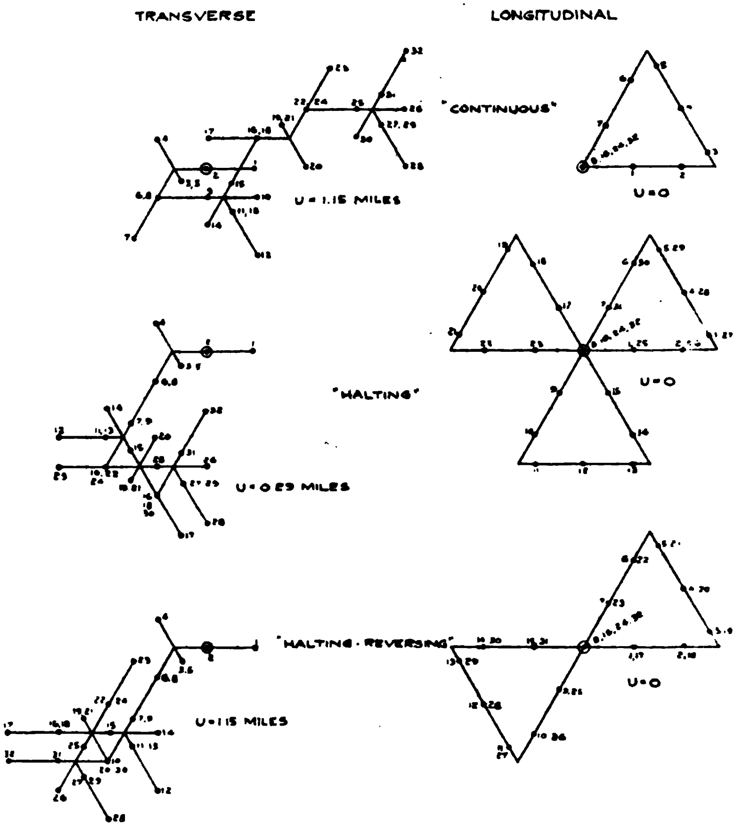
TR NO. 66.

T.R No. 66.

P I C NO 367
9-19-16

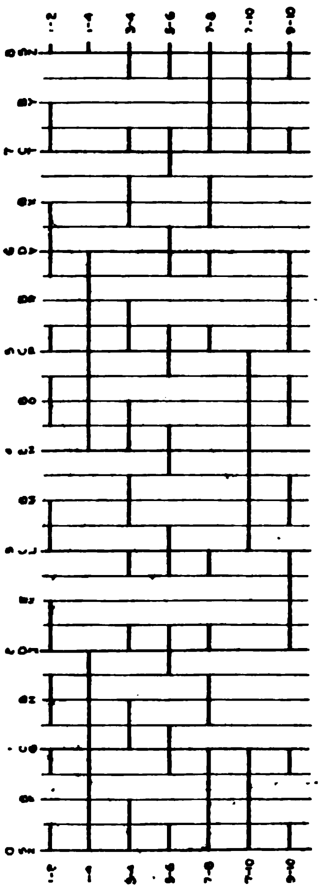
UNBALANCED EXPOSURE CHARTS
INDUCTION FROM BALANCED
THREE PHASE VOLTAGES AND CURRENTS
STANDARD TELEPHONE TRANSPOSITIONS-CIRCUIT 1-2
TWO MILE POWER BARRELS, CONTINUOUS,
HALTING AND HALTING-REVERSING

SCALE. 1 INCH = ½ MILE

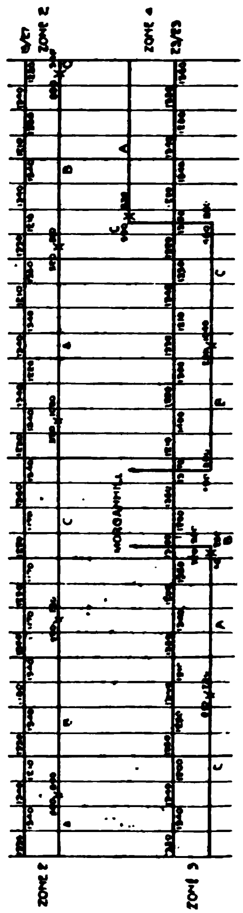


TR NO 66

Pl. C No. 368.
9-25-16

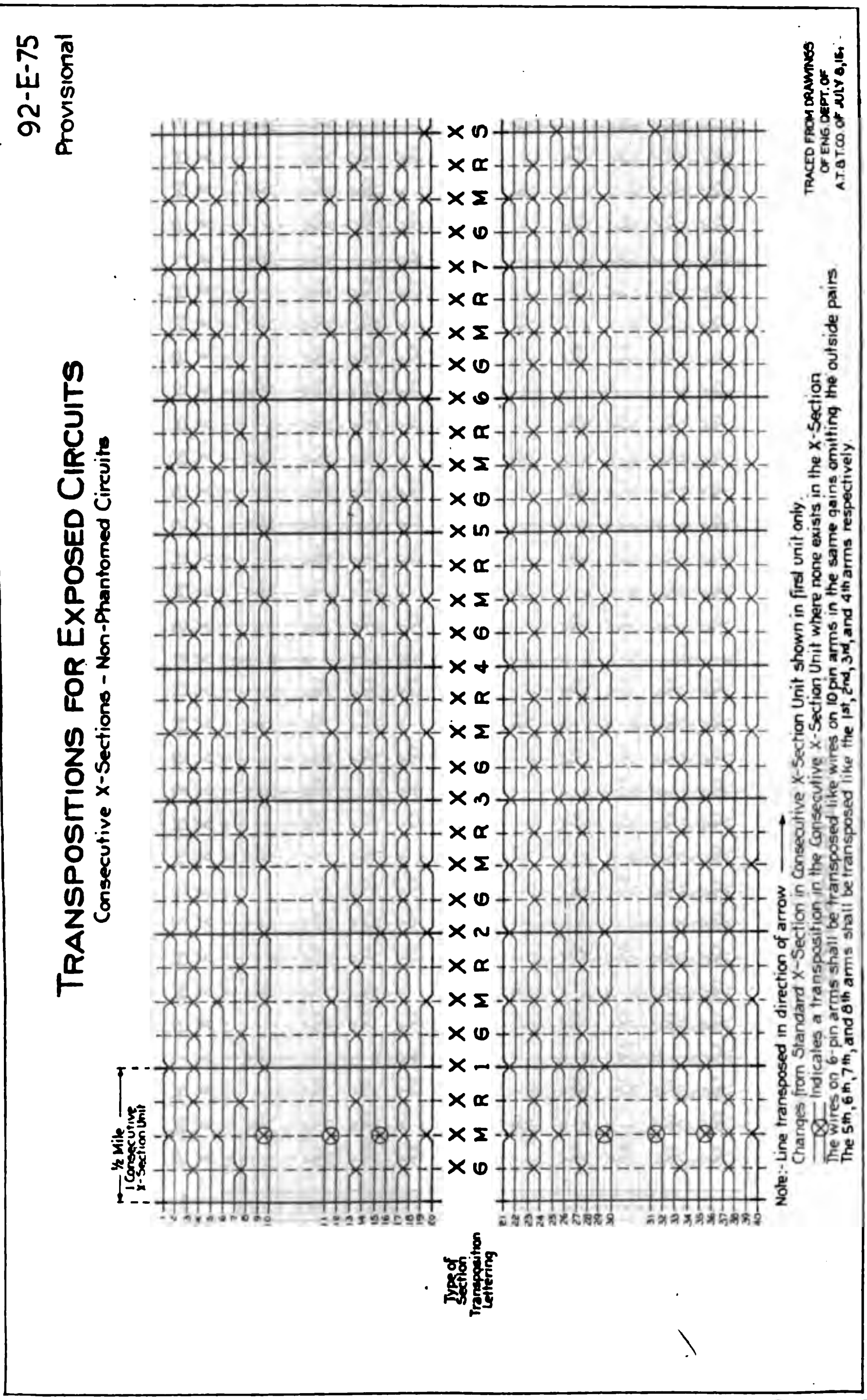


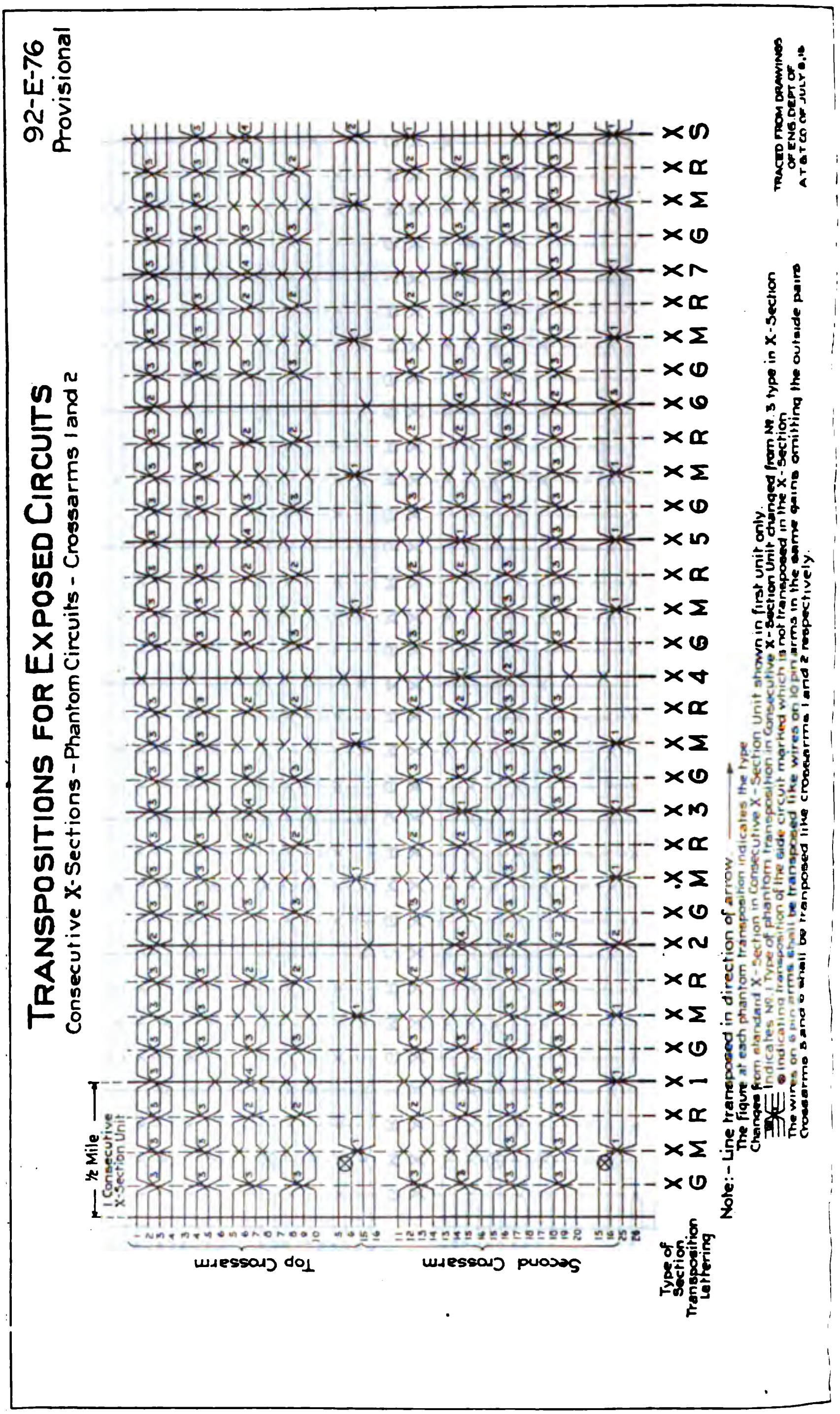
TELEPHONE TRANSPOSITION DIAGRAM
FIG. 1



TRANSPOSITION DISTANCE CHART
SAN JOSE - GILROY PARALLEL
FIG. 2

T.R. No. 86.

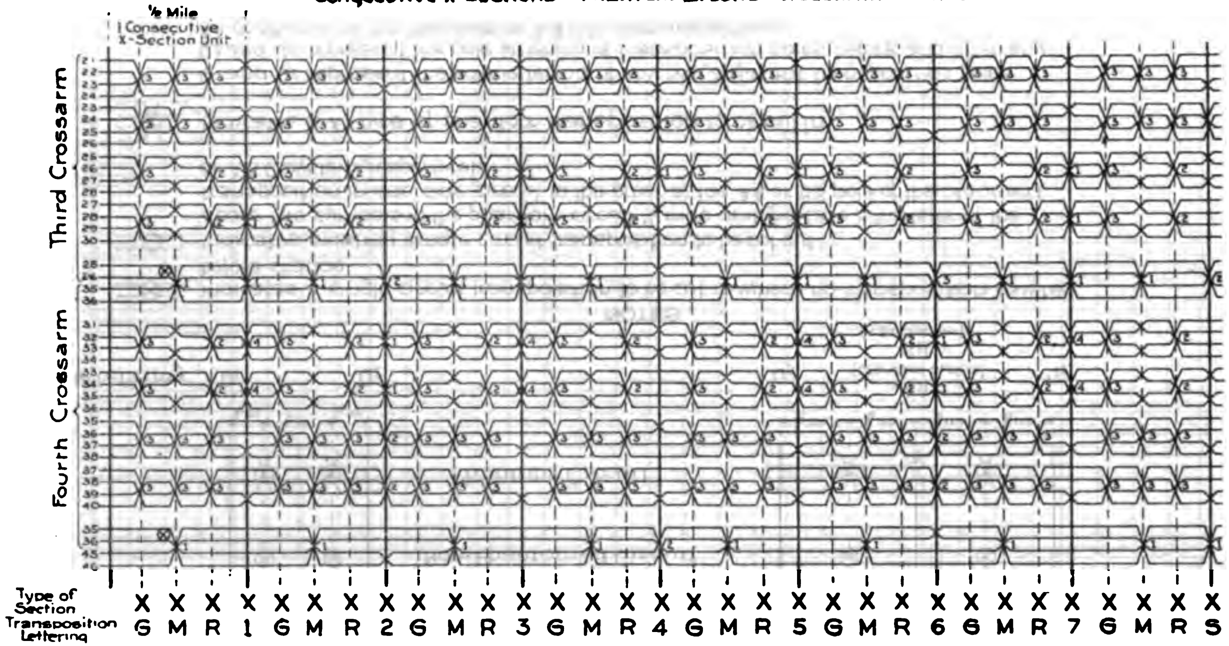

92-E-75
Provisional
TRANSPOSITIONS FOR EXPOSED CIRCUITS
Consecutive X-Sections - Non-Phantomed Circuits
½ Mile
1 Consecutive X-Section Unit
Type of Section
Transposition Lettering
6 M R 1 6 M R 2 6 M R 3 6 M R 4 6 M R 5 6 M R 6 6 M R 7 6 M R S
Note:- Line transposed in direction of arrow
Changes from Standard X-Section in Consecutive X-Section Unit shown in first unit only.
⊗ Indicates a transposition in the Consecutive X-Section Unit where none exists in the X-Section
The wires on 6-pin arms shall be transposed like wires on 10 pin arms in the same gains omitting the outside pairs
The 5th, 6th, 7th, and 8th arms shall be transposed like the 1st, 2nd, 3rd, and 4th arms respectively.
TRACED FROM DRAWINGS OF ENG. DEPT. OF A.T.&T. CO. OF JULY 6, 15.


TRANSPOSITIONS FOR EXPOSED CIRCUITS
Consecutive X-Sections - Phantom Circuits - Crossarms 1 and 2
92-E-76
Provisional
½ Mile
1 Consecutive X-Section Unit
Top Crossarm
Second Crossarm
Type of Section
Transposition Lettering
Note:- Line transposed in direction of arrow.
The figure at each phantom transposition indicates the type
Changes from standard X-Section in Consecutive X-Section Unit shown in first unit only.
Indicates No. 1 Type of phantom transposition in Consecutive X-Section Unit changed from No. 3 type in X-Section
Indicating transposition of the side circuit marked which is not transposed in the X-Section
The wires on 6 pin arms shall be transposed like wires on 10 pin arms in the same gains omitting the outside pairs
Crossarms 5 and 6 shall be transposed like crossarms 1 and 2 respectively.
TRACED FROM DRAWINGS OF ENG. DEPT OF A.T.&T. CO OF JULY 8, 15

TRANSPOSITIONS FOR EXPOSED CIRCUITS

Consecutive X-Sections - Phantom Circuits - Crossarms 3 and 4

92-E-77
Provisional

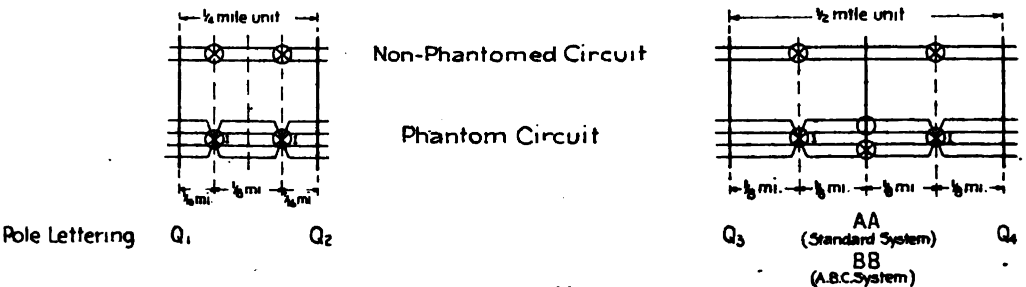


Note:- Line transposed in direction of arrow ⟶
The figure at each phantom transposition indicates the type.
Changes from standard X-Section in Consecutive X-Section Unit shown in first unit only
Indicates Nº 1 type of phantom transposition in Consecutive X-Section Unit changed from Nº 3 type in X-Section.
⊗ Indicating transposition of the side circuit marked which is not transposed in the X-Section
The wires on 6 pin arms shall be transposed like wires on 10 pin arms in the same gains omitting the outside pairs
Crossarms 7 and 8 shall be transposed like crossarms 3 and 4 respectively.

TRACED FROM DRAWINGS OF ENG DEPT OF A T & T CO OF JULY 8, 15

93-E-2
Provisional

Whole Line Transposition Units



Notes

Indicates a single circuit transposition to be cut in where no transposition existed before change

Indicates existing single circuit transposition to be cut out.

Above two symbols in the phantom circuit ½ mile unit indicate a transfer of the transposition in one side circuit to the other at the AA or BB pole of the Standard or A.B.C. System, respectively.

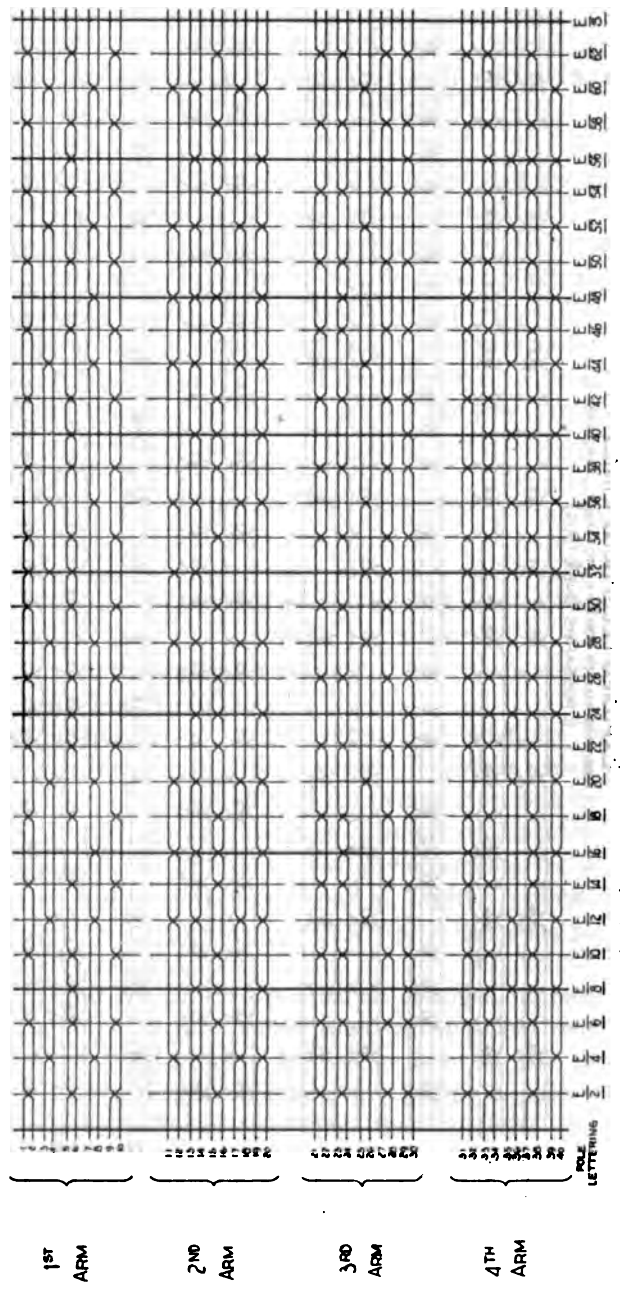
Indicates a #1 type of phantom transposition to be cut in.

Q_1 and Q_2 represent the pole lettering of any two adjacent transposition poles.
Q_3 and Q_4 represent the pole lettering of transposition poles before and after any AA or BB pole of the Standard or A.B.C System respectively.
No changes should be made in the regular transpositions except for the transfer of side circuit transpositions in the case of the ½ mile unit.

TRACED FROM DRAWINGS OF ENG. DEPT. OF A.T.&T. CO. OF NOV 23, 15

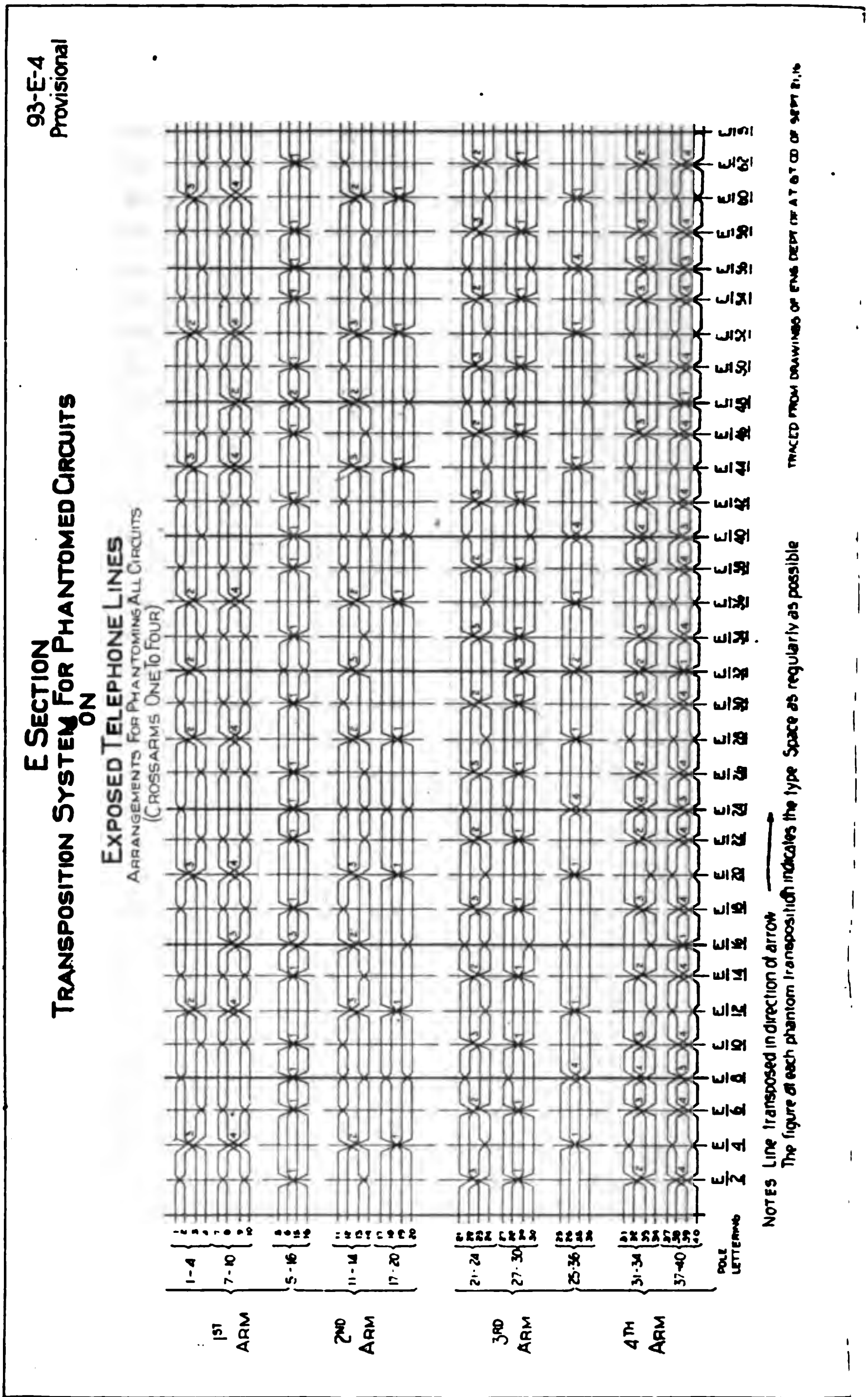
93-E-3
Provisional

E SECTION
TRANSPOSITION SYSTEM FOR NON-PHANTOMED CIRCUITS
ON
EXPOSED TELEPHONE LINES
(CROSSARMS ONE TO FOUR)



NOTES: - Line transposed in direction of arrow
Space as regularly as possible

TRACED FROM DRAWINGS OF ENG DEPT OF A.T.&T CO. OF SEPT 21,16.


93-E-4
Provisional
E SECTION
TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUITS
ON
EXPOSED TELEPHONE LINES
ARRANGEMENTS FOR PHANTOMING ALL CIRCUITS
(CROSSARMS ONE TO FOUR)
1ST ARM
2ND ARM
3RD ARM
4TH ARM
1-4
7-10
5-16
11-14
17-20
21-24
27-30
25-36
31-34
37-40
POLE LETTERING
NOTES Line transposed in direction of arrow
The figure at each phantom transposition indicates the type Space as regularly as possible

Technical Report No. 67.

August 14, 1917.

NOTES ON THE TRANSPOSITION OF POWER CIRCUITS, AND PRIVATE TELEPHONE CIRCUITS.*

Introduction.

Discussions in technical reports Nos. 39, 46, 51, 66 and elsewhere have dealt with: (1) the effect of transpositions in controlling the phase of induction due to balanced currents and voltages, and in balancing the capacitances of the several conductors of the power circuit to earth; and (2) systems of telephone transpositions. The purpose of this report is to consider some details of power-circuit transpositions, and to discuss transposition systems for the private telephone circuits of power companies, often carried on the same lines with the power circuits.

Transposition Systems for Power Circuits.

The ordinary method of transposing a three-phase circuit is to divide a section of uniform configuration into three parts of equal length by two transpositions at the third points, such that each conductor occupies the three conductor positions in succession as shown in Fig. 1, of drawing No. 373. Thus the condition for a barrel† in a three-wire circuit is satisfied with two transpositions. The transpositions of Fig. 1-b are "reversed" relative to those of Fig. 1-a.

With a larger number of transpositions an indefinite variety of arrangements may be used to form a barrel. Ways in which this may be done in practice are illustrated by drawings Nos. 361 and 362 of technical report No. 66, showing plans proposed for particular parallels.

Fig. 2 of drawing No. 373 shows two arrangements for securing a barrel with three ordinary transpositions at "sixth" points. Fig. 2-a has the advantage of having the power wires in the same positions at both ends of the barrel.

Fig. 3 of drawing No. 373 shows a barrel obtained by means of five transpositions at the "sixth" points, each transposition requiring the interchange of two conductors only, the position of the third being unaltered. Such an arrangement may prove useful in cases where it is found simpler to interchange two wires than to make the ordinary cyclic transposition of Fig. 1. While an ordinary cyclic transposition changes the phase of the induction from balanced voltages and currents by 120 degrees, the interchange of two wires reverses the phase of

*Supplementing technical report No. 66.

†A barrel is defined as an arrangement of a uniformly configured section of power circuit within which each conductor occupies each of the conductor positions for equal distances.

the induction with respect to the component from the wire whose position is not altered. This method of transposing a line is not in general to be recommended.

The rearrangement of conductors at crossovers necessary in order that the effect of the crossover on induction may be nullified, is discussed in technical report No. 66, page 1028. At the crossover each conductor is changed to a symmetrical position on the opposite side of the supporting structure, it being assumed that the separation of power and communication lines is the same in the two directions. In cases where it is desired that the crossover have the effect of an ordinary transposition, a special transposition is used, which is equivalent to an ordinary transposition superposed on this rearrangement. If the configuration is such that two of the conductors are symmetrically located with respect to the supporting structure, the position of one of these conductors, at such a transposition point, remains unaltered.

The application of these principles to the unsymmetrical horizontal configuration is shown by Fig. 4 of drawing No. 373. Capacitance balance to earth is secured in each half of the total length shown. By inserting the transposition "M" at the center, a barrel, that is, a section balanced to a communication wire T_1 is secured in the total length, while for communication wire T_2 which crosses the power circuit at each transposition, balance is obtained in each half of the total length.

The arrangements of three-wire circuits on drawing No. 373 cause mutually neutralizing effects in a neighboring communication wire in the length of section shown, for all balanced voltages and currents associated with the conductors, that is, for those whose vector sum is zero. This includes single-phase components employing two wires of a three-phase circuit as well as the components 120 degrees apart in phase. Thus the same transposition system is effective for any three-wire circuit whether three-phase, two-phase or single-phase.

In general, if there are "n" like conductors, by transposing them in cyclic fashion they form a barrel divided into "n" equal parts by "n — 1" transpositions and "n" mutually neutralizing fields are produced in the "n" parts of the barrel. Each transposition changes the time phase by $\frac{360}{n}$ degrees for induction from components equal in magnitude and $\frac{360}{n}$ degrees apart in phase, and also provides mutually neutralizing fields from the single-phase components confined to two or more of the conductors. This leaves only the residuals, having the group of conductors as one side and earth as the other side of their circuit, to produce unneutralized fields of influence.

This principle may be applied in the case of a four-wire, three-phase circuit where the neutral wire is like the phase-wires, by transposing the neutral wire as well as the three phase-wires, thereby taking account of the current in the neutral wire and of its voltage to ground (if isolated therefrom). This arrangement is shown in Fig. 5 of drawing No. 373. Obviously, such transpositions also equalize the capacitances to earth of the several conductors. It is not necessary, however, to transpose a neutral wire for the sake of capacitance balance of the three line wires to earth.

In technical reports Nos. 51 and 65 it is shown that great advantage in reducing capacitance unbalance and in lessening the intensity of the inductive effects of balanced voltages and currents of twin circuits may be gained by properly interconnecting them or by otherwise fixing the phase relations of the conductors of the two circuits. The vertical and nearly vertical configurations are much employed for twin-circuit lines. To minimize capacitance unbalance the method* of connection of the wires should be upper to upper, middle to lower, and lower to middle. For minimum induction the interconnection* should be in most cases, upper to lower, middle to middle, and lower to upper. (Refer to technical report No. 65 for exceptions). Fig. 1 of drawing No. 374 shows the transpositions to provide the best arrangement both within and without a parallel. The several wire-arrangements are shown in cross-section at the top of the figure. The transpositions in the second circuit are reversed with respect to those in the first circuit. At each end of the parallel an additional transposition is necessary in the second circuit, to change the wire-arrangement from the best for capacitance balance to the best for mitigation of induction. Otherwise, transpositions should not be installed at junctions of barrels.

Transposition Systems for Private Telephone Circuits.

The private telephone circuits of power companies, especially when carried on the same lines with power circuits are subject to inductive effects of great intensity† and require very careful transposition in conjunction with the transposition of the power circuits, in order to adequately control the inductive effects. As the number of such telephone circuits on one pole line is few and "phantoming" (common

*These methods apply when ground wires are not used. Special investigation is required when such are present.

†The intensity of the inductive effect depends upon the relative positions of the conductors of both power and telephone circuits and their proximity to the earth. Slight changes in position may very materially change the magnitude of the inductive effects. Methods for studying the variation of the induction with the relative positions of the circuits are given in technical report No. 64, while the results of an extended study of this character are given in technical report No. 65, though not for conditions of such close proximity of the two classes of circuits as is here assumed. To minimize the difference in inductive effects on the two sides of a telephone circuit, the spacing of the telephone conductors should, in general, be the minimum consistent with the avoidance of short circuits.

practice with commercial telephone circuits) is seldom or never employed, the elaborate designs described in technical reports Nos. 39, 46 and 66 are unnecessary. Care should be taken, however, to transpose the telephone circuits frequently and in such a way as to minimize the cumulative effects of phase-change and attenuation.† To accomplish the last mentioned result there should be (1) an even number of telephone transpositions in each third of a three-phase barrel (half of a single-phase barrel) or (2) transpositions should be placed in the telephone circuit opposite the power circuit transpositions; the first alternative being preferable. A system involving simply telephone transpositions opposite mid-points of thirds of a three-phase barrel (halves of a single-phase barrel) is especially undesirable in allowing the unbalances due to phase-change and attenuation to accumulate.

Fig. 2 of drawing No. 374 presents three systems of transpositions for telephone circuits, properly coordinated with the transpositions of one barrel, in a three-phase circuit, and suitable for the transposition of a power company's private telephone circuit. Inspection of the drawing will show that the average position of each side of each telephone circuit with respect to the power circuit is the same in each one-third of a barrel in the three-phase circuit or in each one-half of a barrel in the single-phase circuit. In other words, both sides of a telephone circuit so transposed are equally "exposed" or "balanced" with respect to the power circuit. This, in general, is the condition to be sought in any transposition system. In the three systems shown this condition of balance obtains in each eighth part of the section of barrel above mentioned. For the most effective results an integral number of power-circuit barrels with the accompanying co-ordinated telephone transposition systems should be installed between successive discontinuities.*

The telephone transpositions as shown are adapted for use with barrels in a three-phase circuit ranging approximately from six to twelve miles in length and with barrels in a single-phase circuit ranging approximately from four to eight miles in length. If the lengths are from three to six for three-phase or from two to four miles for single-phase, one-half the systems as shown will ordinarily be sufficient. In such an event, if system 3 is used, an additional transposition should be installed opposite each power-circuit transposition.

Owing to the great intensity of the induction to which these private telephone circuits are subjected, every reasonable effort should be made to have the actual locations of the transpositions coincide with their theoretical locations. The deviations from the theoretical locations should not ordinarily exceed one hundred feet. Rather than sacrifice

†See discussion on page 1020 of technical report No. 66.

*The term "discontinuity" means any abrupt change in the relative positions of a power and communication circuit, or any abrupt change in configuration, line impedance or load along either such circuit.

accuracy in the spacing of the transpositions it may be preferable to employ a less elaborate system than would otherwise be used.

To prevent "cross-talk" between different telephone circuits carried on the same pole-line, the three transposition systems shown are different and mutually balanced. System 1 is recommended when there is only one telephone circuit and systems 1 and 2 when there are two circuits. It is assumed that these telephone circuits are not to be "phantomed."

Methods of Obtaining Clearance at Power Transpositions.

Many different arrangements have been proposed and used for interchanging conductors at transposition points of power circuits. Two general classes are to be distinguished: (1) those in which the interchange requires a span or more of line, with poles spaced normally or nearly so, and (2) those in which the conductors are dead-ended and rearranged at one supporting structure (or two very close together).

The simplest case is that of the triangular configuration where often no special construction whatever is necessary, though it is desirable that the span in which the twist is made be moderately short.

Various methods, employing extra cross-arms, extensions on the regular cross-arms, "buck" arms, extra insulators on the regular cross-arms, etc., are illustrated by the figures on drawing No. 375. Different methods are available, depending on the configuration, and upon whether pin or suspension insulators are used. No attempt is here made to discuss the details of materials required, since they depend so much on the strength, style of construction and voltage. The methods shown for the horizontal configuration are used also for the triangular configuration. One of the figures of vertical lines illustrates twin circuits transposed as shown on Fig. 1 of drawing No. 374. For simplicity, the twin circuit has been omitted from the other drawings.

Within parallels it is an advantage to have the transpositions complete within as short a space as practicable, to minimize the extent of the disturbance of the normal configuration. The three-span type is especially objectionable from this standpoint.

Many different situations arise in practice, methods for the treatment of which may be suggested by the examples given.

Respectfully submitted.

(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

ATTACHMENTS: P. I. C. Drawings Nos. 373, 374, 375.

Approved: August 20, 1917.

(Signed) R. W. MASTICK,
Field Engineer.

Approved: August 24, 1917.

SUBCOMMITTEE ON TESTS,
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(Signed) ARTHUR F. BRIDGE,
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August 24, 1917.

F.I.C. 373

FIG. 1

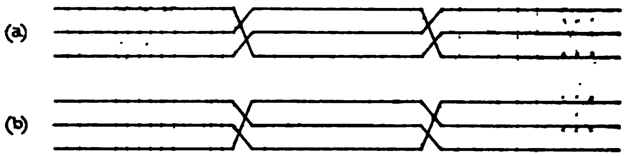


FIG. 2

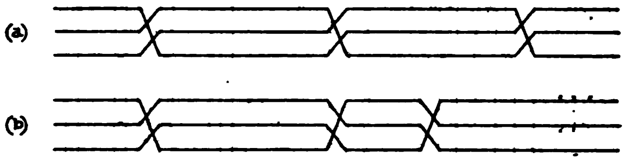


FIG. 3

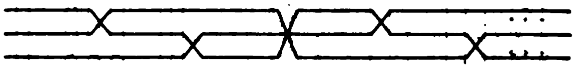


FIG. 4

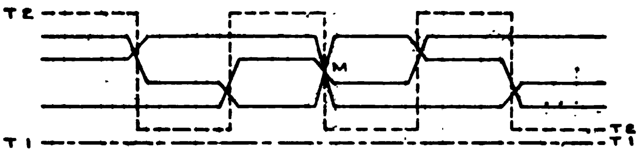
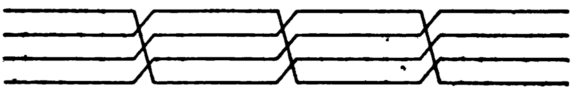
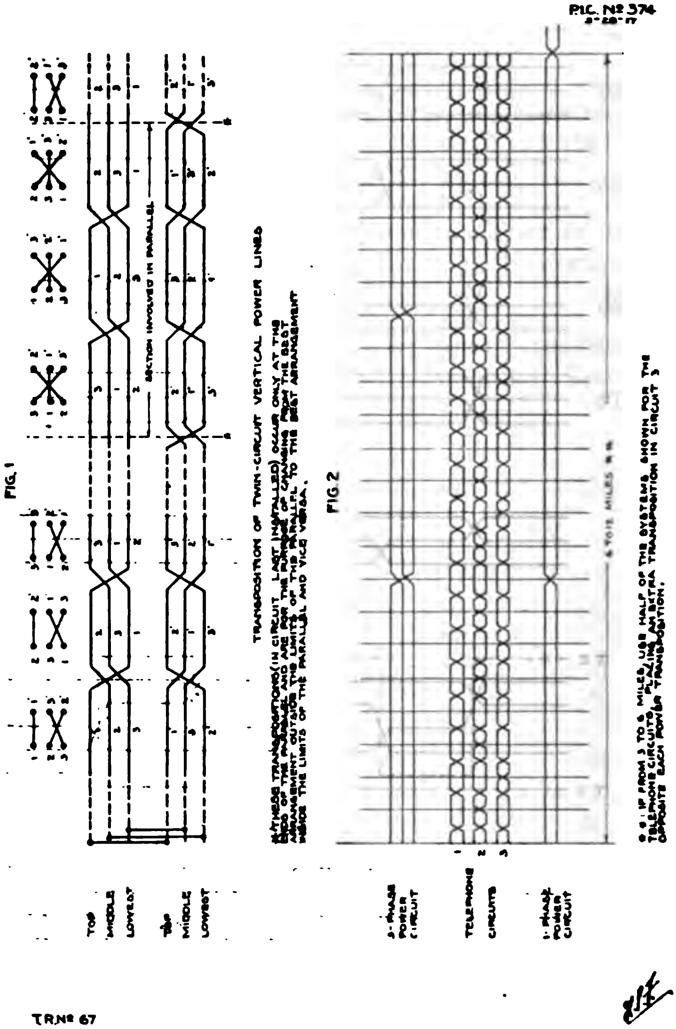


FIG. 5



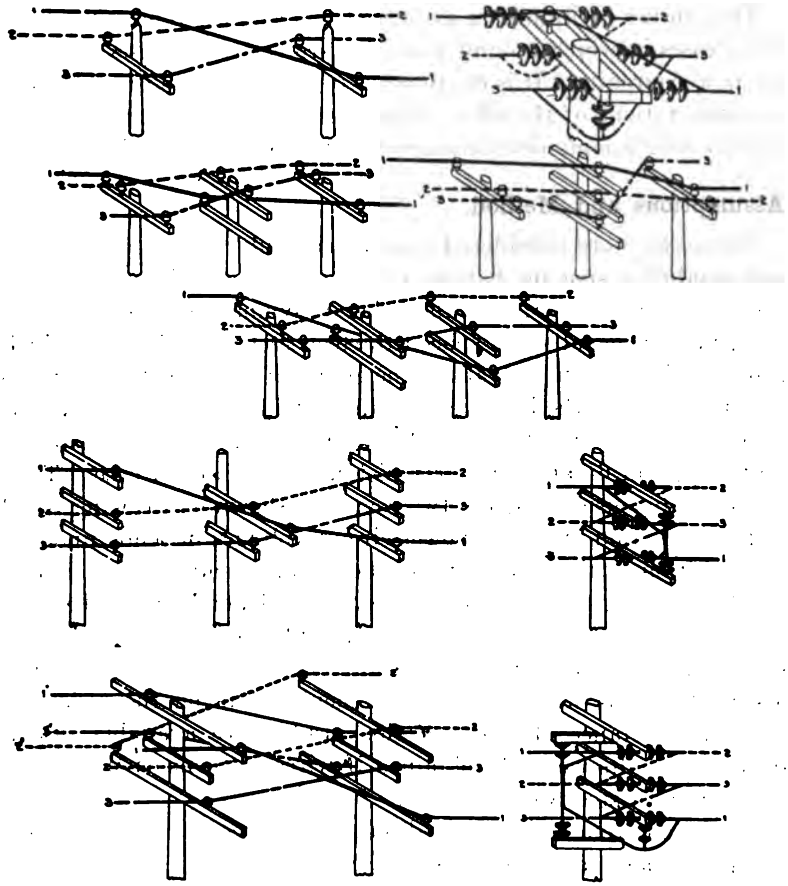
T R. Nº 67

396


P.I.C. No 374
FIG. 1
TOP
MIDDLE
LOWEST
TOP
MIDDLE
LOWEST
SECTION INVOLVED IN PARALLEL
TRANSPOSITION OF TWIN-CIRCUIT VERTICAL POWER LINES
A - THESE TRANSPOSITIONS (IN CIRCUIT LAST INSTALLED) OCCUR ONLY AT THE ENDS OF THE PARALLEL AND ARE FOR THE PURPOSE OF CHANGING FROM THE BEST ARRANGEMENT OUTSIDE THE LIMITS OF THE PARALLEL TO THE BEST ARRANGEMENT INSIDE THE LIMITS OF THE PARALLEL AND VICE VERSA.
FIG. 2
3-PHASE POWER CIRCUIT
TELEPHONE CIRCUITS
1
2
3
1-PHASE POWER CIRCUIT
6 TO 12 MILES
B - IF FROM 3 TO 6 MILES, USE HALF OF THE SYSTEMS SHOWN FOR THE TELEPHONE CIRCUITS, PLACING AN EXTRA TRANSPOSITION IN CIRCUIT 3 OPPOSITE EACH POWER TRANSPOSITION.
TR No 67

METHOD OF TRANSPOSING
THE CONDUCTORS OF POWER CIRCUITS

PIC. Nº 375
AUG. 4. 1917.



TR. Nº 67

Technical Report No. 68.

June 28, 1917.

EFFECT OF PROTECTIVE GROUND WIRES OF POWER LINES ON INDUCTION IN PARALLEL COMMUNICATION CIRCUITS.

Introduction.

The studies of induction presented in technical report No. 65 do not cover cases in which ground wires for protective purposes are used with the power circuits. It is the purpose of this report to present the results of computations of the effect of such ground wires on the induction in neighboring communication circuits for two relatively simple cases.

Assumptions and Method.

Two cases were considered: one a triangular power circuit with six-foot conductor spacing, having a $\frac{3}{8}$-inch steel ground wire six feet above the vertex conductor; and the other a symmetrical horizontal circuit with adjacent conductors spaced thirteen feet and with two $\frac{1}{2}$-inch steel ground wires seven feet above the plane of the power conductors and equidistant from the intermediate and outside conductors. A grounded communication circuit was assumed 30 feet and 600 feet distant in the first case, and 65 and 1040 feet distant in the second case. The cross-sectional dimensions for the two cases are shown on drawing No. 376.

The induction from the power-circuit voltages depends upon the size and position of the ground wires, but is independent of their material. Induction from the power-circuit currents depends upon the resistance and magnetic permeability of the ground wires as well as upon their size and position. Ordinary grade of galvanized steel strand was assumed in this study, since experimental data on the resistance and internal reactance of wires of this grade were available. Somewhat different results would be obtained if wires of Siemens-Martin or extra high-strength steel were assumed.

Induction from both balanced and residual components of the power-circuit voltages and currents was considered, and also the induction from balanced voltages and characteristic residual voltage acting together. The effect of the latter combination is equivalent to that of balanced voltages between power conductors on the assumption of no transpositions and uniform construction throughout the entire length of a short power circuit. In computing the induction from balanced current, both possible directions of phase rotation of the currents were considered.

The theory and method for computing induction has been described in technical report No. 64. The general case of a single-circuit power line with one ground wire or a symmetrical twin-circuit power line with one or two symmetrically placed ground wires is quite complicated, requiring the solution of determinants of the fourth order. Such cases are presented by the vertical configuration. The cases chosen are relatively simple because of symmetrical relations whereby the solution is reduced to that of third-order determinants. To facilitate the computations, forms were developed similar to those described in technical report No. 64. Calculating machines were used for all the arithmetical operations. To lessen the numerical work the dimensions were chosen to accord with certain cases computed for technical report No. 65.

Results of Computations.

The results of the computations are given in Tables I and II. Table I gives the values of induction in volts per volt and millivolts per ampere-mile (at 60 cycles), both with and without the presence of the ground wires. Table II gives the values with the ground wires present in per cent of the values which would exist were the ground wires not present.

Discussion of Results.

The coefficients of induction from residual voltage and current are in all cases reduced by the presenee of the ground wires, the amount of the reduction being, in general, greater for induction from residual voltage than for induction from residual current, due largely to the resistance of the ground wires which limits the current induced by the residual current and alters its phase from complete opposition to the residual current. For induction from residual current the percentage reduction is practically independent of the separation from the power circuit. In the cases studied the reduction for residual voltage increases slightly with increase of horizontal separation.

TABLE I.

Effect of Ground Wires on Induction from Three-Phase Power Circuits.

Configuration	Characteristic residual voltage		Horizontal separation, feet	Coefficients of induction									
				Balanced voltages		Residual voltage		Balanced voltages and characteristic residual voltage		Balanced currents		Residual current	
	Without	With		Without	With	Without	With	Without	With	Without	With	Without	With
	2.376	2.862	30	12.57	12.85	55.0	47.3	12.74	13.21	20.86	17.52 25.83*	381.1	365.4
			600	0.0605	0.0303	0.326	0.255	0.0685	0.0232	0.795	0.146 1.282*	58.68	51.54
	6.772	2.411	65	15.31	12.62	31.58	23.85	14.89	12.30	40.62	37.11 44.46*	299.0	281.3
			1040	0.0139	0.0310	0.1740	0.1219	0.0872	0.0282	0.925	0.668 1.305*	26.35	24.79

*Magnitude depends upon phase rotation of balance currents. The smaller value occurs when current in conductor nearest communication circuit leads current in intermediate conductor by 120°.

NOTE.—Coefficients of induction are expressed thus:
Balanced voltages—volts per kilovolt between power conductors.
Residual voltage—volts per kilovolt residual.
Balanced currents—millivolts per ampere-mile at 60 cycles.
Residual current—millivolts per ampere-mile at 60 cycles.
Characteristic residual voltage is expressed in per cent of balanced three-phase voltage between conductors.

TABLE 11.

Effect of Ground Wires on Induction from Three-Phase Power Circuits.
Induction With Ground Wires in Per Cent of Induction Without Ground Wires.

Configuration	Characteristic residual voltage	Horizontal separation, feet	Induction				
			Balanced voltages	Residual voltage	Balanced voltages and characteristic residual voltage	Balanced currents	Residual currents
•	120	90	102	86.0	104	84.0 124*	95.9
• • • •		600	50.1	78.2	33.9	18.4 161*	95.9
• • •	35.6	65	82.4	75.6	82.6	91.4 120*	94.1
• • •		1040	223	70.1	392	74.4 141*	94.1

*Magnitude depends upon phase rotation of balanced currents. The smaller value occurs when current in conductor nearest communication circuit leads current in intermediate conductor by 120°.

The ground wires may either increase or decrease the induction from balanced voltages and currents, depending upon the relative position of the two classes of circuits; and doubtless, also, on the position of the ground wires relative to the power conductors. This latter variation was not studied.

When the ground wires are present the magnitude of the induction from balanced currents depends upon the phase rotation of the currents or the side of the power circuits on which the communication circuit is situated. For the cases studied the smaller value occurs when the current in the conductor nearest the communication circuit leads the current in the intermediate or vertex conductor by 120°. The induction in the one case is greater than when the ground wires are not present and in the other less. The cause of this dissymmetry in the field about the power circuit is the resistance of the ground wires, on account of which their currents are not in quadrature with their induced voltages. For any symmetrical configuration the field, due to the balanced voltages, is symmetrical with respect to the power circuit.

The addition of the ground wires in the cases studied slightly increases the characteristic residual voltage of the triangular circuit and greatly reduces that of the horizontal circuit.

The results obtained in this brief study are sufficient to indicate the important effect which ground wires may have in either increasing or decreasing the induction otherwise caused by a power circuit. Though only single-circuit lines were considered, equally important effects may be expected in the case of twin-circuit power lines.

Respectfully submitted.

(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

ATTACHMENT: P. I. C. Drawing No. 376.

Approved: August 20, 1917.

(Signed) R. W. MASTICK,
Field Engineer.

Approved: August 24, 1917.

SUBCOMMITTEE ON TESTS,
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Chairman.

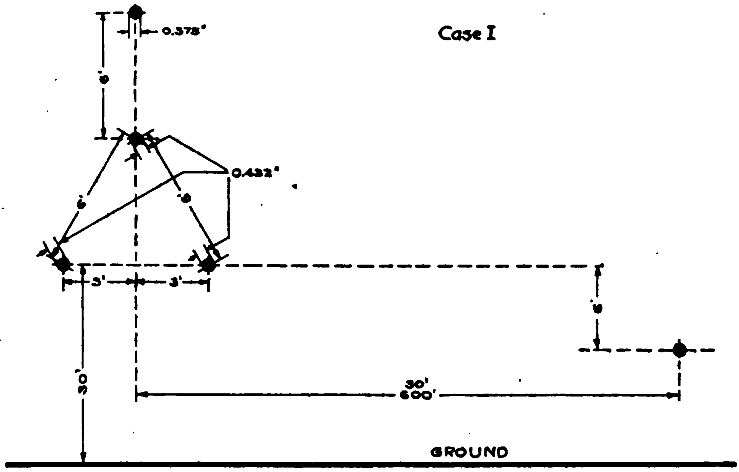
JOINT COMMITTEE ON INDUCTIVE INTERFERENCE,
(Signed) ARTHUR F. BRIDGE,
Secretary.

August 24, 1917.

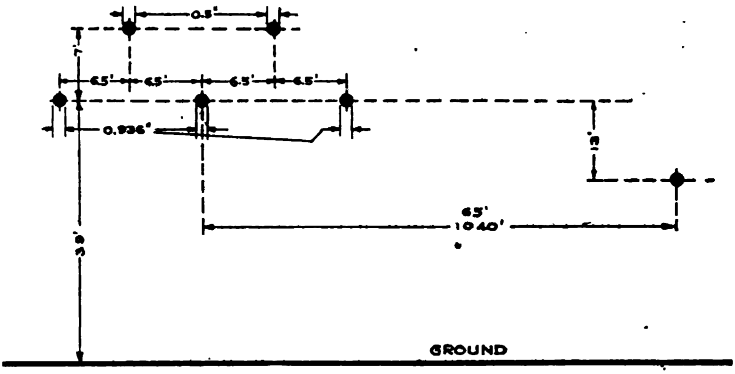
P.I.C. No. 376
6-28-17

EFFECT OF PROTECTIVE GROUND WIRES OF POWER LINES ON INDUCTION IN PARALLEL COMMUNICATION CIRCUITS.

Case I



Case II



Technical Report No. 69.

July 19, 1917.

RELATION OF CURRENTS IN TERMINAL APPARATUS OF TELEGRAPH CIRCUITS TO INDUCED VOLTAGES AND LOCATION OF PARALLEL.

Introduction.

The purpose of this report is to describe and to illustrate by examples the method of computing the current in the terminal apparatus of a telegraph circuit in terms of the induced voltage, given the position of the parallel, the length and constants of the line and the constants of the terminal apparatus. Methods of computing induced voltages in terms of the voltages and currents of parallel power circuits and the dimensions of the parallel are described in technical report No. 64. The coefficients of induction for most practical cases of parallelism with single-circuit power lines are available from the curves of technical report No. 65. The two reports cited together with this one furnish means of determining the induced extraneous current in the terminals of telegraph circuits in terms of the voltages and currents of parallel power circuits and the various dimensions and electrical constants involved.

Methods of Computation.

In computing induced current, any short parallel involving a grounded communication circuit may be represented by the equivalent network shown on drawing No. 377. The electrically induced voltage, E_e, is represented by a fictitious generator, whose open-circuit voltage is E_e, in series with the open-circuit impedance, Z_p, of the section of line within the parallel and connected between line and ground at the middle of the parallel. The magnetically induced voltage, E_l, is represented by two generators each with open-circuit voltage equal to $\frac{1}{2}E_l$ and connected in series with the line. Z_1 and Z_2 are the impedances to ground of the sections of line in the two directions from the middle of the parallel, including the effect of the terminal apparatus at the ends of the line. It is assumed that Z_p is large compared with the other impedances.

Expressing the currents at the two ends of the parallel in terms of the induced voltages and the impedances,

$$I_e' = \frac{Z_2}{Z_1 + Z_2} \cdot \frac{E_e}{Z_p} \qquad (1)$$

$$I_e'' = -\frac{Z_1}{Z_1 + Z_2} \cdot \frac{E_e}{Z_p} \qquad (2)$$

$$I_l = \frac{E_l}{Z_1 + Z_2} \qquad (3)$$

The induced current I_l, due to magnetic induction, is directly proportional to the voltage E_l, induced along the conductors. This voltage, E_l, is proportional to the length of the parallel and the induced voltage per unit length, determined by the cross-sectional dimensions of the parallel and the power-circuit currents. The induced current, I_e, due to electric induction is directly proportional to the induced voltage, E_e, and also to the length of the parallel, since Z_p is inversely proportional to the length. E_e is independent of the length of the parallel and is determined by the cross-sectional dimensions of the parallel and the power-circuit voltages. Thus the current due to magnetic induction is proportional to the corresponding total induced voltage and that due to electric induction to the product of the corresponding induced voltage and the length of parallel. Therefore, the volt may be taken as a measure of magnetic induction and the volt-mile as a measure of electric induction. The relative amounts of current corresponding to these units are given below for the numerical examples.

Where there are several conductors on the line it is necessary to take account of their mutual "shielding" effects. If the separation from the power circuit is moderate or large the voltages induced in the several conductors are very nearly equal. Assuming, then, that the various circuits have equal impedances, the induced voltages and currents of the several conductors at any point along the line are practically equal.

Thus, allowing for other conductors, the impedance Z_p is determined by the *direct* capacitance to ground of the conductor within the parallel.

$$Z_p = -j \frac{10^6}{2\pi f C S_p} \qquad (4)$$

Where f is the frequency in cycles per second, C is the direct capacitance of the conductor in microfarads per unit length and S_p is the length of the parallel.

In computing the impedances Z_1 and Z_2 of the line in the two directions from the parallel the direct capacitance and direct leakage per unit length and an equivalent inductance per unit length are used. This equivalent inductance is equal to the sum of the self-inductance of the conductor and the mutual inductances between the conductor and all other conductors on the line. Thus

$$L = L_a + M_{ab} + M_{ac} + \dots M_{an} \tag{5}$$

In most cases telegraph circuits are sufficiently long that it is desirable to compute their impedances using formulas which take account of the distributed constants. The formulas which follow, given in terms of hyperbolic functions, are based upon this assumption. Their development is given in numerous textbooks.

Let the constants of the telegraph circuit be:

R = resistance—ohms per mile.
L = inductance (defined as above)—henrys per mile.
G = direct leakage conductance—micromhos per mile.
C = direct capacitance—microfarads per mile.
$Z = R + j\,2\pi\,f\,L$ = impedance—ohms per mile.
$Y = (G + j2\pi\,f\,C)\,10^{-6}$ = admittance—mhos per mile.
$P = \sqrt{ZY}$ = propagation constant—radians per mile.
$Z_0 = \sqrt{\frac{Z}{Y}}$ = characteristic impedance—ohms.
Z_r = terminal impedance—ohms.
S_1, S_2 = lengths of line in the two directions from the parallel—miles.

Then:

$$Z_1 = Z_0 \frac{Z_r + Z_0 \operatorname{Tanh} PS_1}{Z_0 + Z_r \operatorname{Tanh} PS_1} \tag{6}$$

$$Z_2 = Z_0 \frac{Z_r + Z_0 \operatorname{Tanh} PS_2}{Z_0 + Z_r \operatorname{Tanh} PS_2} \tag{7}$$

If the line is long and the attenuation great so that Tanh PS is nearly unity, or if the terminal impedance Z_r is nearly equal to the characteristic impedance, the line impedance is then practically equal to the characteristic impedance.

Formulas (1), (2) and (3) give the induced currents at the ends of the parallel in terms of the induced voltages and the impedances. To determine the currents at the terminals of the circuit it is further necessary to compute the attenuation between the parallel and the terminal apparatus.

Let the total currents at the two ends of the parallel be I′ and I″, and let I_r' and I_r'' be the currents in the terminal apparatus at the two ends of the circuit. Then,

$$I' = I'_e + I_1 \tag{8}$$
$$I'' = I''_e + I_1 \tag{9}$$

$$I_r' = I' \frac{Z_0}{Z_0 \operatorname{Cosh} PS_1 + Z_r \operatorname{Sinh} PS_1} \tag{10}$$

$$I_r'' = I'' \frac{Z_0}{Z_0 \operatorname{Cosh} PS_2 + Z_r \operatorname{Sinh} PS_2} \tag{11}$$

In combining the currents due to electric and magnetic induction, a knowledge of the phase relations of the power-circuit voltages and currents is required and also the phase of the induced voltages referred to the voltage or current of the power circuit. However, the attenuation may be computed separately for the currents due to electric and magnetic induction. This is the method employed in the illustrative examples discussed in the following section.

Numerical Examples.

As an illustration of such computations, the results are given of a study of the relation of induced current to induced voltage and position of parallel for a representative telegraph line. The constants of this line are as follows:

Length (S)	400 miles
Number of conductors	6
Spacing	1 foot
Height	20 feet
Size and material	No. 12 N.B.S.G. copper
Resistance (R)	6 ohms/mile
Inductance (equivalent) (L)	0.0138 henrys/mile
Capacitance (direct) (C)	0.00373 microfarads/mile
Leakage conductance (direct) (G)	0, 0.35 and 1.4 micromhos/mile
Terminal impedance at 60 cycles (Z_r)	600 + j 110 ohms

The values of leakage conductance assumed correspond approximately to values of insulation resistance of infinity, 1 and 1/4 megohm per mile as ordinarily measured from one wire to ground with the others grounded. Three positions of the parallel were considered; at each end and at the middle of the line. The results are given in Tables I and II. The percentage values are plotted on drawing No. 378, showing the variation of the current in the terminal apparatus with the distance to the parallel.

TABLE I.

Induced Currents in Telegraph Apparatus.

Parallel—miles from terminal \ G	Electric induction, microamperes per volt-mile			Magnetic induction, microamperes per volt		
	0	0.35	1.4	0	0.35	1.4
0	1.21	1.18	1.10	244	277	368
200	0.802	0.727	0.563	249	242	225
400	0.243	0.215	0.156	284	251	184

In Table II the values are given in per cent of the corresponding value when the parallel is adjacent to one terminal of the circuit.

TABLE II.

Induced Currents in Telegraph Apparatus.

Per Cent of Current When Parallel is Adjacent to Terminal.

Parallel—miles from terminal \ G	Electric induction			Magnetic induction		
	0	0.35	1.4	0	0.35	1.4
0	100	100	100	100	100	100
200	66	62	51	102	87	61
400	20	18	14	116	91	50

The effect of the location of the parallel is much more important for electric induction than for magnetic induction. In all cases, the current at the terminals due to electric induction decreases as the distance to the parallel increases. When the parallel is at the middle of the line the current is from 51 to 66 per cent (depending upon insulation resistance) of the corresponding value when the parallel is adjacent to the terminal. The total current from the parallel due to electric induction is practically independent of the position of the parallel, being controlled by the capacitances to ground of the section of circuit within the parallel. The amounts of current in the two directions are inversely proportional to the impedances. The line impedance increases with increasing distance from the terminal, hence the greater current is in the direction of the shorter length of line and the current in the terminal apparatus therefore decreases as the distance to the parallel increases.

The current due to magnetic induction is proportional to the sum of the impedances of the line in the two directions from the parallel. With high insulation this does not vary greatly with the position of the parallel, and since the attenuation is also small the current at the terminals does not vary greatly with position of the parallel. In this

case when the leakage conductance is negligible, the received current increases slightly as the distance from the terminal to the parallel increases.

Table III shows the relative importance of a volt of magnetic induction and a volt-mile of electric induction for a telegraph circuit of this length and character, giving the number of volt-miles causing the same current in the terminal apparatus as one volt.

TABLE III. Number of Volt-miles Equivalent to One Volt.

G / Parallel—miles from terminal	0	0.85	1.4	Average
0	202	235	334	285
200	311	333	400	375
400	1170	1170	1180	1170

The relative importance of the two factors varies both with insulation resistance and with the position of the parallel, the latter variation being the more important. Considering locations only within the first half of the line, 300 volt-miles may be taken as a representative equivalent of one volt.

Respectfully submitted.

(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

ATTACHED: P. I. C. Drawings Nos. 377 and 378.

Approved: August 20, 1917.

(Signed) R. W. MASTICK,
Field Engineer.

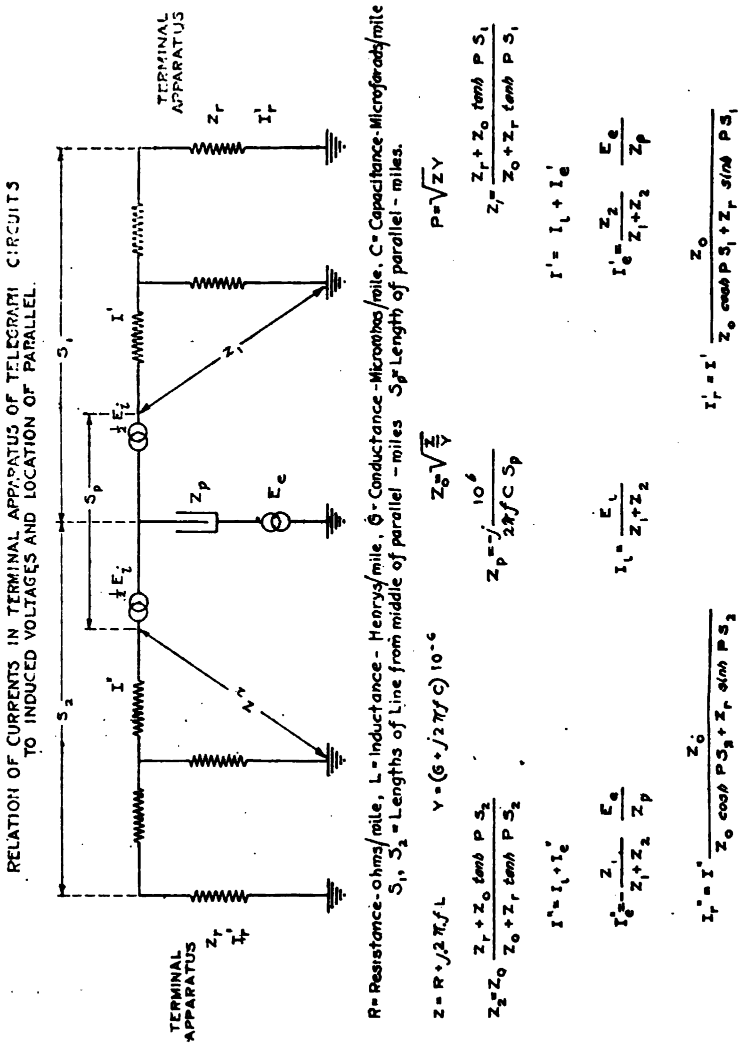
Approved: August 24, 1917.

SUBCOMMITTEE ON TESTS,
(Signed) J. E. WOODBRIDGE,
Chairman.

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE,
(Signed) ARTHUR F. BRIDGE,
Secretary.

August 24, 1917.

P.I.C. No. 377
7-1-17



T.R. No. 69

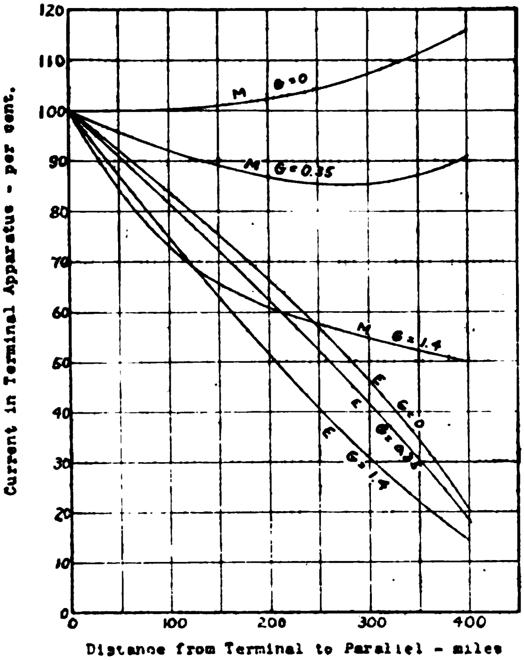
RGM.

P.I.C.No.378
7-18-17.

VARIATION OF INDUCED CURRENT IN TERMINAL
APPARATUS OF TELEGRAPH CIRCUITS WITH
LOCATION OF PARALLEL.

No.12 N.B.S.G. Copper Grounded Telegraph Circuit
400-mile Six-wire Line - Duplex Terminal Apparatus
Frequency of Induced Current - 60 Cycles per Second

G = direct leakage to ground - micromhos per mile
M = Induction from Currents. E = Induction from Voltages.



Technical Report No. 70.

August 17, 1917.

RELATIVE IMPORTANCE OF VOLT-MILE (ELECTRIC INDUCTION) AND VOLT (MAGNETIC INDUCTION) IN CAUSING INTERFERENCE WITH TELEPHONE CIRCUITS.

Introduction.

The relative importance of electric and magnetic induction, in a given case of parallelism involving power and telephone circuits, depends upon: (1) the magnitudes and wave-forms* of the voltages and currents of the power circuit, (2) the coefficients of electric and magnetic induction†, and (3) the relative amounts of current in the telephone receiver caused by unit amounts of induction of the two classes.

It is the purpose of this report to discuss the last-named factor and to derive a representative equivalent of the volt, as a unit of magnetic induction, in terms of the volt-mile as a unit of electric induction.

The current due to magnetic induction is directly proportional to the corresponding voltage induced in the circuit. The current due to electric induction is directly proportional to the product of the corresponding induced voltage and the length of parallel, the voltage due to electric induction being determined by the cross-sectional dimensions of the parallel and the power-circuit voltages, and independent of the length of the parallel. See technical report No. 69, page 1095, for further discussion of these units.

Both the longitudinal‡ induced voltages acting through the telephone circuit unbalances and the transverse‡ induced voltages cause current in the telephone receivers. Different considerations govern the relative importance of electric and magnetic induction in the two cases, and for convenience they are discussed separately.

Longitudinal Voltages.

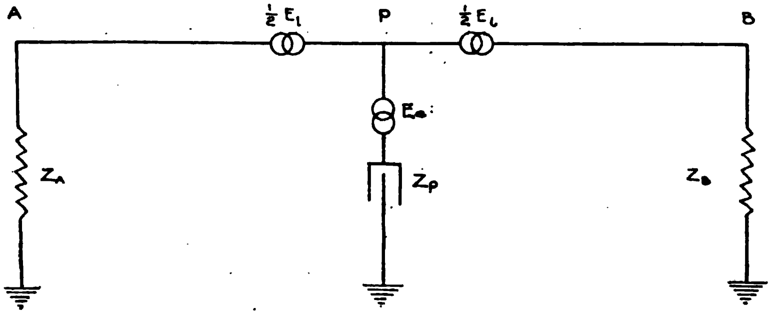
The current in the terminals of a telephone circuit due to longitudinal induction and admittance unbalance to ground is proportional to the product of the unbalance and the voltage to ground at the point where the unbalance occurs. That due to longitudinal induction and series impedance unbalance, is proportional to the product of the unbalance and the current existing in the conductors of the circuit as a group at the point where the unbalance occurs. Outside the parallel this current is proportional to the voltage to ground; hence, the receiver current, in

*See Technical Report No. 71.
†See Technical Report No. 65.
‡See Appendix.

case of either shunt or series unbalance, is proportional to the voltage of the circuit to ground. Thus the relative importance of units of electric and magnetic induction in causing current in the telephone receiver may be ascertained by comparing the voltages to ground, due to the two classes of induction, at points where unbalances occur.

The relative magnitudes of the voltages to ground, due to the two types of induction, vary over a wide range with frequency, length of line, position of parallel and admittance to ground of terminal apparatus. Apparatus connected between the sides of the circuit and without appreciable admittance to ground will usually have a negligible effect on the voltage to ground.

Table I gives the relative importance of units of longitudinal electric and magnetic induction, expressed in volt-miles per volt, for various frequencies and corresponding to several lengths and types of line in the two directions from the parallel. The voltages to ground upon which the ratios are based, were computed from the equivalent network shown below:



The electrically induced voltage is represented by a fictitious generator having an open-circuit voltage E_e in series with the impedance Z_p and connected at the middle of the parallel. The magnetically induced voltage is represented by two generators, each having an open-circuit voltage of $\frac{1}{2}E_l$ and connected in series with the impedances Z_A and Z_B. Z_A and Z_B are the impedances of the line in the two directions from the parallel. Z_p is the open-circuit impedance of the section of line within the parallel.

Since the attenuation of the voltages and currents along the line is the same, whatever their source, the ratio of single-frequency voltages to ground due to the two types of induction at any point along the line outside the limits of the parallel is equal to the ratio of the corresponding voltages to ground at the nearest end of the parallel. This ratio is also equal to the ratio of the currents at the end of the parallel, produced by the two types of induction, since outside the limits of a parallel

the impedances in circuit with the two induced voltages are the same. Thus, the ratio of receiver currents due to longitudinal electric and magnetic induction for any unbalance within the section of line "A" is equal to the ratio of the corresponding currents to ground at the "A" end of the parallel.

In deriving the following formulas it is assumed that Z_p is large compared to Z_A and Z_B, which is true in cases of short parallels.

Let the current in the receiver due to electric induction be I_e, and that due to magnetic induction be I_i; then for an unbalance at any point in section of line "A"

$$\frac{I_i}{I_e} = \frac{E_i}{E_e} \times \frac{Z_p}{Z_B} \tag{1}$$

and for an unbalance at any point in section of line "B"

$$\frac{I_i}{I_e} = \frac{E_i}{E_e} \times \frac{Z_p}{Z_A} \tag{2}$$

For equal receiver currents due to the two classes of induction when the unbalance is in section "A"

$$\frac{E_e}{E_i} = \frac{Z_p}{Z_B} \tag{3}$$

But Z_p is, for short parallels, practically inversely proportional to the length of the parallel. Let L be the length of parallel in miles, and Z'_p the open-circuit impedance of one mile of line. Substituting in (3) and transposing,

$$\frac{E_e L}{E_i} = \frac{Z'_p}{Z_B} \tag{4}$$

being the ratio of volt-miles to volts to cause equal receiver currents for unbalance in Section "A."

For unbalance in Section "B," Z_A is substituted in (4) in place of Z_B.

Equations (3) and (4) have been used in obtaining the ratios of volt-miles to volts given in Table I. It will be observed that the ratios differ greatly, depending upon the length of line, characteristics of terminal apparatus and the section of line in which the unbalance occurs. In practice unbalances will occur in both sections of line. Assuming, as illustrative, an infinite line in one direction from the parallel and one mile of cable in the other, with equal unbalances in each section such

that their effects add in quadrature, the relative importance of the two types of induction is, from Table I:

Frequency; cycles/sec.	60	180	540	796
Ratio; volt-miles/volt	214	86.6	32.5	24.2

or as an approximate average $\frac{300}{n}$ volt-miles per volt where n is the ratio of the frequency to 60 cycles.

TABLE I.

Ratio of Volt-Miles to Volts for Equal Receiver Currents Due to Longitudinal Induction and Unbalances of Telephone Circuit.

Line conditions*		Unbalance located in A				Unbalance located in B			
		Frequency				Frequency			
A	B	60	180	540	796	60	180	540	796
Infinite line	Infinite line	302	121	41.7	28.3	302	121	41.7	28.3
Infinite line	1 mi. cable.	19.3	19.3	19.3	19.3	302	121	41.7	28.3
Infinite line	Cable and composite set	712	167	2	11.3	302	121	41.7	28.3
400 mi. line	400 mi. line				38.4				38.4
400 mi. line	1 mi. cable.				19.3				38.4
100 mi. line	100 mi. line.				39.7				39.7
100 mi. line	1 mi. cable				19.3				39.7
65.7 mi. line	65.7 mi. line			65.6				65.6	
65.7 mi. line	1 mi. cable			19.3				65.6	
44.9 mi. line	44.9 mi. line				41.4				41.4
44.9 mi. line	1 mi. cable.				19.3				41.4
44.9 mi. line	1 mi. cable and composite set				11.3				69.9

*See figure on page 1103.
Line assumed: No. 12 N. B. S. Copper Physical Circuit—6-wire line.
$R = 6$ ohms/mile, $L = 0.0133$ henrys/mile, $C = 0.0037$ microfarads/mile, $G = 0$.
65.7 miles is a quarter wave length at 540 cycles per second; 44.9 miles, at 796 cycles.

Transverse Voltages.

The equivalent network for computing the receiver currents due to the transverse induced voltages is similar to that shown above, the ground being replaced by the other conductor. Z_p, Z_A and Z_B then apply to the metallic-circuit impedances, for the section of line within the parallel and for the sections of line outside the limits of the parallel in the two directions. E_e and E_l are, respectively, the differences of the induced voltages to ground and along the conductors. With these interpretations of the symbols, formulas 3 and 4 above given, hold also for determining the relative importance of unit values of the two types of induction when directly affecting metallic circuits as well as when indirectly affecting them through their unbalances.

Assuming that reflection at the terminals of the circuit is negligible, the line impedances in the two directions from the parallel are the same and equal to the characteristic impedance of the circuit (Z_o), hence

$$\frac{E_eL}{E_1} = \frac{Z'_p}{Z_o} \qquad (5)$$

This formula meets very well the condition of a short parallel with a long section of telephone line in each direction (or where means are taken to eliminate reflection), and has been used in obtaining the ratios of transverse volt-miles to transverse volts, for equal receiver currents, given in Table II, for a number of frequencies and types of telephone circuit.

TABLE II.

Ratio of Volt-Miles to Volts for Equal Receiver Currents Due to Transverse Induction in Telephone Circuits.

	Type of circuit \ Frequency	60	180	540	796
Nonloaded	No. 12 N. B. S. Physical	174	97.3	46.6	33.6
	No. 12 N. B. S. Phantom	189	82.6	47.7	34.0
	No. 8 B. W. G. Physical	258	131	51.3	35.4
	No. 8 B. W. G. Phantom	282	137	51.4	35.3
	Average	226	112	49.2	34.6
Loaded	No. 12 N. B. S. Physical	133	49.4	16.7	11.3
	No. 12 N. B. S. Phantom	136	49.1	16.6	11.3
	No. 8 B. W. G. Physical	139	47.7	15.9	10.8
	No. 8 B. W. G. Phantom	141	48.0	16.0	10.9
	Average	137	48.6	16.3	11.1

The average values from the above table are $\frac{350}{n}$ volt-miles per volt for nonloaded circuits and $\frac{150}{n}$ volt-miles per volt for loaded circuits, where n is the ratio of the frequency to 60 cycles.

Although only a small number of different conditions, regarding length and type of line, type of terminal apparatus and position of parallel which may occur in practice have been considered herein, it is

felt that the values given for the relative importance of the volt and the volt-mile are fairly representative.

Respectfully submitted.

(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

ATTACHED: Appendix.

Approved: August 20, 1917.

(Signed) R. W. MASTICK,
Field Engineer.

Approved: August 24, 1917.

SUBCOMMITTEE ON TESTS,
(Signed) J. E. WOODBRIDGE,
Chairman.

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE,
(Signed) ARTHUR F. BRIDGE,
Secretary.

August 24, 1917.

Appendix.

The terms "longitudinal" and "transverse," as applied to induced voltages, have been used since the beginning of the work of the committee substantially as defined below:

> "Longitudinal Induced Voltage"—Induced voltage from conductors to ground (electric induction) or along the conductors (magnetic induction).
>
> "Transverse Induced Voltage"—The difference of induced voltage from conductors to ground (electric induction) or difference of induced voltages along two conductors or group of conductors (magnetic induction).

It is recognized that such use of the adjectives "longitudinal" and "transverse" is not in accord with their common meaning. "Longitudinal" is fairly expressive of the magnetically induced voltage along a circuit, but "transverse" does not satisfactorily express the difference of the magnetically induced voltages along the two sides of a circuit. On the other hand, "transverse" is fairly expressive of the electrically induced voltage either between two conductors or from conductors to ground, while "longitudinal" does not satisfactorily express the electrically induced voltage from conductors to ground.

No thoroughly satisfactory substitute for these terms has been suggested thus far. The word "differential" is perhaps the most suitable word that has been suggested to express the idea involved in "transverse," but an equally suitable word has not been suggested to express the idea involved in "longitudinal."

Technical Report No. 71.

August 10, 1917.

THE INFLUENCE OF WAVE FORM ON THE DETRIMENTAL EFFECT OF INDUCTION.

Introduction.

Interference with the operation of a communication circuit obviously results when the extraneous current induced in the circuit is of appreciable magnitude compared to the current used in the normal operation of the circuit, and of approximately the same rate of variation, or frequency. The detrimental effect increases as the frequency approaches the normal operating frequency, or range of frequencies, of the communication circuit.

Telephone circuits employ currents having frequencies of 100 to 4000 cycles per second, 800 cycles (about the 13th harmonic of 60 cycles) being an approximate average, and the frequency at which telephone apparatus is most sensitive. Consequently, induction resulting from the higher harmonics of the voltages and currents of power systems is most important in causing interference to telephone circuits.

The important frequencies of the current used in the operation of telegraph circuits are usually less than 300 cycles per second, consequently induction corresponding to the fundamental and low harmonics of parallel power circuits is most important in causing interference to such circuits.

The influence of wave-form on the detrimental effect of induction is discussed separately for the two types of communication circuits.

Effect on Induction in Telephone Circuits.

Tests to determine the effect of extraneous current of single frequency on the intelligibility of telephone conversation were carried out in New York by the American Telephone and Telegraph Company at the request of the Joint Committee. A report was rendered in December, 1914, giving a description of the tests and the results obtained.* The method used was to determine the decrease in the efficiency of the circuit, and thus the increase in the equivalent length, corresponding to different magnitudes and frequencies of extraneous current in the receiver. It was found that the increase in the equivalent length of the circuit is practically proportional to the magnitude of the current, for the range investigated. These tests did not include the determina-

*For data of these and later tests see "Review of Work of Sub-committee on Wave Shape Standard of the Standards Committee" (A. I. E. E.), by H. S. Osborne, Proc. A. I. E. E., Jan., 1919.

tion of the resultant effect of a complex current of several frequencies in terms of the effects of the single-frequency components, but as a reasonable approximation it is assumed that the resultant detrimental effect is proportional to the square root of the sum of the squares of the separate effects of the several single-frequency components.

For constant magnitudes of extraneous current it was found that the detrimental effect increases very rapidly with frequency; nearly as the square up to about 600 cycles per second, then less rapidly, reaching a maximum at about 800 cycles per second and then decreasing up to 1500 cycles, which was the extreme frequency considered in the tests. The variation with the frequency is shown by curve A of drawing No. 382 attached.

In determining the variation of the detrimental effect with the frequency of the voltages and currents of the power circuit, it is necessary to consider also the variation with frequency of the induced voltage, the induced current, and the attenuation of the induced current between the parallel and the receiver. The variation of the last two named will depend somewhat upon the length and type of telephone circuit and type of terminal apparatus.

For constant magnitudes of the power-circuit voltages and currents in a short parallel, the electrically induced voltage is practically independent of the frequency and the magnetically induced voltage directly proportional to the frequency. For constant magnitudes of induced voltage it may be assumed that the induced current arising from electric induction is approximately directly proportional to the frequency and that arising from magnetic induction independent of the frequency. Assuming as a rough approximation that the attenuation of the induced current between the parallel and the receiver is independent of the frequency, the induced current in the receiver is, in both cases, approximately directly proportional to the frequency of the inducing voltages and currents. This relation is shown by curve B of drawing No. 382. The variation of the detrimental effect with frequency of the power-circuit voltages and currents is therefore roughly proportional to the product of the corresponding ordinates given by curves A and B on drawing No. 382. This product, shown by curve C, is approximately proportional to the cube of the frequency up to about 800 cycles per second, then increases less rapidly and reaches a maximum at about 1500 cycles.

If the magnitudes of the various harmonics of the power-circuit voltages and currents at the parallel be known, curve C furnishes a means of determining the detrimental effect as compared to that of a pure 60-cycle wave. The magnitudes of the various harmonics are multiplied by the corresponding ordinates of curve C. These products

are combined by obtaining the square root of the sum of their squares. This combination, divided by the effective value of the wave, gives its noise-producing power as compared to that of a pure 60-cycle wave. This quantity may be called a wave-factor. By the use of this wave-factor and a knowledge of the length and cross-sectional dimensions, different parallels may be compared and their relative severities determined. The noise-producing power may also be expressed as that of an equivalent single-frequency wave, having the same magnitude and detrimental effect as those of the actual complex wave.

The preceding discussion shows the extreme importance, in minimizing induction in telephone circuits, that the voltage and current wave forms of parallel power circuits should approximate a sine wave as closely as practicable. Improvements in practice in this respect will materially lessen the other remedial measures necessary, under present conditions, to sufficiently limit the detrimental effects of parallels.

Effect on Induction in Telegraph Circuits.

Tests were carried out in New York, jointly, by the Western Union Telegraph Company, the Postal Telegraph-Cable Company and the American Telephone and Telegraph Company at the request of the Joint Committee, to determine the detrimental effect of single-frequency extraneous currents on telegraph transmission. The data obtained in this investigation were transmitted to the committee in a report dated September 14, 1916. The tests were made with currents of 25, 60 and 180 cycles per second. The method used was to determine the maximum speed of operation possible with various values of extraneous current, using an automatic transmitter and Wheatstone receiver. The reduction in maximum speed corresponding to the different magnitudes and frequencies of the extraneous current was thus determined. The tests were made on an artificial line designed to represent a 400-mile No. 12, N. B. S. G. copper open-wire circuit.

For the range of frequencies investigated the detrimental effect, as measured by the reduction in maximum speed of operation, is roughly inversely proportional to the frequency.

To determine the variation in the detrimental effect with the frequency of the voltages and currents of a parallel power circuit, it is necessary to consider the various factors which connect the voltage and current of the power circuit and the resulting current in the telegraph apparatus. The same assumptions and approximations made in the discussion regarding telephone circuits apply also with respect to telegraph circuits. Thus the current in the telegraph apparatus is approximately directly proportional to the frequency of the power-circuit voltages and currents.

Since the detrimental effect is nearly inversely proportional to the frequency of the induced current and the induced current nearly directly proportional to the frequency of the power-circuit voltages and currents, the detrimental effect is nearly independent of the wave form of the power-circuit voltages and currents, and dependent only on their effective values. As a rule, the magnitudes of the higher harmonics are small as compared to the fundamental and third harmonic, and their effects in the telegraph circuit are correspondingly negligible.

Wave-Form Standard.

In fixing upon a standard of wave shape, the aim should be to restrict departures from a pure sine-wave in so far as the detrimental effects occasioned by further departure justifies the cost of avoidance, all elements of the problem being considered. In other words, the wave-form of power apparatus should be improved until the cost of effecting further improvement exceeds the benefit gained. This presupposes a measure of the detrimental effects of different wave-forms, in terms of the detrimental effect of the fundamental sine-wave, which is the ideal to be approached. Such a measure of relative detrimental effects, termed a wave-factor, requires the assignment of weights to the different harmonics present, in proportion to their individual detrimental effects, and the combination of the weighted values of the harmonics in the same manner as their detrimental effects combine to give the total detrimental effect of the complex wave. The relative importance to be assigned to the detrimental effects in power and communication circuits should depend on the relative amounts of detriment caused, which, of course, can only be roughly estimated.

The present standard of wave-form of the American Institute of Electrical Engineers and the method of determining the departure from a pure sine wave are admittedly unsatisfactory, in that the different harmonics are not penalized in accordance with their detrimental effects in either power or communication circuits or a properly determined resultant based on the effects in both classes of circuits, and because the actual determination of the rating or "deviation" of a wave by this method is laborious and subject to considerable error. Furthermore, the "deviation" of a wave is greatly influenced by the phase relations of the different harmonics, whereas the total detrimental effect is probably little influenced by this feature.

The Joint Committee has no data concerning the variation of detrimental effect in power circuits with frequency or the relative importance to be assigned to the detrimental effects in power and communication circuits. Since the effects of the higher harmonics on telegraph

circuits are practically negligible, the curve of variation of total detrimental effect with frequency need be based only on effects in power and telephone circuits.

Concerning variation with frequency of detrimental effects in telephone circuits curve C on drawing No. 382 may be considered to apply to wave forms of the power circuits at the parallel. To obtain a curve applicable to the open-circuit voltage wave-form of rotating machinery, requires a knowledge of the relation of wave-form at no load to that under the normal operating load, the resulting current wave-form and the distortion of voltage and current wave-forms between the machine and the parallel. Little information is available concerning these factors and they probably vary over a wide range. However, for the present, it is considered permissible to neglect them and to assume that curve C represents the variation of detrimental effects in telephone circuits with frequency of the harmonics present in the voltage wave-forms of rotating machinery.

Assuming that the various weights, based on the detrimental effects of the different harmonics in both power and communication circuits, have been obtained, it is necessary to determine the law by which the weighted magnitudes of the different harmonics present in the wave-form should be combined in order to properly represent the aggregate effect of all the harmonics. As previously stated, the detrimental effects of several harmonics on telephone circuits have been assumed to combine in accordance with the square root of the sum of the squares of the individual effects. This assumption is probably not unreasonable in reference to the detrimental effects of several harmonics on a power circuit. Such a law possesses marked advantages in permitting the development of apparatus for quickly and accurately determining wave-factors by experimental means.

Assuming the "root-summed-square" law of combination to hold and magnitudes M_1, M_3, . . . M_n and weights W_1 (unity) W_3 . . . W_n, the wave-factor may be expressed as follows:

$$\text{Wave-factor} = \sqrt{\frac{(M_1)^2 + (W_3M_3)^2 + \ldots + (W_nM_n)^2}{M_1^2 + M_3^2 + \ldots + M_n^2}}$$

To compute wave-factors in accordance with this formula, requires a knowledge of the magnitudes of the different harmonics in addition to their weights. This is a laborious process since an analysis of the wave into its constituent harmonics is required. It may be arranged, by the use of a network of admittances, that for a given frequency the reading of a meter in one branch of the network per volt across the network terminals will be proportional to the rating applicable to this frequency. Thus, for a complex wave, the reading of the meter per volt across the terminals of the network will be proportional to the wave factor. Such a network designed to accord with the frequency-

weighting curve C of drawing No. 382 was described in a discussion before the Portland Convention of the A. I. E. E. in September, 1916.*

In order to determine proper limiting values of wave-factors based upon the foregoing principles, to be used as standards, the wave-factors of existing machines determined in accordance with this method should be considered together with the cost of improvement and the benefits to be derived.

In the design of rotating machinery to meet a given wave-form standard, in order to obtain an economical solution, it is necessary to consider both the frequency-weighting curve and the relative costs of reducing the various harmonics a given amount. In general, the cost of a further reduction in the magnitude of a given harmonic will increase as the magnitude of the harmonic is decreased. Obviously, that harmonic should be reduced which will give the largest reduction in the wave-factor, for a given expenditure. The most economical solution is reached when all the harmonics have such magnitudes that the same reduction in wave-factor for a given cost is obtainable by reducing any one harmonic. At this point the magnitudes of the various harmonics are directly proportional to the cost of a unit reduction and inversely proportional to the square of the weighting (based on detrimental effect). Thus, it follows that, if it is necessary to modify a given design so as to reduce the wave-factor, the harmonics to be reduced should be those for which the product of the magnitude and square of weighting, divided by the cost of reduction, is greatest.

It will be apparent that the discussion here given is based on broad assumptions with respect to factors concerning which information is meager. The aim has been to present an outline of the problem. It is recognized that much study is necessary in order to determine practical working data.

Respectfully submitted.

(Signed) LIVINGSTON P. FERRIS,
Assistant Field Engineer.

ATTACHED: P. I. C. Drawing No. 382.

Approved: August 20, 1917.

(Signed) R. W. MASTICK,
Field Engineer.

Approved: August 24, 1917.

SUBCOMMITTEE ON TESTS,
(Signed) J. E. WOODBRIDGE,
Chairman.

JOINT COMMITTEE ON INDUCTIVE INTERFERENCE,
(Signed) ARTHUR F. BRIDGE,
Secretary.

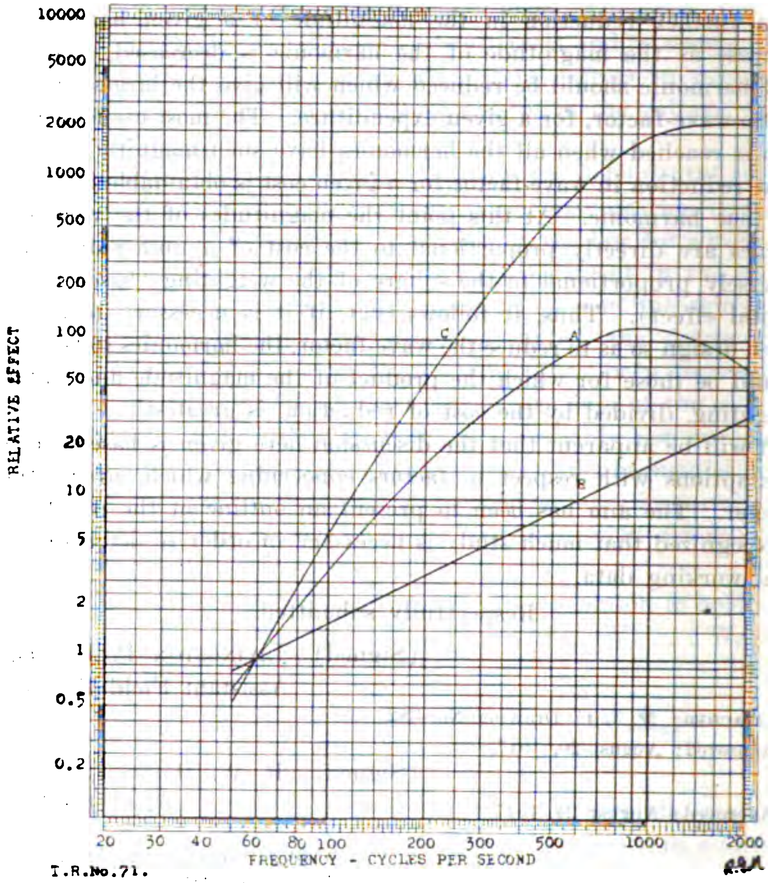
August 24, 1917.

*H. S. Osborne's discussion of Bedell's paper on the "Characteristics of Admittance Type of Wave-Form Standard."

P.I.C.NO.382
7-19-17.

VARIATION OF THE DETRIMENTAL EFFECT OF INDUCTION WITH FREQUENCY.

A - Variation of detrimental effect with frequency of current in telephone receiver
B - Variation of induced current with frequency of voltage and current of parallel power circuit
C - Variation of detrimental effect with frequency of voltage and current of parallel circuit = A x B



T.R.No.71.

EXHIBIT "A."

CONDITIONS CONSTITUTING A PARALLEL.

PREFACE.

Exhibit "A" presents the results of an attempt to set forth the physical relations of power and communication lines which constitute parallelism. This exhibit was not included in the Final Report of the Joint Committee on Inductive Interference to the Railroad Commission, owing to the inability of the committee to come to an agreement regarding the results contained therein.

The exhibit is, however, published here, together with other work of the Joint Committee, with the specific understanding that the results as set forth therein are not results agreed upon by the Joint Committee on Inductive Interference, representing the power and communication interests of California and the Railroad Commission; and, further, that the Railroad Commission takes no responsibility for its contents. It is published mainly for the purpose of placing before those interested the work which has been done, and to indicate a method of attack for the problem.

The curves included in the charts of the exhibit are based on certain assumptions which have not been determined or checked by tests by the Joint Committee. Also, as pointed out in the exhibit, in some cases the assumptions underlying the curves will not even approximately represent the actual conditions.

No claim is made that the exhibit is a complete study in itself; but rather it represents merely a start on a very involved and complicated subject. Tests checking the theoretical assumptions and further extensive study will be required to obtain information that will be sufficiently accurate and comprehensive for the deduction of general rules.

It must be borne in mind that before any rule can be set down stating at what point a condition of parallelism is created, it is first necessary to specifically define the allowable magnitude of extraneous induced current in the communication circuit. In this exhibit, assumptions were made as to the allowable magnitude of such current, which can not be considered as final or authoritative. Moreover, the allowable induced current will vary greatly under conditions met with in practice.

In view of these considerations this document can not be recommended for acceptance except as a preliminary study and a guide to those undertaking further studies of this nature.

EXHIBIT "A"

CONDITIONS CONSTITUTING A PARALLEL.*

The several factors which determine the inductive effects of power circuits on communication circuits vary over an exceedingly wide range under the different conditions which occur in practice. These factors include not only the physical relations between the two classes of lines, which are known in any particular case, but also various properties or characteristics of each system, some of which are usually not known even approximately, especially in cases of new power lines. These considerations, together with the complexity of the relations among these various factors, make it impossible to formulate an accurate and simple definition for general use, showing just what limiting conditions of proximity between the two classes of circuits will in all cases produce interference.

The principal factors which affect inductive interference are enumerated below. By assigning definite values to some of these factors, the resulting relationships of voltage, current, configuration of power circuit, and length and separation of lines in the section of proximity, corresponding to a given amount of induction, have been shown in the form of curves. These curves, presented herewith, are based on assumed values of these factors from within the range of the corresponding actual values commonly occurring in practice. They can be used as an aid in determining the limiting conditions of parallelism, it being understood that suitable correction should be made wherever information necessary therefor is available, so that the result reached will conform as closely as possible with the circumstances of the individual case.

Unless such correction is made, results derived from the curves are not even approximately correct for all cases, on account of the wide variation in magnitude of some of the factors for which it has been necessary to assume constant values. However, as the curves have been drawn in the light of all the relevant information available, it is believed that in general, much better results will be obtained by using them as a guide, even where no specific data are known by which to make allowance for local conditions, than would be possible by the parties concerned following their unaided judgment.

*See definition of "Parallel", page 44.

The commonly known factors which affect the magnitude of induction from a given parallel are:

1—Length of parallel.
2—Separation of lines.
3—Configuration of lines.
4—Normal operating voltages and currents of power circuits; their magnitudes and fundamental frequency.

Besides these the following factors are important in determining the magnitude of the induction and the resulting interference with the operation of communication circuits, but information concerning them is generally lacking in specific cases and may be impossible to secure in case of new construction.

5—Magnitude of residual voltage and current.
6—Wave-form of both balanced and residual voltages and currents, involving the magnitude and frequency of all harmonics.
7—Unbalances of communication circuits; their character, magnitude and location.
8—Terminal apparatus and lengths of unexposed communication line.
9—Voltages and currents of power circuits under abnormal conditions.

In considering the conditions of association of the two classes of lines giving rise to interference, allowance must be made for the cumulative effect of a number of parallels along the same communication line.

A basic element of the conditions constituting a parallel is the amount of extraneously induced current in the receiving instruments which will produce interference. The permissible amount of extraneous current depends upon the sensitiveness of the apparatus, the detriment caused, the importance of the service and the cost of remedial measures. In the case of telephone circuits, particularly important circuits of such length that the margin of transmission is small, it is assumed that the extraneously induced current in the telephone receivers should not exceed, in noise-producing value, the effect of 10 microamperes (millionths of an ampere) at a frequency of 240 cycles per second. On unimportant circuits and where there is a surplus volume of transmission, as on short-distance circuits, current somewhat in excess of the above mentioned limit is permissible.

In the corresponding case of telegraph circuits, it is assumed that the extraneously induced current at the circuit terminals should not exceed 2 milliamperes (thousandths of an ampere) at a frequency of 60 cycles per second, or its equivalent in disturbing effect at any other frequency (1 milliampere at 25 cycles) or combination of frequencies.

In case of unimportant lines, where high-speed transmission is not employed, somewhat greater extraneous currents are permissible.

The curves given herein are intended to be consistent with the foregoing assumptions as to the permissible amounts of extraneous current for important circuits, though it is not feasible to establish a definite and positive relationship. The amounts of induction upon which the curves are based assume the concurrence of the several disturbing factors ordinarily present; balanced and residual voltages and currents, and the cumulative effect of several parallels of average severity along the one communication line.

Some parallels involve interference only at times when the power line is in an abnormal condition. But comparatively few of the rules apply to such parallels. Most of the rules apply only in case of parallels which produce interference under normal conditions of power line operation, which are termed, for brevity, "normal parallels."

Parallelism as Determined by Abnormal Conditions of Power Circuits.

In general the most severe induction, for any given relationship of power and communication circuits, results from abnormal conditions on the power circuit. Hence the question of whether or not a given construction constitutes a parallel, in the unrestricted sense of the term, will ordinarily be determined by the presence or absence of interference at times when the power circuit is grounded or otherwise in an abnormal condition and thus will be determined whether any precautionary measures whatsoever need be observed.

Under abnormal conditions power circuits generally become unbalanced causing large residual voltages and currents and greatly extending the zone of possible interference with neighboring communication circuits. Two different effects are to be considered: (1) the transient disturbance at the instant of occurrence and discontinuance of the fault, which may cause hazardous voltages and acoustic shocks to telephone users; and, (2) the steady disturbance which persists after the initial transient has subsided and continues until the fault is cleared. In general a degree of association of power and communication circuits which will not result in interference from the induced currents and voltages due to a continuance of the fault, will not occasion transient voltages sufficient to operate the protective devices of the communication circuits, although momentary disturbances may be noticeable.

The worst unbalances of power circuits from the standpoint of inductive interferences usually occur when one conductor of a circuit becomes grounded. Such a condition frequently does not preclude continued operation of power circuits which are normally isolated from ground. On power circuits with neutral grounded at one or more points, an

accidental ground on one phase usually interrupts the service. On the latter type of system the open-circuiting of one conductor may be suffered without discontinuance of three-phase service if both supply and receiving stations have grounded neutrals. Under such conditions a residual current corresponding to the load current of the open phase, and under some circumstances a residual voltage equal to the normal voltage from one conductor to ground, are set up.

No charts are given for parallelism as determined by abnormal power circuit conditions, but it may be assumed that such parallelism will be brought about by a less close proximity of power and communication circuits than would require the transposition of a power circuit within a parallel. (See Charts 1, 2, 3 and 4).

Conditions Pertaining to Normal Parallels.

For parallels of the class known as "normal parallels" there are to be considered two different cases which involve the necessity for different remedial measures, as follows:

1. *Power-Circuit Transpositions.*

   The necessity for power-circuit transpositions in parallels is dependent upon the magnitude of the induction from the balanced voltages and currents under normal operating conditions. In some cases the effects of the balanced voltages and currents may be negligible and as the induction is then due to abnormal transients or to residuals under normal operating conditions, transposition of the power circuit is unnecessary, except as required by IV (c)*.

2. *Control of Residuals.*

   Residual voltages and currents are nonessential to power circuits, and have relatively large induction-producing powers (which can not be controlled by transpositions), hence they should be limited in so far as is practicable and necessary to prevent interference.

In the following, these two cases of normal parallelism are treated independently, the various conditions calling for the foregoing remedial measures being given by the curves of Charts 1 to 6. On account of the large variations in value of the factors underlying these curves and because in particular cases the conditions respecting interference may be different from those herein assumed, the curves should not always be followed implicitly, but may be departed from in cases where they indicate the application of remedial measures, which either on the one hand are insufficient, or on the other hand are unnecessarily extreme, due regard being given to the costs and benefits thereof.

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*Recommended Rules, page 47.

Case I.

Conditions Requiring Power-Circuit Transpositions.

The principal object of transpositions in power circuits within the limits of parallels, is to cause the inductive effects due to balanced voltages and currents of the power circuits to neutralize one another in neighboring lengths of the communication circuits, thereby reducing the resultant induced voltages between the communication circuits and ground. It is assumed herein that the direct inductive effects between the sides of metallic communication circuits, such as telephone circuits, are cared for by the transpositions in such circuits, although in some cases it might be more economical to install some power-circuit transpositions for this purpose. The communication-circuit transpositions, however, do not reduce the voltages induced between communication circuits and ground, which produce disturbances because of unavoidable small unbalances, *i. e.*, inequalities or irregularities in resistance, inductance, capacitance and insulation resistance of the communication circuit wires. In those parallels where the length, separation and other conditions are such that the balanced voltages and currents would not cause material inductive effects, it is unnecessary to install power-circuit transpositions. It is assumed that the balancing of the capacitances of the power conductors to earth, as provided in IV (*c*)* has been cared for, and accordingly, that function of power-circuit transpositions is not considered here.

Charts 1, 2, 3 and 4 give information, concerning the length of parallel, horizontal separation of lines, configuration, and operating voltage and current of the power circuit, which is recommended as a guide in predicting in specific cases whether or not power-circuit transpositions should be installed. As previously stated, these charts should not be followed too implicitly without making appropriate corrections for each specific case, owing to the large variations in value of some of the important basic factors which influence the inductive effects, particularly the wave-form. The actual effect experienced under operating conditions must remain the determining factor. The charts are intended to indicate the advisability of transposing the power circuit in the majority of cases where the need of transpositions may be in doubt.

Charts 1 and 2 apply for parallels involving telephone lines and Charts 3 and 4 to parallels involving telegraph lines. Charts 1 and 3 are based upon the inductive effect of the balanced voltages while 2 and 4 are based upon the inductive effect of the balanced currents. The curves of the charts are so drawn that for all combinations of conditions indicated, the inductive effects on the communication circuits will be

*Recommended Rules, page 47.

approximately constant. In determining the curves shown on the charts, representative heights and sizes of power conductors and heights of telephone and telegraph conductors were assumed. The spacing of power conductors typical of 60-kV. circuits (about 6 feet) was assumed. As the inductive effects of balanced voltages and currents are approximately proportional to the power-conductor spacing, the application of these curves affords relatively less protection in cases of parallels involving power lines of greater conductor spacing (characteristic of higher voltage lines) which are more difficult to transpose, and relatively more protection is afforded in cases of lower voltage lines which are more easily transposed.

The values of induction per parallel taken as a basis of the curves are as follows:

For Telephone Circuits.

Due to Balanced Voltages—
Product of length of parallel, in miles, and induced voltage between conductors and ground, in volts, equal to 200 volt-miles.

Due to Balanced Currents—
Average induced voltage along conductors equal to 0.5 volt at 60 cycles.

For Telegraph Circuits.

Due to Balanced Voltages—
Product of length of parallel, in miles, and induced voltage between conductors and ground, in volts, equal to 300 volt-miles.

Due to Balanced Currents—
Induced voltage along the conductors equal to 1 volt at 60 cycles.

The above values of induction for parallels involving telephone circuits are based on the assumption that the voltage and current waves of the power circuit contain harmonics in such proportions as to be equivalent in noise-producing power to waves of about 140 and 160 cycles, respectively, or about 20 and 30 times as detrimental as pure 60-cycle waves.

When applying the charts the effects of the voltages and currents should be separately considered. Transpositions are advisable in case either the product of the length of parallel and power-circuit voltage or the product of the length of parallel and power-circuit current exceeds the value given by the appropriate curve. Also, if both products are but slightly less than the values given by the curves, transpositions may be advisable since the induction effects of the two factors are liable to be more or less additive.

As an example of the use of the charts let it be assumed that a proposed 22-kV., 20-ampere, 60-cycle, triangular three-phase circuit will parallel a telephone line for a mile and three-quarters at a separation

of 50 feet. From the curve of Chart 1 for "Triangular" power circuits it is found that the product of the power-circuit voltage and the length of parallel, corresponding to a horizontal separation of 50 feet, is 34 kilovolt-miles. Dividing this by 22 (the given value of power-circuit voltage expressed in kilovolts between conductors) the permissible untransposed length is found to be 1.5 miles, considering only the effect of the balanced voltages. From the curve of Chart 2 for "Triangular" power circuits, it is found that the product of the power-circuit current and length of parallel, corresponding to a horizontal separation of 50 feet is 42 ampere-miles. Dividing this by 20 (the given value of power-circuit current expressed in amperes per phase) the permissible untransposed length is found to be 2.1 miles, considering only the effect of the balanced currents. Hence, in this assumed case the transposition of the power circuit is advisable, for the permissible untransposed length fixed by the balanced voltages, which in this case predominate, is less than the length of the proposed parallel. Other conditions remaining the same, it would be unnecessary to transpose the power circuit if the length of parallel could be made less than a mile and a half, or if the separation could be increased to 60 feet or more.

While Charts 1, 2, 3 and 4 have been designed particularly for three-phase power circuits, they may also be used as rough guides to judge the necessity of transpositions in single-phase and two-phase circuits.

Where a power line carries two or more circuits the charts (based on a single-circuit power line) may be used as rough guides but care should be taken, as provided in IV (b)* and discussed in the exhibit†, to fix the phase relations among the several circuits by interconnection or otherwise so as to minimize the intensity of the induction in parallel communication circuits. If this is not done, the inductive effects of the several power circuits may be cumulative and transpositions may be needed, though not so indicated by the charts.

For parallels of 25 or 50-cycle power circuits and telephone lines the values of the product of the power-circuit voltage or current and length of parallel should be taken as 3 or 1.25 times. respectively, the corresponding values as shown by Charts 1 and 2, which are designed primarily for 60-cycle power circuits. For parallels involving telegraph circuits and 25 or 50-cycle power circuits, the curves of Charts 3 and 4 apply without correction.

In cases where a parallel includes portions at widely different separations, it is not permissible to use the arithmetical average separation, but the length of each part of the parallel at approximately uniform separation should be calculated as a fraction of the permissible length

*Recommended Rules, page 46.
†Page 53.

at that separation. If the sum of these several fractions exceeds unity, power-circuit transpositions are advisable.

In determining the scheme of transpositions that should be employed where such are called for by the charts, the severity of the parallel must of course be considered. The curves given here are based on the assumption that for any given horizontal separation and configuration, the products of length of parallel and the power-circuit voltage and current are indices of severity. Hence the ratios of the actual kilovolt-miles and ampere-miles of the parallel to the corresponding values shown by the appropriate curves for the given horizontal separation, should be considered in judging the amount of transposing which is justified.

CHART 1

CONDITIONS CALLING FOR POWER-CIRCUIT TRANSPOSITIONS
FOR
PARALLELS OF 60-CYCLE THREE-PHASE POWER LINES AND TELEPHONE LINES
EFFECT OF BALANCED VOLTAGES

Where the product of the voltage between power conductors (in kilovolts) and the length of parallel (in miles) exceeds the value given by the appropriate curve, power-circuit transpositions will ordinarily be advisable. See also Chart 2 for effect of balanced currents.

These curves are to be applied as described in IV-d and in the text of Exhibit A.

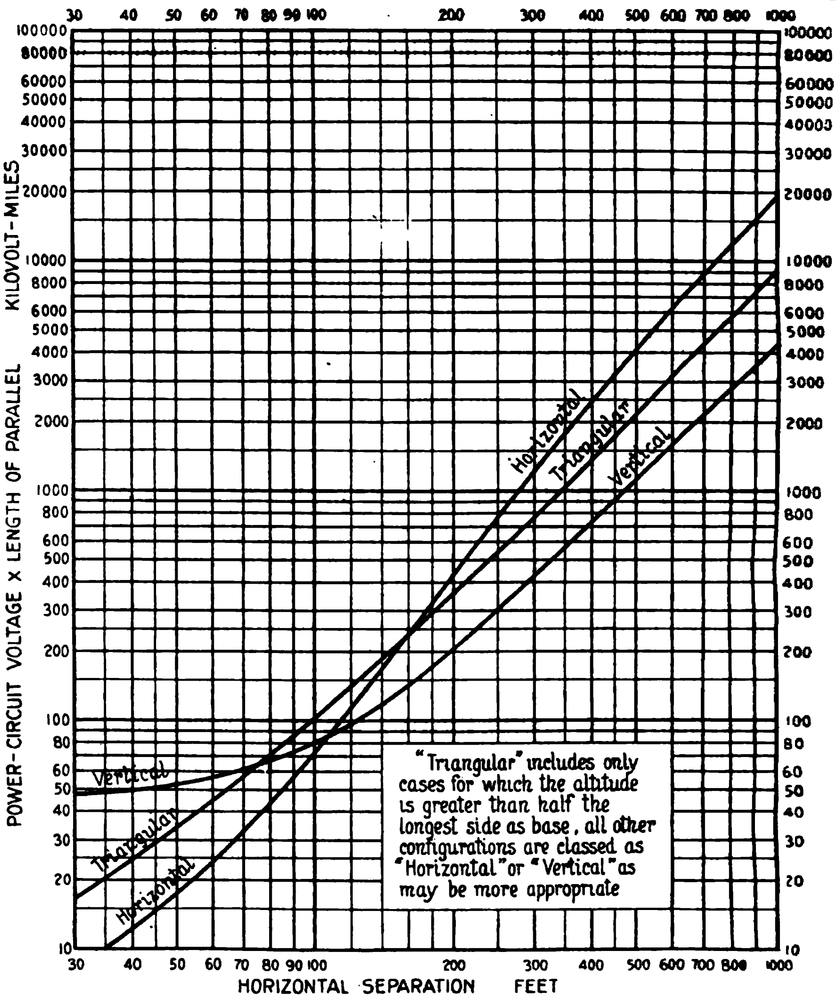


CHART 2

CONDITIONS CALLING FOR POWER-CIRCUIT TRANSPOSITIONS
FOR
PARALLELS OF 60-CYCLE THREE-PHASE POWER LINES AND TELEPHONE LINES
EFFECT OF BALANCED CURRENTS

Where the product of the power-circuit current per phase (in amperes) and the length of parallel (in miles) exceeds the value given by the appropriate curve, power-circuit transpositions will ordinarily be advisable. See also Chart I for effect of balanced voltages.

These curves are to be applied as described in IV-d and in the text of Exhibit A.

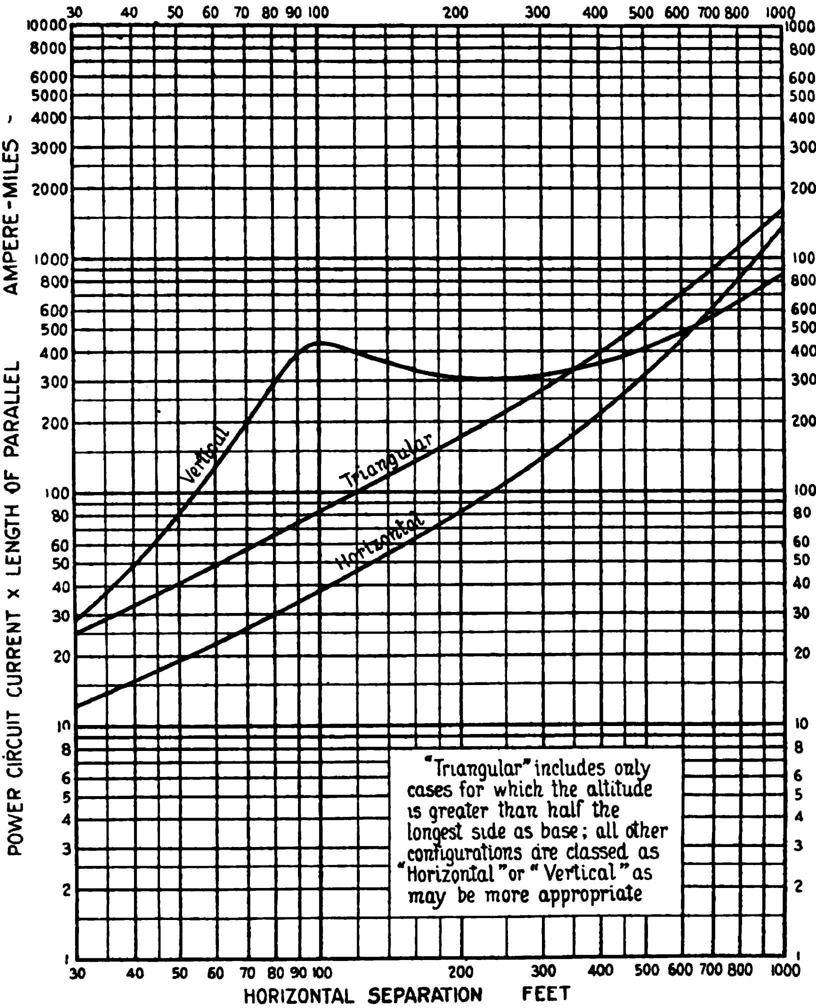


CHART 3

CONDITIONS CALLING FOR POWER-CIRCUIT TRANSPOSITIONS
FOR
PARALLELS OF 60-CYCLE THREE-PHASE POWER LINES AND TELEGRAPH LINES
EFFECT OF BALANCED VOLTAGES

Where the product of the voltage between power conductors (in kilovolts) and the length of parallel (in miles) exceeds the value given by the appropriate curve, power-circuit transpositions will ordinarily be advisable. See also Chart 4 for effect of balanced currents.

These curves are to be applied as described in IV-d and in the text of Exhibit A.

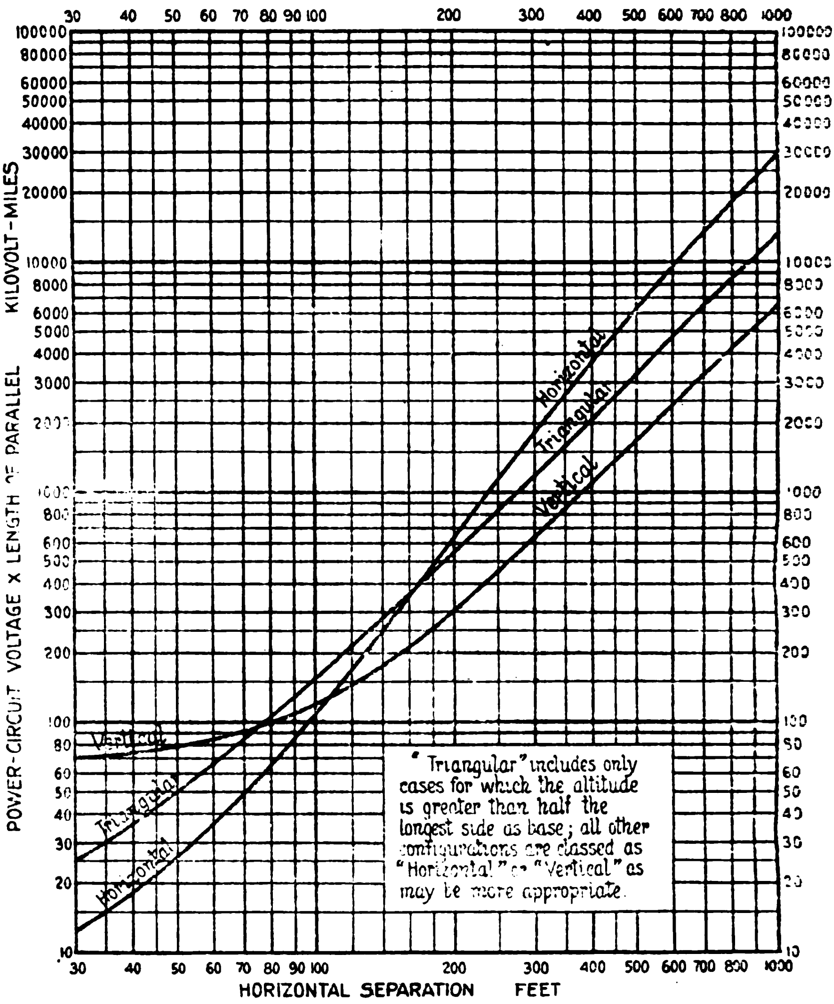


CHART 4

CONDITIONS CALLING FOR POWER-CIRCUIT TRANSPOSITIONS

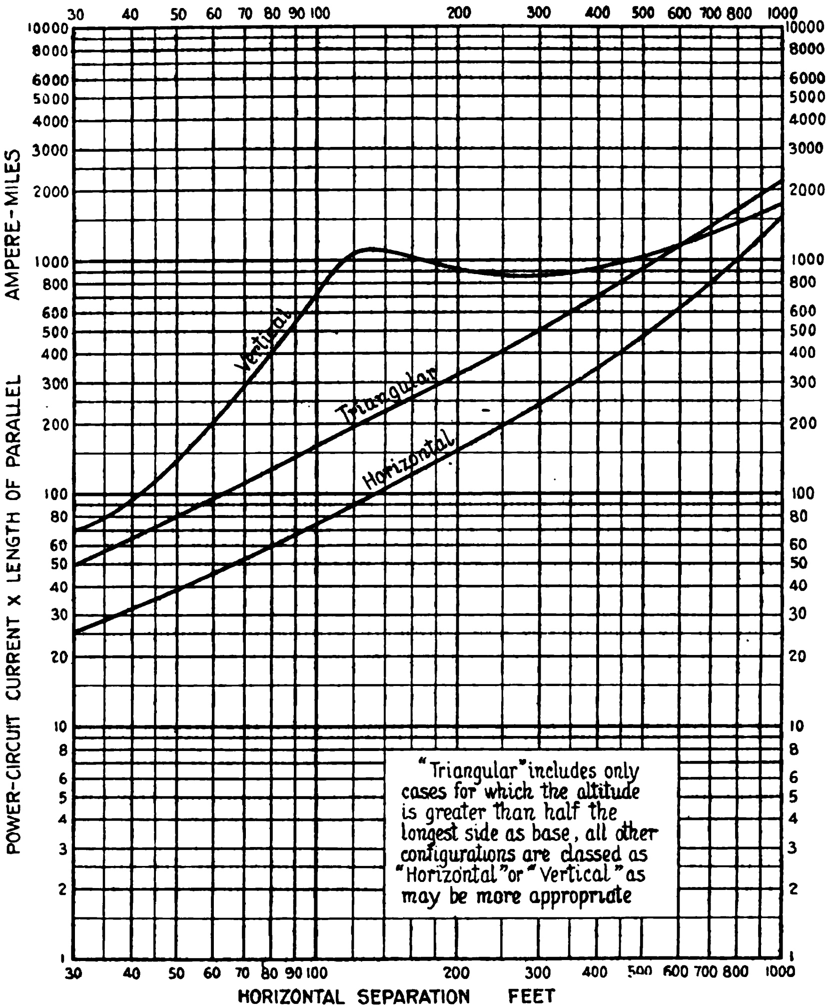
FOR

PARALLELS OF 60-CYCLE THREE-PHASE POWER LINES AND TELEGRAPH LINES

EFFECT OF BALANCED CURRENTS

Where the product of the power-circuit current per phase (in amperes) and the length of parallel (in miles) exceeds the value given by the appropriate curve, power-circuit transpositions will ordinarily be advisable. See also Chart 3 for effect of balanced voltages

These curves are to be applied as described in IV-d and in the text of Exhibit A



Case II.

*Control of Residuals.**

The chief causes of residual voltages and currents are: (1) unequal capacitances and conductances of the several conductors to ground, (2) load and transformer unbalances to neutral, and (3) cyclic variation of permeability of transformer iron. The last two causes are concerned only with circuits having grounded neutrals, while the first mentioned cause gives rise to residual voltages and currents in both isolated and grounded-neutral circuits.

Residual voltages and currents which arise from unbalances, (1) and (2) above, contain the same harmonics as those present in the balanced voltages and currents, though of different relative magnitudes. The variation of permeability of transformer iron, in star-connected three-phase systems with neutral grounded, introduces a series of harmonic residuals, odd multiples of three times the fundamental frequency. Generators with star-connected windings may, under some conditions, introduce residuals of the latter type.

Charts 5 and 6 show the relation of length of parallel, horizontal separation of power and communication lines, and the amounts of residual voltage or residual current which, together with the induction from balanced voltages and currents, the cumulative effect of other parallels and the other conditions herein assumed, will ordinarily result in interference. In case of long parallels at highway separations, it may be difficult, under conditions of present practice, to reduce the residuals to the values indicated, particularly for parallels of grounded-neutral power circuits and telephone circuits. When such cases are proposed the parties concerned should carefully consider whether the values given by the curves are properly applicable under the circumstances, and if some departure from these values is considered justifiable, they should co-operate in determining the most feasible and economical means of reducing, as far as practicable, the interference which may arise.

Because of the difficulties in evaluating the harmonics, the limits are expressed in terms of the resultant effective value (as measured by a meter) of all frequencies present in the residuals and are chosen with regard to wave-forms of residuals that have been observed and are assumed to take into account the conditions ordinarily encountered in practice. Large variations of wave-forms are to be expected. If in specific cases definite information concerning the wave-form is available, it should be taken into account, and appropriate corrections made to the values determined from the curves.

*See definitions in II-l and II-m, page 45.

The values given are independent of the operating voltage, line current and configuration of the power circuit.

Though primarily for 60-cycle power circuits, Charts 5 and 6 may be used to determine corresponding values of residual voltage and current for 25- and 50-cycle power circuits. For parallels involving telegraph lines the values obtained from the charts apply without correction. For parallels involving telephone lines the values for 25 and 50-cycle power circuits are, respectively, 3 times and 1.25 times the corresponding 60-cycle value as determined from the charts.

Residual Voltage.

Chart 5 shows the relation of, (1) the average permissible residual voltage within a parallel in volts, (2) the length of parallel in miles, and (3) the horizontal separation of power and communication lines in feet. This chart applies to parallels involving either a telephone or telegraph line, a curve being given for each. The curve for parallels involving telephone lines is based on a value of the product of the length of parallel and the induced voltage between the conductors and ground of 150 volt-miles; assuming a residual voltage wave of the power circuit containing harmonics in such proportions as to be equivalent in noise-producing power to a wave of about 200 cycles per second, or about 50 to 60 times as detrimental as a pure 60-cycle wave. The curve for parallels involving telegraph lines is based on a value of 300 volt-miles.

The curves represent the relations obtaining with heights of power and communication conductors, configuration and size of power conductors, typical of ordinary practice.

The values of residual voltage determined from Chart 5 apply to both grounded-neutral and isolated power circuits. Grounded-neutral circuits meeting the values of residual current determined from Chart 6 will in general meet the residual voltage limits.

As an example of the use of Chart 5, let it be assumed that it is proposed to parallel a telephone line for 10 miles at a separation of 60 feet. From the chart it is found that the value of the product of length of parallel and residual voltage corresponding to a horizontal separation of 60 feet is 6 kilovolt-miles. Dividing by 10, the given length of parallel, the residual voltage permissible under the conditions assumed is found to be 600 volts (effective value).

Residual Current.

Chart C shows the relation of, (1) the average permissible residual current in amperes, (2) length of parallel in miles, and (3) horizontal separation in feet. Two curves are given, one applying to parallels involving telephone lines and the other to parallels involving telegraph

lines. In both cases the curves apply to power circuits having grounded neutrals. No curve is given for power circuits isolated from ground, as the effect of residual current in such circuits will usually be small in comparison with that of residual voltage, unless two-wire branches or extremely long barrels are employed in the power circuit. The curve for parallels involving telephone circuits is based on an induced voltage along the conductors of 0.20 volt at 60 cycles. It is assumed that the residual current, which actually contains components of several frequencies in proportion differing greatly in different cases, is equivalent, in respect to noise-producing power, to a single-frequency current of about 270 cycles, at which frequency the induced voltage corresponding to the curve is 0.9 volt. Experience has shown that residual currents sometimes have much greater detrimental effects (on telephone circuits) than the detrimental effect here assumed, which is about 125 times that of pure 60-cycle current. The curve for parallels involving telegraph circuits is based upon a value of 1 volt at 60 cycles.

As an example of the use of Chart 6, assume that it proposed to parallel a telephone line for two miles at a separation of 60 feet with a star-connected grounded-neutral power circuit. From the chart it is found that the value of residual ampere-miles corresponding to a horizontal separation of 60 feet is 0.75. Dividing this by 2, the given length of parallel in miles, the residual current permissible under the conditions assumed is found to be 0.37 ampere (effective value).

CHART 5

RESIDUAL VOLTAGE

FOR PARALLELS OF 60-CYCLE THREE-PHASE POWER LINES AND TELEPHONE OR TELEGRAPH LINES

When the product of residual voltage (in kilovolts) and length of parallel (in miles) for a given horizontal separation, exceeds the value given by the appropriate curve, interference may ordinarily be expected, under the assumptions of Case II

These curves are to be applied as described in IV-c and V-c and in the text of Exhibit A

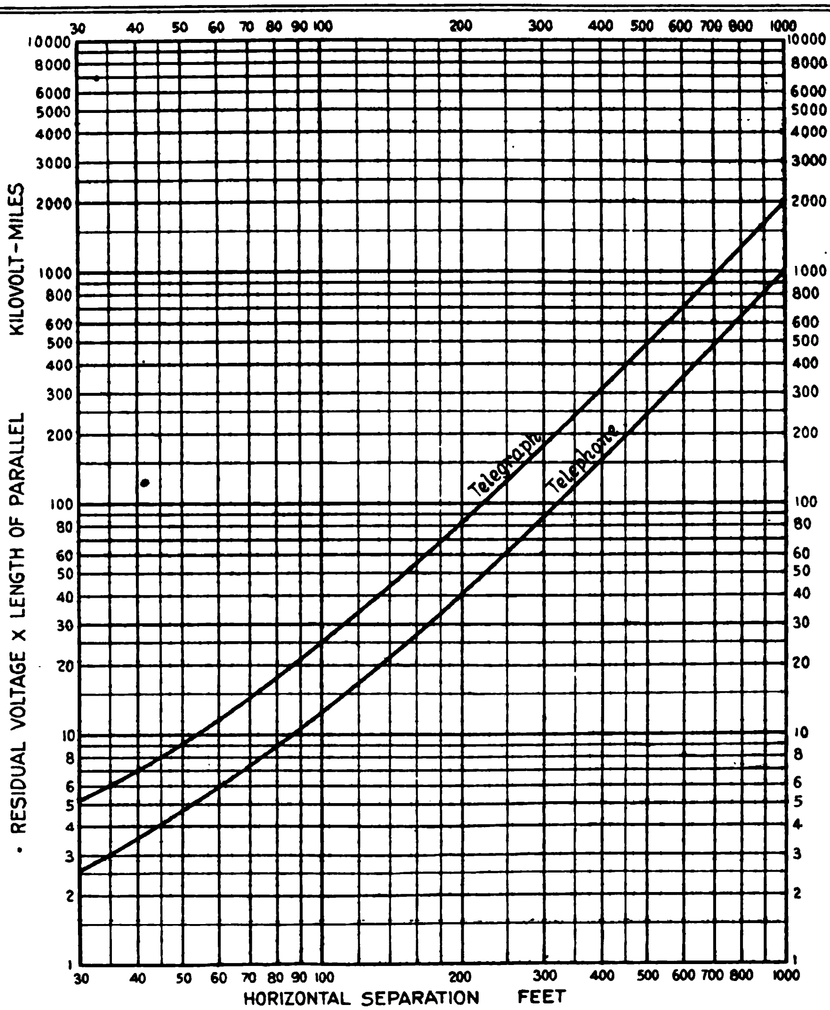


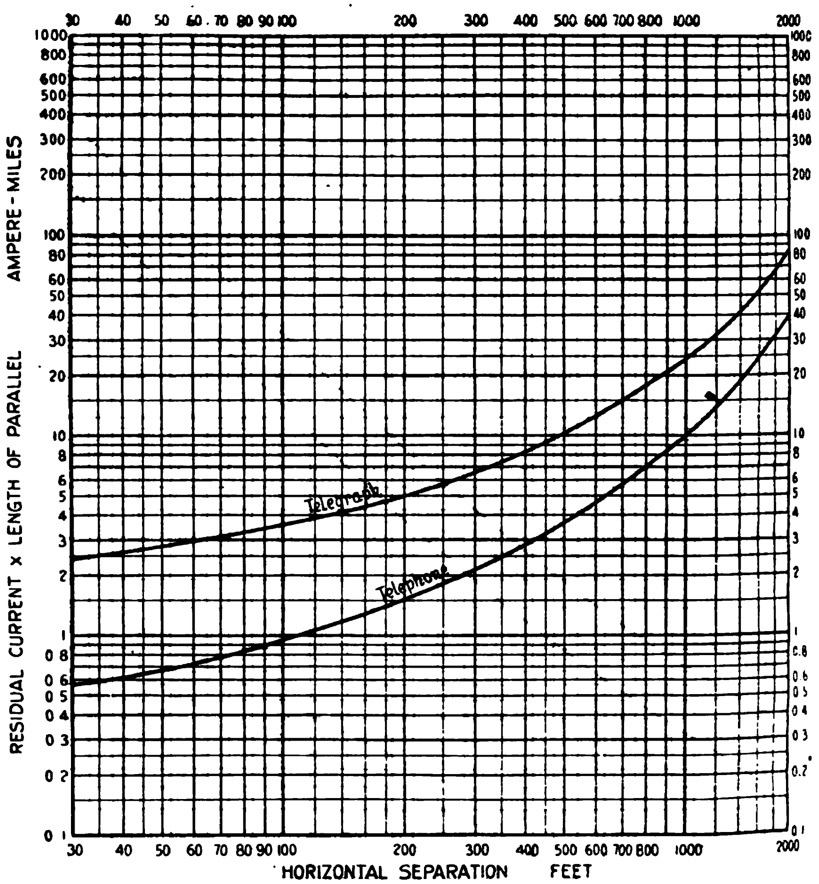
CHART 6

RESIDUAL CURRENT

FOR PARALLELS OF 60-CYCLE THREE-PHASE GROUNDED-NEUTRAL POWER LINES AND TELEPHONE OR TELEGRAPH LINES

When the product of residual current (in amperes) and length of parallel (in miles) for a given horizontal separation, exceeds the value given by the appropriate curve, interference may ordinarily be expected, under the assumptions of Case II

These curves are to be applied as described in IV-c and V-c and in the text of Exhibit A



COMMENTS ON EXHIBIT ... informa- ... the

This exhibit has been prepared to meet, as well ... demand for a specific statement of the conditions wh... parallel as defined in II (f). Based on certain assumptions ... of the variable factors, there are given, in the form of cha... relations of voltage, current, and configuration of the power cir... length of parallel construction, separation of lines and type of communication circuit, which define approximately under the given assumptions two of the most important conditions of parallelism: (1) the conditions calling for power-circuit transpositions within the parallel and (2) the conditions calling for limitation of residual voltages and currents. An effort was also made to include charts defining in a similar manner the relations of power and comunication circuits which would produce interference when the power circuit is in an abnormal condition, but owing to the greater complexity of this case and the difficulties encountered in reaching satisfactory conclusions, this idea was finally abandoned.

In preparing the charts, the relationships of power-circuit voltages or currents and the voltage induced in a parallel communication circuit, were obtained from technical report No. 65, choosing representative dimensions of lines. The amounts of induction assumed as the basis of the different curves are such as would be obtained with circuits isolated and open-circuited within the parallel. This is a definite and convenient condition of reference for which results are most readily obtained either by computations or experiment. However, these amounts of induction were not chosen without due consideration and allowance for the mutual shielding effects which occur under operating conditions with a number of circuits on the same pole-line.

The inductive effect of a given parallel is made up of the component effects of balanced and residual voltages and currents. These component effects, in case of both telephone and telegraph circuits are assumed, as an average approximation, to combine as the square root of the sum of their squares. Separate charts are devoted to each of the above factors. In case of telephone circuits, the residuals are permitted somewhat larger effects than the balanced components because the residuals are the more difficult to control. This may not be apparent from a comparison of the amounts of induction in the corresponding cases, without taking into account the more detrimental wave-forms of the residual voltages and currents.

In general, allowance must be made for the cumulative effect of a number of parallels distributed more or less irregularly along the communication line. The effects of different parallels along the same

ane are not directly additive and it has been assumed, as a ...nable average approximation, that the resultant effect of a number of parallels is equal to the square root of the sum of the squares of the individual effects. This assumption, together with a consideration of the attenuation of the high-frequency induced currents, makes it unnecessary to assume a specific number of parallels affecting the telephone line, in view of the preponderating influence of the few parallels nearest a terminal and the lack of exactitude involved in the other assumptions. For telegraph circuits the conditions are somewhat different. The attenuation of the important induced currents is less, because of their lower frequency; also for a considerable number of parallels the component effects would be directly additive at frequent intervals.

As stated in the exhibit and elsewhere in this report, the wave-forms of the voltages and currents of the power circuit have an enormous influence on the magnitude of the disturbance produced in telephone circuits. In ordinary practice, wave-forms have widely different detrimental effects, and it is particularly difficult to determine representative values for this factor. Were the wave factor easily obtainable in specific cases, it would probably be better to treat this factor on the charts as a variable. Possibly future developments of the art may make such a method practicable.

The condition of telephone circuits with respect to unbalance, which is resultant of an exceedingly large number of small distributed irregularities and inequalities of the electrical constants of the two sides of the circuits, has a direct bearing on the allowable amount of induction in the telephone conductors with reference to ground, the resulting disturbance being directly proportional to both the induction and the unbalance. This relationship of unbalance, induction and resulting noise has not been experimentally investigated by this Committee so as to determine how much noise would be caused in the receiver of a well balanced telephone circuit by one volt magnetic induction, for example, or 100 volt-miles electric induction, between the telephone conductors and ground. On this point, the Committee has been guided largely by the estimates and experience of telephone engineers, who, however, have not made a thorough study of this complex problem. In the case of (grounded) telegraph circuits, the unbalance is of course 100 per cent, and does not enter to complicate the problem of determining the relationship of induced voltage and disturbing current in the terminal apparatus of the circuit. Before setting the limits of induction for telegraph circuits this subject was investigated by the Committee, the results being given in part in technical report No. 69.

Experience with the application of these charts and further information, particularly concerning the wave-form of power circuits and the unbalance of telephone circuits, may indicate the desirability of modification of this exhibit at some future time. To be of value as a guide to such modification, however, this experience must extend over a considerable period, and include a sufficient number of cases to indicate the general results of applying the limits herein suggested under representative conditions, to lines involved in parallels.

NOTES CONCERNING ALTERNATIVE FORMS OF CHARTS 1, 2, 3 AND 4.

Charts 1, 2, 3 and 4 in their alternative form, given herewith, differ from the corresponding charts of the foregoing draft of Exhibit A, in that on each chart a table of correction factors is given to take account of the variation in the spacing of the conductors of power circuits.

The charts in Exhibit A, based on a spacing of about 6 feet between adjacent conductors, make no allowance for variations of induction with variations of spacing, which cover a very wide range in practice. The reason for this omission was to compensate for the relative difficulties of transposing lines of different voltages and for simplicity.

If the full effect of spacing were allowed for, the charts would call for transpositions on the basis of approximately the same amount of induction, irrespective of the relative difficulties of reducing this induction by transpositions. This, of course, is undesirable.

The correction factors of the alternative charts give results intermediate with respect to these extremes.

If these alternative charts are used, the following changes should be made in the text of Exhibit A:

For last sentence of third paragraph of Case I, substitute the following:

> "The inductive effects of balanced voltages and currents are approximately proportional to the power-conductor spacing. On the other hand, the difficulty of transposing power conductors increases with the voltage and hence with the spacing of the conductors. The correction factors on the charts, for variations of power-conductor spacing, give weight of both to these considerations, being proportional to the square-root of the power-conductor spacing. The application of these curves, with the given correction factors, affords relatively less protection in cases of parallels involving power lines of greater conductor-spacing, which are more difficult to transpose, and relatively more protection in cases of lower voltage lines which are more easily transposed."

For sixth paragraph of Case I, giving an example of the use of the charts, substitute the following:

> "As an example of the use of the charts, let it be assumed that a proposed three-phase 22-kV., 20-ampere, 60-cycle equilateral triangular power circuit with 3-foot conductor-spacing, will parallel a telephone line for 2.5 miles at a separation of 50 feet. From the curve of Chart 1 for 'Triangular' power circuits it is found that the product of the power-circuit voltage and length of parallel, corresponding to a horizontal separation of 50 feet, is 34 kilovolt-miles. Multiplying by 1.4, the correction factor for 3-foot spacing, and dividing by 22 (the given value of power-circuit voltage expressed in kilovolts between conductors) the permissible untrans-

posed length is found to be 2.2 miles, considering only the effect of the balanced voltages. From the curve of Chart 2 for 'Triangular' power circuits, it is found that the product of the power-circuit current and length of parallel, corresponding to a horizontal separation of 50 feet, is 42 ampere-miles. Multiplying by 1.4, the correction factor for 3-foot spacing, and dividing by 20 (the given value of power-circuit current expressed in amperes per phase) the permissible untransposed length is found to be 2.9 miles, considering only the effect of the balanced currents. Hence, in this assumed case, the transposition of the power circuit is advisable, since the permissible untransposed length fixed by the balanced voltages, which in this case predominate, is less than the length of the proposed parallel. Other conditions remaining the same, it would be unnecessary to transpose the power circuit if the length of parallel could be reduced to two miles or less or if the separation could be increased to 60 feet or more."

CHART 1

CONDITIONS CALLING FOR POWER-CIRCUIT TRANSPOSITIONS
FOR
PARALLELS OF 60-CYCLE THREE-PHASE POWER LINES AND TELEPHONE LINES
EFFECT OF BALANCED VOLTAGES

Where the product of the voltage between power conductors (in kilovolts) and the length of parallel (in miles) exceeds the value given by the appropriate curve, power-circuit transpositions will ordinarily be advisable. See also Chart 2 for effect of balanced currents.

These curves are to be applied as described in IV-d and in the text of Exhibit A.

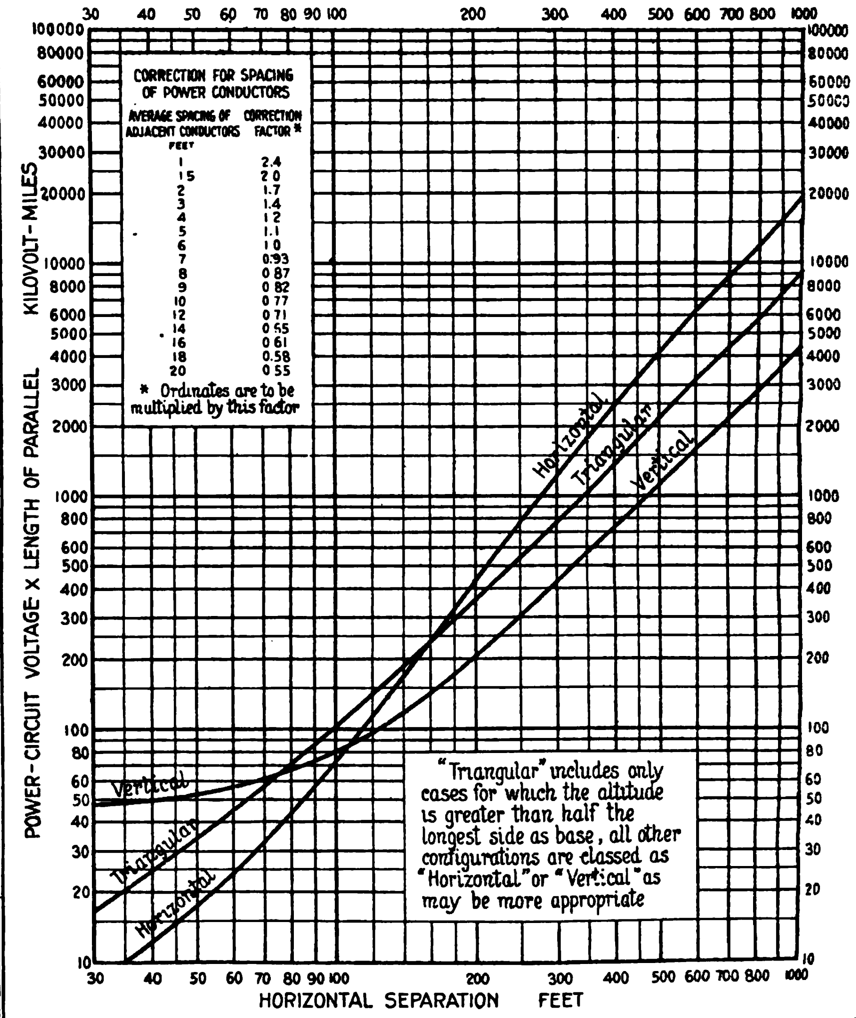
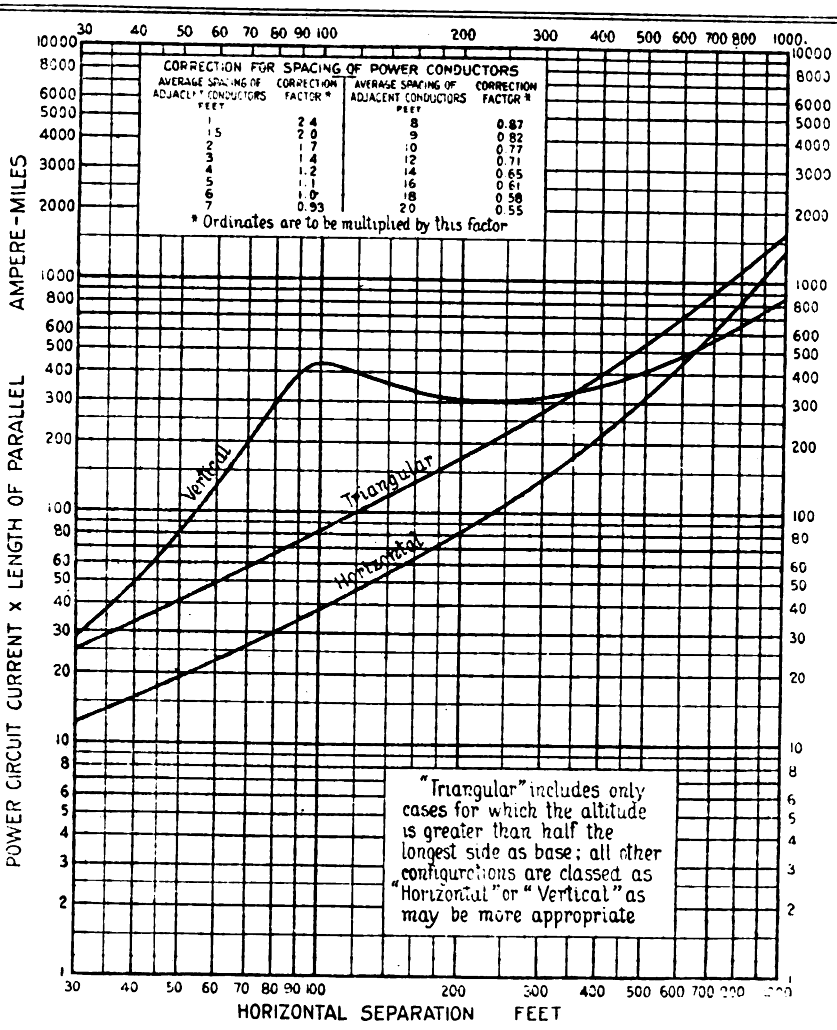


CHART 2

CONDITIONS CALLING FOR POWER-CIRCUIT TRANSPOSITIONS
FOR
PARALLELS OF 60-CYCLE THREE-PHASE POWER LINES AND TELEPHONE LINES
EFFECT OF BALANCED CURRENTS

Where the product of the power-circuit current per phase (in amperes) and the length of parallel (in miles) exceeds the value given by the appropriate curve, power-circuit transpositions will ordinarily be advisable. See also Chart 1 for effect of balanced voltages

These curves are to be applied as described in IV-d and in the text of Exhibit A



CORRECTION FOR SPACING OF POWER CONDUCTORS

AVERAGE SPACING OF ADJACENT CONDUCTORS FEET	CORRECTION FACTOR *	AVERAGE SPACING OF ADJACENT CONDUCTORS FEET	CORRECTION FACTOR *
1	2.4	8	0.87
1.5	2.0	9	0.82
2	1.7	10	0.77
3	1.4	12	0.71
4	1.2	14	0.65
5	1.1	16	0.61
6	1.0	18	0.58
7	0.93	20	0.55

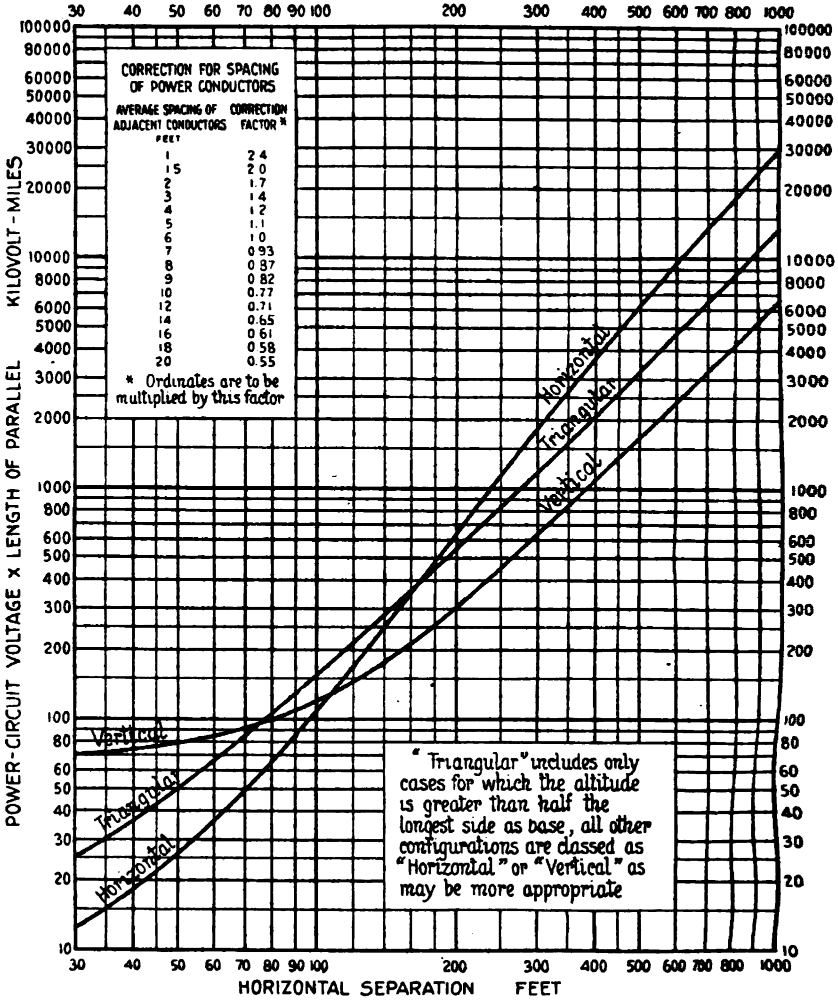
* Ordinates are to be multiplied by this factor

CHART 3

CONDITIONS CALLING FOR POWER-CIRCUIT TRANSPOSITIONS
FOR
PARALLELS OF 60-CYCLE THREE-PHASE POWER LINES AND TELEGRAPH LINES
EFFECT OF BALANCED VOLTAGES

Where the product of the voltage between power conductors (in kilovolts) and the length of parallel (in miles) exceeds the value given by the appropriate curve, power-circuit transpositions will ordinarily be advisable. See also Chart 4 for effect of balanced currents

These curves are to be applied as described in IV-d and in the text of Exhibit A.



CORRECTION FOR SPACING OF POWER CONDUCTORS

AVERAGE SPACING OF ADJACENT CONDUCTORS FEET	CORRECTION FACTOR *
1	2.4
1.5	2.0
2	1.7
3	1.4
4	1.2
5	1.1
6	1.0
7	0.93
8	0.87
9	0.82
10	0.77
12	0.71
14	0.65
16	0.61
18	0.58
20	0.55

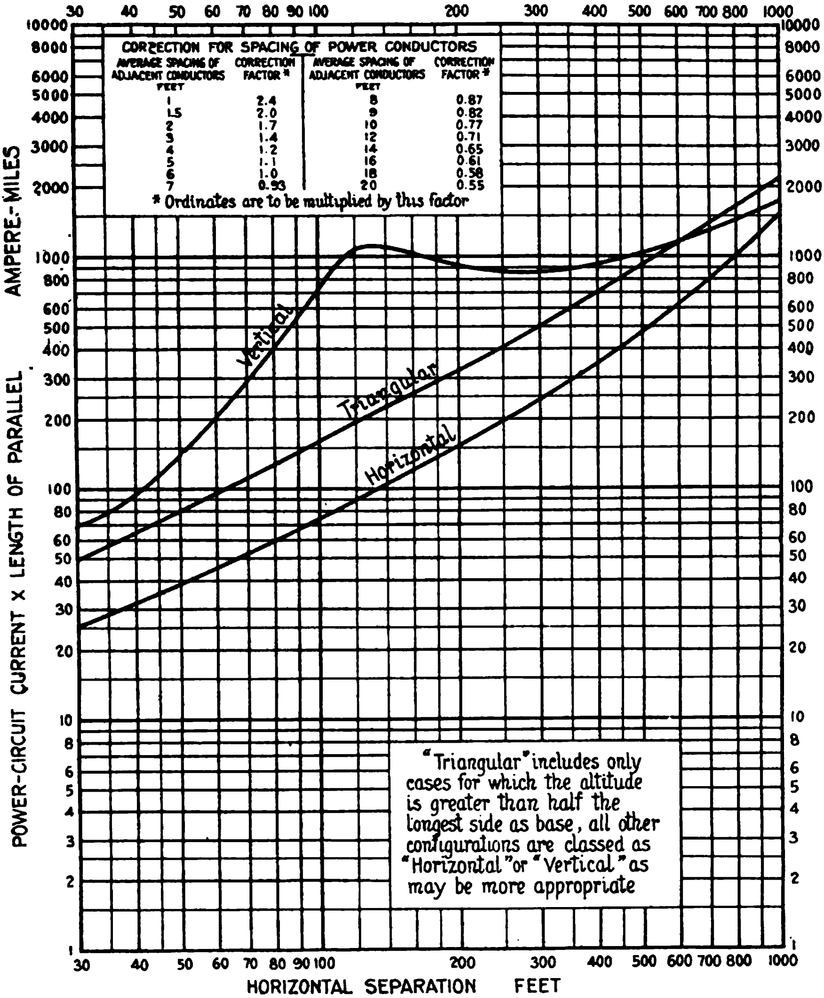
* Ordinates are to be multiplied by this factor

CHART 4

CONDITIONS CALLING FOR POWER-CIRCUIT TRANSPOSITIONS
FOR
PARALLELS OF 60-CYCLE THREE-PHASE POWER LINES AND TELEGRAPH LINES
EFFECT OF BALANCED CURRENTS

Where the product of the power-circuit current per phase (in amperes) and the length of parallel (in miles) exceeds the value given by the appropriate curve, power-circuit transpositions will ordinarily be advisable. See also Chart 3 for effect of balanced voltages

These curves are to be applied as described in IV-d and in the text of Exhibit A



CORRECTION FOR SPACING OF POWER CONDUCTORS

AVERAGE SPACING OF ADJACENT CONDUCTORS FEET	CORRECTION FACTOR *	AVERAGE SPACING OF ADJACENT CONDUCTORS FEET	CORRECTION FACTOR *
1	2.4	8	0.87
1.5	2.0	9	0.82
2	1.7	10	0.77
3	1.4	12	0.71
4	1.2	14	0.65
5	1.1	16	0.61
6	1.0	18	0.58
7	0.93	20	0.55

* Ordinates are to be multiplied by this factor

TOPICAL INDEX.

The numerous incidental references to many topics scattered through the reports make it undesirable to give page references for all. Accordingly, in such cases only the principal references have been given spcifically. Where a subject is referred to often throughout a report, reference is here made to the first page. It is recommended that the individual tables of contents accompanying many of the reports be made use of where this has been done. It should also be noted that the discussion of a given topic extends through several pages following the reference given.

*Practically the entire text deals with induction from alternating current circuits.

*For rules concerning specific questions, e.g., Transpositions in power circuits, refer to appropriate headings in index, e.g., "Transpositions, power circuit."

*Since consolidated with Southern California Edison Company.

*For rules concerning specific questions, e.g., transpositions in power circuits, refer to appropriate headings in index, e.g., "Transpositions, power circuit."

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